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Parallelism in Numerical Analysis

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New possibilities of multiprocessor computers (MC) and the effort for more effective expression of the structure of many algorithms brought about the introduction of parallelism in computation. The paper is a summary of reports [1] and [2] which contain besides some results in study of parallelism in computational processes also detailed survey of literature.

Let $A_i[x_i, s_i, F_i(w_i)] = y_i$ be an operator, which to the each $x_i \in X_i$ uniquely sets $y_i \in Y_i$, where X_i and Y_i are its input and output space, s_i is its starting function which determines the moment of beginning its execution, w_i is its weight giving in some units the total time necessary for its execution, F_i is operation of A_i which executes given operator, determines the order of execution of other operators and desides about further way of computation if A_i contains some predikats.

The mathematical method M we consider as its decomposition on operators $A_i, i = 1, 2, \dots, n$ which creats the set $A = \{A_i[x_i, s_i^{(j)}, F_i(w_i)] = y_i, i = 1, 2, \dots, n, j = 1, 2, \dots, p_i\}$ where p_i gives the number of necessary repeating of A_i in M and $s_i^{(j)}$ is starting function of A_i for its j-th repetition. By introduction of order on A which determines the order of execution of given operators we get from A the computational process V. V is a mapping which to the each $x \in I$ uniquely sets the element $y \in O$ where I and O are input and output space of V. The order of operators on A can be carry out by means of finite, acyclic, weighted and oriented graph G representing functional and control function of V. The execution of A in order given by G is going on MC in the discrete real time.

Let in A to the each A_i exists in accordance with G the set of time values $W_i = \{T_i^{(j)}, j = 1, 2, \dots, p_i, T_i^{(s)} < T_i^{(s+1)}, s = 1, 2, \dots, p_i - 1\}$. Let $W = \bigcup_{i=1}^{n} W_i$. By ordering elements of W we get the sequence $t_0 < t_1 < \dots < t_r$. Let for the each t_j there exists the set $Q_{t_j} = \{A_k[x_k, s_k^{(j)}, F_k(w_k)] = y_k, k = k_m^{(j)}, m = 1, 2, \dots, d_j\}$ where $k_m^{(j)}$ are chosen indices from the set $\{1, 2, \dots, n\}$ for which exists integer q such that $T_{k_m^{(j)}}^{(q)} = t_j$ and $s_{k_m^{(j)}}^{(q)} = 1$ whereby $x_k \in X_k \subset I_j$ and $y_k \in Y_k$ where I_j is the given sequence of spaces $I_0 = I, I_{j+1} = I_j \otimes O_j, j = 0, 1, \dots, r$ and $O_j = Y_{k_1^{(j)}} \otimes \otimes Y_{k_2^{(j)}} \otimes \dots \otimes Y_{k_{d_j}^{(j)}}$. The sequence of sets $\{Q_{t_j}\}, j = 0, 1, \dots, r$ we call the computational process V_t in real time. Expressing O_j as $MV_j \otimes R_j$ where MV_j and R_j is space of interresults and results of operators from Q_{t_j} then $O = R_0 \otimes R_1 \otimes \otimes \dots \otimes R_r$.

If at execution of V_t in each time moment works only one operator from A then V_t is sequential process, in reverse case V_t is the parallel computational process (PCP). In this case decomposition M on A we call as desequention which always must be construct in such a way that s.c. degree of parallelism of V_t was maximal.

The execution V_t on MC we consider as the function $V_t = H(A, G, N, MC, OS, P)$ where N is the array of variables on which operate operators from A, OS is the operational system of given MC and P is a programm for computation of operators from A. Since the state of arguments of H depends on passing time then depending on the time t_0 of beginning of computation V_t on MC the great variability of possibilities for execution of V_t is given. The choice of optimal execution of V_t at given t_0 produces s.c. asynchronous programming. In [1] we investigate all arguments of H. The asummed properties of fictitious MC, its OS and asynchronous programming are formulated whereby their functions in the execution of the given V_t are divided. The basic characteristics of PCP are defined which are important at its construction, clasification and comparison. The synchronous, asynchronous and chaotic execution of PCP is described and the questions of efectivity of these executions are investigated.

Except PCP in real time also PCP with eliminated time with immediate execution of operators is described, whereby the possibilities coding of information about order of execution of operators is given. For determining order of operators three method are described and compared. Method of analysis of matrices which represent given graph G, method of computation of exact time moment of execution of each operator and method of construction starting functions is described.

As long as of algorithms of numerical analysis we can say that in structure of many numerical algorithms there are the feature of parallelism and that for these algorithms parallel information processing and asynchronous computations are more natural and simply than sequential. The aim of research in the construction of parallel algorithms of numerical analysis is in the searching of hidden parallelism in until now sequential used algorithms and in construction of new highly parallel stable algorithms.

We divided parallelism in numerical algorithms on inherent (trivial or non-trivial), artificial (redundant or rational) and constructed (desequential or original). According of this division a survey of up-to-date results in construction of parallel algorithms of numerical analysis is given in [2].

For the description of parallel numerical algorithms we suggested the symbolic description $PARALGOL$ from which is clear the order of execution of all operators of given algorithm.

References

[1] Mikloško, J.: Theoretical Principles of General Parallel Computational Processes, Theoretical Foundations of 4-th Generation Computers, Report Z-20/140 — 1973, p. 6—41, Bratislava (1973).
[2] Mikloško, J.: Parallelism in Numerical Mathematics (The same report, p. 54—68.)