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# A Parallel Method of Solution of the Differential Equations by the Formulas Containing Higher Derivatives

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The paper deals with the solution of the initial problem of the ordinary differential equations. Formulas containing higher derivatives from predictor-corrector method for the parallel calculation are used.

(1) Let us consider the differential equation

$$y' = f(t, y)\,, \quad t > 0\,, \quad y(0) = y_0\,. \tag{1}$$

The mesh points are $t_n = (n - 1)\,h$, $n = 1, 2, 3, \ldots, h$ is increment, $y_n$ denotes an approximation of solution $y$ at point $t_n$, ${}^py_n$, ${}^py'_n$, ${}^py''_n$, ... are the predicted values and ${}^cy_n$, ${}^cy'_n$, ${}^cy''_n$, ... are the corrected values of $y_n, y'_n, y''_n, \ldots$ .

Let us accept the formulas containing higher derivatives for the solution of the differential equation (1), for example [1]:

$$\left.\begin{aligned}
&P_{12}: {}^py_{n+1} = 2\,{}^cy_n - {}^cy_{n-1} - h({}^cy'_n - {}^cy'_{n-1}) + \frac{1}{2}\,h^2(3\,{}^cy''_n + {}^cy''_{n-1}) \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad [0{,}041 \ldots h^5] \\
&Q_{12}: {}^cy_{n+1} = 2\,{}^cy_n - {}^cy_{n-1} + \frac{3}{8}\,h({}^py'_{n+1} - {}^cy'_{n-1}) - \\
&\quad - \frac{1}{24}\,h^2({}^py''_{n+1} - 8\,{}^cy''_n + {}^cy''_{n-1}) \qquad\qquad [0{,}000\,016 \ldots h^8]
\end{aligned}\right\} \tag{2}$$

The sequence of computation by the formulas (2) is

$$\to {}^py_{n+1} \to {}^py'_{n+1} \to {}^py''_{n+1} \to {}^cy_{n+1} \to {}^cy'_{n+1} \to {}^cy''_{n+1} \to$$

This is illustrated in the Schema 1.

By using the formulas (2) in form of a parallel calculation:

$${}^py_{n+1} = 2\,{}^py_n - {}^cy_{n-1} - h({}^py'_n - {}^cy'_{n-1}) + \frac{1}{2}\,h^2(3\,{}^py''_n - {}^cy''_{n-1})$$

$${}^cy_n = 2\,{}^cy_{n-1} - {}^cy_{n-2} + \frac{3}{8}\,h({}^py'_n - {}^cy'_{n-2}) - \frac{1}{24}\,h^2({}^py''_n - 8\,{}^cy''_{n-1} + {}^cy''_{n-2})$$

the sequence of computation is

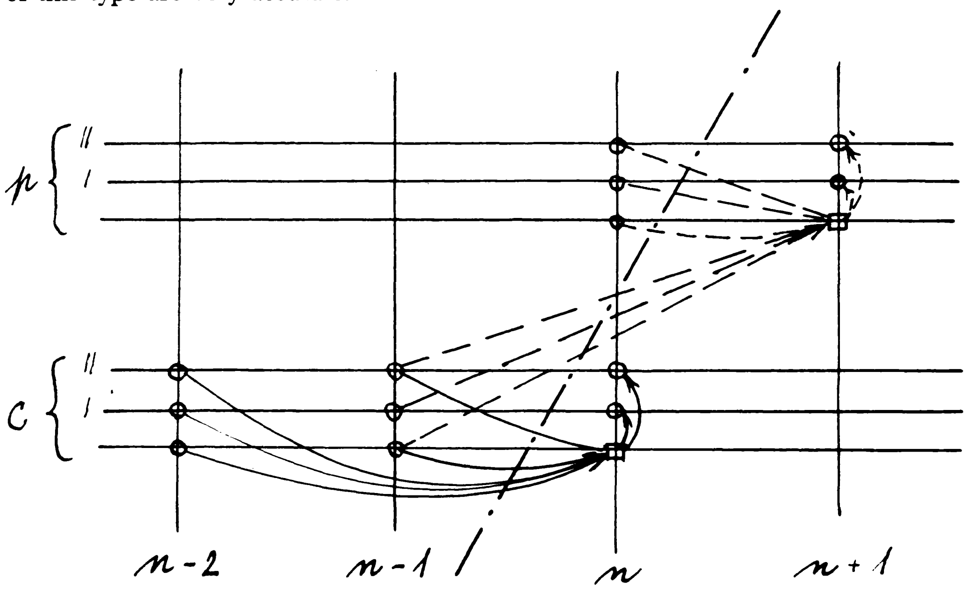
$$\to {}^cy_n \to {}^cy'_n \to {}^cy''_n \to \ldots$$

and

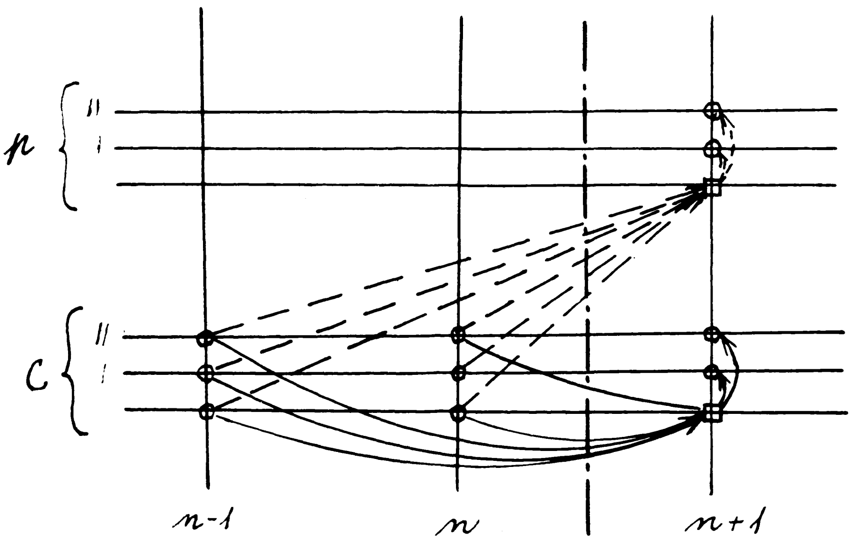
$$\to {}^py_{n+1} \to {}^py'_{n+1} \to {}^py''_{n+1} \to \ldots\,.$$

This is illustrated in the Schema 2.

The computing by using formulas containing the three, four, etc., derivatives and values in more the mesh points proceeds analogically. It is clear that the formulas of this type are very accurate.




*Scheme* 1.




*Scheme* 2.

(2) In general, formulas for parallel calculation for two processors are in the following form:

$$
\begin{aligned}
{}^{p}y_{n+1} &= \sum_{i=0} a_i y_{n-i} + h \sum_{i=0} b_i y'_{n-i} + h^2 \sum_{i=0} c_i y''_{n-i} + \dots \\
{}^{c}y_{n} &= \sum_{i=1} a'_i y_{n-i} + h \sum_{i=0} b'_i y'_{n-i} + h^2 \sum_{i=0} c'_i y''_{n-i} + \dots
\end{aligned}
$$

($a_i$, $a'_i$, $b_i$, $b'_i$, $c_i$, $c'_i$, ... are constants).

For four processors:

$$
\begin{aligned}
{}^{p}y_{2n+2} &= \sum_{i=2} \alpha_i y_{2n-i} + h \sum_{i=0} \beta_i y'_{2n-i} + h^2 \sum_{i=0} \gamma_i y''_{2n-i} + \dots \\
{}^{p}y_{2n+1} &= \sum_{i=2} \alpha'_i y_{2n-i} + h \sum_{i=0} \beta'_i y'_{2n-i} + h^2 \sum_{i=0} \gamma'_i y''_{2n-i} + \dots \\
{}^{c}y_{2n} &= \sum_{i=3} \alpha''_i y_{2n-i} + h \sum_{i=0} \beta''_i y'_{2n-i} + h^2 \sum_{i=0} \gamma''_i y''_{2n-i} + \dots \\
{}^{c}y_{2n-1} &= \sum_{i=3} \alpha'''_i y_{2n-i} + h \sum_{i=0} \beta'''_i y'_{2n-i} + h^2 \sum_{i=0} \gamma'''_i y''_{2n-i} + \dots
\end{aligned}
$$

($\alpha_i$, $\alpha'_i$, $\alpha''_i$, $\alpha'''_i$, $\beta_i$, $\beta'_i$, $\beta''_i$, $\beta'''_i$, $\gamma_i$, $\gamma'_i$, $\gamma''_i$, $\gamma'''_i$, ... are constants).

## References


[1] JANKOVIČ, V.: Formulas for Numerical Solution of $y' = f(t, y)$, Containing Higher Derivatives. Aplikace matematiky 10 (1965).