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The Amplitude Curves of Seismic Waves at Short Epicentral Distances

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In the interpretation of seismic measurements a great importance corresponds to the dynamic parameters of seismic body waves, such as their amplitude curves, spectra etc. A theoretical investigation can be helpful in several ways. *First*: We are in position of making a theoretical evaluation as to which of the waves will have amplitudes of a high order of magnitude at certain distances from the source, and, consequently, concentrate our principal attention upon these waves when the experiment is both outlined and interpreted. At the same time, it is possible to determine the waves which are so weak that there is little probability of identifying them among the noises. By interpreting other types of waves which have not been used so far it is possible to reduce the ambiguous character of the interpretation to a considerable degree. *Second*: The amplitude curves can prove very helpful during the identification of the individual types of waves which also makes for an unambiguous interpretation. *Third*: Amplitude curves can also be used for determination of additional parameters of the medium (such as the logarithmic decrement of the absorption) and, in certain cases, for determination of the depths of reflecting interfaces, of the velocities of seismic waves etc.

The present paper is concerned with the amplitude curves of a series of waves which originate on an interface the parameters of which are close to the parameters of the Mohorovičić discontinuity. On the basis of the following results we shall be able to appreciate which of the waves have a dominant character in seismic records and which of them, on the other hand, are so weak that their determination in the records can be regarded as very improbable. For the sake of simplicity let us suppose that the Earth's crust of 30 km thickness is homogeneous, with a velocity of compressional waves $v_{P1} = 6.4$ km/sec, the velocity of compressional waves below Mohorovičić discontinuity $v_{P2} = 8$ km/sec; the ratio of compressional and shear waves velocities being $\sqrt{3}$ inside the crust and below it. Since the amplitude curves are not noticeably influenced by the ratio of densities in the Earth's crust and below it, it is possible, without any appreciable detriment to a general applicability to assume that the densities in the crust and below it are the same. The source is assumed to be symmetric and harmonic with a frequency of 6.4 cps and to be situated in the proximity of the Earth's surface. By wave amplitudes there are meant the amplitudes of their vertical displacement components. As we are not interested in the details of the amplitude curves, it is possible to use the methods of geometric ray theory for the calculations. It should, however, be born in mind that the formulae lack precision in the vicinity of the critical points (the amplitudes in critical regions are given by dashed lines in the figures which follow). Besides, the reflected and head waves interfere beyond the critical points, the resulting interference waves having properties entirely different from those of individual waves. The length of the interference zones has been calculated for a sinusoidal pulse of a length of one period. At the point where the dashed line changes into a continuous one, there is an end of the interference zone.

Compressional incident wave

In the case of an incidence of a P wave on the interface 4 waves are produced: PP and PS reflected waves together with PPP and PPS head waves (Fig. 1). The critical point of the PPP head wave is situated at the epicentral distance of 80 km,

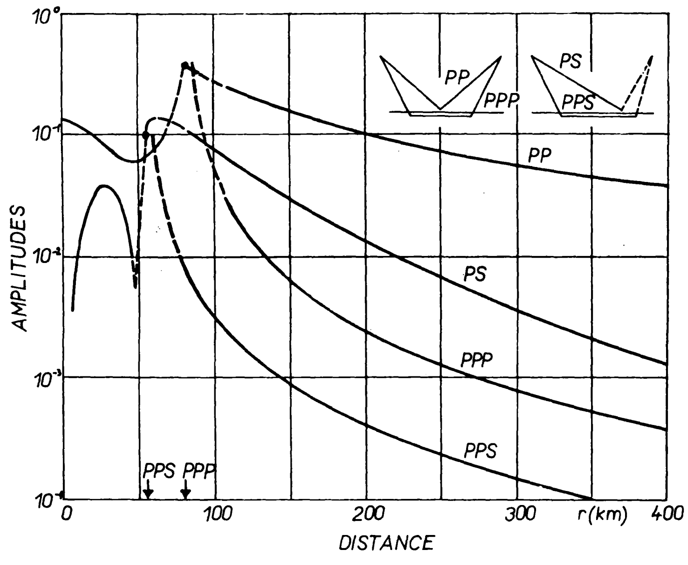

Fig. 1. Amplitude curves of reflected PP and PS waves and of head PPP and PPS waves.

the critical point of the PPS wave at the epicentral distance of 56 km (see the arrows in Fig. 1). In the regions of the critical points, where the formulae of the geometric ray theory lack precision, the amplitude curves are given by dashed lines. In the Fig. 1 we can see that the dominant wave is the PP reflected wave. Only at epicentral distances between 50–70 km the PS reflected wave reaches higher amplitudes, while at the majority of the epicentral distances the PS wave is noticeably weaker. Both the head waves are even weaker than the PS wave; at the ends of the interference zones they are, in both cases, approximately one order of magnitude weaker than the corresponding reflected waves. During the interpretation of seismograms certain waves observed are often regarded as PPS type of head waves. Fig. 1 shows that the PPS wave is about 5–10 times weaker than the PPP head wave and some 2–3 orders of magnitude weaker than the reflected PP wave. It is, consequently, improbable that it could be found in the records.

Incident shear wave

In the case of the incidence of a S wave on an interface, 5 waves are produced: SS and SP reflected waves together with SSS, SPP and SPS head waves. The critical point of the SSS head wave lies at 80 km from the source, that of the SPP head wave at 56 km from the source and that of the SPS head wave at 31 km from the source (see arrows in Fig. 2). The dominant wave over the whole range of epicentral distances is the reflected SS wave, with the exception of very short

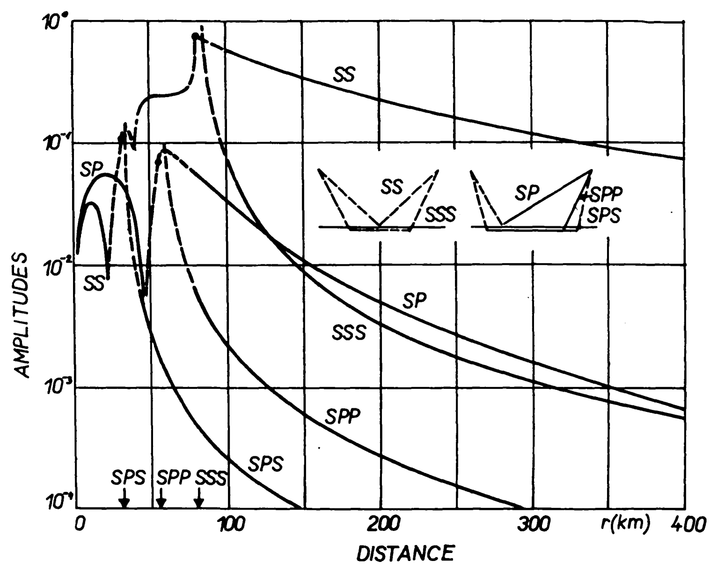

Fig. 2. Amplitude curves of reflected SS and SP waves and of head SSS, SPS and SPP waves.

distances from the source (from 10 to 30 km), where it is commesurate with the SP reflected wave. Otherwise the SP reflected wave is considerably weaker than the SS wave. The head waves are again about one order of magnitude weaker than the corresponding reflected waves at the ends of the interference zones. The largest amplitudes, as far as head waves are concerned, corresponds to the SSS head wave, the SPP wave being about one order of magnitude weaker and the SPS wave being considerably weaker still than the latter one. There is thus very little probability that the SPP and SPS waves might exceed the noise level.

Ratio of the amplitudes of reflected PP and head PPP waves

The compressional PP and PPP waves have the greatest importance for interpretation of seismic records. The knowledge of their amplitude curves and of the mutual amplitude ratios is, therefore, of a great importance. The calculations made

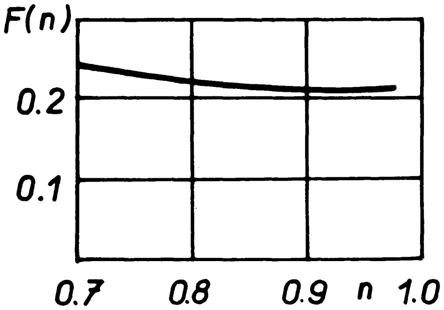

Fig. 3. Values of F(n).

up to date have not taken into consideration the interference zone of both waves. It is pointless to investigate the amplitude ratio of both waves inside the interference zone, neither on the basis of approximate formulas of the geometric ray theory, nor on the basis of more accurate formulas, since both waves are not yet separated from each other there and it is not possible to divide them in this region. An investigation of the amplitude ration of both waves begins to be of a reasonable utility only at the point where both waves present themselves as independent from each other, that is beyond their interference zone. Of considerable importance is a determination of the amplitude ratio of both waves directly at the point where both waves divide, i. e. at the end of their interference zone. At the end of the interference zone we obtain a very simple formula for the amplitude ratio of both waves:

$$A_{PPP}/A_{PP} = F(n)\,(H/\lambda)^{-1/4}. \qquad (1)$$

In the formula (1) n denotes refraction index ($n = v_{P1}/v_{P2}$), the values of F(n) are given in Fig. 3. It is assumed, that $v_{P1}/v_{S1} = v_{P2}/v_{S2} = \sqrt{3}$). From Fig. 3 it is seen that F(n) depends only to a slight extent on the refraction index n. So we obtain the following approximate relation:

$$A_{PPP}/A_{PP} = 0.22\,(H/\lambda)^{-1/4} \qquad (2)$$

The Tab. 1 gives the values of the ratio A_{PPP}/A_{PP} for different values of H/λ. It can be seen that the said amplitude ratio depends only very slightly at the end of the interference zone on the value of H/λ (and thus also on the frequency). The general result is, therefore, that the PPP head wave has, at the end of the interference zone, amplitudes at least about one order weaker than the reflected PP wave. The same conclusion we also obtain for refraction indices n very near to one. It is often claimed that for weak interfaces (n near to one) the head wave is the dominant one in the records, the reflected wave being meanwhile so weak that it cannot be found in the records. It is evident that this fact is not in the agreement with the theoretical conclusions. In most cases the dominant wave has probably the character of interference reflected - head wave, as the length of the interference zone is very great in the case of n near to one. Where both the reflected and the head wave are registered at the same record, the reflected wave will theoretically have always a dominant character even in the case of refraction index very near to one.

Tab. 1. *Values of A_{PPP}/A_{PP} for different values of H/λ*

H/λ	10	20	30	40	50	60	70	80	90	100
A_{PPP}/A_{PP}	0.12	0.10	0.09	0.09	0.08	0.08	0.08	0.07	0.07	0.07

As a result of theoretical calculations, the PPP head wave should always be at least one order of magnitude weaker than the PP reflected wave. It would, therefore, be extremely difficult to register the head wave at all. Besides, as can easily be shown, there should predominate lower frequencies in the spectra of a reflected wave. The amplitude ratio of head and reflected waves determined experimentally is, however, larger as a rule than a ratio determined theoretically. Besides, in the spectra of head waves of the PPP type produced on the Mohorovičić discontinuity, a predominance of higher frequencies in comparison with the spectra of PP reflected waves was found by experiments. These discrepancies can be accounted for by different reasons. It is claimed, for example, that the mechanism which can increase the ratio of the amplitudes of PPP head waves and PP reflected waves is an absorption. It is assumed that below the Mohorovičić discontinuity the coefficient of absorption is lower than that in the Earth's crust, with a resulting relative intensification of the head wave amplitudes. The absorption may influence the situation only to a certain extent. Much more important are, as shown by the calculations, other factors which even may bring about an increase of the amplitudes of the PPP head waves at certain epicentral distances. It is well known that the geometric ray theory in its approximation of zero order gives for the reflected wave non-zero amplitudes, but zero amplitudes for head waves. The head wave is only an effect of the first order. This is due to the fact that the head wave is propagated through the second medium in a horizontal direction along the interface. However, as soon as the ray of a head wave has arrived 1 or 2 wavelength from the interface, a wave of the first order is converted into a wave of the zero order and its amplitudes being thereby noticeably increased. Its amplitude reach then non-zero values even for zero wavelength. No changes in the kinematics of the PPP wave can meanwhile be observed.

Immersed waves

There may be a number of reasons which account for the deviation of a ray of head wave from the interface. If, for example, there is a positive velocity gradient below the Mohorovičić discontinuity, the ray of the PPP wave does not lie in a direction parallel to that of the interface, but immerses bellow it (see Fig. 4). The amplitudes of the PPP waves are very intensely influenced by this immersion even in the case of very low gradients of compressional velocity of the order of magnitude

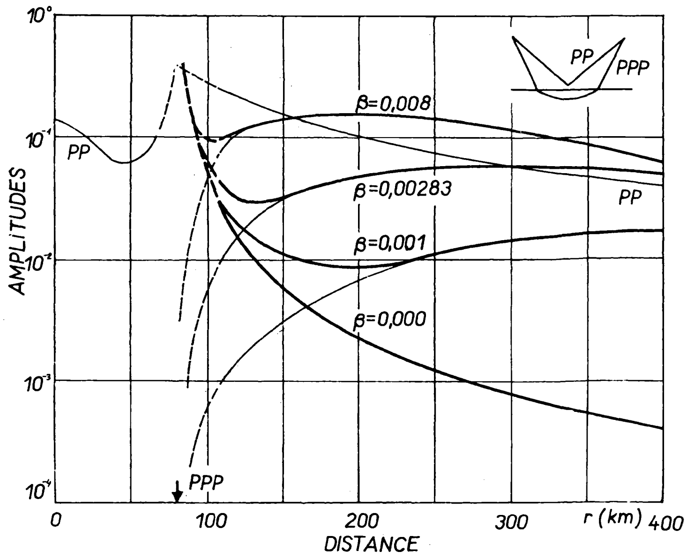

Fig. 4. Amplitude curves of immersed waves for different values of β. For details see text.

of 10 m/sec/km. The wave just described has quite different amplitude properties than the head wave. Moreover, the origin mechanism of this wave is also quite different that the origin mechanism of head wave. We shall call this wave immersed wave.

Fig. 4 gives the amplitude curves of immersed waves of PPP type for three velocity gradients β : $\beta = 0.001$ km^{-1}; $\beta = 0.00283$ km^{-1} and $\beta = 0.008$ km^{-1}. By velocity gradient β here is meant the value β in the velocity-depth equation below Moho discontinuity

$$v(z) = 8\,[1 + \beta\,(z - H)] \tag{3}$$

where H is the depth of the Mohorovičić discontinuity. Fig. 4 further gives an amplitude curve of a reflected PP wave and of a PPP head wave originating in the same medium without a gradient ($\beta = 0$). From the Fig. 4 it is seen that at larger epicentral distances the amplitudes of immersed waves are about 1 or 2 order of magnitude stronger than the amplitudes of head waves. At certain epicentral distances their amplitudes even grow with the increasing epicentral distance. Thin lines in the figure indicate amplitude curves of immersed waves calculated on the basis of zero order approximation of geometric ray theory. The higher the gradient, the earlier the amplitude curves of the immersed waves merge with their asymptotic values. Conversely, the lower the gradient, the less accurate are the zero order values (particularly in the critical region). For short distances from the critical point it is possible to use for the calculation of the immerse wave the formulae for head waves. In the transition zone, where neither of the above formulae, can be used, the formulae of Chekin [1] were used for the connection of both curves.

So even for very little gradients about 0.001 km^{-1} the amplitudes of immersed waves are by 1 or 2 orders stronger than the amplitudes of head waves at epicentral distances of 200 km or more. It is sufficient when the ray immersed into the other medium about 3 wavelengths, that is, in our case about 4 km, to make the amplitude increase by one order of magnitude.

A similar effect can be exerted also by the Mohorovičić discontinuity curvature due to the Earth's curvature or to other factors. Already at epicentral distances of about 400 km the curvature of the Earth will noticeably influence the amplitudes of the PPP waves, as the ray of a PPP wave penetrates about 3 km below Mohorovičić discontinuity. It is further possible to obtain much stronger amplitudes of the PPP waves if under the basic interface there is situated another interface, even a weak one, at not too a great distance. In this case a wave reflected on the second interface is much stronger than the head wave produced on the first interface, while in the kinematics these waves differ only very little.

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