P. Holan; J. Kvasil; F. Štěrba; M. Smrčková

The splitting of the states and p-n interaction in odd-odd deformed nuclei

Acta Universitatis Carolinae. Mathematica et Physica, Vol. 23 (1982), No. 2, 3--13

Persistent URL: http://dml.cz/dmlcz/142491

1982 ACTA UNIVERSITATIS CAROLINAE - MATHEMATICA ET PHYSICA VOL. 23. NO. 2.

The Splitting of the States and p—n Interaction in Odd—odd Deformed Nuclei

P. HOLAN, J. KVASIL, F. ŠTĚRBA, M. SMRČKOVÁ*)

Department of Nuclear Physics, Faculty of Mathematics and Physics, Charles University, Prague**)

Received 19 April 1982

The expressions for proton-neutron residual interaction in odd-odd deformed nuclei are calculated. Nuclear potential including Wigner, Bartlett, Majorana, Heisenberg and tensor forces is considered. Expressions permitting numerical calculations are derived using δ-function, oscillator, square-well, Gaussian and Yukawa radial dependences for proton-neutron potential.

V práci jsou odvozeny vztahy pro proton-neutronovou zbytkovou interakci v licho-lichých deformovaných jádrech. Při výpočtu byl uvažován potenciál, zahrnující Wignerovy, Bartlettovy, Majoranovy, Heisenbergovy a tenzorové síly. Vztahy, umožňující numerické výpočty, byly odvozeny pro radiální závislost potenciálu, odpovídající δ-funkci, harmonickému oscilátoru, pravoúhlé jámě, Gaussovu a Yukawovu potenciálu.

В работе даются формулы для остаточного протон-нейтронного взаимодействия в нечетно-нечетных деформированных ядрах. Учитавается потенциал включающий Вигнера, Бартлэтта, Майорана, Гейзенберга и тензорные силы. Выражения удобные для нумерических вычислений получены для радиальной зависимости потенциала в виде δ-функции, гармонического осциллатора, прямоуголной ямы и потенциалов Гаусса о Юкавы,

Residual proton-neutron interaction in odd-odd deformed nuclei is calculated. Nuclear potential including Wigner, Bartlett, Majorana, Heisenberg and tensor forces are considered. Expressions permitting numerical calculations are derived using δ-function, oscillator, square-well, Gausian and Yukawa radial dependences for proton-neutron potential.

1. Introduction

In last twenty five years the understanding of the structure of defoımed nuclei has ıeached rather big progress. Many excited states were succesfully interpreted in framework of unified model based on the single particle motion in Nilsson or Woods-

*) Participated partly in time of preparation of diploma thesis.

**) 180 00 Praha 8, Pelc-Tyrolka, Povltavská ul., Czechoslovakia.

Saxon potentials with pairing interaction included [1]. Nevertheless, some substantial difficulties were met if the states in odd-odd deformed nuclei were interpreted. They are connected mainly with residual interaction between odd proton and odd neutron, which affects strongly individual quasiparticle states in odd-odd nuclei. As a result the intrinsic states with $K = |\Omega_p \pm \Omega_n|$ have different energy. The splitting, ΔE, is rather sensitive to the form of the potential describing the $p-n$ residual interaction.

Role of $p-n$ interaction in heavy odd-odd deformed nuclei was examined in relatively few papers (e.g. [2–5]). Analysis with rather general form of the $p-n$ potential was carried out in Ref. [2], but only the "odd-even shift" in $K = 0$ rotational bands was discussed. Splitting of the $K = |\Omega_p \pm \Omega_n|$ states was discussed by Pyatov [3], however only potential for zero-range forces was considered. More detailed calculations were performed by Jones et al. [4] and Lassjo et al. [5] who took general form of the $p-n$ potential, but calculations are limited to the Gaussian form of the radial dependence.

In the present work are derived the expressions permitting numerical calculation of the splitting in odd-odd deformed nuclei. The $p-n$ potential, V_{pn}, including Wigner, Bartlett, Majorana, Heisenberg and tensor forces is taken into account. Influence of different radial dependence of central part of V_{pn} on the $K = |\Omega_p \pm \Omega_n|$ splitting is examined and formulas for δ-function (zero-range), oscillator, square-well, Gaussian and Yukawa dependences are expressed in form convenient for numerical calculations.

2. General formulation of problem

Total hamiltonian of odd-odd deformed nucleus can be written in the form [1]

$$H = H_p + H_n + V_{pn} + H_R + H_{CI} . \tag{1}$$

Here H_p and H_n are the hamiltonians for proton (p) and neutron (n) single particle motion respectively, H_R represents rotation of the nucleus as a whole and H_{CI} describes coupling between rotational and intrinsic motion (Coriolis interaction). V_{pn} is potential of residual $p-n$ interaction which is assumed to be fully responsible for the splitting of the $K = |\Omega_p \pm \Omega_n|$ states in the nucleus.

Neglecting H_{CI} term the wave function of unperturbed hamiltonian $H_0 = H_p + H_n + H_R$ can be written in the form [6]

$$|IK, \Omega_p\Omega_n\gamma\rangle = \left(\frac{2I+1}{16\pi^2}\right)^{1/2} (1 + R_1)\, \mathscr{D}^I_{MK}\Phi_K \tag{2}$$

with intrinsic wave function defined as

$$\Phi_K = \chi_{\Omega_p}\chi_{\Omega_n} , \quad K = |\Omega_p \pm \Omega_n| , \quad \gamma = 0 \tag{3}$$

for $K > 0$ and

$$\Phi_{K=0} = \tfrac{1}{2}(\chi_{\Omega_p}\chi_{-\Omega_n} + \gamma_{-\Omega_p}\chi_{\Omega_n}) , \quad \gamma = \pm 1 \tag{4}$$

for $K = 0$. The single particle wave functions χ_Ω are further considered as calculated from the Nilsson potential [1, 6].

The splitting, ΔE, of the states in odd-odd deformed nucleus can be then defined as[a])

$$\Delta E = \langle IK_1, \Omega_p\Omega_n\gamma| \, V_{pn} |IK_1, \Omega_p\Omega_n\gamma\rangle - \\ - \langle IK_2, \Omega_p\Omega_n\gamma| \, V_{pn} |IK_2, \Omega_p\Omega_n\gamma\rangle \tag{5}$$

where $K_1 = \Omega_p + \Omega_n$ and $K_2 = |\Omega_p - \Omega_n|$ correspond to the parallel and antiparallel coupling of Ω – s.

3. The residual p—n interaction

Expressions for the splitting, ΔE, were calculated with proton-neutron potential V_{pn} including Wigner (W), Bartlett (B), Majorana (M), Heisenberg (H) and tensor (T) types of nuclear forces. As the calculation with customary form of potential V_{pn} is very complicated and unclear more convenient form of V_{pn} was used and V_{pn} was written as

$$V_{pn} = V(|\vec{r}_p - \vec{r}_n|) \sum_{i=1}^{5} \alpha_i O_i \tag{6}$$

α_i are parameters connected with strength parameter V_k, $k \equiv W, B, M, H, T$ (deepness parameter) for individual types of p—n potential and are equal

$$\alpha_1 = V_W + \frac{V_B}{2}, \quad \alpha_2 = \frac{V_B}{2} \tag{7}$$

$$\alpha_3 = V_M + \frac{V_H}{2}, \quad \alpha_4 = \frac{V_H}{2}, \quad \alpha_5 = V_T .$$

Operators O_i were used in the form

$$O_1 = 1, \quad O_2 = (\vec{\sigma}_p\vec{\sigma}_n), \quad O_3 = P_M, \quad O_4 = (\vec{\sigma}_p\vec{\sigma}_n) P_M \tag{8}$$

$$O_5 = S_{pn} = \frac{3((\vec{r}_p - \vec{r}_n)\vec{\sigma}_p)((\vec{r}_p - \vec{r}_n)\vec{\sigma}_n)}{|\vec{r}_p - \vec{r}_n|^2} - (\vec{\sigma}_p\vec{\sigma}_n)$$

which made it possible to express simply individual parts, $(V_{pn})_k$, of the total p—n potential V_{pn} and simultaneously simplifies substantially the calculations. For radial

[a]) If calculated splitting for the $K = 0$, $\Omega_p = \Omega_n = 1/2$ states is compared with experimental one the diagonal matrix elements of H_{Cl} has to be considered.

part, $V(|\vec{r}_p - \vec{r}_n|)$, of the potential the δ-function (d), Gaussian (g), harmonic oscillator (h), square-well (w) and Yukava (y) dependences in the forms

$$V_d(r) = -\frac{4\pi}{r^2}\,\delta(r) \tag{9}$$

$$V_g(r) = -\frac{2}{\pi^{1/2} r_g}\,\mathrm{e}^{-(r/r_g)^2} \tag{10}$$

$$V_w(r) = \begin{cases} -\dfrac{1}{r_w} & \text{for } r \leqq r_w \\ 0 & \text{for } r > r_w \end{cases} \tag{11}$$

$$V_h(r) = \begin{cases} -\dfrac{3}{2r_h}\left[1 - \left(\dfrac{r}{r_h}\right)^2\right] & \text{for } r \leqq r_h \\ 0 & \text{for } r > r_h \end{cases} \tag{12}$$

$$V_y(r) = -\frac{1}{r}\,e^{-r/r_y} \tag{13}$$

respectively were considered. Here $r = |\vec{r}_p - \vec{r}_n|$, r_j, $j \equiv g, h, w, y$ is the range parameter. The energy splitting (5) can be then rewritten in more explicite form

$$\Delta E = \sum_{i=1}^{5} \alpha_i (A_i^p - A_i^a) \tag{14}$$

in which A_i are diagonal matrix elements of operators O_i . $V(r)$

$$A_i = \langle IK, \Omega_p\Omega_n\gamma | V(r)\, O_i | IK, \Omega_p\Omega_n\gamma\rangle \tag{15}$$

for fixed form of radial dependence $V(r)$ given by Eq. (9)–(13). Indexes "p" and "a" in (14) refer to the parallel and antiparallel coupling of Ω-s respectively. Derivation of general expressions is now connected with explicite evaluation of A_i for corresponding form of p–n potential.

Structure of operators in (15) made it possible to separate the space and spin dependence in matrix elements A_i. It is therefore convenient to express Nilsson functions χ_Ω in intrinsic wave function Φ_K (Eq. (3) and (4)) as a product of the space and spin parts. Intrinsic wave function Φ_K can be then rewritten in term of Nilsson coefficients $a_{Nl\Lambda}$ for proton and neuton parts [7] and corresponding Clebsch-Gordon coefficients. After rather long calculations matrix elements A_i can be expressed in the form

$$A_i = Q_{IK\gamma} \sum_a RB_i\,, \quad i = 1, 2, 3, 4, 5 \tag{16}$$

where $Q_{IK\gamma}$ is equal

$$Q_{IK\gamma} = 1 + \delta_{K,0}\,\frac{\prod_p \prod_n (-1)^I \gamma - 1}{2} \tag{17}$$

Factor R is composed from Clebsch-Gordon coefficients for coupling of proton and neutron orbital and spin moments and from corresponding Nilsson coefficients $a_{Nl\Lambda}$. Addition in (16) is carried out through indexes $a \equiv (N_p l_p N_n l_n L N'_p l'_p N'_n l'_n\ L' \Lambda_p \Lambda_n \cdot \Sigma_p \Sigma_n \lambda \Lambda'_p \Lambda'_n \Sigma'_p \Sigma'_n \lambda' s \sigma s' \sigma')$ for which relations

$$\Lambda_i + \Sigma_i = \Omega_i\,, \quad i = p, n \tag{18}$$

$$\lambda = \Lambda_p \pm \Lambda_n\,, \quad \sigma = \Sigma_p - \Sigma_n$$

are valid. By stroke are distinguished the indexes related to both wave functions in matrix elements A_i. It should be noted that if the $\Delta N \neq 0$ interaction in nucleus is neglected the addition through N falls off.

Matrix elements B_i in (16) break down in a few paits as a result of the term $(1 + R_1)$ in wave function (2) and can be written in the form

$$B_i = \tfrac{1}{2}\{[C_i^{++} + (-1)^{L+s+L'+s'} C_i^{--}] \pm \pm (-1)^{I-L-s} \delta_{K,0} [C_i^{+-} + (-1)^{L+s+L'+s'} C_i^{-+}]\} \tag{19}$$

for $K = |\Omega_p \pm \Omega_n|$ respectively. The expressions $C_i^{\varphi_p\varphi_n}$ are matrix elements

$$C_i^{\varphi_p\varphi_n} = \langle N'_p l'_p N'_n l'_n;\ L' \varphi_p \lambda' | \langle \tfrac{1}{2}\,\tfrac{1}{2};\ s' \varphi_p \sigma' |\ V(r)\, O_i | \tfrac{1}{2}\,\tfrac{1}{2};\ s \varphi_n \sigma \rangle\, | N_p l_p N_n l_n;\ L \varphi_n \lambda \rangle\,. \tag{20}$$

The indexes "φ_p" and "φ_n" are equal to "+" or "−" and are connected with development of the wave functions.

Matrix elements $C_i^{\varphi_p\varphi_n}$ have to be calculated independently for each operator O_i (8). Calculations are rather complicated and very tedious and are similar foi all O_i (except for O_5, for which different kind of terms appears). Therefore we present here as an illustrative example of used method only evaluation of the matrix elements for O_2 operator in more details. For other opeiators, including O_5 one, only the resulting expressions are given.

First we rewrite matrix elements $C_2^{\varphi_p\varphi_n}$ (20) as a product of space $(D_2^{\varphi_p\varphi_n})$ and spin $(\mathscr{D}_2^{\varphi_p\varphi_n})$ parts

$$C_2^{\varphi_p\varphi_n} = \mathscr{D}_2^{\varphi_p\varphi_n} D_2^{\varphi_p\varphi_n}\,. \tag{21}$$

Further we will evaluate only the teim with $\varphi_p = \varphi_n = +$. Corresponding term $\mathscr{D}_2^{++}$ can be rewritten with respect to the properties of the $\vec{S}^2$ operator in simple form

$$\mathscr{D}_2^{++} = \delta_{s',s} \delta_{\sigma',\sigma} [2s(s+1) - 3]\,. \tag{22}$$

For evaluation of space matrix element D_2^{++} it is convenient to transform the expressions to new coordinates

$$\vec{r}_t = \frac{\vec{r}_p + \vec{r}_n}{2^{1/2}}\,, \quad \vec{r}_r = \frac{\vec{r}_p - \vec{r}_n}{2^{1/2}}\,. \tag{23}$$

The wave function $|N_p l_p N_n l_n, L\lambda\rangle$ can be then rewritten using the Talmi-Moshinski coefficients $(N_p l_p N_n l_n |L| N_r l_r N_t l_t)$ [8, 9] and is

$$|N_p l_p N_n l_n; L\lambda\rangle = \tag{24}$$
$$= \sum_{N_r l_r N_t l_t} (N_p l_p N_n l_n |L| N_r l_r N_t l_t) |N_r l_r N_t l_t; L\lambda\rangle .$$

Now the operator $V(r)$ acts only on new coordinate r_r. This made it possible, after rearrangement of wave functions, to express matrix element D_2^{++} in more insight form

$$D_2^{++} = \delta_{L',L}\delta_{\lambda',\lambda} Z \tag{25}$$

with matrix element Z defined as

$$Z = \sum_{N_r l_r N_t l_t N_r'} (N_p' l_p' N_n' l_n' |L| N_r' l_r N_t l_t) \times \tag{26}$$
$$\times (N_p l_p N_n l_n |L| N_r l_r N_t l_t) F(N_r', l_r, N_r, l_r) .$$

Here $F(N', l', N, l)$ is radial integral calculated in coordinate system r_r

$$F(N', l', N, l) = \int_0^{+\infty} R_{N'l'}(r) V(2^{1/2} r) R_{Nl}(r) r^2 \, dr \tag{27}$$

which is explicitely dependent on the shape of the potential $V(r)$. The evaluation of $F(N', l', N, l)$ for all five types of considered radial dependences (9)–(13) of potential will be given in part 3.1.

Substituting (25) and (22) into (21) the matrix element C_2^{++} (20) can be expressed in definitive form

$$C_2^{++} = \delta_{L',L}\delta_{\lambda',\lambda}\delta_{s',s}\delta_{\sigma',\sigma}[2s(s+1) - 3] Z . \tag{28}$$

Similar evaluation of other $C_2^{\varphi_p \varphi_n}$ and substitution into (19) made it possible to express matrix element B_2.

Further evaluation of matrix element A_i (16) is simplified if addition through indexes Λ_p, Λ_n, Σ_p, Σ_n, λ, Λ_p', Λ_n', Σ_p', Σ_n', λ', s, σ, s', σ' is carried out. Properties of B_2, together with explicite form of R made it possible to rewrite finally matrix element A_2 as

$$A_2 = Q_{IK\gamma} \sum_{N_p' l_p' N_n' l_n' L N_p l_p N_n l_n} Z S^* . \tag{29}$$

Here S^* is rather complex expression

$$S^* = \varrho_{++}' \varrho_{++} + \varrho_{+-}'(2\varrho_{-+} - \varrho_{+-}) + \varrho_{-+}'(2\varrho_{+-} - \varrho_{-+}) + \tag{30}$$
$$+ \varrho_{--}' \varrho_{--} \mp (-1)^{I-L} \delta_{K,0}[\varrho_{++}' \varrho_{--} + \varrho_{+-}'(2\varrho_{+-} - \varrho_{-+}) +$$
$$+ \varrho_{++}'(2\varrho_{-+} - \varrho_{+-}) + \varrho_{--}' \varrho_{++}]$$

for $K = |\Omega_p \pm \Omega_n|$ respectively. Nevertheless, the components $\varrho_{\varphi_p\varphi_n}$ are constructed from Nilsson and Clebsch-Gordon coefficients only and have the form

$$\varrho_{\varphi_p\varphi_n} = a_{N_p l_p \Lambda_p} a_{N_n l_n \Lambda_n} (l_p \Lambda_p l_n \Lambda_n \mid L \Lambda_p + \Lambda_n) \tag{31a}$$

$$\Lambda_p + (\varphi_p \tfrac{1}{2}) = \Omega_p\,, \quad \Lambda_n + (\varphi_n \tfrac{1}{2}) = \Omega_n$$

for $K = \Omega_p + \Omega_n$,

$$\varrho_{\varphi_p\varphi_n} = a_{N_p l_p \Lambda_p} a_{N_n l_n \Lambda_n} (l_p \Lambda_p l_n - \Lambda_n \mid L \Lambda_p - \Lambda_n) \tag{31b}$$

$$\Lambda_p + (\varphi_p \tfrac{1}{2}) = \Omega_p\,, \quad \Lambda_n - (\varphi_n \tfrac{1}{2}) = \Omega_n$$

for $\Omega_p - \Omega_n \geqq 0$ and

$$\varrho_{\varphi_p\varphi_n} = a_{N_p l_p \Lambda_p} a_{N_n l_n \Lambda_n} (l_p - \Lambda_p l_n \Lambda_n \mid L - \Lambda_p + \Lambda_n) \tag{31c}$$

$$\Lambda_p - (\varphi_p \tfrac{1}{2}) = \Omega_p\,, \quad \Lambda_n + (\varphi_n \tfrac{1}{2}) = \Omega_n$$

for $\Omega_n - \Omega_p > 0$.

Evaluation of matrix elements A_i (16) for $i = 1, 3, 4$ can be carried out in a similar way as used for A_2. Only evaluation of matrix element A_5 is more complicated because the tensor operator S_{pn} in (8) has to be explicitely expressed. It can be done if expression similar to Eq. (1.91) in Ref. [10] for S_{pn} is used. Further method of evaluation of A_5 is then similar to that for other A_i.

After rather complicated and tedious calculations the matrix elements A_i can be expressed in definitive form

$$A_1 = Q_{IK\gamma} \sum_{N_p l_p N_n l_n L N_p' l_p' N_n' l_n'} ZS \tag{32}$$

$$A_3 = Q_{IK\gamma} \sum_{N_p l_p N_n l_n L N_p' l_p' N_n' l_n'} Z^* S \tag{33}$$

$$A_4 = Q_{IK\gamma} \sum_{N_p l_p N_n l_n L N_p' l_p' N_n' l_n'} Z^* S^* \tag{34}$$

$$A_5 = Q_{IK\gamma} \sum_{N_p l_p N_n l_n L N_p' l_p' N_n' l_n' L'} YT \tag{35}$$

Here S^* and Z aıe expressions (30) and (26), Z^* and Y are defined as

$$Z^* = \sum_{N_r l_r N_t l_t N_r'} (-1)^{l_r} (N_p' l_p' N_n' l_n' \mid L \mid N_r' l_r N_t l_t) \times \tag{36}$$

$$\times (N_p l_p N_n l_n \mid L \mid N_r l_r N_t l_t) F(N_r', l_r, N_r, l_r)$$

$$Y = (-1)^{L+1} (2L+1)^{1/2} \sum_{N_r l_r N_t l_t N_r' l_r'} (2l_r' + 1)^{1/2} \times \tag{37}$$

$$\times (N_p' l_p' N_n' l_n' \mid L' \mid N_r' l_r' N_t l_t) F(N_r', l_r', N_r, l_r) \times$$

$$\times (N_p l_p N_n l_n \mid L \mid N_r l_r N_t l_t) (l_r' \, 0 \, 2 \, 0 \mid l_r \, 0) \times$$

$$\times \begin{pmatrix} l_r' & l_t & L' \\ L & 2 & l_r \end{pmatrix}$$

and S and T are again complex coefficients expressed through $\varrho_{\varphi p \varphi n}$ (Eq.(31a)–(31c))

$$S = \varrho'_{++}\varrho_{++} + \varrho'_{+-}\varrho_{+-} + \varrho'_{-+}\varrho_{-+} + \varrho'_{--}\varrho_{--} \mp \tag{38}$$
$$\mp (-1)^{I-L}\,\delta_{K,0}(\varrho'_{++}\varrho_{--} + \varrho'_{+-}\varrho_{-+} + \varrho'_{-+}\varrho_{+-} + \varrho'_{--}\varrho_{++})$$

$$T = \varrho'_{++}[2(LK-1\,2\,0\,|\,L'K-1)\,\varrho_{++} + 6^{1/2}(LK\,2-1\,|\,L'K-1)\times \tag{39}$$
$$\times(\varrho_{+-}+\varrho_{-+}) + 2\,.\,6^{1/2}(LK+1\,2-2\,|\,L'K-1)\,\varrho_{--}] - (\varrho'_{+-}+\varrho'_{-+})\times$$
$$\times[6^{1/2}(LK-1\,2\,1\,|\,L'K)\,\varrho_{++} + 2(LK\,2\,0\,|\,L'K)(\varrho_{+-}+\varrho_{-+}) +$$
$$+ 6^{1/2}(LK+1\,2-1\,|\,L'K)\,\varrho_{--}] + \varrho'_{--}[2\,.\,6^{1/2}(LK-1\,2\,2\,|\,L'K+1)\times$$
$$\times\,\varrho_{++} + 6^{1/2}(LK\,2\,1\,|\,L'K+1)(\varrho_{+-}+\varrho_{-+}) +$$
$$+ 2(LK+1\,2\,0\,|\,L'K+1)\,\varrho_{--}] \mp (-1)^{I-L'}\,\delta_{K,0}\{\varrho'_{++}\times$$
$$\times[2\,.\,6^{1/2}(L-1\,2\,2\,|\,L'\,1)\,\varrho_{++} + 6^{1/2}(L0\,2\,1\,|\,L'\,1)\times$$
$$\times(\varrho_{+-}+\varrho_{-+}) + 2(L1\,2\,0\,|\,L'\,1)\,\varrho_{--}] - (\varrho'_{+-}+\varrho'_{-+})\times$$
$$\times[6^{1/2}(L-1\,2\,1\,|\,L'\,0)\,\varrho_{++} + 2(L0\,2\,0\,|\,L'\,0)\times$$
$$\times(\varrho_{+-}+\varrho_{-+}) + 6^{1/2}(L1\,2-1\,|\,L'\,0)\,\varrho_{--}] + \varrho'_{--}\times$$
$$\times[2(L-1\,2\,0\,|\,L'-1)\,\varrho_{++} + 6^{1/2}(L0\,2-1\,|\,L'-1)\times$$
$$\times(\varrho_{+-}+\varrho_{-+}) + 2\,.\,6^{1/2}(L1\,2-1\,|\,L'-1)\,\varrho_{--}]\}\,.$$

The note "$\pm$" in expressions (38) and (39) corresponds again to $K = |\Omega_p \pm \Omega_n|$ respectively.

3.1 Radial integrals

For evalution of integrals (27) radial wave function, $R_{nl}(r)$, has to be explicitely expressed. Using properties of degenerate hypergeometric functions $R_{nl}(r)$ can be expressed as [7]

$$R_{nl}(r) = \sum_{k=0}^{n} \beta_k(l, n)\, \nu^{l/2+k+3/4} e^{-\nu r^2/2} r^{2k+l} \tag{40}$$

where $2n = (N-1)$, $\beta_k(l, n)$ are normalization coefficients and ν is numerical factor of dimension m^{-2} which is proportional to $A^{-1/3}$. After substitution (40) into (27) radial integral $F(N', l', N, l)$ can be expressed in form

$$F(N', l', N, l) = \sum_{k'=0}^{n'} \beta_{k'}(l', n') \sum_{k=0}^{n} \beta_k(l, n) \times \Phi\left(\frac{l+l'}{2} + k + k' + 1, \nu\right) \tag{41}$$

through new integrals $\Phi(m, \nu)$

$$\Phi(m, \nu) = \int_0^{+\infty} (\nu r^2)^m e^{-\nu r^2}\, V(2^{1/2} r)\, \nu^{1/2}\, dr\,. \tag{42}$$

Integrals $\Phi(m, v)$ depends on radial shape of nuclear potential $V(r)$ and can be evaluated with use of auxilliary expressions

$$K(m, x) = \int_0^x r^{2m} e^{-r^2} dr = x^{2m+1} \sum_{k=0}^{+\infty} \frac{(-1)^k x^{2k}}{k!(2m + 2k + 1)}, \quad m \geqq 1 \tag{43}$$

$$L(m) = \lim_{x \to +\infty} K(m, x) = \int_0^{+\infty} r^{2m} e^{-r^2} dr = \frac{(2m - 1)!! \pi^{1/2}}{2^{m+1}}, \quad m \geqq 1 \tag{44}$$

$$M(b, m) = \int_0^{+\infty} r^{2m-1} e^{-r(r+b)} dr = \frac{(2m - 1)!!}{2^{2m}} \mu(b, m), \quad m \geqq 1 \tag{45}$$

Here $\mu(b, m)$ are defined as

$$\mu(b, 1) = 2\left[1 - be^{b^2/4}\left(\frac{\pi^{1/2}}{2} - G\left(\frac{b}{2}\right)\right)\right] \tag{46a}$$

$$\mu(b, 2) = \frac{6 + b^2}{3} \mu(b, 1) - \tfrac{4}{3} \tag{46b}$$

$$\mu(b, m + 1) = \frac{8m - 2 + b^2}{2m + 1} \mu(b, m) - \frac{8(m - 1)}{2m + 1} \mu(b, m - 1), \quad m \geqq 2 \tag{46c}$$

and $G(x)$ is known integral [11]

$$G(x) = \int_0^x e^{-r^2} dr = \sum_{k=0}^{\infty +} \frac{(-1)^k x^{2k+1}}{k!(2k + 1)}. \tag{47}$$

(Evaluation of integral $\Phi(m, v)$ (42) can be done if integration "per partes" and value of integral (47) are used).

Substituting into (42) fixed radial shape of nuclear potential $V(r)$ (in one of forms (9)–(13)), integrals $\Phi(m, v)$ for each shape $V(r)$ can be expressed through (43)–(45) and are of definitive form

$$\Phi_d(m, v) = -2^{1/2} \pi v^{3/2} \delta_{m,1} \tag{48}$$

$$\Phi_g(m, v) = -\frac{2}{\pi^{1/2} r_g \left(1 + \frac{2}{v r_g^2}\right)^{m+1/2}} L(m) = $$

$$= -\frac{(2m - 1)!!}{r_g 2^m \left(1 + \frac{2}{v r_g^2}\right)^{m+1/2}} \tag{49}$$

$$\Phi_w(m, v) = -\frac{1}{r_w} K(m, v^{1/2} r_w 2^{-1/2}) =$$

$$= - \frac{r_w^{2m} v^{m+1/2}}{2^{m+1/2}} \sum_{k=0}^{+\infty} \frac{(-1)^k r_w^{2k} v^k}{2^k k! (2m + 2k + 1)} \tag{50}$$

$$\Phi_h(m, v) = - \frac{3}{2r_h} \left[K(m, v^{1/2} r_h 2^{-1/2}) - K(m + 1, v^{1/2} r_h 2^{-1/2}) \frac{2}{v r_h^2} \right] = \tag{51}$$

$$= \frac{3 r_h^{2m} v^{m+1/2}}{2^{m+1/2}} \sum_{k=0}^{+\infty} \frac{(-1)^k r_h^{2k} v^k}{2^k k! (2m + 2k + 1)(2m + 2k + 3)}$$

$$\Phi_y(m, v) = - v^{1/2} 2^{-1/2} M(2^{1/2} v^{-1/2} r_y^{-1}, m) = \tag{52}$$

$$= - \frac{v^{1/2}(2m - 1)!!}{2^{2m+1/2}} \mu(2^{1/2} v^{-1/2} r_y^{-1}, m) .$$

Radial integrals $F(N', l', N, l)$ can be now eplicitely expressed from Eq. (41). After substituting (41) into Eqs. (26), (36) and (37) for Z, Z^* and Y respectively matrix elements A_i (Eqs. (29), (32)–(35)) for different types of nuclear forces and different radial shapes of proton-neutron potential can be evaluated. The $K = |\Omega_p \pm \Omega_n|$ splitting in odd-odd deformed nuclei, ΔE, is then calculated directly from Eq. (14).

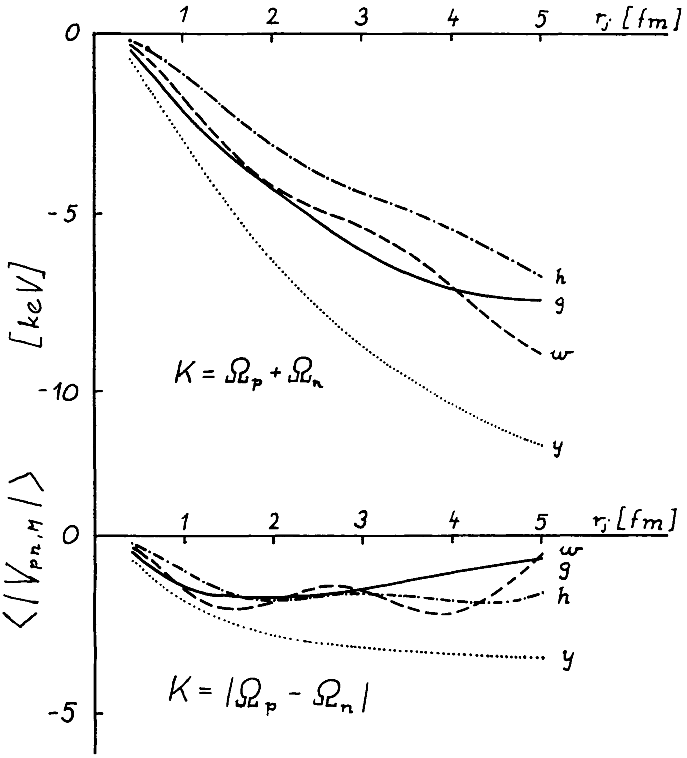

Fig. 1. Dependence of matrix elements for Majorana part of the p—n residual interaction in odd-odd deformed nuclei on radial shape of nuclear potential.

4. Illustrative example

Example of application of theoretical expressions for calculation of splitting in odd-odd deformed nuclei is presented in Fig. 1 for Majorana part of p – n interaction between proton and neutron in 5/2 + [402] and 3/2 – [521] Nilsson states respectively. Both matrix elements $\langle|(V_{pn})_M|\rangle$ from (5) are given as function of radial parameter r_j, $j \equiv g, h, w, y$ for different radial shapes (10)–(13) of p – n potential (for δ-function shape (9) matrix elements are constant). Calculation was performed with strength parameter of Majorana forces equal $V_M = 1$ MeV. Rather strong dependence of matrix elements (and simultaneously of corresponding part of splitting, ΔE, in odd-odd deformed nucleus) on radial parameter r_j and especially on radial shape of the p – n potential is clearly expressed.

References

[1] BOHR A., MOTTELSON B. R.: Nuclear Structure, Vol. II. Benjamin Inc., 1975.
[2] NEWBY N. D. jr.: Phys. Rev. *125* 1962, 2063.
[3] PJATOV N. I.: Izv. AN SSSR (ser. fiz.) *27* 1963, 1436.
[4] JONES H. D. et al.: Phys. Rev. *C3* 1971, 529.
[5] LASSIJO R. S. et al.: Atom Indonesia *3* 1977, 21.
[6] SOLOVLEV V. G.: Tĕorija složnych jadĕr. Nauka, Moskva 1971.
[7] NILSSON S. G.: Mat. Fys. Medd. Dan Vid. Selsk. *29* 1955, No. 16.
[8] MOSHINSKI M.: Nucl. Phys. *13* 1959, 104.
[9] TRLIFAJ L.: Phys. Rev. *C5* 1972, 1534.
[10] BOHR A., MOTTELSON B. R.: Nuclear Structure, Vol. I. Benjamin Inc., 1969.
[11] REKTORYS K. et al.: Přehled užité matematiky. SNTL, Praha 1963.