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SOME PROBLEMS OF COMETARY PHYSICS INVESTIGATED ON THE BASIS OF PHOTOMETRIC DATA

PART TWO

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PREFACE TO PART TWO

This paper is the continuation of the thorough analysis of cometary problems, the first part of which was published in this journal last year.

Part One contained the portions as follows:

(1) Preface to Part One.

(2) Chapter One. History and Evolution of Photometry of Comets.

(3) Chapter Two. A Comet Dust-Gas Model. Fundamental Methods of Determining Its Physical Parameters.

(4) Chapter Three. Influence of the Dust on the Photometric Properties of Comets. Solution of the Dust-Gas Model in Detail.

(5) Chapter Four. Some Applications of a Comet Dust-Gas Model. Comets with Strong Continuous Spectra.

(6) Chapter Five. Interaction between a Comet and Dust Constituent of Interplanetary Matter.

(7) Chapter Six. Development of the Cometary Atmosphere during the Approach of a Comet to the Sun.

(8) Chapter Seven. General Results and Conclusions (Part One).

This continuation comprises next four chapters:

(1) Chapter Eight. Changes of Physical Characteristics of Comets Due to the Changes of Solar Activity.

(2) Chapter Nine. Systematic Variations in Brightness Connected with the Comet's Interior Structure.

(3) Chapter Ten. General Results and Conclusions (Part Two).

(4) Chapter Eleven. A Catalogue of Physical Characteristics of Comets from the Years 1610 to 1954.

In the first of the four chapters the influence on comets of the periodicity of the solar activity is discussed in detail. I am comparing various results reached so far and trying to make a hypothesis consistent with the most abundant observational material available at present. The classification of cometary characteristics is also given according to the form of the curve during an eleven-year solar cycle.

Irregular short-term variations of the comet brightness connected with the solar activity (Beyer's method) are studied together with ,,solar" periodic changes. Short-term fluctuations in the colour-index of the Arend-Roland comet and the perihelion asymmetry of the light curves of comets are discussed separately in the second of the four chapters.

On principle I try to give the observational material used, and if it is not possible because of the extent of the paper I consistently indicate the reference. The synopsis of physical characteristics of comets distributed according to the solar cycles is appended (the last chapter). It may serve for contingent further statistical investigations.

CHAPTER EIGHT

CHANGES OF PHYSICAL CHARACTERISTICS OF COMETS DUE TO THE CHANGES OF SOLAR ACTIVITY

8.1. Introduction

Many quantities characterizing in any way whatever changes of the cometary activity within the eleven-year solar cycle were the subject of studies carried out by various authors. The number of discovered comets in dependence on the phase of the solar cycle, for instance, was studied by LINK and VANÝSEK (1947), LINK (1952), and later on by DOBROVOLSKY (1954, 1958), the variations of the absolute magnitude of short-period comets were studied by DOBROVOLSKY (1957), and the variations of the dispersion of the observed brightnesses of comets by BEYER (1933, 1937a, 1937b, 1938, 1942, 1947, 1950a, 1950b, 1955), and on the basis of his material also by DOBROVOLSKY (1958). Some relations within the brightness of the Encke comet associated with the solar activity were studied by LINK (1948). The variation of the diameter of the coma of the Encke comet during the eleven-year cycle was investigated in BOUŠKA's and ŠVESTKA's work (BOUŠKA, ŠVESTKA, 1949), and the change of many physical parameters of the comet 1943 I with the solar activity during several months was analyzed by BOUŠKA (1950a).

All these works specialized in the solution of a certain part of the problem, and the results are, on the whole, rather divergent. Until now, the best agreement has been obtained in the study of the number of discovered comets in dependence on the phase of the solar cycle. Here, it became apparent that the curve of cometary discoveries reveals during the cycle a double-wave. This fact clearly results from all the mentioned studies (LINK, VANÝSEK, 1947, LINK, 1952, DOBROVOLSKY, 1954, 1958), and also is fully confirmed in this paper.

The present chapter has three main parts, which deal with the following aspects of the problem, respectively:

1. The systematic course of cometary characteristics within the eleven-year cycle, their long-term changes in the odd and even cycles, and the way in which the eighty-year period of solar activity reveals itself in cometary statistics.

2. The variation of the absolute brigtness, fluctuations of the dispersion

of the observed brightness estimates and variation of the head diameter of the Encke comet as related to the changes of solar activity in the eleven-year cycle.

3. Discussion of the Beyer method of cometary brightness dispersion as a criterion of cometary activity.

8.2 The statistical dependence of the cometary characteristics on the solar activity in the eleven-year cycle

First, 563 comets are studied, comprised in VSEKHSVIATSKY's Catalogue of absolute magnitudes of comets (VSEKHSVIATSKY, 1956, 1958), which were observed within the interval of 1610 till 1954, that is, during 31 eleven-year solar cycles, and which had not been looked up in the ephemeris. This criterion is very important, since short-period comets (with an orbital period of an order smaller than 100 years) which, particularly in the last decennia, were looked up almost exclusively in the ephemeris, have quite different physical parameters.

In this part of the study, on the whole eleven cometary characteristics are investigated which, with regard to their essential features, may be classified into three groups:

A) characteristics of the comet as a whole;
B) characteristics of the cometary head;
C) characteristics of the cometary tail.

The characteristics under A) are as follows:

a) N_y — number of comets discovered per year;
b) τ_y — value of the function of the visual importance of the tails of the comets observed within one year; the tail visible with naked eye has $\tau = 2$, that visible with a telescope $\tau = 1$, and a comet without tail has $\tau = 0$;
c) r — the heliocentrical distance of the comet at the time of discovery;
d) Δ — the geocentrical distance of the comet at the time of discovery;
e) H_{10} — the absolute magnitude of the comet according to VSEKHSVIATSKY's definition, see VSEKHSVIATSKY (1956, 1958);
f) m — the apparent magnitude of the comet at the time of discovery;
g) n — the seasonal index introduced by DOBROVOLSKY (1957).

The characteristics under B) are as follows:

h) D — the maximum apparent coma diameter;
i) D_0 — the maximum linear coma diameter.

Finally, the characteristics under C) are as follows:

j) C — the maximum apparent length of the cometary tail;

k) S — the maximum linear length of the cometary tail.

As can be seen, some of the characteristics under A) are in a certain relation to the group B), eventually C) as well. This is particularly the function of the visual importance of tails. We, however, consider this quantity as a characteristic of the appearance of the comet and classify it, therefore, under A). Similarly, the magnitudes m, and H_{10} which approach the characteristics of the cometary coma are, as tradition requires, classified under A), too.

For each of the 563 comets, moreover, the phase-shift Φ with regard to the minimum of the sunspot numbers R has been determined, which is defined by the formula:

$$\Phi = \frac{t_0 - T}{P}, \tag{8.1}$$

P is the length of the respective eleven-year cycle of solar activity, T the time of minumum solar activity, and t_0 the time of measurement of the given characteristic. In the case of quantities N_y, r, Δ, and m it is the time of discovery of the comet, in the case of quantity H_{10}, the middle from the interval of the observation of the comet, in the case of quantities D_0 and S it is, as a rule, the moment close to the time of the passage of the comet through the perihelion, and in the case of quantities τ_y, D and C, in addition, the time of minimum geocentrical distance of the comet plays a part. Generally, from the succession of moments T we select such a moment that

$$T \leqq t_0 < T + P.$$

The dependence of each of the mentioned eleven characteristics on the phase of the eleven-year solar cycle has been studied both for all cycles together and separately for the odd and even cycles. Out of the 563 comets, 259 are available in odd cycles and 304 in even.

Table 39 is a synopsis of the distribution of the total number of comets under consideration according to the place of discovery.

Table 39. Distribution of comet discoveries

Region	Discoveries
West Europe	50.3 per cent
North America	24.5 per cent
South Africa	8.7 per cent
East Europe	8.1 per cent
Far East	3.1 per cent
Australia and New Zealand	2.7 per cent
South America	2.3 per cent
Near and Middle East	0.3 per cent

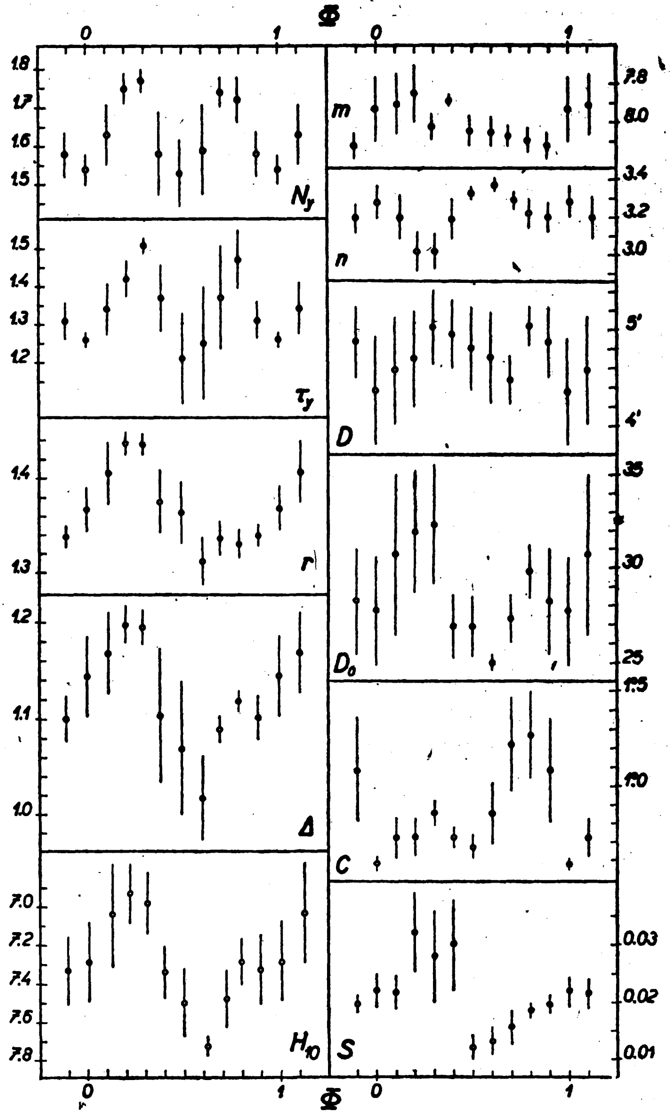

Fig. 33. Curves of cometary characteristics in an average eleven-year cycle (weighted values).

Quantitatively, the approximate form of the curves of each of the studied characteristics (generally already designed as X) will be described by the following parameters:

1. $\Phi_i(X)$ — position of the individual extremes, $i = 1, 2, \ldots$;
2. $A_i(X)$ — semi-amplitudes of the individual extremes;
3. $\bar{X}$ — mean value of quantity X within the cycle.

In the following, index i, if odd, stands for the minimum, if even, for the maximum.

The course of each characteristic from group A) with the phase of the average solar cycle has been determined both by weighted and not-weighted values. The weighted values, in which each cycle is represented by a weight equal to the number of actually studied comets, somewhat overvaluate the latest solar cycles; the not-weighted values, in which all solar cycles are represented by an equal weight, rather considerably overvaluate the old cycles.

The course of the characteristics from groups B) and C), as well as the course of all characteristics within the odd and even cycles, is derived — owing to less abundant material — only from weighted values.

8.2.1. Statistics of the number of discovered comets, and function of the visual importance of cometary tails

The number of discovered comets, reduced to one year, N_y, reveals within the eleven-year cycle a characteristic double-wave which is very well apparent both in the weighted values (Figure 33, Table 40) and in the not-weighted values (Table 40). This fact fully confirms the previous investigations carried out in this respect. Quite analogous is also the course of the function of the visual importance of tails τ_y, as follows from Figure 33 and Table 41. Parameters of the two magnitudes are included in Table 42.

In the odd and even cycles, too, the courses of quantities N_y and τ_y are entirely analogous, as can be seen from Figure 34 and Tables 43—46, so that their appearance may be described together.

Table 40

Statistics of N_y

int Φ	Weighted values		Not-weighted values	
	Φ	N_y	Φ	N_y
0.951—0.250	0.106	1.63 ± 0.08	0.102	1.51 ± 0.14
0.051—0.350	0.202	1.75 ± 0.04	0.201	1.84 ± 0.09
0.151—0.450	0.292	1.77 ± 0.03	0.296	1.79 ± 0.10
0.251—0.550	0.380	1.58 ± 0.11	0.392	1.73 ± 0.13
0.351—0.650	0.490	1.53 ± 0.09	0.493	1.56 ± 0.06
0.451—0.750	0.606	1.59 ± 0.12	0.597	1.64 ± 0.08
0.551—0.850	0.695	1.74 ± 0.04	0.697	1.69 ± 0.06
0.651—0.950	0.788	1.72 ± 0.06	0.800	1.71 ± 0.05
0.751—0.050	0.886	1.58 ± 0.06	0.894	1.45 ± 0.12
0.851—0.150	0.998	1.54 ± 0.04	0.998	1.50 ± 0.14

Table 41
Statistics of τ_y

int Φ	Weighted values		Not-weighted values	
	Φ	τ_y	Φ	τ_y
0.951—0.250	0.111	1.34 ± 0.07	0.106	1.35 ± 0.15
0.051—0.350	0.207	1.42 ± 0.05	0.205	1.65 ± 0.05
0.151—0.450	0.297	1.51 ± 0.02	0.300	1.61 ± 0.06
0.251—0.550	0.385	1.37 ± 0.09	0.396	1.44 ± 0.14
0.351—0.650	0.495	1.21 ± 0.12	0.497	1.15 ± 0.10
0.451—0.750	0,611	1.25 ± 0.15	0.601	1.21 ± 0.15
0.551—0.850	0.700	1.37 ± 0.14	0.701	1.27 ± 0.14
0.651—0.950	0.793	1.47 ± 0.08	0.804	1.39 ± 0.08
0.751—0.050	0.891	1.31 ± 0.05	0.898	1.15 ± 0.08
0.851—0.150	0.003	1.26 ± 0.02	0.002	1.25 ± 0.13

Table 42
Parameters of curves N_y and τ_y

	Weighted values		Not-weighted values	
	N_y	τ_y	N_y	τ_y
Φ_1	0.975	0.988	0.931	0.923
Φ_2	0.255	0.286	0.236	0.241
Φ_3	0.487	0.532	0.512	0.532
Φ_4	0.730	0.785	0.754	0.783
A_1	0.12	0.10	0.25	0.25
A_2	0.17	0.17	0.17	0.36
A_3	0.11	0.18	0.10	0.26
A_4	0.17	0.13	0.15	0.08
$\overline{N}_y, \overline{\tau}_y$	1.64	1.35	1.64	1.35

Table 43
Statistics of N_y in odd and even cycles

int Φ	Φ	N_y	
		odd cycles	even cycles
0.951—0.250	0.11	1.43 ± 0.09	1.82 ± 0.13
0.051—0.350	0.20	1.67 ± 0.10	1.82 ± 0.13
0.151—0.450	0.29	1.58 ± 0.12	1.95 ± 0.11
0.251—0.550	0.38	1.45 ± 0.18	1.71 ± 0.10
0.351—0.650	0.49	1.40 ± 0.14	1.66 ± 0.12
0.451—0.750	0.61	1.47 ± 0.15	1.69 ± 0.14
0.551—0.850	0.70	1.71 ± 0.04	1.77 ± 0.12
0.651—0.950	0.79	1.63 ± 0.04	1.79 ± 0.11
0.751—0.050	0.89	1.51 ± 0.11	1.64 ± 0.03
0.851—0.150	0.00	1.47 ± 0.09	1.61 ± 0.02

a) in the even cycles, on the whole, more comets have been discovered than in the odd cycles—see also LINK (1952) — and τ_y attains, on the average, higher values in the even cycles;

b) the mean amplitude is in the odd cycles deeper than in the even cycles;

c) in the odd cycles both maxima (in $\Phi \approx 0.3$ and $\Phi \approx 0.8$) are approximately equally high, while in the even cycles the first maximum is somewhat higher;

Table 44

Statistics of τ_y in odd and even cycles

int Φ	Φ	τ_y	
		odd cycles	even cycles
0.951–0.250	0.11	1.21 ± 0.08	1.47 ± 0.12
0.051–0.350	0.21	1.39 ± 0.05	1.46 ± 0.12
0.151–0.450	0.30	1.38 ± 0.05	1.64 ± 0.08
0.251–0.550	0.39	1.27 ± 0.12	1.46 ± 0.10
0.351–0.650	0.50	1.01 ± 0.12	1.41 ± 0.11
0.451–0.750	0.61	1.09 ± 0.17	1.42 ± 0.11
0.551–0.850	0.70	1.26 ± 0.18	1.49 ± 0.10
0.651–0.950	0.79	1.39 ± 0.10	1.54 ± 0.07
0.751–0.050	0.89	1.19 ± 0.09	1.43 ± 0.01
0.851–0.150	0.00	1.17 ± 0.08	1.34 ± 0.05

Table 45

Parameters of curves N_y in odd and even cycles

	Odd cycles	Even cycles
Φ_1	0.07	0.96
Φ_2	0.22	0.28
Φ_3	0.49	0.51
Φ_4	0.72	0.76
A_1	0.15	0.18
A_2	0.16	0.22
A_3	0.13	0.10
A_4	0.20	0.06
$\overline{N_y}$	1.53	1.75

d) the phases of the individual extremes are practically independent on the cycle-type, with the sole exception of, perhaps, the minimum in $\Phi \approx 0$.

8.2.2. Statistics of the heliocentrical and geocentrical distances of comets at the time of discovery

The mean heliocentrical and geocentrical distances at the moment of discovery of the comets reveal, in distinction to the curves N_y and τ_y,

Table 46

Parameters of curves τ_v in odd and even cycles

	Odd cycles	Even cycles
Φ_1	0.98	0.99
Φ_2	0.25	0.30
Φ_3	0.53	0.54
Φ_4	0.78	0.78
A_1	0.08	0.14
A_2	0.19	0.17
A_3	0.28	0.07
A_4	0.17	0.08
$\overline{\tau_v}$	1.24	1.47

Table 47

Statistics of r

int Φ	Weighted values			Not-weighted values	
	Φ	r	N	Φ	r
		A. U.			A. U.
0.951−0.250	0.105	1.405 ± 0.033	166	0.090	1.374 ± 0.038
0.051−0.350	0.202	1.436 ± 0.011	179	0.197	1.382 ± 0.033
0.151−0.450	0.293	1.435 ± 0.011	181	0.301	1.372 ± 0.030
0.251−0.550	0.381	1.375 ± 0.034	162	0.386	1.312 ± 0.034
0.351−0.650	0.489	1.363 ± 0.033	157	0.489	1.353 ± 0.038
0.451−0.750	0.604	1.311 ± 0.026	161	0.600	1.294 ± 0.039
0.551−0.850	0.694	1.335 ± 0.019	177	0.689	1.330 ± 0.034
0.651−0.950	0.788	1.329 ± 0.016	174	0.796	1.302 ± 0.022
0.751−0.050	0.886	1.338 ± 0.012	160	0.898	1.310 ± 0.017
0.851−0.150	0.997	1.367 ± 0.024	157	0.994	1.339 ± 0.033

Table 48

Statistics of Δ

int Φ	Weighted values			Not-weithted values	
	Φ	Δ	N	Φ	Δ
		A. U.			A. U.
0.951−0.250	0.105	1.168 ± 0.043	166	0.090	1.153 ± 0.066
0.051−0.350	0.202	1.197 ± 0.020	179	0.197	1.181 ± 0.047
0.151−0.450	0.293	1.195 ± 0.019	181	0.301	1.184 ± 0.049
0.251−0.550	0.381	1.103 ± 0.071	162	0.386	1.067 ± 0.080
0.351−0.650	0.489	1.069 ± 0.071	157	0.489	1.075 ± 0.080
0.451−0.750	0.604	1.017 ± 0.045	161	0.600	0.987 ± 0.044
0.551−0.850	0.694	1.089 ± 0.014	177	0.689	1.043 ± 0.008
0.651−0.950	0.788	1.118 ± 0.010	174	0.796	1.064 ± 0.020
0.751−0.050	0.886	1.101 ± 0.023	160	0.898	1.041 ± 0.031
0.851−0.150	0.997	1.144 ± 0.042	157	0.994	1.113 ± 0.056

within the eleven-year cycle a single wave the phase of which, in respect to the sunspot-number curve, is shifted in phase. At the point, where in the characteristics N_y and τ_y the secondary maximum was located (i. e. in $\Phi \approx 0.8$), now only a slight local increase of the values may be found, eventually an inflexion. Again, there is no basic difference in the course of the curves obtained from the weighted and not-weighted values (Figure 33 and Tables 47—49).

Within the odd as well as even cycles, the mean heliocentrical and geocentrical distances also reveal single waves, as can be seen in Figure 34.

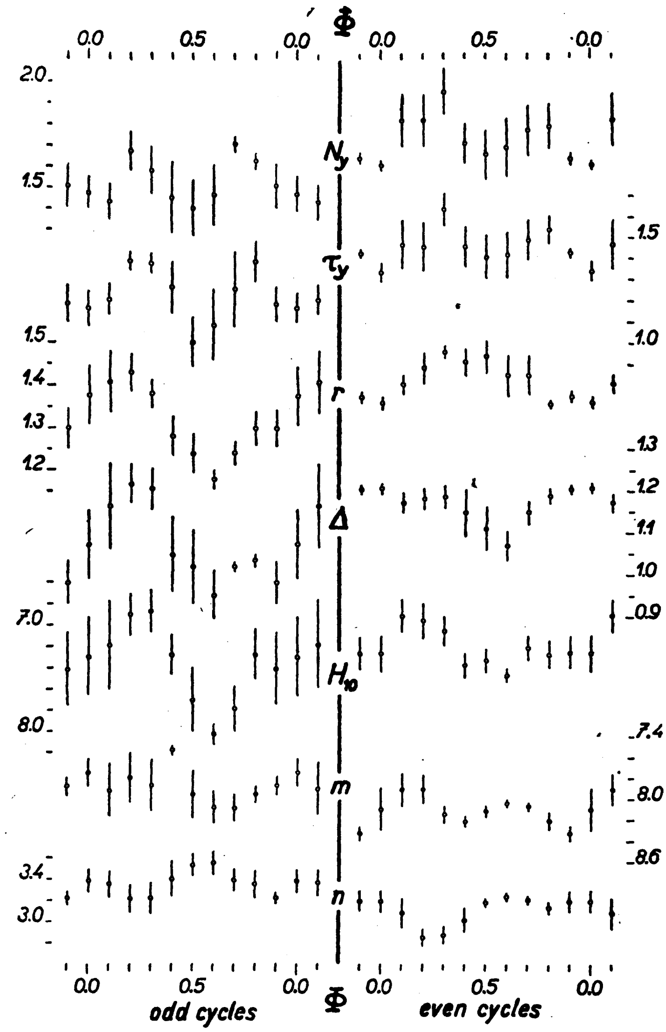

Fig. 34. Curves of characteristics of comets as a whole in odd and even cycles.

Table 49

Parameters of curves r and Δ

	Weighted values		Not-weighted values	
	r	Δ	r	Δ
Φ_1	0.633	0.584	0.602	0.600
Φ_2	0.245	0.242	0.190	0.250
A_1	0.060	0.119	0.045	0.105
A_2	0.076	0.082	0.046	0.111
$\overline{r}, \overline{\Delta}$	1.370	1.122	1.337	1.092

Table 50

Statistics of r in odd and even cycles

int Φ	Φ	Odd cycles		Even cycles	
		r	N	r	N
		A. U.		A. U.	
0.951 – 0.250	0.11	1.408 ± 0.075	72	1.403 ± 0.023	94
0.051 – 0.350	0.20	1.430 ± 0.046	85	1.441 ± 0.037	94
0.151 – 0.450	0.29	1.381 ± 0.035	81	1.478 ± 0.016	100
0.251 – 0.550	0.38	1.280 ± 0.047	74	1.455 ± 0.032	88
0.351 – 0.650	0.49	1.237 ± 0.047	71	1.468 ± 0.041	86
0.451 – 0.750	0.60	1.178 ± 0.019	73	1.422 ± 0.049	88
0.551 – 0.850	0.69	1.239 ± 0.027	84	1.422 ± 0.047	93
0.651 – 0.950	0.79	1.299 ± 0.040	80	1.354 ± 0.007	94
0.751 – 0.050	0.89	1.299 ± 0.044	75	1.372 ± 0.013	85
0.851 – 0.150	0.00	1.375 ± 0.070	73	1.360 ± 0.015	84

Table 51

Statistics of Δ in odd and even cycles

int Φ	Φ	Odd cycles		Even cycles	
		Δ	N	Δ	N
		A. U.		A. U.	
0.951 – 0.250	0.11	1.163 ± 0.103	72	1.172 ± 0.022	94
0.051 – 0.350	0.20	1.214 ± 0.051	85	1.182 ± 0.024	94
0.151 – 0.450	0.29	1.206 ± 0.051	81	1.187 ± 0.024	100
0.251 – 0.550	0.38	1.047 ± 0.091	74	1.150 ± 0.057	88
0.351 – 0.650	0.49	1.020 ± 0.091	71	1.109 ± 0.055	86
0.451 – 0.750	0.60	0.952 ± 0.060	73	1.071 ± 0.035	88
0.551 – 0.850	0.69	1.021 ± 0.002	84	1.151 ± 0.028	93
0.651 – 0.950	0.79	1.036 ± 0.012	80	1.189 ± 0.019	94
0.751 – 0.050	0.89	0.983 ± 0.049	75	1.206 ± 0.011	85
0.851 – 0.150	0.00	1.072 ± 0.083	73	1.207 ± 0.011	84

In the odd cycles, the variation of both distances is about twice as prominent as in the even cycles. At the same time, the mean value of both distances is in the odd cycles smaller, so that their maxima lie in both types of cycles on the same level. As for the phase-shifts of extremes, they attain approximately the same values in both cycle-types; in the even cycles, however, the extremes are flatter and consequently difficult to define, which is so, first of all, in the case of the maximum of the mean geocentrical distance (Tables 50—53).

Table 52
Parameters of curves r in odd and even cycles

	Odd cycles	Even cycles
Φ_1	0.59	0.82
Φ_2	0.18	0.30
A_1	0.14	0.08
A_2	0.13	0.06
$\overline{r}$	1.31	1.42

Table 53
Parameters of curves Δ in odd and even cycles

	Odd cycles	Even cycles
Φ_1	0.59	0.57
Φ_2	0.23	0.26
A_1	0.12	0.04
A_2	0.15	0.05
$\overline{\Delta}$	1.07	1.16

8.2.3. Statistics of the absolute magnitudes of comets

The dependence of the absolute brightness of comets on the solar activity may — from the physical point of view — be considered the most important, since it provides us with an idea of the average state in the number of the radiating particles in the coma within the individual phases of the solar cycle. In the total absolute magnitude, of course, the brightnesses of both coma constituents take a part, namely, the gaseous and dust components. Actually, in the changes of H_{10}, particularly the changes of the gaseous coma are reflected. In the visual region, in fact, the solar constant reveals minute variation which, of course, are true also of the light scattered on the dust particles in the coma. Therefore, the only factor that may induce changes in the dust-coma brightness during the cycle is a systematic dependence of the number of photometrically effective dust particles in the coma on the phase Φ. The gaseous component, on the other hand, may reveal changes in the brightness owing to systematic changes both in the number of gas molecules in the coma and in the ratio of the radiating molecules to the total number of molecules during the cycle. This second cause precisely is associated to the fluctuations of the ultra-violet solar radiation ($\lambda \approx 900$ Å), which is the exciting radiation for the gas molecules constituting the coma.

Table 54
Statistics of H_{10}

int Φ	Weighted values			Not-weighted values	
	Φ	H_{10}	Σw	Φ	H_{10}
0.951–0.250	0.128	$7^{m}.04 \pm 0^{m}.27$	312	0.104	$7^{m}.05 \pm 0^{m}.29$
0.051–0.350	0.224	6.93 ± 0.16	346	0.211	6.83 ± 0.15
0.151–0.450	0.312	6.98 ± 0.16	363	0.314	6.83 ± 0.16
0.251–0.550	0.398	7.34 ± 0.14	323	0.400	7.35 ± 0.26
0.351–0.650	0.504	7.50 ± 0.18	306	0.502	7.60 ± 0.28
0.451–0.750	0.623	7.73 ± 0.05	313	0.614	7.83 ± 0.15
0.551–0.850	0.715	7.48 ± 0.15	353	0.702	7.54 ± 0.14
0.651–0.950	0.802	7.29 ± 0.12	344	0.809	7.12 ± 0.15
0.751–0.050	0.898	7.33 ± 0.18	308	0.911	7.25 ± 0.23
0.851–0.150	0.014	7.29 ± 0.21	284	0.008	7.11 ± 0.24

Table 55
Parameters of curve H_{10}

	Weighted values	Not-weighted values
Φ_1	0.607	0.596
Φ_2	0.233	0.262
A_1	$0^{m}.51$	$0^{m}.64$
A_2	$0^{m}.36$	$0^{m}.52$
$\overline{H}_{10}$	$7^{m}.28$	$7^{m}.24$

From the material we obtain the course of H_{10} with a phase of cycle as presented in Figure 33 and Table 54. The not-weighted values, again, reveal a very similar course to the weighted values. Furthermore, in Figure 33 and Table 55 it can be seen that similarly as in the case of quantities r and Δ, the mean absolute magnitude, too, reveals, on the whole, a single wave which again is phase-shifted relative to the curve of sunspot numbers for about the same value as both preceding curves.

Within the odd cycles, the mean absolute magnitude of comets reveals a very similar course as do the heliocentrical and geocentrical distances. In the even cycles, in distinction to r and Δ, the curve H_{10} has a rather

Table 56

Statistics H_{10} in odd and even cycles

int Φ	Φ	Odd cycles		Even cycles	
		H_{10}	Σ_w	H_{10}	Σ
0.951 – 0.250	0.13	$7^m19 \pm 0^m42$	138	$6^m92 \pm 0^m15$	174
0.051 – 0.350	0.22	6.90 ± 0.19	162	6.96 ± 0.16	184
0.151 – 0.450	0.31	6.87 ± 0.19	156	7.06 ± 0.14	207
0.251 – 0.550	0.40	7.28 ± 0.18	145	7.38 ± 0.11	178
0.351 – 0.650	0.50	7.70 ± 0.30	135	7.34 ± 0.11	171
0.451 – 0.750	0.62	8.02 ± 0.10	142	7.48 ± 0.06	171
0.551 – 0.850	0.72	7.78 ± 0.21	165	7.22 ± 0.12	188
0.651 – 0.950	0.80	7.28 ± 0.23	153	7.29 ± 0.14	191
0.751 – 0.050	0.90	7.41 ± 0.34	141	7.27 ± 0.15	167
0.851 – 0.150	0.01	7.30 ± 0.37	136	7.27 ± 0.17	148

Table 57

Parameters of curves H_{10} in odd and even cycles

	Odd cycles	Even cycles
Φ_1	0.62	0.60
Φ_2	0.27	0.16
A_1	0^m65	0^m31
A_2	0^m62	0^m33
$\overline{H_{10}}$	7^m37	7^m21

sharp maximum. As for the size of the amplitudes, their phases and mean value of absolute magnitude, the picture is quite analogous to that of the heliocentrical and geocentrical distances (Figure 34 and Tables 56, 57).

8.2.4. Statistics of the apparent magnitudes of comets at the time of discovery

In the determination of the dependence of the mean apparent magnitude of comets at the time of discovery on the phase of solar activity, a rather great obstacle consists in the considerable inaccuracy with which the individual apparent magnitudes are determined. While in the preceding paragraph we analysed a material in which the intrinsic errors of the individual absolute magnitudes ranged on the average from $\pm 0^m3$ to $\pm 0^m5$, the apparent magnitudes comprise errors of at least $\pm 1^m$. The deformation of the curves due to observational errors, too, is a probable reason for the finding that the apparent magnitude has been until now the only quantity studied in the present paper, in which a considerable quantitative disagreement is found in the course of the weighted and not-weighted values,

Table 58
Statistics of *m*

int Φ	Weighted values			Not-weighted values	
	Φ	m	N	Φ	m
0.951 – 0.250	0.105	$7^{m}91 \pm 0^{m}16$	168	0.090	$7^{m}82 \pm 0^{m}16$
0.051 – 0.350	0.202	7.85 ± 0.15	181	0.197	7.65 ± 0.17
0.151 – 0.450	0.293	8.02 ± 0.07	181	0.301	7.83 ± 0.16
0.251 – 0.550	0.381	7.89 ± 0.03	162	0.386	7.82 ± 0.15
0.351 – 0.650	0.489	8.04 ± 0.08	157	0.489	8.31 ± 0.18
0.451 – 0.750	0.604	8.05 ± 0.08	162	0.600	8.28 ± 0.19
0.551 – 0.850	0.694	8.07 ± 0.06	179	0.689	8.24 ± 0.19
0.651 – 0.950	0.788	8.09 ± 0.07	176	0.796	7.97 ± 0.04
0.751 – 0.050	0.886	8.12 ± 0.07	161	0.898	7.98 ± 0.03
0.851 – 0.150	0.997	7.93 ± 0.17	159	0.994	7.77 ± 0.12

Table 59
Parameters of curve *m*

	Weighted values	Not-weighted values
Φ_1	0 853	0.538
Φ_2	0.177	0.194
A_1	$0^{m}15$	$0^{m}46$
A_2	$0^{m}18$	$0^{m}32$
$\overline{m}$	$8^{m}00$	$7^{m}96$

Table 60
Statistics of *m* in odd and even cycles

int Φ	Φ	Odd cycles		Even cycles	
		m	N	m	N
0.951 – 0.250	0.11	$7^{m}91 \pm 0^{m}26$	73	$7^{m}91 \pm 0^{m}16$	95
0.051 – 0.350	0.20	7.79 ± 0.26	86	7.91 ± 0.15	95
0.151 – 0.450	0.29	7.86 ± 0.26	80	8.14 ± 0.08	101
0.251 – 0.550	0.38	7.52 ± 0.03	74	8.21 ± 0.05	88
0.351 – 0.650	0.49	7.95 ± 0.23	71	8.12 ± 0.05	86
0.451 – 0.750	0.60	8.07 ± 0.17	74	8.04 ± 0.01	88
0.551 – 0.850	0.69	8.08 ± 0.14	86	8.07 ± 0.03	93
0.651 – 0.950	0.79	7.95 ± 0.07	82	8.21 ± 0.09	94
0.751 – 0.050	0.89	7.87 ± 0.09	76	8.33 ± 0.07	85
0.851 – 0.150	0.00	7.74 ± 0.14	75	8.10 ± 0.21	84

both, in fact, in the phase of extremes and in their amplitudes (Figure 33, Tables 58, 59).

In the odd and even cycles, the course of the apparent magnitude with the cycle-phase is even so irregular (Figure 34, Tables 60, 61) that it cannot

Table 61
Parameters of curves m in odd and even cycles

	Odd cycles	Even cycles
Φ_1	0.65	0.88
Φ_2	0.38	0.15
A_1	$0\overset{m}{.}24$	$0\overset{m}{.}24$
A_2	$0\overset{m}{.}36$	$0\overset{m}{.}30$
$\overline{m}$	$7\overset{m}{.}88$	$8\overset{m}{.}10$

be characterized by some current curve. For the mentioned reasons, the curve of apparent magnitudes cannot, for the time being, be detailedly investigated.

8.2.5 Statistics of the seasonal indices

In his work analyzing the short-period variations of absolute magnitudes of comets with shortest orbital periods DOBROVOLSKY (1957) tries to prove that these variations are not correlated to the sunspot number nor to the values of the Holetschek criterion of the conditions of visibility of comets (HOLETSCHEK, 1916), but to the so called seasonal index which for comets with $q \leqq 1$ A. U. is introduced in an opposite way than for comets with $q > 1$ A. U. In its essence, the seasonal index characterizes, properly speaking, the inclination of the zodiac to the horizon which, thus, according to Dobrovolsky, is the chief factor determining the value of the absolute magnitude established from observations. For comets with $q \leqq 1$ A. U., it attains the highest value in January, for comets with $q > 1$ A. U., in July. Its values, introduced by DOBROVOLSKY definitorically in a scale of 0 to 6 are listed in Table 62. In 17 out of the total number of 33 comets

Table 62
Seasonal index

Month	$q \leqq 1$ A.U.	$q > 1$ A.U.
January	6	0
February	5	1
March	4	2
April	3	3
May	2	4
June	1	5
July	0	6
August	1	5
September	2	4
October	3	3
November	4	2
December	5	1

with an orbital period shorter than 15 years, DOBROVOLSKY finds short-period fluctuations of the brightness, and for each of them he constructs correlations between the absolute brightness and the seasonal index. The degree of this correlation may be best judged in the case of the Encke comet in Figures 1—2 of his study, where, in addition, two other quantities, the curve of the Holetschek criterion and that of the sunspot numbers, are presented. From these pictures one obtains the impression that all three criteria approximately equally well correlate with the absolute brightness of the Encke comet (for more details see Paragraph 8.11.1.). DOBROVOLSKY refers in the beginning of his work already to KONOPLEVA's work (KONOPLEVA 1954) who pointed at the fact that within the interval of the years of 1901—1934, the brightness of the Encke comet and the sunspot numbers did not reveal any correlation, and similar correlations cannot be found, according to her, in other short-period comets, either.

DOBROVOLSKY, moreover, constructs in his work dependences of the absolute magnitudes on the seasonal index for some more of the 17 short-period comets. These graphs have been constructed even in the case of comets in which the absolute brightness is known only from four returns and, moreover, one cannot find that its secular decrease would have been reduced off, so that in some of the cases it is difficult to differentiate the bend due to this decrease from that due to the short-period fluctuation. From four or six values, in fact, the secular decrease cannot be derived with sufficient reliability. If we take into consideration, moreover, that the fluctuations of the brightness are of the same order as the intrinsic errors in the absolute magnitudes of VSEKHSVIATSKY's catalogue, we must arrive at the conclusion that a study of these fluctuations is justified only just in the case of the Encke comet, in which a sufficiently long series of H_{10} values is available.

Table 63
Statistics of seasonal indices

int Φ	Weighted values			Not-weighted values	
	Φ	n	Σw	Φ	n
0.951—0.250	0.128	3.20 ± 0.12	312	0.104	3.19 ± 0.10
0.051—0.350	0.224	3.02 ± 0.11	346	0.211	3.02 ± 0.11
0.151—0.450	0.312	3.02 ± 0.10	363	0.314	2.98 ± 0.08
0.251—0.550	0.398	3.19 ± 0.11	323	0.400	3.18 ± 0.13
0.351—0.650	0.504	3.33 ± 0.04	306	0.502	3.34 ± 0.06
0.451—0.750	0.623	3.37 ± 0.04	313	0.614	3.45 ± 0.04
0.551—0.850	0.715	3.29 ± 0.05	353	0.702	3.41 ± 0.03
0.651—0.950	0.802	3.22 ± 0.08	344	0.809	3.26 ± 0.13
0.751—0.050	0.898	3.20 ± 0.08	308	0.911	3.18 ± 0.11
0.851—0.150	0.014	3.28 ± 0.09	284	0.008	3.17 ± 0.10

In this paper the course is given of the mean values of the seasonal index during the solar cycle, determined from the 563 investigated comets. This course is presented in Figure 33 and Table 63, from which it is evident that the course of the mean seasonal index of the analyzed comets is precisely opposite to that of the mean absolute brightness (see Table 64, too). Thus, from a material about ten times more abundant, we obtain a result which is in absolute conflict with the result obtained by Dobrovolsky.

The curves of the seasonal index within the odd and even cycles presented in Figure 34 and Table 65, again attain their minimum almost precisely

Table 64

Parameters of curves of seasonal indices

	Weighted values	Not-weighted values
Φ_1	0.268	0.277
Φ_2	0.593	0.627
A_1	0.25	0.29
A_2	0.18	0.25
$\bar{n}$	3.21	3.22

Table 65

Statistics of seasonal indices in odd and even cycles

int Φ	Φ	Odd cycles		Even cycles	
		n	Σw	n	Σw
0.951—0.250	0.13	3.36 ± 0.12	135	3.08 ± 0.14	177
0.051—0.350	0.22	3.22 ± 0.13	162	2.85 ± 0.08	184
0.151—0.450	0.31	3.22 ± 0.15	156	2.87 ± 0.08	207
0.251—0.550	0.40	3.40 ± 0.17	145	3.01 ± 0.11	178
0.351—0.650	0.50	3.53 ± 0.11	135	3.18 ± 0.04	171
0.451—0.750	0.62	3.55 ± 0.11	142	3.23 ± 0.01	171
0.551—0.850	0.72	3.39 ± 0.11	165	3.20 ± 0.01	188
0.651—0.950	0.80	3.35 ± 0.13	153	3.13 ± 0.05	191
0.751—0.050	0.90	3.22 ± 0.06	138	3.19 ± 0.10	170
0.851—0.150	0.01	3.38 ± 0.11	133	3.19 ± 0.10	151

Table 66

Parameters of curves n in odd and even cycles

	Odd cycles	Even cycles
Φ_1	0.27	0.26
Φ_2	0.57	0.62
A_1	0.20	0.32
A_2	0.24	0.14
$\bar{n}$	3.36	3.09

at the phase in which the curves r, Δ and H_{10} (brightness) reveal a maximum, and vice versa (Table 66). Besides, in the odd cycles there appears a second prominent minimum (at $\Phi \approx 0.9$) which in the even cycles is almost indiscernible.

8.2.6. Statistics of the maximum apparent and linear coma diameters

For each comet of our statistics in which, according to VSEKHSVIATSKY (1958), the dimensions of the coma were measured, from the series of its measures the maximum value was selected, in the case that the coma was of an elliptic shape, again its maximum diameter was taken into account. The material, prior of being dealt with, was submitted to a statistical analysis, in the course of which the comets with $D > 15'$, observed from distances $\Delta \leqq 0.2$ A. U., and all comets observed from distances $\Delta \leqq 0.10$ A. U. were eliminated. These values, of which there were 12, in fact, would have undoubtedly distorted the statistics of $D = D(\Phi)$, though their magnitude associated neither with the solar activity, nor with the meteorological conditions, but with the extraordinary small geocentrical distance of the comet.

Since the literature on 133 comets did not offer the values D, our statistics comprised a total of 418 comets. The way of treatment was entirely analogous to that in the case of the characteristics of group A), with the sole difference, that the course of the characteristic was derived only from the weighted system of values. The dependence of the size of the maximum apparent diameter of the cometary coma on the phase of the solar cycle was, again, studied both for all cycles together and separately for odd and even cycles. From the results given in Figures 33, 35 and Tables 67—69

Table 67
Statistics of D

int Φ	Weighted values		
	Φ	D	N
0.951—0.250	0.1	4.58 ± 0.57	118
0.051—0.350	0.2	4.70 ± 0.52	124
0.151—0.450	0.3	5.02 ± 0.40	127
0.251—0.550	0.4	4.95 ± 0.38	121
0.351—0.650	0.5	4.80 ± 0.45	122
0.451—0.750	0.6	4.71 ± 0.49	127
0.551—0.850	0.7	4.48 ± 0.26	140
0.651—0.950	0.8	5.04 ± 0.22	136
0.751—0.050	0.9	4.87 ± 0.38	124
0.851—0.150	0.0	4.36 ± 0.57	115

Table 68
Statistics of D in odd and even cycles

int Φ	Φ	Odd cycles		Even cycles	
		D	N	D	N
0.951 – 0.250	0.1	4'.66 ± 0'.58	52	4'.51 ± 0'.70	66
0.051 – 0.350	0.2	4.60 ± 0.54	59	4.77 ± 0.57	65
0.151 – 0.450	0.3	5.55 ± 0.34	55	4.62 ± 0.57	72
0.251 – 0.550	0.4	5.18 ± 0.28	56	4.74 ± 0.75	65
0.351 – 0.650	0.5	5.10 ± 0.30	58	4.52 ± 0.86	64
0.451 – 0.750	0.6	4.78 ± 0.08	62	4.63 ± 0.84	65
0.551 – 0.850	0.7	5.06 ± 0.23	67	3.95 ± 0.37	73
0.651 – 0.950	0.8	5.64 ± 0.27	59	4.59 ± 0.19	77
0.751 – 0.050	0.9	5.71 ± 0.26	54	4.22 ± 0.47	70
0.851 – 0.150	0.0	4.77 ± 0.61	54	4.00 ± 0.48	61

Table 69
Parameters of curves D

	Cycles		
	all	odd	even
Φ_1	0.03	0.16	0.98
Φ_2	0.32	0.35	0.33
Φ_3	0.69	0.60	0.70
Φ_4	0.85	0.88	0.84
A_1	0'.39	0'.51	0'.46
A_2	0'.29	0'.49	0'.37
A_3	0'.25	0'.33	0'.51
A_4	0'.32	0'.64	0'.19
$\overline{D}$	4'.75	5'.11	4'.46

it is apparent that the dependence $D = D(\Phi)$ reveals a suggestion of a double-wave which, however, owing to the large dispersion of values D is rather problematic.

The dispersion in the values of the maximum linear diameters of the comas available of 444 comets is substantially smaller than in the case of the apparent diameters. While the ratio between the mean and highest values was

$$\overline{D} : D_{\max} = 1 : 25,$$

it now is, in the linear diameters, only

$$\overline{D}_0 : D_{0\max} = 1 : 12.$$

The disturbing effect of different geocentrical distances is in this case eliminated, too, so that the material is ready for direct treatment; the results are contained in Figures 33, 35 and Tables 70 to 72. In them, the values D_0

Table 70
Statistics of D_0

int Φ	Weighted values		
	Φ	D_0	N
		E.U.	
0.951 — 0.250	0.1	30.7 ± 4.3	121
0.051 — 0.350	0.2	31.9 ± 3.3	128
0.151 — 0.450	0.3	32.3 ± 3.2	136
0.251 — 0.550	0.4	26.9 ± 1.7	131
0.351 — 0.650	0.5	26.9 ± 1.6	135
0.451 — 0.750	0.6	25.0 ± 0.4	136
0.551 — 0.850	0.7	27.3 ± 1.3	152
0.651 — 0.950	0.8	29.8 ± 1.4	145
0.751 — 0.050	0.9	28.2 ± 2.8	131
0.851 — 0.150	0.0	27.7 ± 2.9	117

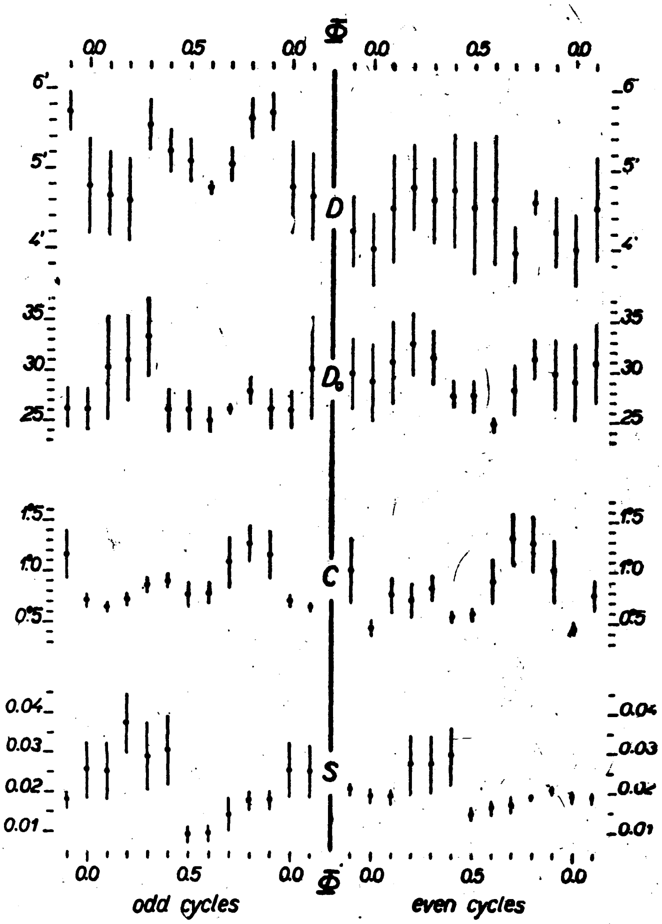

Fig. 35. Curves of characteristics of the cometary head and tail in odd and even cycles.

Table 71

Statistics of D_0 in odd and even cycles

int Φ	Φ	Odd cycles		Even cycles	
		D_0	N	D_0	N
		E.U.		E.U.	
0.951–0.250	0.1	30.3 ± 5.2	55	30.9 ± 4.2	66
0.051–0.350	0.2	31.0 ± 4.4	63	32.7 ± 3.2	65
0.151–0.450	0.3	33.3 ± 4.0	60	31.4 ± 2.8	76
0.251–0.550	0.4	26.1 ± 2.1	59	27.6 ± 1.7	72
0.351–0.650	0.5	26.1 ± 2.0	62	27.6 ± 1.7	73
0.451–0.750	0.6	25.0 ± 1.4	65	24.8 ± 0.8	71
0.551–0.850	0.7	26.2 ± 0.4	74	28.2 ± 2.7	78
0.651–0.950	0.8	28.0 ± 1.4	65	31.2 ± 2.2	80
0.751–0.050	0.9	26.2 ± 2.1	59	29.8 ± 3.7	72
0.851–0.150	0.0	26.1 ± 2.2	56	29.0 ± 4.0	61

Table 72

Parameters of curves D_0

	Cycles		
	all	odd	even
Φ_1	0.59	0.62	0.59
Φ_2	0.28	0.27	0.19
A_1	3.6	2.8	4.6
A_2	3.8	5.7	3.5
$\overline{D}_0$	28.6	27.8	29.3

are expressed in linear radii of the Earth. From these figures and tables it results that the dependence $D_0 = D_0(\Phi)$ is of a similar character as function $H_{10} = H_{10}(\Phi)$, that is, actually a single-wave, in this case with a somewhat more pronounced suggestion of a secondary maximum at $\Phi = 0.8$. Thus, in the course of the eleven-year cycle of solar activity, a greater linear diameter of the comet corresponds, too, to a greater absolute brightness.

A certain disagreement with the form of curves $H_{10} = H_{10}(\Phi)$ may be found only in the ratio of their amplitudes in the odd and even cycles which in the case of D_0 are practically equal.

8.2.7. Statistics of the maximum apparent and linear lenght of cometary tails

The maximum angular length of the tails of all 563 comets under observation, if put together, reveals approximately the same dispersion as the maximum apparent coma diameter. The ratio between the mean and maximum value of C is

$$\overline{C} : C_{\max} = 1 : 28.$$

It is for this reason that, first of all, the observational material must be again submitted to discussion. The angular length of the cometary tail depends, in addition to the observational conditions, also on its linear length S, and on its space-orientation relative to the Earth. In the case that the tail is averted directly away from the Sun, which usually is the case, the second condition is reduced to the determination of the mutual position of the members of the system Sun-Earth-comet; then, the length C may be determined from the well-known formula

$$\text{cotg } C = \frac{\Delta}{S} \text{cosec } k + \text{cotg } k \tag{8.2}$$

where k is the phase angle at the comet.

Let us investigate, first of all, the influence exerted on the course of the curve $C = C(\Phi)$ by the dispersion in the actual length S of the cometary tails; this dispersion, in fact, attains the exceptionally high value:

$$\bar{S} : S_{\max} = 1 : 74.$$

From cometary physics it is known that exceptionally powerful tails are observable only in some of the very bright comets having a considerable supply of gas. Although a sudden rise in the solar activity may result in certain anomalies in the tail of such a comet, together with a certain further extension of this tail, the existence itself of such anomalously powerful tails cannot be directly associated to the solar activity or to the observational conditions. Therefore, it is indispensable a priori to eliminate such comets from the statistics. So as to reduce the value of dispersion to the acceptable size of $\bar{S} : S_{\max} = 1 : 10$ we must content ourselves with comets having $S < 0.100$ A. U. In curve $S = S(\Phi)$ in this case the maximum amplitude is $\Delta S = 0.0017$ A. U. at a mean value of $\bar{S} = 0.010$ A. U., which in curve $C = C(\Phi)$ may reveal itself by a maximum relative uncertainty of ± 20

Table 73

Statistics of C (for $S < 0.10$ A.U.)

int Φ	Weighted values		
	Φ	C	N
		° °	
0.951 – 0.250	0.1	0.72 ± 0.11	142
0.051 – 0.350	0.2	0.72 ± 0.11	157
0.151 – 0.450	0.3	0.85 ± 0.07	157
0.251 – 0.550	0.4	0.72 ± 0.06	138
0.351 – 0.650	0.5	0.67 ± 0.07	136
0.451 – 0.750	0.6	0.85 ± 0.17	144
0.551 – 0.850	0.7	1.22 ± 0.25	159
0.651 – 0.950	0.8	1.27 ± 0.23	153
0.751 – 0.050	0.9	1.08 ± 0.28	137
0.851 – 0.150	0.0	0.58 ± 0.04	135

per cent, which, as such, cannot explain the ascertained course of the curve $C = C(\Phi)$, the values of which are given in Figures 33, 35, and Tables 73–75.

Table 74
Statistics of C in odd and even cycles

int Φ	Φ	Odd cycles		Even cycles	
		C	N	C	N
		° °		° °	
0.951–0.250	0.1	0.65 ± 0.03	62	0.77 ± 0.18	80
0.051–0.350	0.2	0.72 ± 0.04	74	0.72 ± 0.18	83
0.151–0.450	0.3	0.86 ± 0.06	69	0.84 ± 0.13	88
0.251–0.550	0.4	0.90 ± 0.06	64	0.56 ± 0.07	74
0.351–0.650	0.5	0.77 ± 0.12	64	0.59 ± 0.07	72
0.451–0.750	0.6	0.78 ± 0.12	68	0.90 ± 0.23	76
0.551–0.850	0.7	1.09 ± 0.26	78	1.33 ± 0.25	81
0.651–0.950	0.8	1.27 ± 0.19	73	1.28 ± 0.29	80
0.751–0.050	0.9	1.16 ± 0.24	67	1.01 ± 0.33	70
0.851–0.150	0.0	0.71 ± 0.07	65	0.45 ± 0.08	70

Table 75
Paremeters of curves C

	Cycles		
	all	odd	even
Φ_1	0.03	0.09	0.02
Φ_2	0.25	0.37	0.23
Φ_3	0.48	0.55	0.45
Φ_4	0.77	0.81	0.74
A_1	0°31	0°25	0°41
A_2	0°01	0°02	0°03
A_3	0°21	0°15	0°33
A_4	0°41	0°38	0°52
$\overline{C}$	0°87	0°90	0°85

The restriction in the linear length of the tails to $S < 0.1$ A. U., of course, offers no security that the statistics of the apparent length of the cometary tails would be devoid of all anomalies. Actually, the changes in curve $C = C(\Phi)$ may be due to the already mentioned exceptional position of the Sun, the Earth and the comet. These effects are added up with meteorological factors, and the resulting curve $C = C(\Phi)$ is, then, their reflexion-picture. Therefore, this curve, though it reveals a suggestion of a double-wave which we would expect in the case of a direct dependence of the tail-lengths on the meteorological factors, cannot, in itself, serve as a proof of the influence of weather on the cometary characteristics.

The maximum linear length of the tail is already freed from the influence of the geometrical situation within the system Sun-Earth-comet, it contains, however, in itself, the effect of the meteorological conditions. This effect, similarly as in the case of the linear coma diameters, cannot be entirely eliminated. We only may reduce it by confining ourselves to comets of an angular length that did not surpass a certain limit on the sky. In the same way, we also shall eliminate comets with abnormally powerful tails. So as to obtain a ratio of $\bar{C} : C_{max} = 1 : 10$ between the average and maximum values, we must confine ourselves to $C \leqq 10°$. Function $S = S(\Phi)$, then, has a course, which with its form again reminds the function $H_{10} = H_{10}(\Phi)$; thus, to a greater absolute brightness corresponds a longer tail (Figure 33, Table 76). Figures 34—35 indicate that agreement with curve $H_{10} =$

Table 76
Statistics of S (for $C \leqq 10°$)

int Φ	Weighted values		
	Φ	S	N
		A. U.	
0.951—0.250	0.1	0.0216 ± 0.0030	153
0.051—0.350	0.2	0.0320 ± 0.0072	169
0.151—0.450	0.3	0.0279 ± 0.0082	165
0.251—0.550	0.4	0.0300 ± 0.0082	146
0.351—0.650	0.5	0.0120 ± 0.0022	139
0.451—0.750	0.6	0.0131 ± 0.0026	146
0.551—0.850	0.7	0.0156 ± 0.0033	162
0.651—0.950	0.8	0.0186 ± 0.0015	158
0.751—0.050	0.9	0.0196 ± 0.0016	146
0.851—0.150	0.0	0.0221 ± 0.0030	146

Table 77
Statistics of S in odd and even cycles

int Φ	Φ	Odd cycles		Even cycles	
		S	N	S	N
		A. U.		A. U.	
0.951—0.250	0.1	0.0251 ± 0.0073	67	0.0189 ± 0.0022	86
0.051—0.350	0.2	0.0374 ± 0.0077	81	0.0271 ± 0.0079	88
0.151—0.450	0.3	0.0288 ± 0.0088	74	0.0272 ± 0.0077	91
0.251—0.550	0.4	0.0305 ± 0.0087	68	0.0295 ± 0.0078	78
0.351—0.650	0.5	0.0092 ± 0.0023	65	0.0145 ± 0.0020	74
0.451—0.750	0.6	0.0096 ± 0.0023	68	0.0162 ± 0.0024	78
0.551—0.850	0.7	0.0143 ± 0.0042	80	0.0169 ± 0.0025	82
0.651—0.950	0.8	0.0180 ± 0.0025	75	0.0191 ± 0.0008	83
0.751—0.050	0.9	0.0180 ± 0.0026	70	0.0209 ± 0.0018	76
0.851—0.150	0.0	0.0254 ± 0.0070	69	0.0191 ± 0.0023	77

$= H_{10}(\Phi)$ is attained also in the ratio of the depth of the amplitudes within the odd and even cycles. A further finding of interest is the fact that the meteorological agents affect the resulting curves fundamentally less than the curves of the linear coma diameters, as well as of the absolute magnitudes.

The course of S in the odd and even cycles is included in Table 77. The parameters of the S-curves are in Table 78.

Table 78
Parameters of curves S

	Cycles		
	all	odd	even
Φ_1	0.54	0.54	0.56
Φ_2	0.28	0.27	0.28
A_1	0.010	0.014	0.007
A_2	0.012	0.016	0.009
$\overline{S}$	0.0214	0.0219	0.0211

8.3. Causalities in the form of the curves of cometary characteristics within the solar cycle

A study of the form of the curves of cometary characteristics revealed that during the solar cycle not all of them change in the same way. Nine cometary characteristics reveal a qualitatively equal form of curves in both cycle-types. From the form of the curves it is apparent that these characteristics are divided into two classes:

1. Characteristics of the first class. The curve reveals within the cycle a double-wave with maxima at $\Phi \approx 0.3$ and $\Phi \approx 0.8$, while, at the same time, both are either equally high, or the second is higher. This class contains the characteristics as follows: cometary discoveries N_y, the function of the visual importance of tails τ_y, the maximum apparent diameters D, and the maximum apparent length of cometary tails C; out of them, the most reliably determined are the first two characteristics, while the remaining may be considered rather only probable elements of this class.

The mutual agreement as well as differences between the individual characteristics of the first class may be assessed from Table 79. In it, for each characteristic, the phases of all four extremes, their weights, the mean relative semi-amplitude expressed in per cent, and the total number of comets used for the computation are given. The agreement in the position of the extremes, determined from the individual characteristics is very good both in the odd, and even cycles separately as well as together. From column $\overline{A}$ it is obvious that the greatest changes within the solar cycle are

Table 79

Comparison of parameters of the characteristics of the first class

Characteristic	Cycles	Φ_1	$w(\Phi_1)$	Φ_2	$w(\Phi_2)$	Φ_3	$w(\Phi_3)$	Φ_4	$w(\Phi_4)$	$\bar{A}$	N
N_v (weighted values)	all	0.98	3.0	0.26	4.9	0.49	1.1	0.73	3.4	9 %	563
	odd	0.07	1.7	0.22	1.6	0.49	0.9	0.72	5.0	11 %	259
	even	0.96	7.2	0.28	2.0	0.51	0.8	0.76	0.5	8 %	304
N_v (not-weighted values)	all	0.93	2.1	0.24	1.9	0.51	3.0	0.75	2.7	10 %	551
	odd	—	—	—	—	—	—	—	—	—	—
	even	—	—	—	—	—	—	—	—	—	—
τ_v (weighted values)	all	0.99	5.0	0.29	8.5	0.53	1.4	0.79	1.6	11 %	563
	odd	0.98	1.0	0.25	3.8	0.53	2.1	0.78	1.5	15 %	259
	even	0.99	2.8	0.30	2.1	0.54	0.6	0.78	1.1	8 %	304
τ_v (not-weighted values)	all	0.92	2.8	0.24	6.5	0.53	2.2	0.78	0.9	18 %	551
	odd	—	—	—	—	—	—	—	—	—	—
	even	—	—	—	—	—	—	—	—	—	—
D (weighted values)	all	0.03	0.7	0.32	0.7	0.69	1.0	0.85	1.1	7 %	418
	odd	0.16	0.9	0.35	1.6	0.60	4.1	0.88	2.5	10 %	192
	even	0.98	1.0	0.33	0.6	0.70	1.4	0.84	0.6	8 %	226
C (weighted values)	all	0.03	5.2	0.25	0.1	0.48	3.0	0.77	1.7	27 %	486
	odd	0.09	8.3	0.37	0.3	0.55	1.3	0.81	2.0	22 %	228
	even	0.02	4.1	0.23	0.2	0.45	4.7	0.74	1.9	38 %	258

revealed in the angular length of the tails, the least in the angular coma diameters. The resulting values of the phases determined from all characteristics of the first class are for all cycles together as well as for the odd and even cycles separately given in Table 80. It becomes evident that in the odd cycles, the basic minimum is shifted for 0.1 period in the direction of the increasing phase with regard to the same minimum in the even cycles. In the remaining three minima, there are no systematic differences in their position in the odd and even cycles.

Table 80

Resulting values of phases of extremes in the curves of the characteristics of the first class

Cycles	Φ_1	Φ_2	Φ_3	Φ_4
all	0.98 ± 0.01	0.27 ± 0.01	0.52 ± 0.02	0.76 ± 0.01
odd	0.08 ± 0.01	0.27 ± 0.02	0.56 ± 0.02	0.78 ± 0.01
even	0.98 ± 0.01	0.29 ± 0.01	0.51 ± 0.04	0.77 ± 0.01

2. Characteristics of the second class. The curve reveals during the cycle, in principle, a single wave with a prominent maximum at $\Phi \approx 0.2$ and a minimum at $\Phi \approx 0.6$; at $\Phi \approx 0.8$, there is either a local maximum with an amplitude substantially smaller than that of the primary maximum, or even only an inflexion. This class, contains first of all, the curve of the absolute brightness H_{10}, then the curves of the heliocentrical r and geocentrical Δ distances of the comet at the time of discovery, as well as the curves of the maximum linear coma diameters D_0, and of the maximum linear length of tails S. The curves of these characteristics reveal altogether higher amplitudes in the odd cycles and, with the exception of S, their mean levels are somewhat higher in the even cycles. The curves are, moreover, characterized by several prominent deviations from the curves of the sunspot numbers:

a) it is, first of all, a considerable phase-shift which in the maxima of both curves exceeds 0.1 of the period and in the minima even almost 0.4 of the period;

b) this phase-shift is of such nature that both to the maximum and minimum in the curve of sunspot numbers corresponds precisely the mean value of each of the characteristics of the second class;

c) while the curve of the sunspot numbers reveals a steeper increase and a slower drop, in our curves just the reverse is the case.

All these deviations from the curve of sunspot numbers R are also fully reflected in the value of the correlation-factor ψ as well, indicating the relationship between the sunspot number and the value of the characteristics of the second class. For the absolute magnitude and both distances at the time of discovery of the comet, the mentioned correlation factor attains in an average cycle the following values:

$$\psi(R, H_{10}) = -0.12 \pm 0.04,$$
$$\psi(R, r) = +0.07 \pm 0.04,$$
$$\psi(R, \Delta) = +0.07 \pm 0.04,$$

in the odd cycles

$$\psi(R, H_{10}) = -0.04 \pm 0.06,$$
$$\psi(R, r) = +0.05 \pm 0.06,$$
$$\psi(R, \Delta) = +0.06 \pm 0.06,$$

and in the even cycles

$$\psi(R, H_{10}) = -0.17 \pm 0.06,$$
$$\psi(R, r) = +0.09 \pm 0.06,$$
$$\psi(R, \Delta) = +0.03 \pm 0.06.$$

Thus, altogether a very low (practically zero) degree of correlation is concerned; therefore, the influence of solar activity on the cometary characteristics of the second class decidedly cannot be understood as a linear relation between them and the sunspot number. The mutual relations between the individual characteristics of the second class can be seen in

Table 81

Comparison of parameters of the characteristics of the second class

Characteristic	Cycles	Φ_1	$w(\Phi_1)$	Φ_2	$w(\Phi_2)$	$\bar{A}$	Φ_0	A_0	N
r (weighted values)	all	0.63	2.6	0.25	6.9	5 %	0.74	0.4 %	558
	odd	0.59	6.5	0.18	2.5	11 %	0.85	0.2 %	256
	even	0.82	8.9	0.30	3.3	5 %	0.91	0.5 %	302
r (not-weighted values)	all	0.60	1.2	0.19	1.4	3 %	0.70	1.1 %	507
	odd	—	—	—	—	—	—	—	—
	even	—	—	—	—	—	—	—	—
Δ (weighted values)	all	0.58	2.9	0.24	4.1	9 %	0.80	0.9 %	558
	odd	0.59	1.9	0.23	2.9	13 %	0.77	3.0 %	256
	even	0.57	2.7	0.26	1.7	4 %	0.95	1.7 %	302
Δ (not-weighted values)	all	0.60	2.4	0.25	2.3	10 %	0.78	1.4 %	507
	odd	—	—	—	—	—	—	—	—
	even	—	—	—	—	—	—	—	—
H_{10} (weighted) values)	all	0.61	7.3	0.23	4.4	39 %	0.82	2.4 %	563
	odd	0.62	6.5	0.27	3.3	61 %	0.81	6.4 %	259
	even	0.60	4.4	0.16	2.2	31 %	0.73	3.8 %	304
H_{10} (not-weighted values)	all	0.60	4.0	0.26	3.4	53 %	0.82	6.4 %	515
	odd	—	—	—	—	—	—	—	—
	even	—	—	—	—	—	—	—	—
D_0 (weighted values)	all	0.59	7.2	0.28	1.2	13 %	0.81	3.5 %	444
	odd	0.62	2.3	0.27	1.4	16 %	0.78	3.7 %	206
	even	0.59	5.1	0.19	1.1	14 %	0.82	4.0 %	238
S (weighted values)	all	0.54	4.2	0.28	1.5	52 %	0.95	1.8 %	510
	odd	0.54	6.1	0.27	1.9	69 %	0.85	0.0 %	239
	even	0.56	3.2	0.28	1.2	38 %	0.90	6.1 %	271

Table 81, where for each of them the phases of the extremes of the single wave and their weights, the mean relative semi-amplitude $\bar{A}$ of the single wave, then, the phase Φ_0 of the local maximum (eventually of the inflexion) on the increasing part of the curve and its relative semi-amplitude A_0 (if $A_0 = 0$, it is a case of an inflexion point), and, finally, the total number of comets used for the computation are given. In the absolute magnitude, the value of amplitudes $\bar{A}$ and A_0 must be understood as variations of the brightness. Column $\bar{A}$ indicates that during the solar cycle the greatest changes may be found just in the absolute brightness of the comets and in the linear length of the tails. The results of Table 81 persuade us, moreover, that the amplitude of the local maximum at $\Phi \approx 0.8$ is of a lower order than that of the principal extremes, since for their ratio we obtain on the average the value 0.08 ± 0.01. As for the phases of the extremes, there is between the individual characteristics again, on the whole, a good agreement, although the dispersion of the values in the even cycles in somewhat greater. The resulting values of the phases of the extremes are presented in Table 82.

Table 82
Resulting values of phases of extremes in the curves of the characteristics of the second class

Cycles	Φ_1	Φ_2	Φ_0
all	0.59 ± 0.01	0.25 ± 0.01	0.80 ± 0.02
odd	0.59 ± 0.01	0.24 ± 0.01	0.81 ± 0.01
even	0.67 ± 0.04	0.25 ± 0.02	0.86 ± 0.03

The most essential difference between the characteristics of the first and second class, encountered up till now, is the unequal period which is reflected in their form: the characteristics of the first class have a period of 5.5 years, those of the second class of 11.

In two characteristics, namely, the apparent brightness of the comet at the time of discovery and the Dobrovolsky seasonal index, qualitative differences in the form of the curves in both cycle-types are encountered. In the apparent brightness, these differences may be explained by the great inaccuracy in the original values, that is, by observational errors. In the seasonal index, they are probably due to the unappropriate, too rough way of defining of this quantity which, in all probability, does not correspond to the actual state of things.

8.4. **Analysis of the material for the study of long-period changes**

In using for the determination of the form of curves of the cometary characteristics within the eleven year solar cycle the method described in Section 8.2., we tacitly assumed that in the results both the individual differences between the comets and the secular changes as well as the differences associated with the unequally intense solar activity in individual cycles will become balanced.

As long as the individual differences do not reveal a maximum relative dispersion from the average value exceeding more than one order — and the characteristics of those comets not satisfying this condition have been eliminated from the statistics — their influence on the resulting form of the curves (provided the material is abundant enough) may be considered as of a secondary nature.

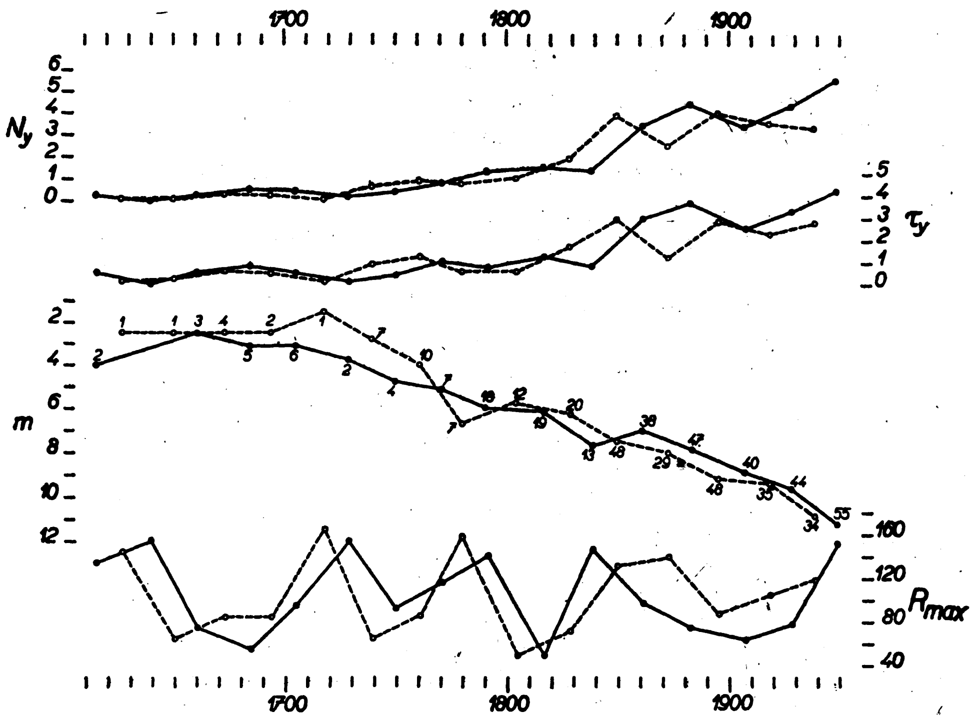

Fig. 36. Secular changes of the cycle-values of the characteristics N_y, τ_y, m.

The influence of the secular changes in the cometary characteristics of group A) on their course within an average eleven-year cycle may be observed by comparing their weighted and not-weighted values; from this comparison it cannot be seen that during the long periods systematic changes in the depth of the amplitudes and in the phases of extremes of the curves would occur. Systematic differences between the weighted and not-weighted values reveal themselves, however, in the average value of the characteristics, $\overline{X}$, during the cycle (quantities N_y and τ_y excepted, in which the average values of the system of weighted and not-weighted values have been definitorically put equal to one another), which further down will be called the cycle-values of the cometary characteristics.

These systematical changes in the cycle-values $\overline{X}$ within the system of weighted and not-weighted values are, of course, a consequence of the continuous progress in observational methods. Important is the fact that in this historic progress participate both the introduction of revolutionary discoveries into the methods of observation (use of telescopes in observations of and scanning for comets, photographs of comets, and the like) and the activity of the so called „comet hunters" which causes jumps to appear in the curves, even if, in addition to this, the evolutional component becomes active, too.

The study of curves of the individual cometary characteristics within the odd and even cycles showed (particularly in those of them which were most reliably determined) that there exist systematical differences between the two cycle-types both in the amplitude of the extremes and in the cycle values of the quantities which now will be more detailedly examined.

The secular changes of the cycle-values of ten cometary characteristics (without the seasonal index) are presented in Figures 36, 37, and 38. For reasons apparent from the following, in each diagram the time course, too, of the maximum monthly sunspot numbers is plotted. In all quantitites, the values for the odd cycles are given in empty circlets and are connected by dashed lines, and those valid for the even cycles in full circlets and full lines. The number at each point indicates the number of comets from which the cycle-value has been computed; in the case of the absolute magnitude, it is the sum total from the estimates of accuracy of the individual values in VSEKHSVIATSKY's scale (VSEKHSVIATSKY, 1956, 1958).

In the curves N_y and τ_y (Figure 36), again with a very similar course, incessantly alternating periods of a length of about 50 ± 30 years are apparent, during which, the sign of the difference of the cycle-value N_y, as well as τ_y, in both cycle-types remains the same. The character of the curve of the apparent magnitudes (Figure 36) may — with regard to their intrinsic errors — be guessed with more justification not earlier than from the period 1800 on. Until 1850, the cycle-values of the apparent brightness of comets are higher in the odd cycles whereupon, from 1850 on, the reverse is true. The deviations, however, are altogether less than 1^m. On the whole, from Figure 36 a certain relation between the course of the differences of the apparent magnitude, one the on side, and of the quantitites N_y and τ_y, on the other, may be ascertained in the odd and even cycles in that respect that with a greater number of discovered comets the average brightness at the time of discovery increases.

From the other characteristics of group A), the time-course of which is presented in Figure 37, an interesting course is revealed only by the absolute magnitude: with the exception of two comparatively short time intervals (30 years on the whole), the absolute brightness of comets is systematically greater in even cycles.

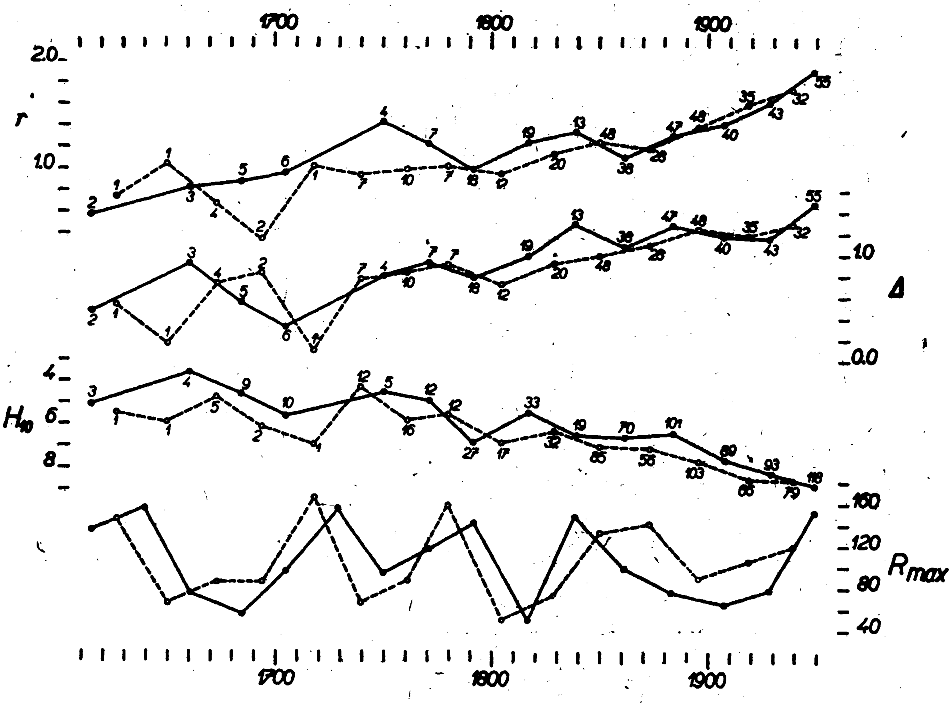

Fig. 37. Secular changes of the cycle-values of the characteristics r, Δ, H_{10}.

The cycle-values of the characteristics of the cometary coma and tail (Figure 38) became observable only from the fifties of the 18th century on. While in all characteristics of the comets as a whole the secular changes could be very well observed thanks to the already mentioned development of observational methods, in these characteristics this effect is either less prominent which fact is strengthened, moreover, by the great dispersion of the cycle-values (quantitites C and S), or it is not visible at all (quantity D_0 and, practically, D, too). This circumstance may be explained by the selection of the material from which the abnormally high individual values have been eliminated in accordance with the criteria mentioned in Paragraphs 8.2.6. and 8.2.7. and which in the old cycles were (in per cent) much more frequent.

Cycle-values of ten cometary characteristics are presented in Table 83, together with the most important parameters of solar cycles (number of the cycle, time-interval, type, period, maximum monthly sunspot number and its average value within the observation-intervals of comets). The values of the cycle characteristics, determined from three or less individual values, are given in parenthesis; column N shows the total number of comets discovered within the cycle. The cycles before 1755 are, in accordance with Gnevyshev and Ol (1948), denoted by zero and negative numbers.

Table 83

List of cycle-values of ten cometary characteristics

Solar cycle						N_ν	τ_ν	m	r	Δ	H_{10}	D	D_0	C	S	N
No	int t	type	P	R_{max}	$\bar{R}$											
								m	A. U.	A. U.	m	′	rad. of Earth	o	A. U.	
−12	1610.8—1619.0	even	8.2	(135)		0.24	0.49	(4.0)	(0.58)	(0.48)	(5.1)	(3.0)	(7)	(5.0)	(0.05)	2
−11	1619.0—1634.0	odd	15.0	(145)		0.07	0.13	(2.5)	(0.74)	(0.54)	(5.5)					1
−10	1634.0—1645.0	even	11.0	(155)		0.00	0.00									0
− 9	1645.0—1655.0	odd	10.0	(65)		0.10	0.20	(2.5)	(1.04)	(0.17)	(5.9)		(27)	(8.0)	(0.04)	1
− 8	1655.0—1666.0	even	11.0	(75)		0.27	0.55	(2.5)	(0.83)	(0.93)	(3.6)	(4.5)	(48)	(6.0)	(0.07)	3
− 7	1666.0—1679.5	odd	13.5	(85)		0.30	0.59	2.5	0.68	0.74	(4.7)	(4.0)	(18)	(0.8)	(0.05)	4
− 6	1679.5—1689.5	even	10.0	(55)		0.50	0.80	3.1	0.88	0.56	4.6			(7.3)	(0.04)	5
− 5	1689.5—1698.0	odd	8.5	(85)		0.24	0.47	(2.5)	(0.35)	(0.84)	(6.1)					2
− 4	1698.0—1712.0	even	14.0	(95)		0.43	0.50	3.1	0.97	0.33	5.6			1.1	0.01	6
− 3	1712.0—1723.5	odd	11.5	(165)		0.09	0.09	(1.5)	(1.03)	(0.11)	(6.9)	(10.5)	(9)	(0.0)	(0.00)	1
− 2	1723.5—1734.0	even	10.5	(155)		0.19	0.10	(3.8)	(2.54)	(1.65)	(2.7)	(1.5)	(29)	(3.8)	(0.01)	2
− 1	1734.0—1745.0	odd	11.0	(65)		0.64	0.91	2.8	0.95	0.78	4.3	(1.0)	(6)	1.7	0.05	7
0	1745.0—1755.2	even	10.2	92.6		0.39	0.39	4.8	1.44	0.81	(4.5)	(5.0)	(75)	(0.7)	0.03	4
1	1755.2—1766.5	odd	11.3	86.5	44	0.88	1.24	4.0	1.00	0.85	5.8	(14.0)	29	2.4	0.02	10
2	1766.5—1775.5	even	9.0	115.8	77	0.78	1.00	5.1	1.24	0.94	4.9	(7.0)	29	0.5	0.06	7
3	1775.5—1784.7	odd	9.2	158.5	61	0.76	0.54	6.6	1.03	0.92	5.5	5.1	36	1.5	0.03	7
4	1784.7—1798.3	even	13.6	141.2	60	1.32	0.74	5.93	1.00	0.79	6.83	5.3	24	0.72	0.018	18
5	1798.3—1810.6	odd	12.3	49.2	23	0.98	0.57	5.71	0.96	0.73	6.84	5.3	25	0.7	0.039	12
6	1810.6—1823.3	even	12.7	48.7	18	1.50	1.26	6.16	1.25	1.00	5.44	7.3	26	0.29	0.045	19
7	1823.3—1833.9	odd	10.6	71.7	33	1.89	1.70	6.23	1.15	0.93	6.30	4.3	22.9	1.02	0.023	20
8	1833.9—1843.5	even	9.6	146.9	48	1.35	0.83	7.64	1.35	1.30	6.48	3.3	17	0.49	0.016	13
9	1843.5—1856.0	odd	12.5	131.6	59	3.84	2.96	7.45	1.25	1.00	7.00	5.10	27.4	0.99	0.014	48
10	1856.0—1867.2	even	11.2	97.9	54	3.39	2.95	6.95	1.11	1.08	6.58	4.46	22.6	1.67	0.012	38
11	1867.2—1878.9	odd	11.7	140.5	54	2.48	1.20	7.94	1.20	1.10	7.05	4.74	27.0	0.32	0.005	29
12	1878.9—1889.6	even	10.7	74.6	32	4.39	3.69	7.80	1.33	1.28	6.40	4.57	34.9	0.62	0.043	47
13	1889.6—1901.7	odd	12.1	87.9	37	3.97	2.85	9.11	1.39	1.25	7.68	5.12	32.8	0.61	0.032	48
14	1901.7—1913.6	even	11.9	64.2	30	3.36	2.56	8.82	1.42	1.18	7.66	5.28	27.0	0.93	0.007	40
15	1913.6—1923.6	odd	10.0	105.4	39	3.50	2.30	9.36	1.60	1.19	8.50	4.94	23.3	0.87	0.030	35
16	1923.6—1933.8	even	10.2	78.1	44	4.31	3.33	9.59	1.61	1.16	8.23	4.22	29.5	0.48	0.013	44
17	1933.8—1944.2	odd	10.4	119.2	61	3.27	2.79	10.79	1.74	1.29	8.59	5.59	29.2	0.80	0.009	34
18	1944.2—1954.4	even	10.2	151.8	85	5.49	4.26	11.17	1.91	1.48	8.79	3.57	28.4	0.45	0.010	56

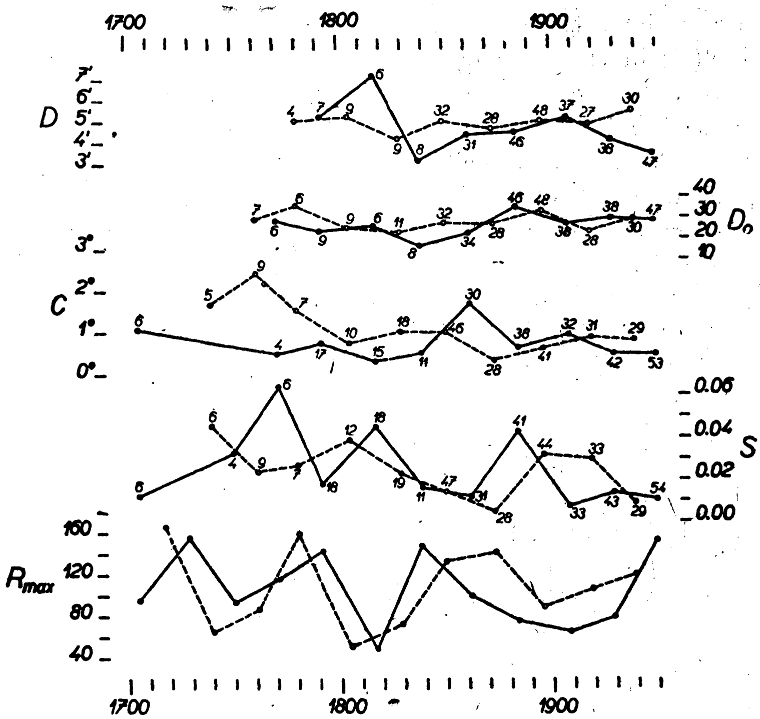

Fig. 38. Secular changes of the cycle-values of the characteristics of the cometary head and tail.

8.5. Method of study of long-term changes in the differences of the cometary characteristics in both cycle-types

Long-term changes in the differences of the cycle-values of the cometary characteristics in the odd and even cycles in dependence on the changes of solar activity in both cycle-types were investigated through the introduction of the values defined by relations (8.3):

$$
(8.3)\quad \left\{
\begin{aligned}
\partial N_y &= 2\,\frac{N_y(o) - N_y(e)}{N_y(o) + N_y(e)},\\
\partial \tau_y &= 2\,\frac{\tau_y(o) - \tau_y(e)}{\tau_y(o) + \tau_y(e)},\\
\partial m &= -[m(o) - m(e)],\\
\partial r &= r(o) - r(e),\\
\partial \Delta &= \Delta(o) - \Delta(e),\\
\partial H_{10} &= -[H_{10}(o) - H_{10}(e)],\\
\partial D &= D(o) - D(e),\\
\partial D_0 &= D_0(o) - D_0(e),\\
\partial C &= C(o) - C(e),\\
\partial S &= S(o) - S(e).
\end{aligned}
\right.
$$

In these equations, the cycle-value of quantity $\bar{X}$ is in the odd cycle designated $X(o)$, in the even cycle $X(e)$. We shall determine the values ∂X, for instance, for every tenth year. In defining quantities ∂N_y and $\partial \tau_y$, with regard to the further procedure of the treatment, a reduction has been carried out in the increase of the differences $|N_y(o) - N_y(e)|$, or $|\tau_y(o) - \tau_y(e)|$, which is roughly proportional to the secular growth of N_y, or τ_y. In the other characteristics, secular changes in the differences $|X(o) - X(e)|$ are not apparent any longer, although the cycle-values as such of these characteristics are affected by the mentioned changes.

Let us denote, furthermore, the maximum monthly sunspot number within the cycle as R_{max}, and let us write analogously

$$\partial R_{max} = R_{max}(o) - R_{max}(e).$$

Now, if we plot against ∂R_{max} successively the differences $\partial N_y, \ldots, \partial S$, we obtain relations, of which some are presented in Figures 39—42. The crosses designate the values from the period 1630—1730, when the number of comets within a cycle was 6 at the utmost, the empty circles the values from the years 1740—1840 (number of comets 4—20). and the full circles the values from the period 1850—1930 (number of comets 29—48). The strong drawn lines in these diagrams are regression lines of the relation in the case that we attribute to all values within the diagrams the same weight (so this is an analogy to the system of not-weighted values), the thinly drawn lines are, then, the regression lines of the relation in the case that we attribute to the values from the years 1630—1730 a zero weight, to those from the years 1740—1840 a weight of 1 and from the period 1850—1930 a weight of 2 (analogy to the system of weighted values). This second case does not consider at all the form of correlation within the period extending to 1730. For each of the ten cometary characteristics, the correlation

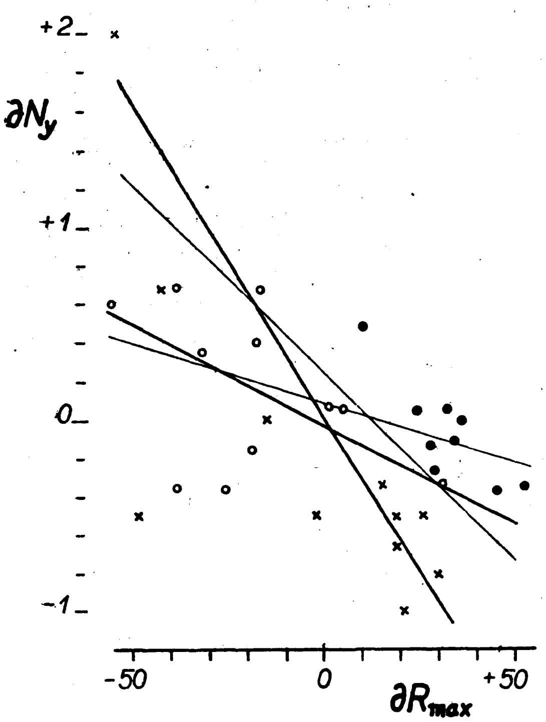

Fig. 39. Correlation between ∂R_{max} and ∂N_y.

coefficient of the dependence of their differences in the odd and even cycles on ∂R_{max} has been computed. Moreover, in the dependence $\partial m = \partial m(\partial R_{max})$ the correlation coefficient also has been determined from the values of the whole period 1630—1730, with the exception of the years 1680 till 1720. In these years, in the odd cycles only 3 comets were discovered, namely, in 1689, 1695 and 1718. The last two of them were at

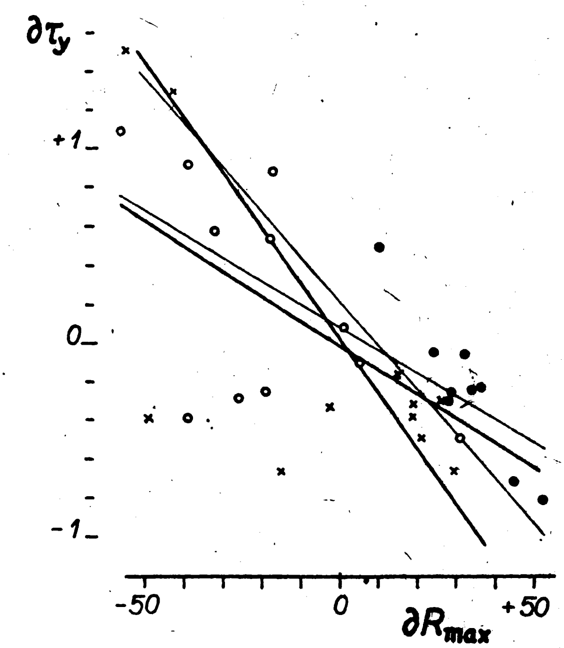

Fig. 40. Correlation between ∂R_{max} and $\partial \tau_y$.

Table 84

Coefficients of correlations $\partial X = f(\partial R_{max})$

$f(\partial R_{max})$	Not-weighted values			Weighted values			$\left(\frac{\psi}{\Delta\psi}\right)$
	ψ	$\frac{\psi}{\Delta\psi}$	N	ψ	$\frac{\psi}{\Delta\psi}$	N	
∂N_y	-0.56 ± 0.12	4.7	31	-0.57 ± 0.15	3.8	20	4.2
$\partial \tau_y$	-0.67 ± 0.10	6.7	31	-0.72 ± 0.11	6.5	20	6.6
∂m	-0.52 ± 0.13 -0.79 ± 0.07	4.0 11.3	31 26	-0.81 ± 0.08	10.1	20	7.7
∂r	$+0.07 \pm 0.18$	0.4	31	$+0.71 \pm 0.11$	6.5	20	1.6
$\partial \Delta$	$+0.44 \pm 0.15$	2.9	31	$+0.29 \pm 0.21$	1.4	20	2.0
∂H_{10}	-0.06 ± 0.18	0.3	31	-0.38 ± 0.19	2.0	20	0.8
∂D	$+0.28 \pm 0.25$	1.1	14	$+0.28 \pm 0.25$	1.1	14	1.1
∂D_0	-0.40 ± 0.21	1.9	16	-0.43 ± 0.20	2.1	16	2.0
∂C	-0.58 ± 0.15	3.9	20	-0.65 ± 0.13	5.0	20	4.4
∂S	-0.23 ± 0.21	1.1	20	-0.19 ± 0.22	0.9	20	1.0

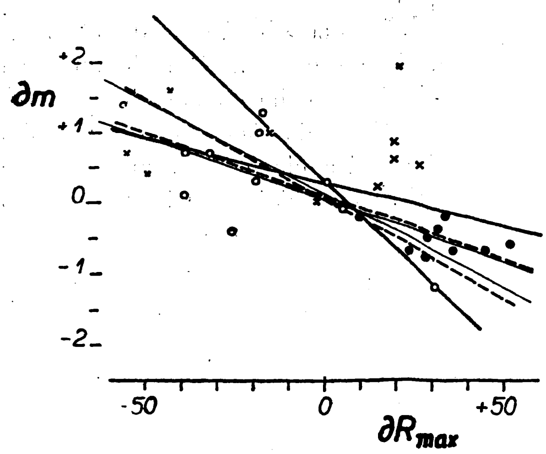

Fig. 41. Correlation between $\partial R_{\max}$ and ∂m.

the time of discovery extraordinarily bright objects and caused, thus, a considerable increase of the value ∂m. After elimination of these five values, we obtain regression lines, drawn in Figure 41 in dashed. Table 84 comprises for each correlation — in addition to its coefficient ψ (with mean error) — also the ratio from the value of this coefficient and the error $\frac{\psi}{\Delta\psi}$, and the number of values from which it has been established, both for the system of weighted and not-weighted values. The last column of the Table shows the geometrical mean from the ratios $\frac{\psi}{\Delta\psi}$.

Each of the values ∂X characterizes in its way the long-term changes, either in the activity of the comets, or in the quality of the observational conditions in the odd and even cycles, and the value $\partial R_{\max}$, as difference in the height of the cycles, is the parameter of the changes in the solar activity. All these quantities are introduced only to the just mentioned purpose, and it would be useless to try to interpret them in a different way, in the same way as it would be senseless to speak of an odd and even cycle at the given time simultaneously.

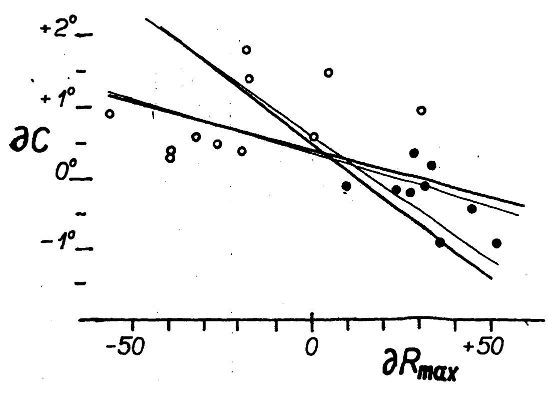

Fig. 42. Correlation between $\partial R_{\max}$ and ∂C.

In analysing the properties of each of the ten dependences in Figures 39—42 we shall assert the following two viewpoints:

a) the reality of the dependence, given, in principle, by the value of the quotient from the size and error of the correlation coefficient;

b) the distribution of values with regard to $\partial X = 0$, contingently the value of the zero-point.

The first criterion is thoroughly analysed in Table 84. According to the size of the coefficients of the individual correlations and ratios $\frac{\psi}{\Delta\psi}$, all quantities under consideration are divided into two classes:

α) the quantities ∂N_y, $\partial\tau_y$, ∂m and ∂C have altogether negative and in the absolute value greater than 0.50 correlation coefficients, and the geometrical mean from ratios $\frac{\psi}{\Delta\psi}$ ranges in them from 4.2 to 7.7; on the average it is 5.8;

β) the quantities ∂r, $\partial\Delta$, ∂H_{10}, ∂D, ∂D_0 and ∂S, on the other hand, reveal a substantially lower degree of correlation; the mean from ratios $\frac{\psi}{\Delta\psi}$ is in them altogether low, ranging from 0.8 to 2.0 and having an average of 1.3.

Of a real relation with the differences of the sunspot number one may speak only in the case of the quantities of class α). The quantities of class

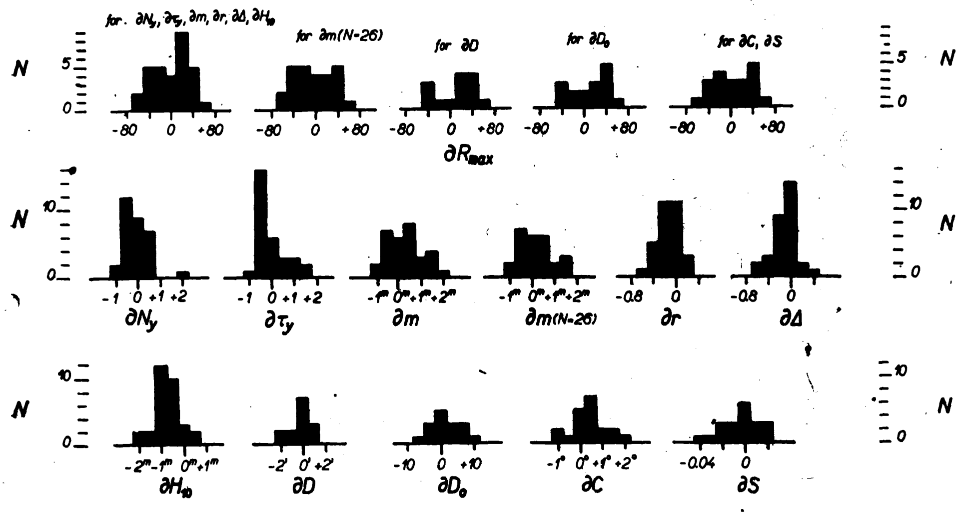

Fig. 43. Frequency distribution of the values ∂X and ∂R_{max} within the system of not-weighted values.

β), either are not a simple function of the sunspot numbers, or their cycle-values are affected by a real dispersion of such degree that the systematical differences are hereby completely offaced.

The investigation of the cometary characteristics from the second point of view will be carried out on the basis of Figures 43 and 44. In the first of them, the frequency-curves of the differences of each of the ten cometary characteristics and of the corresponding differences of the sunspot numbers are presented for the system of not-weighted values, in the second the same curves for the system of weighted values. Since the cycle-values of the characteristics of the cometary coma and tail are determined rather unreliably, the uncertainty in their establishment may unfavourably affect the form, too, of the frequency-curves, so that in our consideration we shall rely, first of all, upon the curves of the characteristics of comets as a whole.

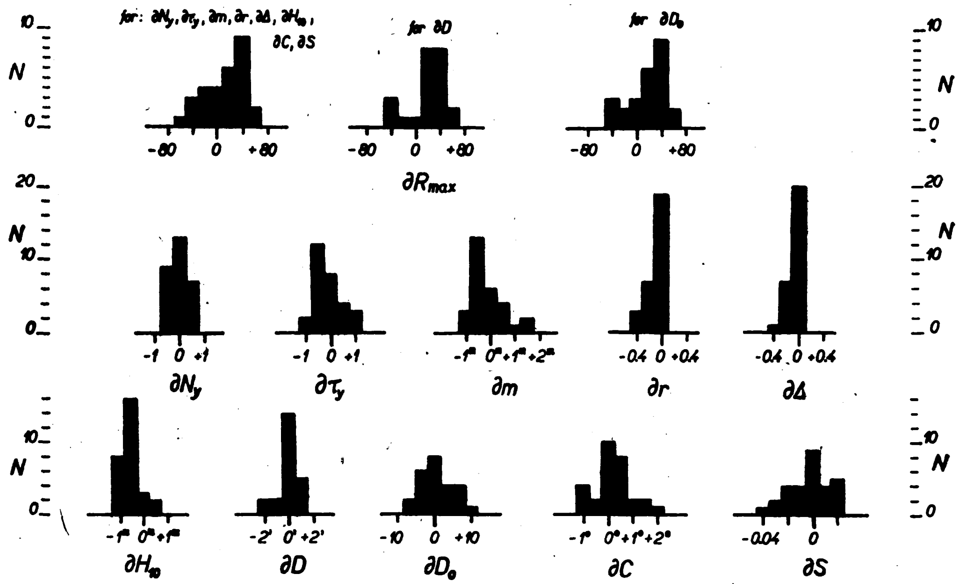

Fig. 44. Frequency distribution of the values ∂X and ∂R_{max} within the system of weighted values.

Let us, first of all, turn our attention at those quantities that were, from the first view-point, classified into the class α). Beside an evident asymmetry of their frequency-curves towards positive values it may be seen that — with the exception of quantity ∂C — the mode lies in the region of negative values. At the same time, there is an altogether good agreement in the position of the modus in both systems of values. The quantities of class β) also reveal an asymmetry of frequency-curves; in distinction to the quantities of class α), however, the curves are now extended in the opposite direction, towards strongly negative values. The modus lies, this time, in the region of zero-values (with the exception of quantity ∂H_{10} in which the influence of curve ∂m may be discerned). With only slight corrections, we thus obtain from the second criterion again a division of the cometary characteristics into two classes α) and β).

8.6. Properties of the curves of the characteristics of classes α) and β)

In order to be able to draw certain conclusions concerning the real correlation established in Section 8.4., the form of the correlation relations must be investigated more in detail:

1. The quantities ∂N_y and $\partial \tau_y$ drop rather steeply with increasing ∂R_{max}; from the position and the mutual gaping of the regression straight lines we may determine the most probable course of the correlation straight line. Thus, to an increase of ∂R_{max} for 10 units corresponds, in the system of not-weighted values, a drop of ∂N_y for 0.21 ± 0.11 and a drop of $\partial \tau_y$ for 0.21 ± 0.08; in the system of weighted values the corresponding decreases are somewhat lower: 0.13 ± 0.07 and 0.17 ± 0.05. In the region of the most frequent values ∂R_{max} (according to Figures 43, 44) the values $\overline{\partial N_y}$ as well as $\overline{\partial \tau_y}$ are strongly negative; from the system of not-weighted values results $\overline{\partial N_y} = -0.43 \pm 0.20$, $\overline{\partial \tau_y} = -0.41 \pm 0.14$, from the system of weighted values -0.28 ± 0.15 and -0.46 ± 0.12. This, consequently, means that in an average even cycle, for 30—40 per cent more comets are being discovered than in an odd cycle. LINK (1952) obtained from the years 1755—1951 a difference of about 25 per cent.

2. The dependece of quantity ∂m on ∂R_{max} reveals the highest value of correlation coefficient. This certainly is a surprising finding, if we recall the fact that during the eleven-year cycle the apparent brightness of comets at the moment of discovery did not reveal any interpretable dependence on its phase. This fact is the more prominent, as the highest values are, by the coefficient of correlation attained in the system of weighted values, that is, in later values which have been determined more reliably, nontheless it classifies the apparent magnitude in this respect into that group of characteristics which are the indicators of night-cloudiness. We accept this relation, as there exist certain presuppositions for it which are absent in any other objective way of explanation; it is, for instance, evident that, from the statistical point of view, the increase of the absolute brightness of comets due to the rate of physical processes taking place in them (which are directly associated to the intensity of the exciting solar radiation) cannot reveal itself under the same observational conditions in the brightness of the comet at the moment of discovery, since, at the same time, the respective heliocentrical and geocentrical distances, too, are increased.

Our process of logic reasoning runs now as follows: the increase of night-cloudiness causes — in addition to the decrease of the number of newly discovered comets — also changes in their average apparent brigthness at the moment of discovery. While the variation of the cometary discoveries may be explained by referring predominantly to a low and continuous cloudiness, we must, in the case of apparent magnitudes, operate with a high cloudiness which affects the „clearness of the sky". Then, the assertion must be true that an increase of high cloudiness results in an increase of the limit of apparent brightness of comets at the moment of discovery. If we presuppose a positive correlation between both types of cloudiness and precipitations as well, we must arrive at the conclusion that in cycles with a higher precipitation activity, the limit of apparent brightness of comets at the moment of discovery must be higher. If we compare this assertion with what has been said in respect to cometary discoveries (which is also true of the cometary tails) we shall find that the correlation coefficient between ∂R_{max} and ∂m must be provided — owing to the definition of ∂m — precisely with an opposite sign than the correlation coefficient between ∂R_{max} and ∂N_y, or $\partial \tau_y$, or ∂C. From Table 84, however, just the contrary may be ascertained.

In the foregoing, the fact has been mentioned that there is not known any other objective way of explanation of this dependence. The paradox that arose between the results following from consideration and those obtained from the material may be explained only if we resort to the subjective factors. We assume that its effect is based on two fundamental and very simple experiences:

a) comets, in distinction to stars, are diffuse objects;

b) the brightness of comets at the moment of discovery has up till now been estimated from comparisons with the surrounding stars.

Under worse observational conditions — particularly owing to light clouds — the eye perceives the brightness of diffused objects considerably more weakened than the brightness of point-objects. Therefore, in order to make the mentioned paradox explicable, we must assume that the effect of the meteorological conditions (as such) on the apparent brightness of the comets at the time of discovery is smaller than the effect of this subjective factor (which, of course, itself is due to the changes of these conditions). Thus, for instance, an increase of high night-cloudiness (it may be understood both in the sense of its extent on the sky and in the sense of duration) would increase, objectively, the average observed apparent brightness of comets at the moment of discovery, at the same time, however, the human eye underrates its value while comparing its brightness with that of the stars for more, no doubt, than for the degree for which it increased owing to the presence of cloudiness. Moreover, the subjective factor may also affect the value of the zero point.

The foregoing consideration is, of course, only of a purely qualitative nature, and it would be highly useful to verify it quantitatively by experiments (for instance, on terrestrial objects).

From the material there results that to a change of ∂R_{max} for $+10$ units corresponds a change of ∂m for $-0^{m}.13$ to $-0^{m}.28$, and that in the region of most frequent values ∂R_{max}, ∂m ranges within the limits from 0^m to -1^m.

3. Quantity ∂C — in spite of the difficulties arisen during the treatment of the material — shows, on the whole nicely, a course analogous to that in the class of quantities ∂N_y and $\partial \tau_y$. To an increase of ∂R_{max} for 10 units corresponds a decrease of ∂C for $0^\circ.26 \pm 0^\circ.13$ in the system of not-weighted values, and for $0^\circ.25 \pm 0^\circ.10$ in the system of weighted values. In the region of most frequent values ∂R_{max} (according to Figures 43 and 44), quantity ∂C acquires the value $-0^\circ.45 \pm 0^\circ.40$ in the system of not-weighted values, and $-0^\circ.40 \pm 0^\circ.20$ in the system of weighted values. In an average even cycle, thus, the most probable angular length of cometary tails in the sky attains a value exceeding the 1.5 multiple of its size in the odd cycles.

4. The quantities of class β), that is ∂r, $\partial \Delta$ ∂H_{10}, ∂D, ∂D_0, and ∂S do not reveal, as already mentioned in Section 8.5., any systematical course with the change of ∂R_{max}, nor does from their frequency curves — with the exception of the absolute magnitude — result any inclination towards positive or negative values. It is just only in the differences of the absolute brightness that a prominent inclination towards negative values becomes apparent. In all probability here, too, the action of the subjective factor, presupposed in the apparent brightnesses, makes itself felt, although, with regard to the fact that H_{10} is frequently being determined from a long series of observations carried out more carefully than in the case of the first orientative value of the brightness, its influence is somewhat reduced. This assertion is in agreement with the fact that the mutual relation between the cycle values of both cycle-types is in the period of the years 1850—1950 in the characteristics m and H_{10} very analogous.

It is interesting to note that the first three quantities revealed during the eleven-year cycle, too, a very low degree of correlation with the course of the sunspot numbers, and that the average value of ratio $\frac{\psi}{\Delta\psi}$ amounted only to about 2.1.

Finally, let us compare the properties of the curves of the individual characteristics during the eleven-year cycle with the just investigated properties of their cycle-values in both cycle-types during a longer period of time. In Section 8.3. we have divided nine cometary characteristics into two classes; into the first class those characteristics have been classified the curves of which in the course of the eleven-year cycle reveal a double wave, into the second, the characteristics the curves of which form a single wave. From Sections 8.4. and 8.5. and from the present it follows that the characteristics belonging into the first class and into class α) are, in principle, identical, and the same is true also in the case of the characteristics belonging into the second class and into class β). If we apply the terminology of Section 8.3., the main conclusion of the study of the differences of the cycle-values of cometary characteristics in both cycle-types for the interval of the last three hundred years is the following: these differences in the characteristics of the first class reveal a real correlation with the difference of the maximum sunspot numbers of both cycle-types in the sense that to a higher value of the characteristics corresponds a lower R_{max}, and that in even cycles, their mean cycle-value is, then, greater than in the odd cycles; in the characteristics of the second class, there does not exist any real correlation between these differences and the differences of the maximum sunspot numbers of both cycle-types, nor can there be discerned a systematical difference between their cycle-values in both cycle-types.

Interesting is the difference of the behaviour of the curves ∂D and ∂C. In Section 8.3., both these characteristics — with certain reservations — have been classified into the first class; while in the case of quantity ∂C this classification has been proved as correct, quantity ∂D — apparently owing to the great dispersion inside the cycle-values — does not correlate with ∂R_{max} and is, thus, the first exception.

The second exception is the quantity ∂m. Its character is, in its substance, quite exceptional. Owing to the reality of the observed relation, it belongs into class α), provided, however, that the consideration in point 2 of the present section is valid, it may be taken for a representative of a special sub-class of the first class of cometary characteristics.

8.7. **Reflex of the eighty-year period of solar activity in cometary statistics**

The relation between the changes of the cycle-values of the cometary characteristics of the first class and the changes of the maximum sunspot numbers of both cycle-types does not reveal directly anything of the effect of long-term periodical changes of solar activity upon cometary characteristics.

There are, however, two circumstances that lead me to the study of the problem of the influence of the eighty-year period of solar activity on the cometary characteristics. It was partly the endeavour for an analogy of the dependences under consideration regardless of the cycle-type, partly the results obtained by Link (1956) on the principle of cometary climatology, when the large four-hundred-year-period of solar activity had been found.

In the search for a relationship between the characteristics ∂X and ∂R_{max}, we were not obliged — when computing the cycle-values — to take into consideration their secular changes (excepting ∂N_y and $\partial \tau_y$), as we were working with relative values. This was the chief merit of the method. Since we work now with absolute values, the disturbing effect of the secular changes must be reduced particularly in order to eliminate the inflexions in the curves of the time-course of cometary discoveries caused by the introduction of new observational methods.

The influence of the eighty-year period will be observed in two statistical sets from 1610 till 1957:

a) in comets, the apparent brightness of which exceeds 5^m for a short interval of time at least;

b) in comets, the tails of which were visible with the naked eye for a short time-interval at least.

In spite of the material-reduction, the course of the number of discovered bright comets with the phase of the eighty-year period of solar activity may be quantitatively observed only in the axis of abscissae, that is, in time, while in the axis of ordinates, the mentioned dependence can be observed only qualitatively. For the determination of the minima in the curves of sets a) as well as b) we shall use the same method as used by Link in his work (Link, 1956); on the axis of abscissae we shall plot the years and on the axis of ordinates the serial number of the discovered comet of the given set. The slope of this monotonously rising curve indicates in each point the rate of growth of the discovered comets and is — besides the reflex of the social factor which, owing to the carried out reduction of material in the course of the period 1610—1957 changes comparatively slowly — an indicator of night-cloudiness in shorter time-intervals.

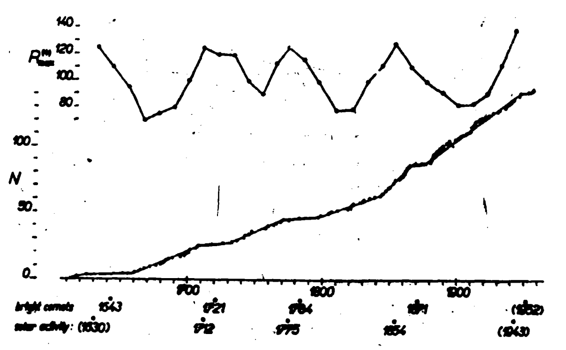

Fig. 45. Curve of the set of comets brighter than 5^m

In this curve, a series of thresholds may be found which represent periods with an exceptionally low number of bright comets eventually of comets with bright tails, that is, periods of a high night-cloudiness. Since, from 1610 on we have at our disposal values of the sunspot numbers of cycles (Waldmeier, 1955) — even if, at the beginning, only approximate values — we may observe the correlation between the eighty-year period of solar activity determined by smoothed out values of the sunspot numbers and between the assumed climatic variations, characterized by the cometary thresholds.

The curve of the set of comets brighter than 5^m is shown in Figure 45, the curve of the set of comets with tails visible with the naked eye, in Figure 46. In the upper part of both graphs, the form of the eighty-year period is plotted, determined from the smoothed out maximum sunspot numbers R_{max}; below the axis of abscissas are two rows of black points, of which the first indicates the time-moments of the centre of thresholds in the curve of the represented cometary set, and the second the time-moments of the maxima of the eighty-year period resulting from the sunspot numbers.

Prior to proceeding to the evaluation of the results, let us briefly remark that in the curves of both cometary sets, there are, in addition to these most

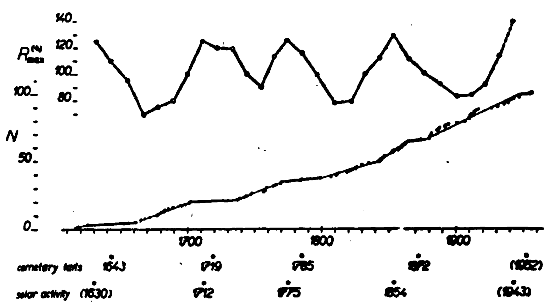

Fig. 46. Curve of the set of comets with tails visible with naked eye.

prominent thresholds, many others of them which correspond to the eleven-year solar cycles. These fluctuations have been studied quantitatively in both axes in Section 8.2. By the method of cometary thresholds, however, in the mentioned work by LINK (1956), the change of the length of the eleven-year cycles in the period from —235 till 1948 were studied.

From Figures 45 and 46 it can be seen that the prominent thresholds in the curves of both cometary sets reveal an eighty-year periodicity that, however, from the view-point of time occur with a certain retardation following the maximum of the eighty-year solar period. The properties of the curves of both sets are in detail presented in Table 85, the individual columns of which comprise:

T_0 — the epoch of the maximum of the eighty-year period of solar activity;

T_1 or T_2 — the epoch of the centre of the threshold of the set of comets brighter than 5^m, eventually of the set of comets with tails visible with the naked eye;

ΔT_1 or ΔT_2 — the phase retardation: $\Delta T_1 = T_1 - T_0$, $\Delta T_2 = T_2 - T_0$;

P_0 — the difference in time of two successive maxima of the eighty-year period of solar activity;

P_1, or P_2 — the difference in time of the centres of two successive thresholds in the curve of the set of comets brighter than 5^m, or of the set of comets with tails visible with the naked eye.

Table 85
Eighty-year periodicity of the prominent thresholds in the curve of the set of comets brighter than 5^m and of comets with tails visible with the naked eye

	T_0	T_1	T_2	ΔT_1	ΔT_2	P_0	P_1	P_2
I	(1630)	1643	1643	(+13)	(+13)	(82)	78	76
II	1712	1721	1719	+ 9	+ 7	63	63	66
III	1775	1784	1785	+ 9	+10	79	87	87
IV	1854	1871	1872	+17	+18	(89)	(81)	(80)
V	(1943)	(1952)	(1952)	(+ 9)	(+ 9)			
Average				+11	+11	78	77	77

Regardless of a certain retardation, at the epoch of the maxima of the eighty-year period a certain increase of night-cloudiness may be observed; we found a similar phenomenon by another method in the eleven-year cycle, and the same relation was found by LINK in the mentioned 410-year period of solar activity as well.

By comparing the results in Section 8.2. and 8.7. and the results obtained by LINK (1956), the mutual relation between the phase-retardation may be established:

a) of the secondary minimum (in $\Phi \doteq 0.5$) in the curve of the cometary characteristics of the first class following the maximum of solar activity in the eleven-year cycle;

b) of the centre of the thresholds in the curves of both sets of prominent comets following the maximum of solar activity in the eighty-year period and

c) of the beginning of increase of cloudiness following the beginning of the rise of solar activity found by comparison of the frequency of cometary discoveries and of the frequency of aurora borealis in the large 410 year solar period.

All these retardations, in fact, amount to about one seventh of the respective period.

The eighty-year periodicity of cometary thresholds may be in Figures 2—4 of LINK's work observed as far as into the period round the year 500 A. D. Prior to this date, the situation was complicated by a rapid decrease of records on cometary discoveries. This problem, however, is not any longer within the sphere of the research of the present study, since, prior to 1600, the periodicity of cometary thresholds with the eighty-year periodicity of solar activity cannot be compared.

8.8. Interpretation of the form of curve $H_{10} = H_{10}(\Phi)$. Basic equation of energy balance

From the physical point of view the most important quantity of cometary characteristics of the second class is the absolute magnitude, H_{10}, which is therefore taken as a subject of study. As the variation of the brightness of the gaseous part of the coma mainly contributes to the variation of the total absolute brightness during an eleven-year cycle (Section 8.2.), the main subject of our study is the brightness of the gaseous coma and its dependence on the solar activity.

The most available way to solve the problem of the dependence of the brightness of the gas part of cometary coma on the phase of a solar cycle, Φ, is to determine the balance of the changes in a number of radiating molecules in the coma. At every infinitesimal interval of time the number of them, N, increases with increasing intensity of exciting solar radiation, $I_{\odot}$, and decreases according to which percentual part of them is dissociated per unit of time, so that it is possible to write in general

(8.4) $$\mathrm{d}N(\Phi) = a \,.\, \mathrm{d}(I_{\odot}^{j}) - bF(\Phi) \,.\, N(\Phi)\, \mathrm{d}\Phi,$$

where a, b are positive constants and $F(\Phi)$ is the function giving the change of photo-dissociation of molecules during a cycle. To apply formula (8,4) to the material, exponent j must be ascertained and the second term on the right side of the eqaation must be replaced by the expression with known quantities. Exponent j may be derived from the dependence of radiating (more exactly, evaporated) molecules of gas on the heliocentric distance. If we assume the invariability of the solar constant during observations of a comet and leave out of account effects connecting with the drop of the concentration of gaseous molecules in surface layers of a cometary nucleus and the thermal inertia of the process of evaporation, we may apply Levin's formula (Levin, 1943, 1948) according to which the brightness of gaseous coma changes at the neighbourhood of heliocentric distance of 1 A. U. as follows:

(8.5) $$I \sim \exp\left[\left(\frac{1}{4} + \frac{1}{2}\,\frac{L}{R_0 T_0}\right) \ln \frac{1}{r}\right],$$

where L ist the heat necessary for evaporation of 1 Mol of gas, R_0 the gas constant, T_0 the absolute temperature of the surface of cometary nucleus at $r = 1$ A. U. As the intensity of the solar radiation changes with the heliocentric distance according to formula

$$I_{\odot} \sim r^{-2},$$

it results

(8.6) $$j = \frac{1}{8} + \frac{1}{4}\,\frac{L}{R_0 T_0}$$

in proportion $I \sim I_{\odot}^{j}$. For non-period and long-period comets forming the desicive majority of material used it follows from OORT's, SCHMIDT's and VANÝSEK's studies (OORT, SCHMIDT, 1951, SCHMIDT, 1951, VANÝSEK, 1952) on the average $L \doteq 3500$ cal/Mol; as for other quantities in (8.6) we can admit the following approximate values: $R_0 = 2$ cal/Mol . grad, $T_0 = 350$ °K, exponent j is equal to

$$j = \frac{11}{8}. \tag{8.7}$$

Therefore we do not make a great mistake if we assume the linear dependence between I and $I_{\odot}$ to a first approximation.

Let us assume further that the average life-time of gaseous molecules, i. e. the interval during which they are able to radiate in cometary atmosphere, is constant during the whole solar cycle. Then, whether the photodissociation of molecules occur in the cometary head region or in the tail, the relative part of them, which does not contribute to the radiation of the coma, can be in the simplest form expressed by the relation

$$-\frac{dN}{N} \sim \beta, \ \beta > 0, \tag{8.8}$$

so that equation (8.4) has the form:

$$\frac{dN}{d\Phi} = a\frac{dI_{\odot}}{d\Phi} - \beta N(\Phi). \tag{8.9}$$

This formula is correct even in the case of variability of the average life-time of molecules, if it reachs such values that the dissociation occurs out of the coma region. Otherwise, the average life-time, τ, must be introduced into equation (8.4) as a further agent affecting the total number of radiating molecules in the coma, regarding the relation

$$dN \sim d\tau. \tag{8.10}$$

Considering (8.7), (8.8), and (8.10) and replacing the number of molecules, N, by the brightness of (gaseous) coma, I_{10}, we can write

$$dI_{10} = \alpha I_{\odot} - \beta I_{10}\, d\Phi + \gamma'\, d\tau. \tag{8.11}$$

This is the basic equation for the study of relation $H_{10} = H_{10}(\Phi)$. Coefficients α, β, and γ' depend on the choice of the units of I_{10}, $I_{\odot}$, τ and Φ. According to VANÝSEK (1960) we can approximately write

$$\tau \,.\, I_{\odot}^{1^1/_2} = a', \tag{8.12}$$

where a' is a constant. The sunspot number is taken as a parameter characterizing the solar radiation. The relative intensity of that is assumed in the form of

$$I_{\odot} = 1 + kR, \tag{8.13}$$

which was successfully used by ALLEN (1946, 1948) and HULBURT (1955), in studying the influence of the short-wave solar radiation on the critical frequency of individual ionospheric layers. By inserting (8.12) and (8.13) into differential equation (8.11) we obtain after integration

$$I_{10}(\Phi) = \exp[-\beta\Phi] \cdot \left\{ k \int_0^{\Phi} \left(\alpha - \frac{1}{2} a'\gamma' I_{\odot}^{-3/2} \right) \frac{\mathrm{d}R}{\mathrm{d}\Phi} \exp[\beta\Phi]\,\mathrm{d}\Phi + \right. \qquad (8.14)$$
$$\left. + \text{const} \right\},$$

where the constant is equal to the average absolute brightness of comets at the time of minimum solar activity.

8.9. Discussion and application of the balance formula to the material

General equation (8.14) cannot be used for the calculation of coefficients α, β, and γ', while very uncertain values of $\mathrm{d}H_{10}/\mathrm{d}\Phi$ hamper an application of differential form (8.11) to the material. Further on, we shall therefore attempt to solve the problem in an approximate way neglecting individual factors on the right side of equation (8.11) and investigate the following special cases:

I. If the change of exciting solar radiation has the main influence on the change of cometary brightness, then

$$I_{10} = \text{const} + \alpha k R, \qquad (8.15)$$

which is in absolute variance with the preceding results of this chapter.

II The assumption that the influence of excitation or ejection out of coma of a constant part of a total number of molecules on the change of the absolute brightness is negligible in comparison with the influence of the two other factors, i. e. $\beta \approx 0$, gives the relation (we put $\gamma = \frac{1}{2} a'\gamma'$):

$$I_{10} = \text{const} + \alpha I_{\odot} + 2\gamma I_{\odot}^{-1/2}. \qquad (8.16)$$

In this case, the form of the I_{10}-curve is not equal to that of the curve of sunspot numbers; however, only one corresponding I_{10} exists to a given sunspot number. But Fig. 34 shows that two different values of I_{10} correspond to each sunspot number, so that this assumption is inacceptable, too.

III. The assumption neglecting the influence of the length of life-time of molecules in the form of the I_{10}-curve gives the following expression:

$$I_{10} = \text{const}_1 + \text{const}_2\, e^{-\beta\Phi} + \alpha k R - \alpha\beta k \int_0^{\Phi} R(\Phi)\, e^{\beta\Phi} \mathrm{d}\Phi. \qquad (8.17)$$

Its validity is confined to a long life-time of radiating molecules and it must be characterized by a low value of coefficient γ in general equation

(8.11), which is not fulfilled; expression (8.17) cannot therefore be considered as the correct form of the dependence we are looking for.

IV. Let us consider finally that the change of the average life-time of molecules during the solar cycle contributes mainly to the change of absolute brightness I_{10}

$$dI_{10} \sim d\tau, \tag{8.18}$$

so that the following expression we can obtain from equation (8.11):

$$I_{10} \sim \frac{dR}{d\Phi}. \tag{8.19}$$

The fact that the mean values of the characteristics of the second class correspond to the maximum and minimum solar activity is considered as one of the main properties of these curves (Section 8.3). Connecting this empirical fact with relation (8.19) we arrive at the formula

$$I_{10} = X_0 + Y_0 \frac{dR}{d\Phi}, \tag{8.20}$$

where X_0 is the mean from the cycle-values of absolute brightness, and

$$Y_0 = \frac{\alpha k}{\beta}. \tag{8.21}$$

Now we shall prove that this assumption corresponds best of all with observations. At the same time we must note down that relation (8.18) gives quite a different variation of the average life-time of molecules than formula (8.12).

Sunspot number R is determined from Chvojková's immaterially modified interpolation formula (Chvojková, 1952, 1956):

$$R(\Phi) = R_m + \frac{1}{2}(R_M - R_m)\left(1 - \cos\frac{2\pi\Phi}{a + (1-a)\Phi}\right), \tag{8.22}$$

where R_m and R_M are minimum and maximum monthly sunspot numbers respectively, and constant a connects with the asymmetry of the curve:

$$a = \frac{\Phi_M}{1 - \Phi_M},$$

Φ_M is the phase-distance between a maximum and preceding minimum of solar activity. The last ten cycles (from years 1843—1954), during which the majority of comets of our statistics were discovered, are taken for the determination of averages of parameters R_m, R_M, and a. Results are included in Table 86.

The secondary maximum (or point of inflexion) in curves of cometary characteristics of the second class can be explained by the influence of observational conditions (Section 8.2.). It is reasonable to eliminate these

Table 86

Parameters of solar cycles from years 1843 to 1954

	Odd cycles	Even cycles	Average cycle
R_m	5.1 ± 1.1	4.3 ± 0.7	4.7 ± 0.6
R_M	116.9 ± 6.4	93.3 ± 10.4	105.1 ± 6.3
a	0.542 ± 0.031	0.694 ± 0.057	0.608 ± 0.033
Φ_M	0.352 ± 0.013	0.410 ± 0.020	0.378 ± 0.013

changes by the assumption that curve $H_{10} = H_{10}(\Phi)$ is the sum of a simple sine curve (influence of solar radiation on the processes in coma) with semi-amplitude B and a double sine curve (meteorological influences) with semi-amplitude C. If we denote a phase of the minimum in a simple and double waves as Φ_1 and Φ_2 respectively we get

$$H_{10} = A + B \cos 2\pi(\Phi - \Phi_1) + C \cos 4\pi(\Phi - \Phi_2). \tag{8.23}$$

In equation (8.20) brightness I_{10} is then equal to

$$I_{10} = \exp\left[-\frac{0.4}{\text{mod}} M_{10}\right], \tag{8.24}$$

where

$$M_{10} = H_{10} - C \cos 4\pi(\Phi - \Phi_2). \tag{8.25}$$

In Fig. 47 the values of M_{10} are plotted by full circles and those of H_{10} by open circles. Relation (7.20) is represented in Fig. 48 separately for odd and even cycles. The method of least squares gives the resulting values of parameters as follows:

$$\begin{aligned} X_0 &= (+1.14 \pm 0.02) \,.\, 10^{-3}, \\ Y_0 &= (+1.10 \pm 0.07) \,.\, 10^{-6} \end{aligned} \tag{8.26}$$

in the odd cycles, and

$$\begin{aligned} X_0 &= (+1.30 \pm 0.01) \,.\, 10^{-3}, \\ Y_0 &= (+0.95 \pm 0.06) \,.\, 10^{-6} \end{aligned} \tag{8.27}$$

in the even cycles. Correlation coefficients are 0.93 ± 0.03 and 0.95 ± 0.02 respectively. A guarantee of expanding the curve $H_{10} = H_{10}(\Phi)$ in a simple and double waves must be shown by an agreement of value A from (8.23) with the average cycle-value of absolute magnitude of comets (Table 57); analogously the validity of formula (8.20) is checked on an agreement of X_0 from (8.26) or (8.27) with the average cycle-value of absolute brightness. A comparison of cycle-value O derived directly from material with both just-mentioned quantities is given in Table 87.

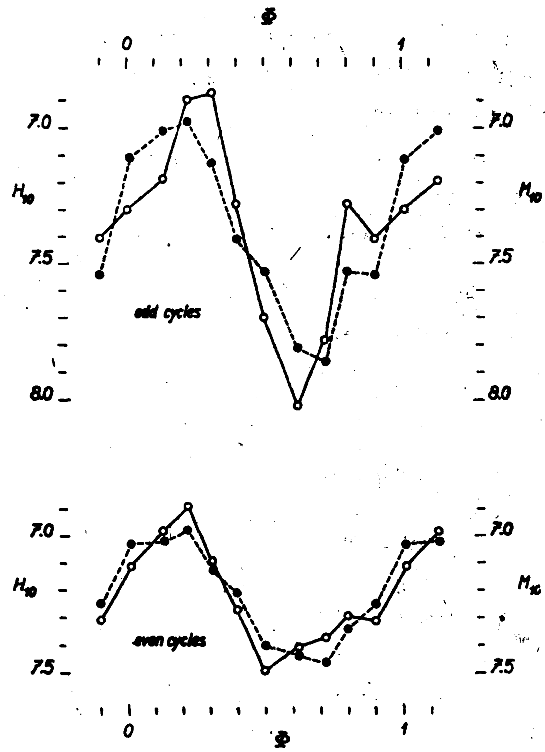

Fig. 47. Variation of the average absolute magnitude of comets in odd cycles (at the top) and in even cycles (at the bottom); H_{10} are the magnitudes obtained directly from the material, M_{10} are those corrected for a double-wave.

For the determination of ratio β/α it is necessary to know the value of coefficient k from relation (8.13), which can be derived from the average solar cycle by applying a general form of balance equation (8.11). By inserting of (8.12), (8.13), and (8.24) into (8.11) we obtain the balance equation in the form of:

Table 87

Average absolute magnitude of comets during solar cycle determined in different ways

Cycles	$\overline{H_{10}}$		
	O	A	X_0
odd	$7^m.37 \pm 0^m.08$	$7^m.40 \pm 0^m.03$	$7^m.36 \pm 0^m.02$
even	7.21 ± 0.04	7.23 ± 0.01	7.21 ± 0.01

$$\text{(8.28)} \qquad \frac{dH_{10}}{d\Phi} = A_1 + \frac{dR}{d\Phi} \exp\left[\frac{0.4}{\text{mod}} M_{10}\right] . \{A_2 + A_3(1 + kR)^{-\frac{1}{2}}\},$$

where

$$\text{(8.29)} \quad \begin{cases} A_1 = +2.5 \text{ mod } \beta, \\ A_2 = -2.5 \text{ mod } \alpha k, \\ A_3 = +2.5 \text{ mod } \gamma k. \end{cases}$$

By the method of least squares we determine sums of squares of differences between empirical and computed derivatives $dH_{10}/d\Phi$ for various selected values of parameter k and then we plot them against k. The minimum gives the most probable value of k at once. In this way we get

(38.0) $$k = 0.0080 \pm 0.0001,$$

so that the variation of the „monochromatic" solar constant of exciting radiation during the average cycle yields

(8.31) $$I_{\odot M} : I_{\odot m} = 1.77 \pm 0.05$$

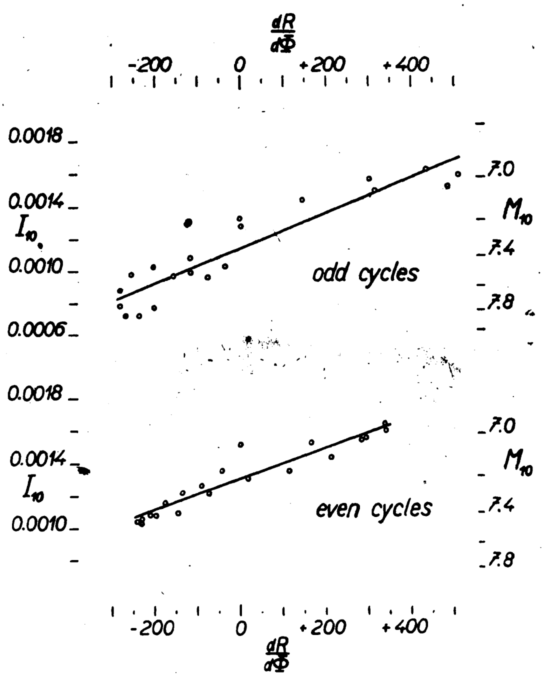

Fig. 48. Dependence of the average absolute brightness of comets on the change of the sunspot number during an eleven-year cycle.

On the assumption that k is independent of the type of a cycle we obtain the following values of ratio β/α:

(8.32a) $$\frac{\beta}{\alpha} = 7270 \pm 470$$

in the odd cycles, and

(8.32b) $$\frac{\beta}{\alpha} = 8420 \pm 540$$

in the even cycles.

8.10. Conclusions from the statistical investigation

From the investigation of eleven cometary characteristics in dependence on the solar activity the following most important conclusions were arrived at:

1. On the basis of the properties within the eleven-year cycle, most characteristics classify themselves into one of two classes I, II. The properties of both classes are described in Table 88.

2. The same classification is arrived at by a study of the long-periodical changes of cometary properties in the odd and even cycles as well (see again Table 88).

Table 88

Properties of both classes of cometary characteristics

Property	Class I	Class II
form of curve in the cycle	double-wave	single wave
period of variations	5.5 years	11 years
sign-characteristic of curve	both maxima equally high, or the second higher	in $\Phi \approx 0.8$ local maximum with amplitude lower in order, or inflexion
ratio of amplitudes of curve in odd and even cycles	in wide ranges, on the average 1 : 1	on the average 2 : 1
degree of correlation of differences ∂X of cycle values with ∂R_{max}	high, as to sign negative	very low
frequency distribution of ∂X	extension to positive values	extension to negative values
mode of frequency distribution	in negative values	round zero
representation of characteristics	N_ν, τ_ν, C, (D?), (m?)	r, Δ, H_{10}, D_0, S
tentative interpretation	indicators of night-cloudiness	indicators of solar activity (direct influence on processes within the comet)

3. DOBROVOLSKY's seasonal index reveals with respect to the absolute brigtness of comets in the statistics of the whole VSEKHSVIATSKY's catalogue (from 1610 on) opposite properties than in the set of short-period comets (DOBROVOLSKY, 1957); from the view-point of correlation with the solar activity it cannot be classified into either class.

4. The eighty-year period of solar activity is reflected in the periodicity of the thresholds in the curve of the set of comets brighter than 5^m and of the set of comets with powerful tails; the centres of the thresholds lag for about one seventh of the period behind the maxima of solar activity.

5. For explaining the form of curves of cometary characteristics of the second class during an eleven-year solar cycle it is necessary to consider generally three constituents of the balance of the molecular cometary radiation: the direct excitation effect of the solar short-wave radiation, the influence of the change of the life-time of excited molecules during a cycle, and the molecular waste-effect.

6. The more detailed comparison of the observational material with the theory shows that the change of the life-time of radiating molecules has the predominant influence on the change of the intensity of the radiation of comets, so that the intensity of a cometary radiation depends linearly on the change of an exciting solar radiation; the mean value of the intensity of a cometary radiation during a cycle derived by this method agrees very well with that derived directly from the observational material.

7. The ratio of coefficients β/α results on an average of about 8000 in the system of units used.

8. The given method makes it possible to derive the variation of the intensity of the exciting solar radiation during a solar cycle from the form of the curve of the absolute brightness of comets assuming the validity of (8.13); its numerical value yields about 1 : 1.8.

As for the interpretation of the dependence of the cometary characteristics of the first class on the solar activity, let us only remark that as early as 1947, LINK and VANÝSEK (1947) pointed to the fact that the characteristic curve of the number of comets discovered per year revealed a correlation with the HELLMANN, well-known period of precipitation in Europe (HELLMANN, 1909), and explained it by the cloudiness accompanying the precipitation. A detailed analysis of this problem is out of the scope of investigation of this study.

8.11. Periodic changes in the activity of the Encke comet

The Φ-curves of all eleven characteristics, investigated in the preceding sections of the present chapter, have been analyzed on the assumption that their forms are not affected by the actual differences between the individual comets of the set. The extent to which this assumption is correct can be verified only by studying the same curves of one comet in which the following basic conditions must be fulfilled:

a) a large enough number of measurements of a given characteristics of the comet must be available over a long enough period of time;

b) the effects associated with the development and „ageing" of the comet must be dependably eliminated.

Only the Encke comet satisfies both these conditions. Physical data of more than 40 returns are at our disposal at present, and the secular changes of this comet are roughly continuous. The influence of solar activity on the Encke comet will be investigated in the following characteristics:

A) in the fluctuation of the absolute brightness of the comet;

B) in the fluctuation of the observed brightness dispersion of the comet;

C) in the fluctuation of the linear diameter of the cometary head.

8.11.1. Variation of the absolute brighness

For the study of the brightness variation of the Encke comet its absolute magnitudes were used as ascertained in 41 returns during 1819—1954. These magnitudes are included in VSEKHSVIATSKY's „Catalogue of Absolute Magnitudes of Comets" again.

The continuity of the secular decrease of the absolute brightness of this comet is evident from Fig. 49; therefore, the dependence of the absolute magnitude, H_{10}, on time may be expressed in the form of a progression. Let us limit it to a quadratic term:

$$H_{10}(t) = a + b\Delta t + c(\Delta t)^2, \qquad (8.33)$$

$\Delta t = t - t_0$, t is the moment of observation, $t_0 = 1900.0$. If Δt is expressed in hundreds of years, the coefficients of equation (8.33) are as follows:

$$\left.\begin{aligned} a &= +9^m\!.98 \pm 0^m\!.06, \\ b &= +2.94 \pm 0.14, \\ c &= +1.09 \pm 0.32. \end{aligned}\right\} \qquad (8.34)$$

The absolute magnitudes H_{10} obtained for each t from the parameters (8.34) are included in column C of Tab. 89. For each return of the comet its phase-shift referred to the preceding minimum of sunspot numbers, Φ, the observed absolute magnitude O, weight in the scale of VSEKHSVIATSKY (1956, 1958), w, and residual $O - C$ are also presented in Tab. 89.

Further, here is discussed the endeavour of DOBROVOLSKY (1957) to prove that the ascertained variations of H_{10} are in no correlation either with the sunspot number R, or with the Holetschek criterion of the conditions of visibility ΔT (HOLETSCHEK, 1916), but that they are correlated with the seasonal index (Paragraph 8.2.5.).

DOBROVOLSKY's study of the correlation was, however, based on the mere appearance of the curves so that it was rather subjective. Moreover, he did not eliminate the secular variations of the absolute magnitude, and he reffered the seasonal index to the time of perihelion passage instead of to the middle of the interval of observations. This caused the values of the seasonal index obtained by DOBROVOLSKY to be turned by ± 1 to ± 2 degrees, and in one case even by 5 degrees.

The correlation between the residual $O - C$ on the one hand, and the sunspot number, the conditions of visibility and the seasonal index on the other, can be objectively established by determining the correlation coefficient ψ. If we denote the seasonal index for the perihelion passage and that for the middle of the observational interval n_p and n respectively, the following correlation coefficients are obtained from the data of Tab. 89 and those applying to the returns in 1786, 1795 and 1805:

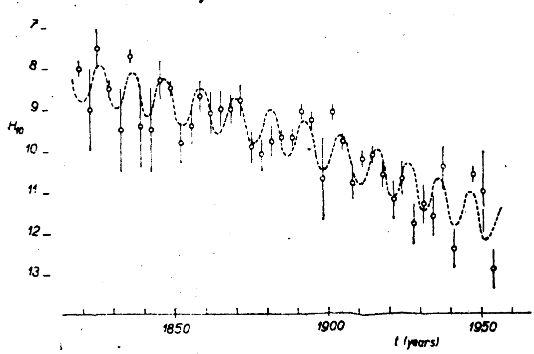

Fig. 49. Course of the absolute magnitude of the Encke comet.

$$
(8.35) \begin{cases} \psi[O - C, R] = -0.39 \pm 0.09, \\ \psi[O - C, \Delta T] = -0.36 \pm 0.09, \\ \psi[O - C, n_p] = -0.18 \pm 0.10, \\ \psi[O - C, n] = +0.08 \pm 0.10. \end{cases}
$$

The relations $O - C = f(R)$ and $O - C = f(\Delta T)$ are the only ones that come into consideration.

In Table 82 it has been shown that during the solar cycle the curves of the cometary characteristics of the second class reach the maximum values at $\Phi_0 = 0.25 \pm 0.01$ with the semi-amplitude corresponding to about $0^m.5$. Thus, let us assume to a first approximation the following time course of the residuals:

$$(8.36) \qquad O - C = d \, . \cos 2\pi(\Phi - \Phi_0).$$

The semi-amplitude of the fluctuations is

$$(8.37) \qquad d = -0^m.38 \pm 0^m.01$$

and the phase of the maximum

$$(8.38) \qquad \Phi_0 = 0.248 \pm 0.005.$$

The smothed-out dependence of the residuals $O - C$ on the phase of the solar cycle is given in Table 90. The corresponding correlation coefficient is

$$\psi[O - C, \cos 2\pi(\Phi - 0.25)] = -0.55 \pm 0.07. \quad (8.39)$$

The form of the smoothed-out curve of the residuals $O - C$ is shown in Fig. 50. These results prove beyond any doubt that the character of the residuals is the same as that of the characteristics of the second class.

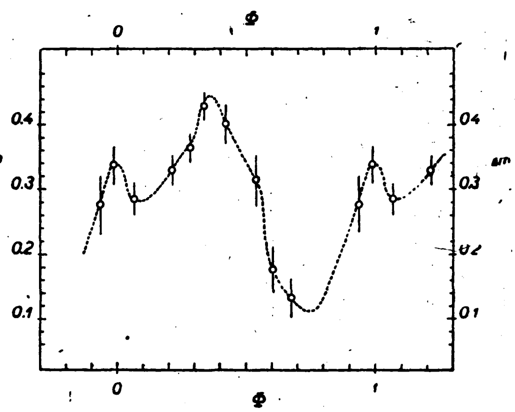

Fig. 50. Residuals $O - C$ as related to the phase of the solar cycle.

Thus, the ascertained brightness fluctuations of the Encke comet can be interpreted as a sign of physical processes within the comet (Section 8.8.).

The analysis of these fluctuations has even a certain bearing for prediction the course of the absolute magnitude. The general formula describing the dependence of the absolute brightness of the Encke comet on time has the following form:

$$H_{10}(t) = a + b\Delta t + c(\Delta t)^2 + d \cos 2\pi(\Phi - \Phi_0), \quad (8.40)$$

where the constants are equal to (Δt is again expressed in hundreds of years):

$$(8.41) \begin{cases} a = +10^m.03 \pm 0^m.05, \\ b = +2.85 \pm 0.12, \\ c = +0.87 \pm 0.27, \\ d = -0.^m50 \pm 0^m.05, \\ \Phi_0 = 0.250 \pm 0.016. \end{cases}$$

Since formula (8.40) is of only approximate character, its applicability for prognostic purposes is limited to about 2000.

8.11.2. Fluctuations of the dispersion of the observed brightness estimates

Beyer (1933, 1937a, 1937b, 1938, 1942, 1947, 1950a, 1950b, 1955) considered the average absolute value of the departures between the brightness estimates and the smoothed-out photometric curve to be a parameter characterizing the observed brightness of a comet. This method though correct in principle, has certain disadvantages which will be discussed in the next section.

Table 89

Residuals of the absolute magnitude of the Encke comet in individual returns

t	Φ	H_{10}				R	ΔT	n_p	n
		O	C	$O-C$	i				
		m	m	m					
1819.0	0.664	8.0	8.31	−0.31	2	28	+2	6	5
1822.5	0.943	9.0	8.36	+0.64	1	7	−6	2	1
1825.6	0.218	7.5	8.40	−0.90	2	22	+2.5	2	1
1828.9	0.525	8.5	8.44	+0.06	2	52	+4	6	4
1832.5	0.864	9.5	8.49	+1.01	1	27	−6	2	1
1835.6	0.174	7.7	8.54	−0.84	1	59	−2.5	1	0
1838.8	0.509	9.4	8.59	+0.81	2	81	+6	5	3
1842.3	0.870	9.5	8.65	+0.85	1	24	−3	3	4
1845.5	0.162	8.3	8.70	−0.40	2	31	−3	1	0
1848.8	0.421	8.5	8.76	−0.26	3	116	+5	4	3
1852.1	0.689	9.8	8.82	+0.98	1	68	0	4	5
1855.6	0.966	9.4	8.89	+0.51	2	2	−5	0	0
1858.7	0.240	8.7	8.95	−0.25	2	72	+4	3	2
1862.0	0.534	9.1	9.02	+0.08	2	66	+1	5	5
1865.3	0.832	9.0	9.09	−0.09	1	34	−6	2	3
1868.6	0.121	9.0	9.16	−0.16	2	33	+2	2	1
1871.8	0.396	8.8	9.24	−0.44	2	94	+6	5	3
1875.2	0.685	9.9	9.32	+0.58	2	26	−3	3	4
1878.6	0.978	10.1	9.40	+0.70	1	0	−4	0	1
1881.8	0.266	9.8	9.48	+0.32	2	58	+5	4	2
1885.1	0.582	9.7	9.56	+0.14	3	54	0	4	5
1888.6	0.906	9.7	9.66	+0.04	3	3	−5	1	1
1891.6	0.169	9.1	9.74	−0.64	3	49	+4	3	1
1895.0	0.442	9.3	9.83	−0.53	2	60	+1	5	5
1898.5	0.734	10.7	9.93	+0.77	1	18	−6	2	1
1901.6	0.995	9.1	10.03	−0.93	3	1	+2	2	1
1904.9	0.265	9.8	10.13	−0.33	3	46	+4	6	4
1908.4	0.565	10.8	10.23	+0.57	2	48	−6	2	1
1911.6	0.835	10.2	10.33	−0.13	3	4	−2	1	1
1914.8	0.118	10.1	10.44	−0.34	2	10	+6	5	3
1918.2	0.455	10.6	10.55	+0.05	2	78	−3	4	5
1921.6	0.800	11.2	10.66	+0.54	2	23	−5	0	1
1924.7	0.108	10.7	10.77	−0.07	2	23	+4.5	4	2
1928.1	0.437	11.8	10.89	+0.91	3	72	+1	5	6
1931.7	0.792	11.3	11.02	+0.28	2	16	−6	1	2
1934.6	0.076	11.6	11.13	+0.47	2	8	+2.5	2	1
1937.8	0.386	10.4	11.25	−0.85	2	100	+6	5	3
1941.2	0.711	12.4	11.38	+1.02	2	41	−3	3	4
1947.8	0.349	10.6	11.63	−1.03	3	159	+5	4	3
1951.1	0.675	11.0	11.77	−0.77	2	83	0	4	6
1954.1	0.973	12.9	11.89	+1.01	2	8	−5	0	5

Nevertheless, let us apply this method to the Encke comet. The material used consisted of 114 brightness estimates from 19 returns of the comet before 1915, collected by Holetschek (1916), 43 estimates from 1937 till 1951 carried out by Beyer (1942, 1950a, 1955), and 23 observations made during 1947, taken over from a few Copenhagen Circulars (Thernoe, 1947, Vinter Hansen, 1947a, 1947b).

Table 91

Average dispersions Δm of the Encke comet

t	Φ	Δm	w	R
		m		
1805.8	0.610	0.26	1	35
1825.6	0.218	0.30	2	22
1828.9	0.525	0.07	2	52
1838.8	0.509	0.16	2	81
1848.8	0.421	0.48	2	116
1852.1	0.689	0.31	1	68
1858.7	0.240	0.53	1	72
1862.0	0.534	0.69	1	66
1868.6	0.121	0.12	1	33
1871.8	0.396	0.46	2	94
1878.6	0.978	0.26	2	0
1881.8	0.266	0.25	2	58
1891.6	0.169	0.20	2	49
1895.0	0.442	0.62	3	60
1898.5	0.734	0.03	1	18
1901.6	0.995	0.42	2	1
1904.9	0.265	0.47	3	46
1914.8	0.118	0.41	1	10
1937.8	0.385	0.11	2	100
1947.8	0.349	0.32	3	159
1947.8	0.349	0.66	3	159
1951.1	0.675	0.07	3	83

The numerical results are listed in Tab. 91; the individual columns give: the middle of the interval of observations, t, the phase-shift referred to the preceding minimum of solar activity, Φ, the average dispersion of the brightness estimates, Δm, its weight, w, and the corresponding average sunspot number, R.

Assuming the dependence $\Delta m = \Delta m(\Phi)$ in the form of

$$\Delta m(\Phi) = \overline{\Delta m} + A(\Delta m) \cos 2\pi(\Phi - \Phi_0) \tag{8.42}$$

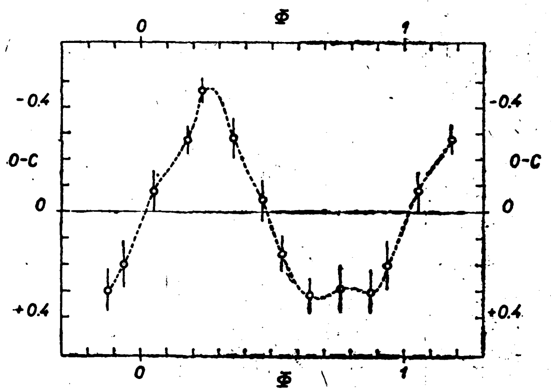

Fig. 51. Average dispersion Δm as related to the phase of the solar cycle.

we obtain

$$(8.43)\quad \begin{cases} \overline{\Delta m} = 0^m.29 \pm 0^m.02, \\ A(\Delta m) = 0^m.14 \pm 0^m.03, \\ \Phi_0 = 0.307 \pm 0.034, \end{cases}$$

so that parameter Φ_0 is in good agreement with the parameters of the cometary characteristics of the second class. Nevertheless, the values (8.43) must be taken with great reserve owing to the defects of the method (Section 8.12.). The curve $\Delta m = \Delta m(\Phi)$ is given in Fig. 51 and Table 92.

Table 90
Dependence $O - C = f(\Phi)$ for the Encke comet

int Φ	Φ	$O - C$	w
		m m	
0.901–0.200	0.051	−0.079±0.079	26
0.001–0.300	0.178	−0.279±0.056	23
0.101–0.400	0.234	−0.465±0.048	28
0.201–0.500	0.355	−0.282±0.077	26
0.301–0.600	0.464	−0.044±0.079	28
0.401–0.700	0.540	+0.158±0.068	28
0.501–0.800	0.644	+0.315±0.067	25
0.601–0.900	0.757	+0.291±0.088	20
0.701–0.000	0.877	+0.302±0.083	25
0.801–0.100	0.936	+0.202±0.095	20

8.11.3. Variation of the cometary-head diameter

On the basis of the material assembled by Bouška and Švestka (1949), and supplemented by a few values taken by the author from the monography of Vsekhsviatsky (1958), the time course of the coma diameter

Table 92
Dependence $\Delta m = \Delta m(\Phi)$ for the Encke comet

int Φ	Φ	Δm	w
		m m	
0.901–0.200	0.065	0.286±0.028	8
0.001–0.300	0.215	0.331±0.026	12
0.101–0.400	0.260	0.404±0.028	17
0.201–0.500	0.334	0.487±0.022	18
0.301–0.600	0.441	0.458±0.040	15
0.401–0.700	0.506	0.378±0.045	12
0.501–0.800	0.579	0.219±0.051	8
0.601–0.900	0.678	0.200±0.058	3
0.701–0.000	0.936	0.278±0.048	5
0.801–0.100	0.987	0.340±0.031	4

of the Encke comet is investigated, reduced to the unit of geocentric distance. From this material it follows that there takes place a gradual decrease of the coma diameter (Tab. 93) which may be assumed in the form analogous to (8.33):

$$D(t) = \alpha + \beta \Delta t + \gamma (\Delta t)^2, \tag{8.44}$$

where again $\Delta t = t - t_0$, $t_0 = 1900.0$. The individual coefficients are as follows (Δt is again expressed in hundreds of years):

Table 93
Residuals of the coma diameter of the Encke comet in individual returns

Designation	D			N
	O	C	O − C	
	′	′	′	
1825 III	2.2	2.70	−0.50	2
1829	3.4	2.68	+0.72	7
1838	1.9	2.59	−0.69	8
1842 I	1.0	2.55	−1.55	1
1848 II	3.9	2.49	+1.41	5
1852 I	2.3	2.46	−0.16	1
1855 III	1.4	2.42	−1.02	6
1858 VIII	1.3	2.39	−1.09	4
1862 I	2.5	2.35	+0.15	20
1868 III	3.9	2.28	+1.62	5
1871 V	1.9	2.25	−0.35	2
1875 II	3.2	2.21	+0.99	3
1878 II	1.7	2.17	−0.47	2
1881 VII	2.9	2.13	+0.77	4
1885 I	1.5	2.09	−0.59	8
1888 II	1.3	2.05	−0.75	3
1891 III	1.7	2.01	−0.31	3
1895 I	2.6	1.97	+0.63	5
1898 III	1.2	1.92	−0.72	2
1901 II	2.0	1.88	+0.12	6
1905 I	2.3	1.84	+0.46	15
1908 I	1.0	1.79	−0.79	5
1914 VI	2.3	1.71	+0.59	4
1918 I	1.3	1.66	−0.36	3
1924 III	1.5	1.57	−0.07	2
1928 II	1.0	1.52	−0.52	5
1934 III	0.5	1.42	−0.92	1
1937 VI	1.7	1.37	+0.33	9
1941 V	0.9	1.32	−0.42	3
1947 XI	1.8	1.22	+0.58	3

$$\left\{ \begin{aligned} \alpha &= +1'.90 \pm 0'.09, \\ \beta &= -1.39 \pm 0.24, \\ \gamma &= -0.30 \pm 0.56. \end{aligned} \right. \tag{8.45}$$

The residuals are listed in Tab. 93. The individual columns give the designation of the Encke comet, the observed coma diameter, that computed according to (8.44), the residual $O - C$, and the number of observations N.

The correlation of the coma-dimension residuals with the sunspot number, with the Holetschek criterion and the Dobrovolsky seasonal index is given by the following correlation coefficients:

$$(8.46)\quad \begin{cases} \psi[O - C, R] = +0.25 \pm 0.11, \\ \psi[O - C, \Delta T] = +0.27 \pm 0.11, \\ \psi[O - C, n] = +0.19 \pm 0.12. \end{cases}$$

Each of these coefficients is too low to indicate an actual degree of correlation; this fact is to a considerable extent due to the uncertainty of the coma dimension estimates. If the departures $O - C$ are again assumed in the form of a sine curve, the general expression of the course of the cometary head diameter is

$$(8.47)\qquad D(t) = \alpha + \beta\Delta t + \gamma(\Delta t)^2 + \delta \cos 2\pi(\Phi - \Phi_0),$$

where

$$(8.48)\quad \begin{cases} \alpha = + 1'.81 \pm 0'.09, \\ \beta = -1.47 \pm 0.23, \\ \gamma = -0.65 \pm 0.56, \\ \delta = +0'.44 \pm 0'.10, \\ \Phi_0 = 0.323 \pm 0.034. \end{cases}$$

The difference between the value of the phase-shift Φ_0 and the values derived in another way is not great enough to exclude the identity of the character of these fluctuations with the cometary characteristics of the second class.

8.11.4. Conclusions

1. The time course of the absolute brightness of the Encke comet may be split into two superimposed curves: the secular decrease, and the periodical fluctustions of a period equal to the length of the eleven-year solar cycle. As to the amplitude and the phase-shift referred to the sunspot-number curve, the curve of the departures $O - C$ perfectly agrees with the curves of the cometary characteristics of the second class (Section 8.10). There is no doubt that the character of both quantities is the same.

2. In addition, a relatively high degree of correlation makes it possible to apply the general formula describing the absolute magnitude variation for prognostic purposes.

3. Beyer's method of the observed brightness-estimation dispersion applied to 21 returns of the comet gives a course of the Δm-dependence

on the phase of the solar cycle that is very similar to that of the cometary characteristics of the second class.

4. An analogy between the variation of the coma diameter and that given under 1. is vague; it is obvious that the ascertained form of the course of the coma dimensions is strongly affected by the conditions of visibility, and by other effects resulting from the non-homogeneity of the material.

8.12. Beyer's method of cometary brightness dispersion as a criterion of cometary activity

Within the interval from 1933 till 1955 BEYER (1933, 1937a, 1937b, 1938, 1942, 1947, 1950a, 1950b, 1955) was publishing the photometric curves of 43 comets constructed on the basis of his measurements of the total coma brightness. The treatment of the material was carried out in the standard manner, i. e. by determining the photometric parameters H_0 and n. The departures of the individual measurements from the smoothed-out straight line are considered by BEYER the product of the activity of a comet, and the average of their absolute values given its certain characteristics.

An undisputed advantage of this method is the fact that all the observations were carried out by the same author and in the same way. On the other hand, this method has several disadvantages which may be summarized as follows:

a) from the papers dealing with the dust-gas model of a comet (e. g. VANÝSEK, 1952, VANÝSEK, HŘEBÍK, 1954, HRUŠKA, VANÝSEK, 1958) and with the statistics of the photometric exponents (VANÝSEK, HŘEBÍK, 1954, HRUŠKA, VANÝSEK, 1958, HRUŠKA, 1957) it follows beyond any doubt that the photometric exponent of any comet is a fuction of heliocentric distance. Since BEYER considers the exponent to be constant, the average dispersion Δm will change; this alternation will be different for various comets because the photometrical exponent depénds also on the intensity ratio between the dust and gas components, as well as on the type of gas present in a cometary head;

b) various comets react in different way on the variation of solar activity (Schwassmann—Wachmann 1 as against a number of absolutely faint comets). There are even instances that the reaction of a certain comet on the change of solar activity differs at various periods. A typical example is the comet Whipple—Fedtke—Tevzadze 1942 gas described by BOUŠKA (1950). Prior to the perihelion passage (1942, December — 1943, February) the comet revealed considerable anomalies in the course of its brightness, while the sunspot number did not surpass 35 over the whole interval, no large sunspot group passed through the Sun's central meridian in the direction towards the comet, and the efficiency of chromospherical flares in the same direction exceeded the value of 100 only once. On the other hand, after the perihelion passage (1943, February — 1943, May), the fluctuations of the comet brightness were much smaller, though the amplitude of the sunspot number variation amounted to about 70, 14 large sunspot groups went through the Sun-comet meridian, and the efficiency of flares once exceeded 200 and several times reached values over 100. The effects of this character seem to occur expecially in the absolutely bright comets;

c) the variation in the transparency of the Earth's atmosphere may considerably affect the observed brightness dispersion, especially if it has a systematic course (the so-called subjective factor, see Section 8.6.);

d) an undetermined part of the resulting dispersion is produced by incidental departures; to give their influence on the value of the average dispersion is a completely insolvable problem.

Each of the given disadvantages of the method is the more prominent, the less abundant and less homogeneous the material used.

When investigating the course of the average brightness dispersion during the solar cycle, the differences between the reactions of various comets on the solar radiation represent the greatest obstacle. Therefore the investigation of the only, as far as possible absolutely faint comet must be relatively the most successful (Section 8.11.). The same dependence may be statistically studied on the basis of the representative material, i. e. of that including a few hundred of comets at least. Such material, however, is not readily accessible.

So far, the material of the brightness dispersion, obtained by Beyer, has been treated in two ways:

a) in dependence on the sunspot number dispersion, ε_R (Beyer, ibid.);

b) in dependence on the phase of the solar cycle, Φ (Dobrovolsky, 1958).

The results of Beyer's study show a certain course of the increase of the average dispersion Δm with increasing dispersion ε_R, some of the studied comets, however, are beyond this dependence so that the resulting correlation coefficient amounts to:

$$\psi[\Delta m, \varepsilon_R] = +0.32 \pm 0.09 \text{ (p. e.)}. \tag{8.49}$$

In his paper Dobrovolsky asserts that these „special" comets are not the exception, but the reflex of the double-wave course of Δm during the eleven-year cycle; according to Dobrovolsky, the curve of $\Delta m = \Delta m(\Phi)$ supports the form of the curve of comets discovered during the solar cycle (Tab. 1 of his work). The dependence $\Delta m = \Delta m(\Phi)$, constructed by Dobrovolsky, gives indeed two maxima; however, the correlation coefficient between Δm and the number of discovered comets N (as a typical cometary characteristic of the first class) leads to the following rather unfavourable result:

(8.50) $$\psi[\Delta m, N] = +0.04 \pm 0.10 \text{ (p. e.)}.$$

Table 94

List of the brightness dispersions of 55 comets of Beyer's observational series

Comet	t	Δm	R	Φ	Comet	t	Δm	R	Φ
		m					m		
1932 V	1932.7	0.07	4	0.89	1948 IX	1948.8	0.18	138	0.45
1932 VI	1933.4	0.06	10	0.96	1948 X	1949.1	0.22	182	0.48
1932 X	1933.2	0.06	22	0.94	1948 XI	1949.1	0.42	182	0.48
1933 I	1933.4	0.10	10	0.96	1949 IV	1949.8	0.13	118	0.55
1935 I	1935.3	0.08	12	0.14	1949 VI	1949.6	0.19	132	0.53
1936 II	1936,4	0.07	70	0.25	1950 I	1950.2	0.17	110	0.59
1937 II	1937.2	0.19	109	0.33	1950 VII	1951.1	0.11	60	0.68
1937 IV	1937.4	0.26	130	0.35	1951 I	1950.4	0.14	91	0.61
1937 V	1937.5	0.11	138	0.36	1951 II	1951.2	0.25	93	0.69
1937 VI	1937.6	0.11	74	0.37	1951 III	1951.1	0.07	60	0.68
1939 I	1939.0	0.10	77	0.50	1951 IV	1951.3	0.17	109	0.70
1939 III	1939.2	0.19	118	0.52	1952 I	1952.4	0.16	23	0.80
1939 V	1939.3	0.06	101	0.53	1952 III	1952.5	0.31	22	0.81
1941 I	1940.7	0.10	68	0.66	1952 V	1952.6	0.31	36	0.82
1941 II	1941.0	0.12	45	0.69	1952 VI	1952.7	0.28	55	0.83
1941 IV	1941.1	0,07	33	0.70	1953 I	1953.0	0.58	34	0.86
1941 VIII	1941.3	0.21	67	0.72	1953 III	1953.5	0.09	13	0.91
1941 VIII	1941.6	0.10	38	0.75	1954 III	1954.2	0.21	11	0.98
1943 I	1943.1	0.48	27	0.89	1954 VI	1954.1	0.41	0	0.97
1946 II	1946.2	0.10	74	0.20	1954 VII	1954.7	0.11	2	0.03
1946 VI	1947.3	0.14	201	0.30	1954 X	1954.2	0.21	11	0.98
1947 I	1947.1	0.17	133	0.28	1955 I	1955.1	0.14	21	0.07
1947 I	1947.6	0.40	164	0.33	1955 III	1955.5	0.11	32	0.10
1947 III	1947.2	0.22	150	0.29	1955 IV	1955.7	0.39	43	0.12
1947 VII	1947.6	0.15	164	0.33	1955 V	1955.7	0.65	43	0.12
1947 XI	1947.6	0.32	164	0.33	1955 VI	1955.7	0.13	43	0.12
1948 I	1948.3	0.24	174	0.40	1956 IV	1956.4	0.48	137	0.18
1948 V	1948.2	0.32	150	0.39	1957 III	1957.3	0.21	175	0.27
1948 V	1948.7	0.15	96	0.44	1957d	1957.7	0.13	236	0.30
1948 V	1949.1	0.15	182	0.48					

Let us add into BEYER's statistics the results of his latest papers (BEYER, 1958, 1959). The complete list is included in Table 94, where individual columns give the designation of the comet, the moment of the middle of observations, the average dispersion Δm, the average sunspot number and the phase-shift of the middle of the observations relative to the preceding minimum of solar activity. The smoothed-out relation $\Delta m = \Delta m(\Phi)$ is presented in Table 95, and in Fig. 52 by full circles. The maximum dispersion Δm coincides with the minimum solar activity, while the minimum of Δm occurs at about 0.2 of a cycle after the maximum of solar activity.

Table 95

Relation $\Delta m = \Delta m(\Phi)$ from Beyer's observational material

int Φ	Φ	Δm	N
		m m	
0.96–0.25	0.079	0.217±0.031	15
0.06–0.35	0.232	0.228±0.024	19
0.16–0.45	0.324	0.208±0.017	19
0.26–0.55	0.400	0.198±0.013	24
0.36–0.65	0.480	0.180±0.015	16
0.46–0.75	0.607	0.156±0.013	19
0.56–0.85	0.715	0.171±0.014	15
0.66–0.95	0.779	0.197±0.024	18
0.76–0.05	0.909	0.229±0.028	15
0.86–0.15	0.002	0.228±0.032	17

The same analysis may be carried out on the basis of a thorough study on the photometrical curves of 45 comets from 1858—1937, published by Bobrovnikoff (1941, 1942). This study comprises a careful analysis of 4447 individual visual observations of comet brightness. Although the measurements were made by 160 observers the obtained results are considered reliable (Levin, 1947). The average dispersions Δm determined for

Table 96

List of the brightness dispersions of 45 comets of Bobrovnikoff's observational series

Comet	t	Δm	R	Φ	Comet	t	Δm	R	Φ
		m					m		
1858 VI	1858.75	0.18	86	0.246	1912 II	1912.91	0.30	4	0.942
1861 II	1861.57	0.30	78	0.497	1913 II	1913.40	0.34	0	0.983
1862 III	1862.64	0.11	63	0.593	1913 IV	1913.76	0.28	3	0.016
1874 III	1874.48	0.20	38	0.622	1913 VI	1913.78	0.23	3	0.018
1881 III	1881.61	0.24	58	0.253	1914 II	1914.42	0.15	8	0.082
1884 I	1882.92	0.26	42	0.376	1914 V	1914.67	0.24	10	0.107
1886 II	1886.22	0.09	57	0.684	1915 II	1915.56	0.22	72	0.196
1886 IX	1886.91	0.31	6	0.749	1917 II	1917.41	0.15	115	0.381
1890 II	1890.81	0.49	11	0.100	1917 III	1917.50	0.27	117	0.390
1893 II	1893.57	0.20	89	0.328	1919 III	1919.70	0.22	55	0.610
1898 I	1898.33	0.36	20	0.721	1921 II	1921.34	0.15	27	0.774
1899 I	1899.34	0.21	11	0.805	1925 I	1925.40	0.21	43	0.176
1900 II	1900.65	0.15	4	0.913	1930 II	1930.03	0.13	65	0.630
1902 III	1902.81	0.12	16	0.093	1930 III	1930.32	0.21	38	0.659
1903 IV	1903.55	0.19	28	0.155	1932 V	1932.69	0.24	4	0.891
1904 I	1904.62	0.13	58	0.245	1932 VI	1933.23	0.09	10	0.944
1906 VII	1906.92	0.03	52	0.439	1932 X	1933.06	0.32	12	0.927
1907 IV	1907.74	0.07	75	0.508	1935 I	1935.17	0.17	22	0.132
1908 III	1908.89	0.12	46	0.604	1936 II	1936.52	0.21	52	0.262
1910 II	1910.17	0.19	26	0.712	1937 II	1937.26	0.18	109	0.333
1911 II	1911.57	0.34	4	0.829	1937 IV	1937.41	0.26	124	0.347
1911 V	1911.77	0.23	3	0.846	1937 V	1937.60	0.11	138	0.365
1911 VI	1911.80	0.20	3	0.849					

45 comets investigated by BOBROVNIKOFF are listed in Table 96. The individual columns give the same quantities as Table 94. The correlation coefficient

$$\psi[R, \Delta m] = -0.20 \pm 0.10 \text{ (p. e.)} \tag{8.51}$$

is again low, but it suggests the course of $\Delta m = \Delta m(\Phi)$ which is similar to that we found from BEYER's supplemented material (Table 97). Fig. 52, in which the smoothed-out course of Δm from BOBROVNIKOFF's material is shown by open circles, proves it quite well. The agreement of both curves is excellent both in the phase-shift and in the amplitude and zero-point.

Table 97

Relation $\Delta m = \Delta m(\Phi)$ from Bobrovnikoff's observational material

int Φ	Φ	Δm	N
		m m	
0.951 – 0.250	0.119	0.227±0.019	13
0.051 – 0.350	0.204	0.213±0.015	15
0.151 – 0.450	0.299	0.189±0.011	15
0.251 – 0.550	0.373	0.190±0.017	12
0.351 – 0.650	0.501	0.164±0.017	12
0.451 – 0.750	0.632	0.192±0.018	12
0.551 – 0.850	0.712	0.205±0.014	15
0.651 – 0.950	0.816	0.226±0.015	15
0.751 – 0.050	0.903	0.237±0.015	13
0.851 – 0.150	0.011	0.240±0.020	13

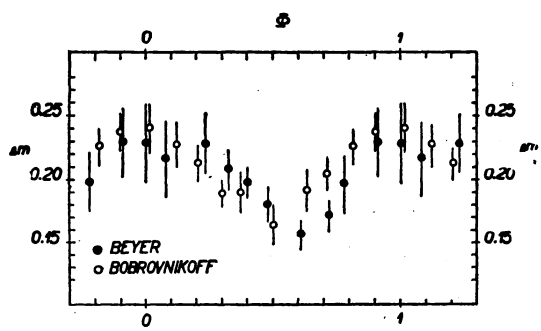

Fig. 52. Course of the cometary brightness estimates dispersion during the eleven-year solar cycle.

The cause of the ascertained course of the dispersion Δm can hardly be determined at present; however, on the basis of a comparison of the forms of these curves with that of the Encke comet (Section 8.11.), and with respect to what has been said of BEYER's method in the present section, it seems probable that the problem consists in the influence of a systematic effect inherent in the observational conditions.

CHAPTER NINE

SYSTEMATIC VARIATIONS IN BRIGHTNESS CONNECTED WITH THE COMET'S INTERIOR STRUCTURE

9.1. Subjects of the study

In Chapter Eight the behaviour of comets has been studied as an indicator of solar activity, regardless of the specific physical features of theirs as of a special group of cosmic bodies. In this chapter we shall deal with two phenomena in the comet's brightness, in which the physical properties of the comet's nucleus are expressed:

1. Short-term changes in the colour-index of comets.
2. Perihelion asymmetry of the photometric curves of comets.

Up to now, no attempt has been made as for the application of any comet model to the colorimetric measurements of a comet head and tail. Here are studied the relations between the colour-indices of the Arend—Roland comet of 1957, and the comet dust-gas model is applied to their physical interpretation.

Concerning the latter question the pure gaseous model has been applied the reason being an endeavour after the simplicity of the mathematical solution of the problem.

9.2. Short-term changes in the colour index of the comet head and tail. Comet 1957 III

The photoelectric observations of the head (diaphragm 4') and tail (distance of about 30' from the nucleus) of the Arend—Roland comet, 1957 III, were performed at the University Observatory in Brno, and the obtained results were published by Vanýsek and Tremko (1958). The measurements were carried out in three different effective wave-lengths included in Table 98.

Table 98
Spectral regions used for photoelectric photometry of the Arend-Roland comet

Magnitude	λ_{eff}	Filter
B	4422 Å	$BG12$ (1 mm) + $GG13$ (2 mm)
V	5510 Å	$GG11$ (2 mm)
P	4800 Å	without filter

Table 99
List of colour-index measurements

Date	$(B-P)$	$(P-V)$	$(B-V)$	Region
	m	m	m	
1957 IV. 27.819	+0.74	+0.07	+0.81	head
27.837	+0.86	−0.14	+0.72	head
27.844	+0.78	−0.08	+0.70	tail
29.860	+0.47	−0.26	+0.21	head
29.872	+0.32	+0.06	+0.38	head
29.879	+0.41	−0.04	+0.37	head
V. 2.930	−0.02	+0.06	+0.04	head
2.936	+0.13	−0.38	−0.25	tail
2.960	−0.15	+0.34	+0.19	head
4.854	+0.47	+0.39	+0.86	head
4.863	+0.41	+0.45	+0.86	head
25.923	+1.02	−0.37	+0.65	head
25.948	+0.42	+0.42	+0.84	tail

The observations extended over the period from April 27th till May 30th, 1957. For our purposes were used 10 measurements of the brightness of the head, and 3 measurements of that of the tail, from which all three colour indices $(B-P)$, $(P-V)$ and $(B-V)$ were computed (however, only two of them are indepedent). The indices are listed in Table 99. If plotted one against the other they give the relations presented in Fig. 53. The full circles stand for values related to the cometary head, the open ones for those related to the tail. The number at each circle indicates the date of observation. The values of the indices in Fig. 53 are concentrated along three straight lines, the first of which corresponds to April 27th, May 4th and May 25th, the second to April 29th and the third to May 2nd. Furthermore, the interpretation is given of the relations ascertained between the individual indices.

9.3. Interpretation of the variation in the colour indices $(B-P)$, $(P-V)$ and $(B-V)$. Results and conclusions

Let us proceed from the conception that the dust-gas model of a comet applies to each of the three spectral regions, i. e. let us assume that both molecular radiation and reflection on the dust particles of the coma and tail take place in each of the studied regions. This assumption is supported by polarization measurements in the integral light (Blaha, Hruška, Švestka, Vanýsek, 1958), and by the results of spectral analysis in the individual spectral regions (Rajchl, 1958).

If we denote the brightness of the comet in the unit of geocentric distance as I_Δ again, we can — according to (2.4), (2.5), (2.6) and (2.12) of Part One — write

(9.1) $$I_\Delta = I_{og}\, r^{-\frac{\alpha}{2}}\, e^{B(1-r^\alpha)} + I_{od} r^{-\eta_\alpha}.$$

Since according to (2.20) the ratio of the brightnesses of both constituents in a given heliocentric distance is

(9.2) $$\psi(r) = k \,.\, r^{\frac{\alpha}{2}-\eta_\alpha} \,.\, e^{B(r^\alpha-1)},$$

formula (9.1) transcribed into the magnitude scale gives:

(9.3) $$H_\Delta = -2.5 \log I_{og} + 1.25\alpha \log r + 1.086B(r^\alpha - 1) - 2.5 \log (1 + \psi).$$

If this equation is written for two effective wave-lenghts λ_i, λ_j, we obtain — after subtraction of the latter from the former — the colour index $(CI)_{ij} = H^{(i)} - H^{(j)}$:

(9.4) $$(CI)_{ij} = 2.5 \log \frac{I^{(i)}_{og}}{I^{(j)}_{og}} + 1.086\,(1 - r^\alpha)[B^{(j)} - B^{(i)}] + 2.5 \log \frac{1 + \psi^{(j)}}{1 + \psi^{(i)}}\,.$$

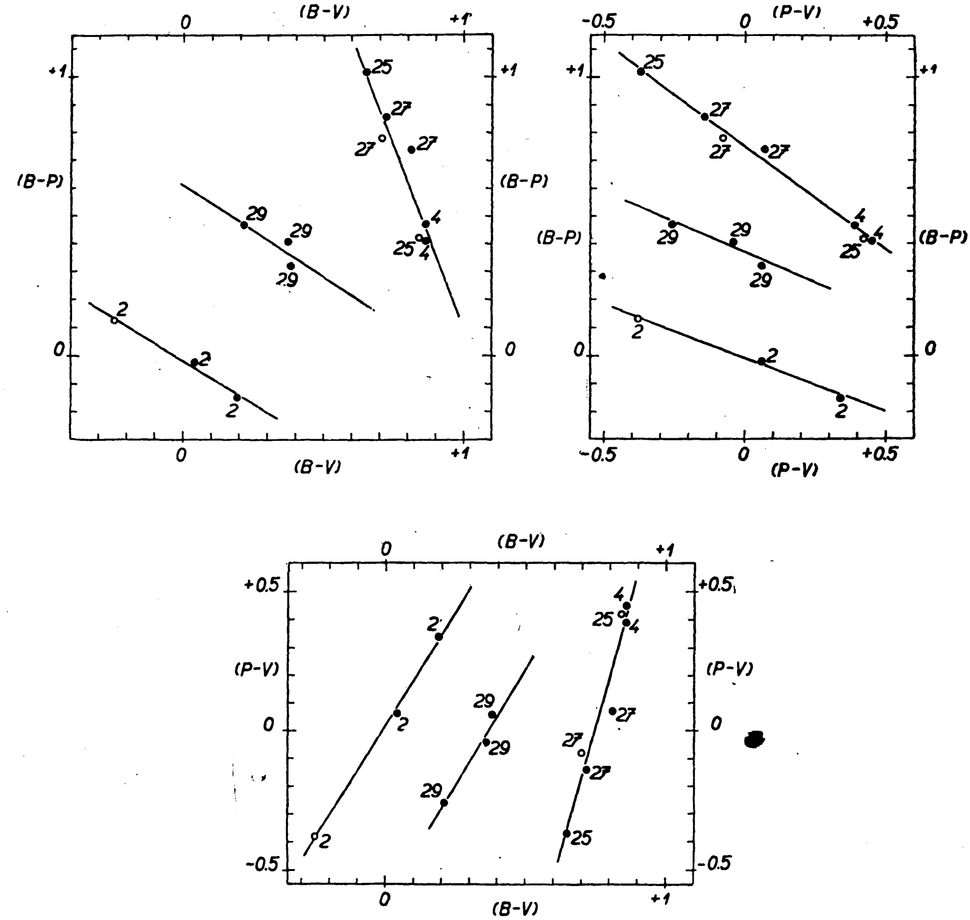

Fig. 53. Relations between three colour-indices of the head and tail of the comet 1957 III.

Table 100

Parameters of the colour-index relations

Date 1957	$(B-V)\sim(B-P)$ α_1	β_1	$(P-V)\sim(B-P)$ α_2	β_2	$(B-V)\sim(P-V)$ α_3	β_3
	m m		m m		m m	
IV. 27 + V. 4 + V. 25	$+2.70 \pm 0.24$	-2.61 ± 0.31	$+0.75 \pm 0.01$	-0.74 ± 0.02	-2.70 ± 0.24	$+3.61 \pm 0.31$
IV. 29	$+0.61 \pm 0.10$	-0.66 ± 0.30	$+0.37 \pm 0.02$	-0.44 ± 0.10	-0.61 ± 0.10	$+1.66 \pm 0.30$
V. 2	-0.02 ± 0.01	-0.62 ± 0.06	-0.01 ± 0.01	-0.38 ± 0.02	$+0.02 \pm 0.01$	$+1.62 \pm 0.06$

An analogous equation may be written for the wave-lengths λ_k, λ_l:

$$(9.5)\quad (CI)_{kl} = 2.5 \log \frac{I^{(l)}_{0s}}{I^{(k)}_{0s}} + 1.086(1 - r^{\alpha}) \,.\, [B^{(l)} - B^{(k)}] + 2.5 \log \frac{1 + \psi^{(l)}}{1 + \psi^{(k)}}.$$

If factor $1.086\,(1 - r^{\alpha})$ is eliminated, the relation between both the indices can be written in the form:

$$(9.6)\quad (CI)_{ij} = \alpha_m + \beta_m (CI)_{kl},$$

where

$$(9.7)\quad \alpha_m = 2.5 \left[\log \frac{I^{(i)}_{0s}(1 + \psi^{(j)})}{I^{(j)}_{0s}(1 + \psi^{(i)})} - \beta_m \log \frac{I^{(l)}_{0s}(1 + \psi^{(l)})}{I^{(k)}_{0s}(1 + \psi^{(k)})} \right]$$

and

$$(9.8)\quad \beta_m = \frac{B^{(j)} - B^{(i)}}{B^{(l)} - B^{(k)}}.$$

Let us denote

$$(CI)_{12} \equiv (B - P),\ (CI)_{13} \equiv (B - V),\ (CI)_{23} \equiv (P - V),$$

so that the indices i, j, k, l pass through 1, 2, 3 and m is equal to that of the indices i, j, k, l, which occurs twice in relation (9.6).

The parameters of the straight lines in Fig. 53 are listed in Table 100. Since the indices $(B - P)$, $(P - V)$ and $(B - V)$ are interrelated by three equations of form (9.6), the following five relations must apply to α_m and β_m, if our interpretation is correct:

$$(9.9)\quad \begin{cases} A_0(\beta_m) = \beta_2\beta_3 - \beta_1 \equiv 0, \\ A_1(\beta_m) = \dfrac{1}{\beta_3} - \beta_2 - 1 \equiv 0, \\ A_2(\beta_m) = \beta_1 + \beta_3 - 1 \equiv 0, \\ A_3(\alpha_m) = \alpha_1 + \alpha_3 \equiv 0, \\ A_4(\alpha_m, \beta_m) = \alpha_1(1 + \beta_2) - \alpha_2 \equiv 0. \end{cases}$$

The values listed in Table 101 were for the parameters (9.9) obtained from the data of Table 100. It is obvious that all the empirical values $A_0, \ldots, A_4$ range very closely about zero, and that all ascertained differences lie within the limits of errors.

Table 101

Empirical values of identities $A_p(\alpha_m, \beta_m) \equiv 0$

Date 1957	IV. 27 + V. 4 + V. 25	IV. 29	V. 2
A_0	-0.06 ± 0.39	-0.07 ± 0.37	0.00 ± 0.07
A_1	$+0.02 \pm 0.03$	$+0.04 \pm 0.15$	0.00 ± 0.03
A_2	0.00 ± 0.44	0.00 ± 0.42	0.00 ± 0.08
A_3	0.00 ± 0.34	0.00 ± 0.14	0.00 ± 0.01
A_4	-0.05 ± 0.08	-0.03 ± 0.08	0.00 ± 0.01

Thus, the obtained results support the interpretation of the experimental relations given by equation (9.6) The variability of the colour index can be regarded as a reflex of the fluctuations in the amount of gas and dust in the head and tail of the comet connected with the release mechanism. These fluctuations are also reflected in the numerical values of the parameters α_m, β_m, which are not constant, not even during a single night. However, since the characteristics of both radiation constituents in the individual spectral regions appear in the coefficients α_m, β_m in the form of a ratio, the above-mentioned fluctuations are minimum. Moreover, the changes in the colour index during one night may be also affected by the shift of the photometer diaphragm with respect to the cometary nucleus, if several measurements are carried out in succession. Owing to this effect, the physical conditions are registered in a few somewhat different parts of the coma in which the instantaneous ratio of both radiation constituents may differ quite considerably.

Parameter β_m, which is a function of the heat of evaporation necessary to release a certain amount of gas, is probably related to the initial velocity v_0 in the cometary tail. A comparison of one of the parameters β_m with v_0 derived in the paper of VANÝSEK, GRYGAR and SEKANINA (1959) from the width of the tail (see Figure 3 of the paper) is given in Table 102. For want of comparable data this relation cannot be verified in detail. Parameter α_m quantitatively demonstrates the amount of gas and dust ejected into the cometary head- and tail- regions. Besides the small number of measurements, it is a range of unknown quantities in the expression

Table 102

Comparison of the variability of parameter β_2 with the initial particle velocity in the cometary tail

Date 1957	β_2	v_0 km/s
IV. 27.9	-0.74 ± 0.02	12.0 ± 0.5
IV. 29.9	-0.44 ± 0.10	6.8 ± 0.8
V. 2.9	-0.38 ± 0.02	7.3 ± 0.5
V. 4.9	-0.74 ± 0.02	>9.0

for α_m that prejudices the analysis of this problem. Thus, the results obtained so far are of a qualitative character. The main result is the finding that problems of multicoloured cometary photometry may be solved on the basis of a comet dust-gas model.

9.4. Irregularities in the comet's brightness near the perihelion passage. Physical considerations concerning the perihelion asymmetry of the light curves of comets

Almost all the formulae used for describing the form of the photometric curve of comets have assumed a symmetry of the curve regarding the perihel on passage. However, a lot of comets indicate a disagreement between the time of maximum brightness and the time of perihelion passage. If we omit irregular short-term fluctuations which are, as a rule, most frequent near the perihelion passage and are in connection with a peculiar structure of a certain comet itself, we may consider two general phenomena which affect the form of the photometric curve, particularly in the vicinity of the perihelion passage:

(a) The concentration drop of molecules in the surface layers of the comet's nucleus.

(b) The thermal inertia of the nucleus blocks in the process releasing particles into the atmosphere.

The former of the two phenomena produces the preceding of the time of maximum brightness regarding the time of perihelion passage. On the basis of his theory Levin (1948) suggested a method of taking into account the successive concentration drop of molecules. The photometric formula of the comet gas model was completed by him by adding a correction term. It was derived by means of numerical quadrature and therefore unsuitable for applying to a current treatment of photometric curves of comets. Moreover, it includes the evaporation heat of molecules, which, being unknown before a treatment of observational data, makes it impossible to enumerate the correction term at all. Only if some observations in larger heliocentric distances are available, an approximate solution may be found by graphical means.

Since 1948 nobody else has attempted to study this phenomenon and, particularly, to compare some hypothetical models with observed photometric curves of comets.

If the latter of the two phenomena has a predominant influence on the form of the photometric curve, then the time of maximum brightness follows the time of perihelion passage. No discussion of observational data with respect to this phenomenon has been provided up to now. Although this effect has been observed in behaviour of many comets, authors have reduced it, if they have done so at all, by adding a date term, as, for example, Gadomski (1947) in the case of the Whipple—Fedtke comet of 1943. Such a date term has no physical meaning and, because it is introduced regardless of the concentration-drop factor, the real value of thermal inertia is in this way underestimated.

The observed position of the brightness maximum on the photometric curve with regard to the perihelion passage is then given by summing up the two phenomena. If its preceding is observed, the concentration drop is more effective than the thermal inertia, while, on the other hand, if the retardation takes place the inertia is of greater importance.

The character of the concentration-drop process is in the mentioned study of Levin based on quite clear theoretical ideas, and no further comments are desirable on it from this point of view.

The problem of the thermal inertia of the processes is more complicated because of its connection with the structure of the surface layer of the comet's nucleus, and such of its properties as thermal conductivity, specific heat, porosity, albedo, way of deposition of gases, etc.

Conduction of heat inside the comet's nucleus was discussed in a few papers of Dobrovolsky (1953, 1956, 1961) and Markovich (1957, 1958, 1959). Some important consequences of these studies will be here derived.

Assuming that all the heat incident upon the comet's surface is spent for heating the nucleus Dobrovolsky (1953) gives the following expression for the shift rate of isotherms:

$$\frac{dx}{dt} = \frac{1}{2}\sqrt{\frac{K_0}{t}} \cdot \exp\left[-\frac{x^2}{4K_0 t}\right] \cdot \left(1 - \frac{2}{\pi}\int_0^{\frac{x}{2\sqrt{K_0 t}}} e^{-\xi^2}\, d\xi\right)^{-1}, \tag{9.10}$$

where x is the depth measured from the nucleus surface, t is the interval of time over which the surface source is emitting the solar heat towards the deeper layers of the nucleus, K_0 is the coefficient of temperature conductivity, defined by the relation

$$K_0 = \frac{K}{\varrho c}, \tag{9.11}$$

where K is the coefficient of thermal conductivity, ϱ the mass density of the medium, c the specific heat.

Assuming that the comet's nucleus is a black body, its period of rotation is P, and axis of rotation perpendicular to the plane of orbit, the average shift rate of isotherms resulting from the thermal-energy balance during the period of rotation is given by (9.10) after inserting $t = P/\pi$. Consequently, the „transfer" of the isotherm from the nucleus surface to the depth X_m will proceed over a period of ΔT according to the relation:

$$X_m = \frac{1}{2}\sqrt{\frac{\pi K_0}{P}} \cdot f(X_m, K_0, P)\, \Delta T, \tag{9.12}$$

where

$$f(X_m, K_0, P) \doteq \exp\left[-\frac{\pi}{16}\frac{X_m^2}{K_0 P}\right]\left(1 - \frac{2}{\sqrt{\pi}}\int_0^{\frac{\sqrt{\pi}}{4}\frac{X_m}{\sqrt{K_0 P}}} e^{-\xi^2} d\xi\right)^{-1}. \tag{9.13}$$

Here ΔT is nothing but the retardation caused by the thermal inertia. Equation (9.12) indicates that the ratio $X_m/\Delta T$ is independent of the heliocentrical distance, or surface temperature, to a first approximation.

However, the condition that all the heat coming from the Sun to the comet's surface is spent for heating the nucleus is never fulfilled because of the thermal radiation of the nucleus and some other losses of energy. Therefore the real depth of the isotherm will not be X_m, but only

$$X = \sigma X_m, \quad \sigma < 1. \tag{9.14}$$

Further, under ΔT we will understand the interval of time during which a certain isotherm will reach the depth of the icy base, X. It means that the release of gases of a certain intensity takes place for ΔT later than it should do if no dust layer existed, and the observed photometric curve is also retarded for ΔT.

Markovich (1957) has studied the thermal balance of the surface layer, taking into consideration the losses of energy due to the thermal radiation of the comet's surface. The flux of solar radiation has been expressed by the Fourier series, and the depth L, defined as the depth where $(dT/dx)_{x=L} = 0$, has been established from a property of the amplitude of the first periodical term as:

$$L = \frac{2}{\text{mod}}\sqrt{\frac{K_0 P}{\pi}}. \tag{9.15}$$

From the physical point of view, L is the thickness of the dispersion dust layer at the nucleus surface, under which the icy base is situated. For comets, in which the existence of such a dust layer can be assumed, the validity of the equality as follows is obvious:

$$L = X,$$

so that the period of rotation is independent of the coefficient of temperature conductivity:

$$P = \alpha \Delta T, \tag{9.16}$$

where

$$\alpha = \frac{\pi}{4}\sigma \bmod . \exp\left[-\frac{1}{4 \bmod^2}\right] . \left(1 - \frac{2}{\sqrt{\pi}}\int_0^{\frac{1}{2 \bmod}} e^{-\xi^2} d\xi\right)^{-1} = 0\,87\,\sigma. \tag{9.17}$$

In addition, with respect to (9.15)

$$X = 2.4\sqrt{\sigma K_0 \Delta T}. \tag{9.18}$$

For the short-period comets, in which the dispersion surface layer has been exhausted, this consideration fails. The coefficient of thermal conductivity of the blocks of the nucleus is greater than that of the dispersion layer of dust, and the dependence of the depth of gaseous supplies on the time of retardation must be studied according to (9.12).

Hence, the parameters of both the concentration-drop process and the thermal-inertia process have now absolutely clear physical meaning.

9.5. Methods of determining the concentration drop and thermal inertia from the form of the photometric curve

In accordance with Levin we will study the two physical phenomena on the basis of a gaseous model, not of a dust-gas model. There is no doubt that a dust-gas model would describe the photometric curve better than a gas model, particularly in comets with strong continuous spectra, but the number of unknown constants would be too great to secure any reliability of their values derived from the observations.

We will start from the Levin formula corrected for a concentration-drop term only (Levin, 1948):

$$H_\Delta(r) = H_0 + \mu a(\sqrt{r} - 1) + F[a(\sqrt{r} - \sqrt{q})]\,\Delta H, \tag{9.19}$$

where H_0 is the initial absolute magnitude, i. e. the magnitude which the comet would have at $r = 1$ A. U., if it had the same supplies of gas as at the time when it started approaching the Sun; ΔH is the decrease of the comet's brightness during the orbital period; μ is the constant

$$\mu = \frac{5}{2} \bmod \frac{5000}{R_0 T_0}, \tag{9.20}$$

R_0 the universal gas constant, T_0 the absolute temperature of the comet's nucleus at $r = 1$ A. U.; and a is the ratio

$$a = \frac{L}{5000}, \tag{9.21}$$

L is the heat of evaporation (in cal/Mol). LEVIN gave no analytical form of function $F[a(\sqrt{r} - \sqrt{q})]$ and I have recently succeeded in finding that no exact expression for the F-function exists in a close form. As it is shown in Fig. 54, the F-function may satisfactorily be approximated by the formula as follows:

$$F[a(\sqrt{r} - \sqrt{q})] \approx \frac{1}{2}[1 \pm a^{1/3}(\sqrt{r} - \sqrt{q})^{1/3}], \tag{9.22}$$

as far as

$$r < \left(\sqrt{q} + \frac{1}{a}\right)^2. \tag{9.23}$$

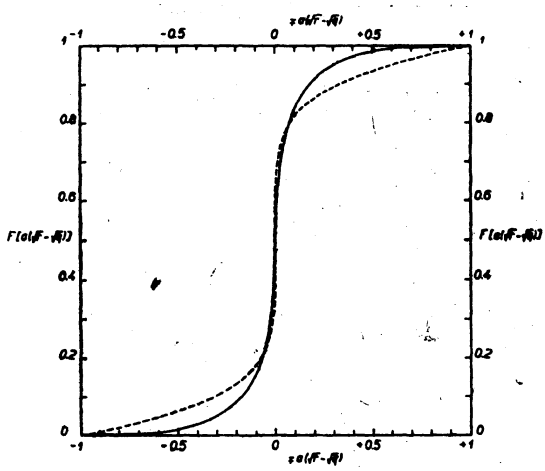

Fig. 54. The curve of concentration drop of molecules according to LEVIN (full line), and according to (9.22) (dash line).

The upper sign in (9.22) is valid for the pre-perihelion period, the bellow sign for the post-perihelion period. The same rule is valid in all the other formulae which follow. In Fig. 54 the Levin F-function is represented by a full line, the approximation (9.22) by a dash line.

In accordance with the results of the discussion of the previous section we assume that the thermal inertia of the evaporation process is constant throughout the comet's orbit, and equal to

(9.24) $$\bar{\tau} = \frac{3\sqrt{2}}{4} k\Delta T,$$

ΔT is the retardation in days, k the Gauss constant. After introducing it into the photometric formula (9.19) the formal appearance of the latter will remain the same, but the heliocentric distance must be taken not at the time of observation, t, but at the time

$$t - \frac{2\sqrt{2}}{3k}\bar{\tau}.$$

Throughout this paper we consider parabolical orbits, which approximate quite satisfactorily most cometary orbits near their perihelion passage. For such an orbit the relation between heliocentric distance and time is:

(9.25) $$r(\tau) = (\tau + \sqrt{q^3 + \tau^2})^{2/3} + (-\tau + \sqrt{q^3 + \tau^2})^{2/3} - q,$$

or

(9.26) $$\tau(r) = \mp \frac{1}{2}\sqrt{r - q}\,(r + 2q),$$

where $\tau = \frac{3\sqrt{2}}{4} k(t - t_0)$, t_0 is the time of perihelion passage.

Heliocentric distance $r\left(t - \frac{2\sqrt{2}}{3k}\bar{\tau}\right)$ may be expressed through $r(t)$ by means of expanding in a series of $\bar{\tau}$:

(9.27) $$r\left(t - \frac{2\sqrt{2}}{3k}\bar{\tau}\right) = r(t)[1 \pm \xi_1(t)\,\bar{\tau} + \xi_2(t)\,\bar{\tau}^2 \mp \xi_3(t)\,\bar{\tau}^3 - \xi_4(t)\,\bar{\tau}^4 \pm ..],$$

where

(89.2) $$\begin{cases} \xi_1(t) = \frac{4}{3} r^{-2}(r - q)^{1/2}, \\ \xi_2(t) = \frac{8}{9} r^{-4}\left(q - \frac{1}{2} r\right), \\ \xi_3(t) = \frac{32}{27} r^{-6}(r - q)^{1/2}\left(q - \frac{1}{3} r\right), \\ \xi_4(t) = \frac{160}{81} r^{-8}\left(q^2 - \frac{7}{6} qr + \frac{7}{30} r^2\right). \end{cases}$$

Similarly we may write:

(9.27′) $$r\left(t + \frac{2\sqrt{2}}{3k}\bar{\tau}\right) = r(t)[1 \mp \xi_1(t)\,\bar{\tau} + \xi_2(t)\,\bar{\tau}^2 \pm \xi_3(t)\,\bar{\tau}^3 - \xi_4(t)\,\bar{\tau}^4 \mp \ldots]$$

The term of the fourth order guarantees — for current values $\bar{\tau}$ — the accuracy greater than to ± 0.001 A. U.

The time of maximum observed brightness of a comet, t_m, is determined by the condition $(dH_\Delta/d\tau)_{\tau_m} = 0$, $\tau_m = \frac{3\sqrt{2}}{4} k(t_m - t_0)$. If we denote $\tau_m(\Delta H)$ the shift of the time of maximum brightness due to the concentration drop, the following important relation is valid among τ_m, $\tau_m(\Delta H)$ and $\bar{\tau}$:

(9.29) $$\tau_m = \tau_m(\Delta H) + \bar{\tau}.$$

If no thermal inertia took place the relation between the magnitude decrease ΔH and the heliocentric distance of maximum brightness would be of the form:

(9.30) $$\Delta H = 10\,\mu a^{4/5}(\sqrt{r_m(\Delta H)} - \sqrt{q})^{4/5},$$

so that

(9.31) $$r_m(\Delta H) = \left[\sqrt{q} + \frac{1}{a}\left(\frac{\Delta H}{10\,\mu}\right)^{5/4}\right]^2,$$

and, owing to the expression

$$\frac{1}{a}\left(\frac{\Delta H}{10\,\mu}\right)^{5/4}$$

Table 103

The time of maximum brightness

$q = 0.500$ A. U., $L = 5{,}000$ cal/Mol, $r < 2.92$ A. U.

ΔH \ ΔT	0^d	5^d	10^d	20^d	40^d
m 0.01	d −0.2 (0.500)	d +4.8 (0.513)	d +9.8 (0.553)	d +19.8 (0.682)	d +39.8 (1.017)
0.1	−0.7 (0.500)	+4.3 (0.511)	+9.3 (0.548)	+19.3 (0.675)	+39.3 (1.008)
0.5	−2.0 (0.502)	+3.0 (0.505)	+8.0 (0.536)	+18.0 (0.657)	+38.0 (0.986)
1.0	−3.1 (0.506)	+1.9 (0.502)	+6.9 (0.527)	+16.9 (0.641)	+36.9 (0.967)
2.0	−4.8 (0.513)	+0.2 (0.500)	+5.2 (0.516)	+15.2 (0.618)	+35.2 (0.940)
3.0	−6.2 (0.522)	−1.2 (0.501)	+3.8 (0.509)	+13.8 (0.599)	+33.8 (0.917)
5.0	−8.7 (0.542)	−3.7 (0.508)	+1.3 (0.501)	+11.3 (0.569)	+31.3 (0.872)
10.0	−14.1 (0.603)	−9.1 (0.544)	−4.1 (0.510)	+ 5.9 (0.520)	+25.9 (0.780)

Table 104

The time of maximum brightness

$q = 1.000$ A. U., $L = 5{,}000$ cal/Mol, $r < 4.00$ A. U.

ΔH \ ΔT	0^d	5^d	10^d	20^d	40^d
m	d	d	d	d	d
0.01	− 0.4 (1.000)	+ 4.6 (1.003)	+ 9.6 (1.014)	+19.6 (1.055)	+39.6 (1.203)
0.1	− 1.7 (1.000)	+ 3.3 (1.002)	+ 8.3 (1.011)	+18.3 (1.049)	+38.3 (1.192)
0.5	− 4.7 (1.003)	+ 0.3 (1.000)	+ 5.3 (1.004)	+15.3 (1.034)	+35.3 (1.166)
1.0	− 7.3 (1.008)	− 2.3 (1.001)	+ 2.7 (1.001)	+12.7 (1.023)	+32.7 (1.144)
2.0	−11.3 (1.019)	− 6.3 (1.006)	− 1.3 (1.000)	+ 8.7 (1.012)	+28.7 (1.113)
3.0	−14.6 (1.031)	− 9.6 (1.014)	− 4.6 (1.003)	+ 5.4 (1.004)	+25.4 (1.090)
5.0	−20.4 (1.059)	−15.4 (1.034)	−10.4 (1.016)	− 0.4 (1.000)	+19.6 (1.055)
10.0	−32.6 (1.143)	−27.6 (1.105)	−22.6 (1.073)	−12.6 (1.023)	+ 7.4 (1.008)

being small,

$$\tau_m(\Delta H) = -\frac{3}{2}\sqrt{2}\, a^{-1/2} q^{5/4} \left(\frac{\Delta H}{10\,\mu}\right)^{5/8}, \tag{9.32}$$

and, finally, according to (20),

$$t_m - t_0 = \frac{2}{k}\left[\frac{\sqrt{2}}{3}\bar{\tau} - a^{-1/2} q^{5/4} \left(\frac{\Delta H}{10\,\mu}\right)^{5/8}\right]. \tag{9.33}$$

Values $t_m - t_0$ as well as corresponding r_m are for $L = 5000$ cal/Mol and a few combinations of q, ΔH and $\bar{\tau}$ given in Tab. 103—105.

In the four following sections four different methods are developed to derive numerical values of ΔH, $\bar{\tau}$, as well as H_0 and L.

9.5.1. Method of expanding in a series (M. E. S.)

Let us study the photometric curve of a comet, the observed magnitudes being plotted against time:

$$H_\Delta = H_\Delta(\tau).$$

Let us expand this function in a series at the time τ_0 and omit the terms of the fourth and higher orders. Then:

$$(9.34)\qquad H_\Delta(\tau) = \sum_{i=0}^{3} A_i(\tau - \tau_0)^i,$$

where, according to (10) and (13), the coefficients are:

$$(9.35)\qquad \begin{cases} A_0 = H_0 + \mu a(\sqrt{r_0} - 1) + \dfrac{1}{2}\left[1 \mp a^{1/5} r_0^{3/10}\left(1 - \sqrt{\dfrac{q}{r_0}}\right)^{1/5}\right]\Delta H, \\ A_1 = \dfrac{2}{3r_0}\left(1 - \dfrac{q}{r_0}\right)^{1/2}\left[\mp \mu a + \dfrac{1}{10}\Delta H a^{1/5} r_0^{-3/5}\left(1 - \sqrt{\dfrac{q}{r_0}}\right)^{-4/5}\right], \\ A_2 = \dfrac{4}{9} r_0^{-5/2}\left[-\mu a\left(1 - \dfrac{3}{2}\dfrac{q}{r_0}\right) \mp \dfrac{1}{100}\Delta H a^{1/5} r_0^{-3/5}\left(1 - \sqrt{\dfrac{q}{r_0}}\right)^{-9/5} \cdot \right. \\ \qquad \left. \cdot f\left(\dfrac{q}{r_0}\right)\right], \\ A_3 = \dfrac{16}{81} r_0^{-4}\left(1 - \dfrac{q}{r_0}\right)^{1/2}\left[\mp \dfrac{5}{2}\mu a\left(1 - \dfrac{21}{10}\dfrac{q}{r_0}\right) + \dfrac{1}{1000}\Delta H a^{1/5} r_0^{-3/5} \cdot \right. \\ \qquad \left. \cdot \left(1 - \sqrt{\dfrac{q}{r_0}}\right)^{-14/5} \cdot g\left(\dfrac{q}{r_0}\right)\right], \end{cases}$$

and

$$(9.36)\qquad \begin{cases} f\left(\dfrac{q}{r_0}\right) = -14 + 10\left(\dfrac{q}{r_0}\right)^{1/2} + 19\dfrac{q}{r_0} - 15\left(\dfrac{q}{r_0}\right)^{3/2}, \\ g\left(\dfrac{q}{r_0}\right) = 406 - 620\left(\dfrac{q}{r_0}\right)^{1/2} - 491\dfrac{q}{r_0} + 1230\left(\dfrac{q}{r_0}\right)^{3/2} - 525\left(\dfrac{q}{r_0}\right)^2. \end{cases}$$

Now we will introduce the inertia effect $\bar{\tau}$ into (9.34). If τ's remain the moments of observation, the form of (9.34) will change into:

$$(9.37)\qquad H_\Delta(\tau) = \sum_{i=0}^{3} A_i[\tau - (\tau_0 + \bar{\tau})]^i.$$

As we do not know the value of $\bar{\tau}$ — we know only that the order of this magnitude is not higher than 10^{-1} — we change the expression in the brackets into

$$[(\tau - \tau_0) - \bar{\tau}].$$

and write

$$(9.38)\qquad H_\Delta(\tau) = \sum_{i=0}^{3} B_i(\tau - \tau_0)^i,$$

where

$$(9.39)\qquad \begin{cases} B_0 = A_0 - A_1\bar{\tau} + A_2\bar{\tau}^2 - A_3\bar{\tau}^3, \\ B_1 = A_1 - 2A_2\bar{\tau} + 3A_3\bar{\tau}^2, \\ B_2 = A_2 - 3A_3\bar{\tau}, \\ B_3 = A_3. \end{cases}$$

We must not forget that B_i are taken at τ_0 (and they result from the observational data), while A_i are taken at $\tau_0 + \overline{\tau}$ (and they result from (9.35)). Now (9.39) represents four equations for four unknown parameters, H_0, L (or a), ΔH and $\overline{\tau}$.

The computation of the parameters must be carried out by means of successive approximations. First we may derive a from the quadratic equation:

(9.40)
$$M\mu^2a^2 + N\mu a + P = 0,$$

where

(9.41)
$$\begin{cases} M = \left(1 - \frac{3}{2}\frac{q}{r_0}\right)^2 + 50\,\dfrac{f\left(\frac{q}{r_0}\right)}{g\left(\frac{q}{r_0}\right)}\left(1 - \sqrt{\frac{q}{r_0}}\right)\left(1 - \frac{21}{10}\frac{q}{r_0}\right)\left\{1 - \frac{3}{2}\frac{q}{r_0} + \right. \\ \qquad \left. + \dfrac{25}{2}\,\dfrac{f\left(\frac{q}{r_0}\right)}{g\left(\frac{q}{r_0}\right)}\left(1 - \sqrt{\frac{q}{r_0}}\right)\left(1 - \frac{21}{10}\frac{q}{r_0}\right)\right\}, \\ N = \pm\frac{81}{8}B_3r_0^4\left(1 - \frac{q}{r_0}\right)^{-1/2}\left\{\left(1 - \frac{q}{r_0}\right)\left[1 - 250\,\dfrac{\left(1 - \sqrt{\frac{q}{r_0}}\right)^2}{g\left(\frac{q}{r_0}\right)}\left(1 - \right.\right.\right. \\ \qquad \left.\left. - \frac{21}{10}\frac{q}{r_0}\right)\right] + 10\left(1 - \sqrt{\frac{q}{r_0}}\right)\dfrac{f\left(\frac{q}{r_0}\right)}{g\left(\frac{q}{r_0}\right)}\left[1 - \frac{3}{2}\frac{q}{r_0} + 25\left(1 - \sqrt{\frac{q}{r_0}}\right)\cdot\right. \\ \qquad \left.\left. \cdot\left(1 - \frac{21}{10}\frac{q}{r_0}\right)\dfrac{f\left(\frac{q}{r_0}\right)}{g\left(\frac{q}{r_0}\right)}\right]\right\}, \\ P = \frac{81}{16}r_0^8\left\{(3B_1B_3 - B_2^2)\,r_0^{-3} + \frac{81}{16}10^2\left(1 - \sqrt{\frac{q}{r_0}}\right)^2B_3^2\left[\left(1 - \right.\right.\right. \\ \qquad \left.\left.\left. - \frac{q}{r_0}\right)^{-1}\left(\dfrac{f\left(\frac{q}{r_0}\right)}{g\left(\frac{q}{r_0}\right)}\right)^2 - \dfrac{2}{g\left(\frac{q}{r_0}\right)}\right]\right\}. \end{cases}$$

Let us point out that parameter B, which is often used instead of L, is simply given through μa:

(9.42)
$$B = \frac{2}{5\,\mathrm{mod}}\,\mu a.$$

Table 105
The time of maximum brightness
$q = 2.000$ A. U., $L = 5{,}000$ cal/Mol, $r < 5.83$ A. U.

ΔH \ ΔT	0^d	5^d	10^d	20^d	40^d
m	d	d	d	d	d
0.01	− 1.0 (2.000)	+ 4.0 (2.001)	+ 9.0 (2.003)	+19.0 (2.014)	+39.0 (2.055)
0.1	− 4.1 (2.001)	+ 0.9 (2.000)	+ 5.9 (2.001)	+15.9 (2.009)	+35.9 (2.047)
0.5	−11.2 (2.005)	− 6.2 (2.002)	− 1.2 (2.000)	+ 8.8 (2.003)	+28.8 (2.031)
1.0	−17.3 (2.011)	−12.3 (2.006)	− 7.3 (2.002)	+ 2.7 (2.000)	+22.7 (2.020)
2.0	−26.7 (2.026)	−21.7 (2.018)	−16.7 (2.010)	− 6.7 (2.002)	+13.3 (2.007)
3.0	−34.6 (2.044)	−29.6 (2.033)	−24.6 (2.023)	−14.6 (2.008)	+ 5.4 (2.001)
5.0	−48.0 (2.083)	−43.0 (2.067)	−38.0 (2.052)	−28.0 (2.029)	− 8.0 (2.002)
10.0	−76.0 (2.200)	−71.0 (2.177)	−66.0 (2.155)	−56.0 (2.112)	−36.0 (2.047)

If we know a, we may compute the brightness decrease per orbital period from the relation:

$$\Delta H = a^{-1/5} r_0^{8/5} . 10^3 \frac{\left(1-\sqrt{\frac{q}{r_0}}\right)^{14/5}}{g\left(\frac{q}{r_0}\right)} \left\{\frac{81}{16} B_3 r_0^4 \left(1-\frac{q}{r_0}\right)^{-1/2} \pm \frac{5}{2}\mu a\left(1-\frac{21}{10}\frac{q}{r_0}\right)\right\}. \tag{9.43}$$

If we know a and ΔH we can compute coefficient A_2 and then

$$\bar{\tau} = \frac{A_2 - B_2}{3B_3}. \tag{9.44}$$

In the first approximation we put $r_0 = r_0(\tau_0)$ in (9.41) and (9.43), i. e. we assume $\bar{\tau} = 0$. With a and ΔH derived in this way we establish a new $\bar{\tau}_1$ from (9.44), determine $r_0(\tau_0 + \bar{\tau}_1)$ according to (9.27′) or (9.25), and compute new values of a and ΔH. They give another value of $\bar{\tau}$, $\bar{\tau}_2$, etc. The approximations are finished when $\bar{\tau}_{j+1} = \bar{\tau}_j$. Finally, the initial absolute magnitude is given by the formula:

$$H_0 = B_0 - \frac{1}{2}\Delta H\left[1 \mp a^{1/5} r_0^{1/10}\left(1-\sqrt{\frac{q}{r_0}}\right)^{1/5}\right] - \mu a(\sqrt{r_0}-1) + A_1\bar{\tau} - A_2\bar{\tau}^2 + A_3\bar{\tau}^3. \tag{9.45}$$

Condition (9.23), of course, must be fulfilled.

9.5.2. Method of symmetric positions (M. S. P.)

The method of expanding in a series requires a comparatively numerous observational material in order that the values of coefficients B_i should be reliable, and accurate enough. Such a material is not, as a rule, available and therefore some other methods must be found to make it possible to establish the asymmetry parameters, ΔH and $\bar{\tau}$.

Let us assume that we know the magnitude, $H_\Delta(r)$, and the photometric exponent $n(r)$, at a certain heliocentric distance r both in the pre-perihelion period and in the post-perihelion period. If index A belongs to the pre-perihelion observations and B to those after the perihelion passage, we may write according to (9.19) and (9.27):

$$n_A(r) = \frac{\sqrt{r}}{5\,\mathrm{mod}}\Big[\mu a(1 + \lambda_1\bar{\tau} + \lambda_2\bar{\tau}^2 - \lambda_3\bar{\tau}^3 - \lambda_4\bar{\tau}^4) - \frac{1}{10}\Delta H a^{1/5} \cdot (\sqrt{r} - \sqrt{q})^{-4/5}\Big], \tag{9.46}$$

$$n_B(r) = \frac{\sqrt{r}}{5\,\mathrm{mod}}\Big[\mu a(1 - \lambda_1\bar{\tau} + \lambda_2\bar{\tau}^2 + \lambda_3\bar{\tau}^3 - \lambda_4\bar{\tau}^4) + \frac{1}{10}\Delta H a^{1/5} \cdot (\sqrt{r} - \sqrt{q})^{-4/5}\Big], \tag{9.47}$$

$$H_{\Delta A}(r) = H_0 + \mu a\sqrt{r}(1 + \eta_1\bar{\tau} + \eta_2\bar{\tau}^2 - \eta_3\bar{\tau}^3 - \eta_4\bar{\tau}^4) - \mu a + {} + \frac{1}{2}\Delta H - \frac{1}{2}\Delta H a^{1/5}(\sqrt{r} - \sqrt{q})^{1/5}, \tag{9.48}$$

$$H_{\Delta B}(r) = H_0 + \mu a\sqrt{r}(1 - \eta_1\bar{\tau} + \eta_2\bar{\tau}^2 + \eta_3\bar{\tau}^3 - \eta_4\bar{\tau}^4) - \mu a + {} + \frac{1}{2}\Delta H + \frac{1}{2}\Delta H a^{1/5}(\sqrt{r} - \sqrt{q})^{1/5}, \tag{9.49}$$

where

$$\lambda_i = (-1)^{i+1}\frac{(i + 1)\,\eta_{i+1}}{\eta_1}, \qquad i = 1, \ldots, 4, \tag{9.50}$$

and

$$\left\{\begin{aligned}
\eta_1 &= \frac{2}{3} r^{-2}(r - q)^{1/2},\\
\eta_2 &= \frac{2}{9} r^{-4}(3q - 2r),\\
\eta_3 &= \frac{4}{81} r^{-6}(r - q)^{1/2}(21q - 10r),\\
\eta_4 &= \frac{2}{243} r^{-8}(231q^2 - 300qr + 80r^2),\\
\eta_5 &= \frac{4}{729} r^{-10}(r - q)^{1/2}(693q^2 - 780qr + 176r^2).
\end{aligned}\right. \tag{9.51}$$

In formulae (9.46) to (9.49) the concentration drop term is not corrected for inertia. If we did so the solution would be much more complicated, but, on the other hand, the influence of the correction would be quite negligible, particularly in distances not too close to the perihelion distance.

If we sum up (9.46) and (9.47), and (9.48) and (9.49) as well, and if we further subtract (9.47) from (9.46), and (9.49) from (9.48) we obtain four other equations, from which the equation of the fourth degree in $\bar{\tau}$ results after the elimination of H_0, ΔH and a. It may be written in the form appropriate for successive approximations:

(9.52)

$$\bar{\tau} = C(n, H_\Delta) \frac{\eta_1 - 3\eta_3\bar{\tau}^2 + 5\eta_5\bar{\tau}^4}{\frac{1}{5}\eta_1^2 - 2\eta_2\left(1 - \sqrt{\frac{q}{r}}\right) - \left[\frac{1}{5}\eta_1\eta_3 - 4\eta_4\left(1 - \sqrt{\frac{q}{r}}\right)\right]\bar{\tau}^2}$$

where

$$C(n, H) = \frac{1}{n_A + n_B}\left[\frac{H_{\Delta A} - H_{\Delta B}}{25 \bmod} - \left(1 - \sqrt{\frac{q}{r}}\right)(n_A - n_B)\right]. \tag{9.53}$$

If r is not close to q the following value may be used as the first approximation suitable for inserting into (9.52):

$$\bar{\tau} = C(n, H_\Delta)\, \eta_1 \left[\frac{1}{5}\eta_1^2 - 2\eta_2\left(1 - \sqrt{\frac{q}{r}}\right)\right]^{-1}. \tag{9.52'}$$

The three other magnitudes are given as follows:

$$a = \frac{5}{2}\frac{\bmod}{\mu} r^{-1/2}\eta_1 \frac{n_A + n_B}{\eta_1 - 3\eta_3\bar{\tau}^2 + 5\eta_5\bar{\tau}^4}, \tag{9.54}$$

$$\begin{cases} \Delta H = 10a^{-1/5}(\sqrt{r} - \sqrt{q})^{4/5}\left[\frac{2\mu a\bar{\tau}}{\eta_1}(\eta_2 - 2\eta_4\bar{\tau}^2) - \frac{5}{2}\bmod r^{-1/2}(n_A - \right. \\ \left. - n_B)\right] = a^{-1/5}(\sqrt{r} - \sqrt{q})^{-1/5}[2\mu a\sqrt{r}\bar{\tau}(\eta_1 - \eta_3\bar{\tau}^2) - (H_{\Delta A} - H_{\Delta B})], \end{cases} \tag{9.55}$$

$$H_0 = \mu a[1 - \sqrt{r}(1 + \eta_2\bar{\tau}^2 - \eta_4\bar{\tau}^4)] - \frac{1}{2}\Delta H + \frac{1}{2}(H_{\Delta A} + H_{\Delta B}). \tag{9.56}$$

A quantitative treatment of real photometric curves of comets does not directly give, as a rule, the photometric quantities H_Δ and n precisely in the same distances prior to and after the perihelion passage. But if the difference between the two distances, r and r', is small, the reduction of the two photometric quantities may be carried out according to the formulae as follows:

$$
(9.57)\quad \begin{cases} n(r') = n(r)\sqrt{\dfrac{r'}{r}}, \\ H_\Delta(r') = H_\Delta(r) + 5 \operatorname{mod} . \, n(r)\left(\sqrt{\dfrac{r'}{r}} - 1\right), \end{cases}
$$

i. e. we assume that the differential influence of the asymmetry magnitudes between r and r' is negligible.

In addition, the photometric exponent may be expressed through $\mathrm{d}H_\Delta/\mathrm{d}\tau$, or $\mathrm{d}H_\Delta/\mathrm{d}t$ according to the formulae:

$$
(9.58)\qquad n(r) = \mp \frac{3}{10 \operatorname{mod}} \cdot \frac{r^2}{\sqrt{r-q}} \left(\frac{\mathrm{d}H_\Delta}{\mathrm{d}\tau}\right)_r,
$$

or

$$
(9.59)\qquad n(r) = \mp \frac{\sqrt{2}}{5k \operatorname{mod}} \cdot \frac{r^2}{\sqrt{r-q}} \left(\frac{\mathrm{d}H_\Delta}{\mathrm{d}t}\right)_r.
$$

9.5.3. Method of four points (M. F. P.)

The advantage of the preceding method is the fact that we need not know the behaviour of a comet in a close vicinity of its perihelion passage. On the other hand, the reliability of the results obtained by that method depends in a high degree on the precision in deriving the photometric exponents n_A and n_B.

If we are not able to reach high enough accuracy in this direction, and if we know the form of the photometric curve in a close neighbourhood of the perihelion passage, namely the shift of the time of maximum brightness with regard to the time of perihelion passage, and the comet's brightness in the perihelion, we may develop another method.

Let us, in addition, know the comet's brightness at a certain heliocentric distance, r, far enough from the perihelion distance, both prior to the perihelion passage and after it. Then four initial points are at our disposal, i. e. three magnitude data and the time shift, so that we can derive all the required quantities of the photometric formula from the equations:

$$
(9.60)\qquad \tau_{\mathrm{m}} = \bar{\tau} - \frac{3}{2}\sqrt{2}\, a^{-1/2} q^{5/4} \left(\frac{\Delta H}{10\,\mu}\right)^{5/4},
$$

$$
(9.61)\qquad H_\Delta(q) = H_0 + \mu a q^{1/2}\left(1 + \frac{2}{9} q^{-3}\, \bar{\tau}^2 - \frac{22}{243} q^{-6} \bar{\tau}^4\right) - \mu a + \frac{1}{2}\Delta H,
$$

and from (9.48) and (9.49).

Each of the initial four data must be, of course, derived from the observational material by means of the compensation computation.

After the elimination of H_0, ΔH and a we obtain the following equation for $\bar{\tau}$:

$$(9.62) \quad \left\{ \begin{aligned} \bar{\tau} = (\eta_1 - \eta_3\bar{\tau}^2)^{-1} \Big\{ 5 \left(\frac{\sqrt{2}}{3}\right)^{2/5} q^{-2} (\bar{\tau} - \tau_m)^{4/5} r^{-2/5} \left(1 - \sqrt{\frac{q}{r}}\right)^{1/5} + \\ + C'(H_\Delta) \left[1 - \sqrt{\frac{q}{r}} + \left(\eta_2 - \frac{2}{9} q^{-3} \sqrt{\frac{q}{r}}\right) \bar{\tau}^2 - \left(\eta_4 - \frac{22}{243} q^{-6} \cdot \right. \right. \\ \left. \left. \cdot \sqrt{\frac{q}{r}} \right) \bar{\tau}^4 \right] \Big\}, \end{aligned} \right.$$

where

$$(9.63) \qquad C'(H_\Delta) = \frac{H_{\Delta A} - H_{\Delta B}}{H_{\Delta A} + H_{\Delta B} - 2H_\Delta(q)}.$$

The solution must again be carried out by means of successive approximations. Then the heat of evaporation is

$$(9.64) \quad \left\{ \begin{aligned} a = \frac{H_{\Delta A} - H_{\Delta B}}{2\mu\sqrt{r}} \Big[\bar{\tau}(\eta_1 - \eta_3\bar{\tau}^2) - 5\left(\frac{\sqrt{2}}{3}\right)^{2/5} q^{-2} (\bar{\tau} - \tau_m)^{4/5} r^{-2/5} \cdot \\ \cdot \left(1 - \sqrt{\frac{q}{r}}\right)^{1/5} \Big]^{-1}, \end{aligned} \right.$$

the magnitude decrease

$$(9.65) \qquad \Delta H = 10\mu \left(\sqrt{\frac{2}{3}}\right)^{2/5} a^{4/5} q^{-2} (\bar{\tau} - \tau_m),$$

and finally, the initial absolute magnitude

$$(9.66) \quad H_0 = \frac{1}{2}(H_{\Delta A} + H_{\Delta B} - \Delta H) + \mu a [1 - \sqrt{r}(1 + \eta_2\bar{\tau}^2 - \eta_4\bar{\tau}^4)].$$

All the symbols used here are the same as earlier in this paper.

9.5.4. Generalized method of photometric exponent (G. M. P. E.)

In the two last mentioned methods we have assumed some photometric properties of a comet to be known in a distance rather far from the perihelion distance both prior to the passage through the perihelion and after it. Let us now assume that a comparatively numerous observational material is available either prior to or after the perihelion passage, and that the either prior to or after the perihelion passage, and that the behaviour of the comet in a close neighbourhood of the perihelion is also known. Then we have the four following equations at our disposal:

$$(9.67) \quad \left\{ \begin{aligned} H_\Delta(r) = H_0 + \mu a \sqrt{r} [1 \pm \eta_1\bar{\tau} + \eta_2\bar{\tau}^2 \mp \eta_3\bar{\tau}^3 - \eta\bar{\tau}^4] - \mu a + \\ + \frac{1}{2}\Delta H [1 \mp a^{1/5}(\sqrt{r} - \sqrt{q})^{1/5}], \end{aligned} \right.$$

$$
(9.68)\quad \begin{cases} n(r) = \dfrac{\sqrt{r}}{5\,\mathrm{mod}}\left[\mu a\left(1 \pm \dfrac{2\eta_2}{\eta_1}\bar{\tau} - \dfrac{3\eta_3}{\eta_1}\bar{\tau}^2 \mp \dfrac{4\eta_4}{\eta_1}\bar{\tau}^3 + \dfrac{5\eta_5}{\eta_1}\bar{\tau}^4\right) \mp \right. \\ \qquad \left. \mp \dfrac{1}{10}\Delta H a^{1/5}(\sqrt{r} - \sqrt{q})^{-4/5}\right], \end{cases}
$$

and (9.60) and (9.61) again. This system of equations represents nothing but a generalized method of photometric exponent (for a gas model), the original form and solution of which for a comet dust-gas model is included in Chapter Two of Part One of this study (SEKANINA, 1962).

The asymmetry parameter $\bar{\tau}$ may be computed from the equation as follows:

$$
(9.69)\quad \begin{cases} \bar{\tau} = \left[\eta_1 - \dfrac{2\eta_2}{\eta_1}C'' - \left(\eta_3 - \dfrac{4\eta_4}{\eta_1}C''\right)\bar{\tau}^2\right]^{-1} . \left[\left(\dfrac{\sqrt{2}}{3}\right)^{8/5} q^{-2}(\bar{\tau} - \tau_{\mathrm{m}})^{8/5} . \right. \\ . \, r^{-2/5}\left(1 - \sqrt{\dfrac{q}{r}}\right)^{-4/5} . \left\{5\left(1 - \sqrt{\dfrac{q}{r}}\right) - C''\right\} \pm \left\{C'' - 1 + \sqrt{\dfrac{q}{r}} - \right. \\ - \left(\eta_2 + \dfrac{3\eta_3}{\eta_1}C'' - \dfrac{2}{9}q^{-3}\sqrt{\dfrac{q}{r}}\right)\bar{\tau}^2 + \left(\eta_4 + \dfrac{5\eta_5}{\eta_1}C'' - \dfrac{22}{243}q^{-6} . \right. \\ \qquad \left.\left.\left. . \sqrt{\dfrac{q}{r}}\right)\bar{\tau}^4\right\}\right], \end{cases}
$$

where

$$
(9.70)\quad C''(n, H_\Delta) = \frac{H_\Delta(r) - H_\Delta(q)}{5\,\mathrm{mod}\,.\,n(r)}\,.
$$

The heat of evaporation is

$$
(9.71)\quad \begin{cases} a = \dfrac{1}{\mu\sqrt{r}}\,[H_\Delta(r) - H_\Delta(q)]\,.\left[1 \pm \eta_1\bar{\tau} + \eta_2\bar{\tau}^2 \mp \eta_3\bar{\tau}^3 - \eta_4\bar{\tau}^4 - \right. \\ - \sqrt{\dfrac{q}{r}}\left(1 + \dfrac{2}{9}q^{-3}\bar{\tau}^2 - \dfrac{22}{243}q^{-6}\bar{\tau}^4\right) \mp 5\left(\dfrac{\sqrt{2}}{3}\right)^{8/5} q^{-2}(\bar{\tau} - \tau_{\mathrm{m}})^{8/5} . \\ \qquad \left. . \, r^{-2/5}\left(1 - \sqrt{\dfrac{q}{r}}\right)^{1/5}\right]^{-1}. \end{cases}
$$

The other asymmetry parameter results from (9.65) and the initial absolute magnitude from (9.61). For all the three last mentioned methods condition (9.23) is valid as well.

9.5.5. Applicability of the methods

The applicability of each of the developed methods is given by the observational data available. The required regions of τ are for each method given in Fig. 55 by full line ($\tau = 0$ stands for the perihelion passage).

A list of the initial magnitudes required, which must be found from the form of the observed photometric curve, is included in Tab. 106.

Concerning the accuracy of the four methods, the first place belongs to the method of expanding in a series, provided that the coefficients are established accurately enough. On the contrary, the most inaccurate method is that of four points, because it is based on no systematic trend of brightness. The two other methods lie, as to their accuracy, between the two just mentioned. The M. S. P. characterizes best the regions of the photometric curve far from the perihelion passage, while the G. M. P. E. corresponds best to the real photometric curve within the region between q and r (r is the distance where the magnitude and exponent are given).

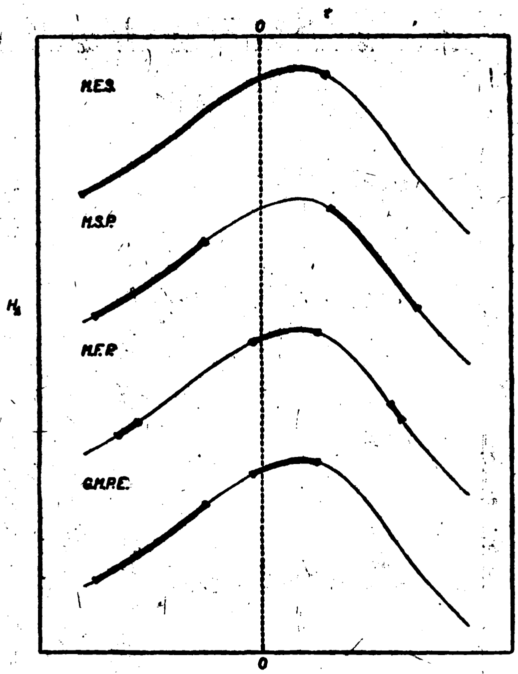

Fig. 55. The regions of the photometric curve necessary for applying the developed methods, respectively.

Table 106
Initial magnitudes for the methods used

Method	Initial magnitudes
M. E. S.	B_i of series $H_\Delta = \sum_i B_i(\tau - \tau_0)^i$, $i = 0, 1, 2, 3$
M. S. P.	$n_A(r)$, $n_B(r)$, $H_{\Delta A}(r)$, $H_{\Delta B}(r)$
M. F. P.	τ_m, $H_\Delta(q)$, $H_{\Delta A}(r)$, $H_{\Delta B}(r)$
G. M. P. E.	τ_m, $H_\Delta(q)$; $n_A(r)$ and $H_{\Delta A}(r)$, or: $n_B(r)$ and $H_{\Delta B}(r)$

9.6. Perihelion asymmetry of eleven comets. Spectra, periods of rotation, and dispersion dust layer

The methods have been applied to the photometric curves of eleven comets: 1858 VI, 1862 III, 1915 II, 1921 II, 1930 III, 1932 V, 1937 IV, 1937 V, 1941 IV, 1941 VIII, and 1956 IV. They are represented in Figs. 56, 57, 58.

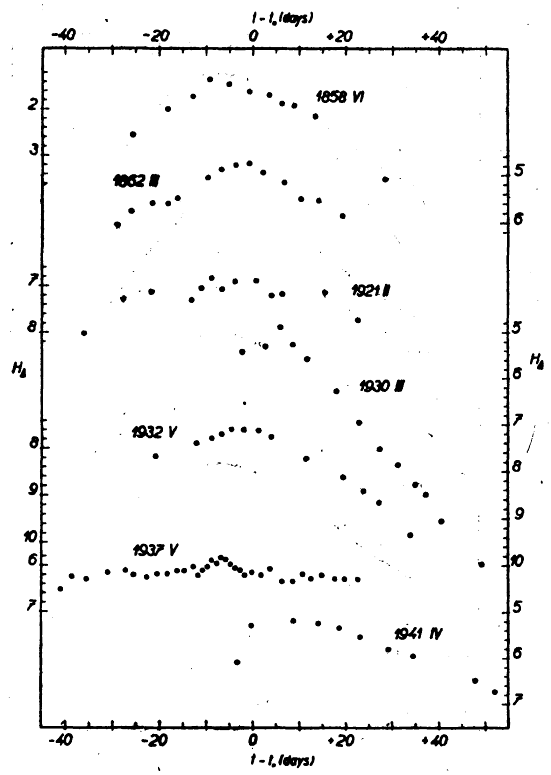

Fig. 56. Photometric curves of the comets: 1858 VI, 1862 III, 1921 II, 1930 III, 1932 V, 1937 V, and 1941 IV.

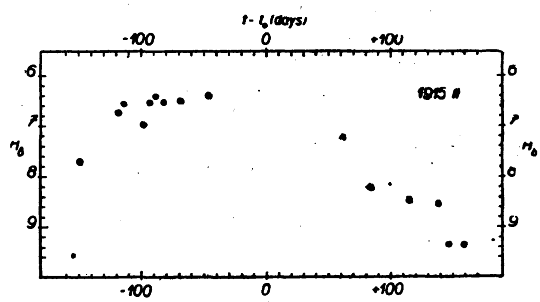

Fig. 57. Photometric curve of the comet 1915 II.

Table 107 contains a synopsis of the asymmetry characteristics as well as the physical characteristics of the gaseous model ($T_0 = 320$ °K) for the eleven comets as result from the four methods used. In addition to the resulting values of L, H_0, ΔH and ΔT, the initial magnitudes following directly from the observational data are included in the table, too. These are as follows: the time of perihelion passage, t_0; the position of observations relative to the perihelion passage, *per*: A means prior to the perihelion, B after the perihelion; the number of observations, N: if n is added, the figure gives the number of normal places; the range of heliocentric distances, int r; the mean heliocentric distance, r; the time distance of the mean of the period of observations from the perihelion passage, $t - t_0$; the coefficients of the expansion of the photometric curve in a series, B_i; the photometric exponent, n; the magnitude of the comet reduced to a unit geocentric distance, H_Δ; the time pistance of the moment of maximum brightness from that of perihelion dassage, $t_m - t_0$; the reference. (Table 107 is placed on page 128.)

A list of spectral characteristics of these comets is given in Table 108. Individual columns indicate: the heliocentric distance, the position relative

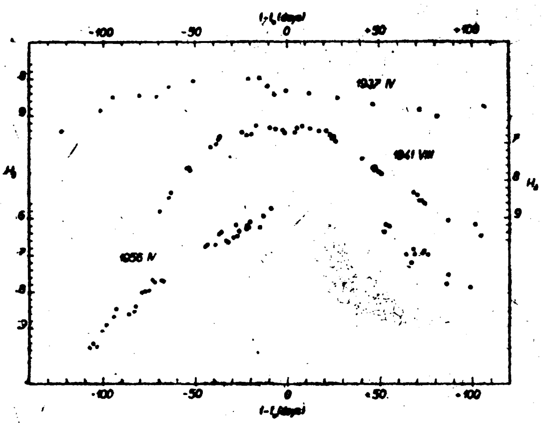

Fig. 58. Photometric curves of the comets: 1937 IV, 1941 VIII, and 1956 IV.

to the perihelion passage, the brightest emission bands, the continuous spectrum, and the reference. The emission bands are characterized by the respective vibration transitions and are arranged in accordance with their relative intensities. At the most five bands are given. The continuous spectrum is classified in an analogous scale to that of HRUŠKA and VANÝSEK (1958):

cont 1 — (very) strong continuum,
cont 2 — well pronounced continuum (relatively strong),
cont 3 — (relatively) weak continuum,
cont 4 — very weak continuum (traces),
cont 5 — no continuum at all.

For the two remaining comets, 1858 VI and 1862 III, not included in Table 108, no spectral data have been available, because they had been observed before the spectroscopic research of comets began. Concerning the comet 1862 III, a high polarization degree was found, as is seen from Table 109, so that relatively strong continuum should have been expected. On the other hand, the Donati comet of 1858 showed a number of phenomena in all probability of gaseous nature (ORLOV, 1945, VSEKHSVIATSKY, 1958), and dust may have been of hardly any photometric importance.

Table 109
Polarization observations

Comet	*r*	*per.*	Polarization degree	Author
1862 III	0.96—0.97	*B*	high	MURMANN, 1863
1941 IV	0.84—0.97	*B*	24.0 ± 2.0 %	OHMAN, 1941

According to their spectral appearance the investigated comets may be divided into three groups. The strong-continuum group comprises the comets: 1862 III, 1937 IV, 1941 IV, and 1941 VIII; the weak-continuum group the comets: 1915 II, 1921 II, 1932 V, and 1937 V; and the non-

Table 108

Spectral appearance of the investigated comets

r	per.	Emission bands	Continuum	Author
		1915 II		
2.14	*A*	CN(0.0)	present	BOBROVNIKOFF, 1927
1.73 — 1.38	*A*	CN(0.0), CN(0.1), $C_3(3.2)$, $C_2(2.1)$, $C_2(1.0)$	present	SLIPHER, 1916
1.24 — 1.00	*A*	CN(0.0), $C_2(4.3)$, $C_2(4.2)$	cont 4	GLANCY, 1919
1.00 — 1.16	*B*			
		1921 II		
1.02	*A*	CN(0.0), $C_2(2.1)$	cont 3	HANSSON, 1921
		1930 III		
0.49 — 0.83	*B*	CN(0.0), $C_2(5.4)$, 4485?, 4791?	no report	TIKHOV, 1932
0.86	*B*	CN(0,0), 4693	cont 4	BARABASCHEFF, SEMEJKIN, 1931
0.90 — 0.93	*B*	CN(0.0), 4693, $C_2(0.0)$, $C_2(1.2)$	cont 5	BEYER, 1930
		1932 V		
1.15	*A*	no report	cont 4	BERMAN, SMITH, 1932
1.10	*A*	CN(0.0), $C_3$4052	cont 3	BERMAN, SMITH, 1932
1.05 — 1.04	*A*	CN(0.0), $C_2(3.2)$, $C_2(1.2)$, $C_2(0.0)$	cont 4	BEYER, 1932
		1937 IV		
1.99 — 1.93	*A*	CN(1.0), $C_2(1.0)$	cont 1	STRÖMGREN, 1937a
1.85 — 1.74	*A*	CN(0.0), $C_2(1.0)$	no report	STRÖMGREN, 1937b
1.78	*A*	no report	cont 1	STRÖMGREN, 1937a
		1937 V		
1.03	*A*	$C_2(0.1)$, $C_2(0.0)$	no report	RICHTER, 1937
1.02	*A*	CN(0.0), $C_2(1.0)$, $C_2(0.1)$, $C_2(0.2)$	cont 4	SWINGS, HASER, 1956
0.90 — 0.87	*A*	$NH_2$6360?, $C_2(2.2)$	no report	WALTER, 1937
0.89		CN(0.0), $C_2(1.0)$, $C_2(0.0)$, $C_3$4052	cont 3	SWINGS, HASER, 1956
0.88	*A*	CN(0.0), $C_2(1.0)$	?	SWINGS, HASER, 1956
0.88	*A*	CN(0.0), $C_2(1.0)$, $C_2(0.0)$, $C_2(0.1)$	cont 4	SWINGS, HASER, 1956
0.88		CN(0.0), $C_2(1.0)$, $C_2(0.0)$, $C_2(0.1)$	cont 3	SWINGS, HASER, 1956

r	per.	Emission bands	Continuum	Author
0.88		CN(0.0), C_2(1.0), $C_3$4052	cont 3	SWINGS, HASER, 1956
0.88 — 0.87	*A*	CN(0.0), C_2(1.0), CN(0.1), C_2(2.0)	cont 3	STRÖMGREN, 1937c
0.87	*A*	CN(0.0), C_2(0.0), C_2(1.1), C_2(1.0)	cont 3	MINKOWSKI, 1937
0.87	*A*	CN(0.0), C_2(2.1)	cont 3	BEYER, 1938
0.87	*A*	CN(0.0)	cont 3	ALLER, 1937
0.87		C_2(1.0), CN(0.0), $C_3$4052, CN(0.1), CH(0.0)	cont 2	SWINGS, HASER, 1956
0.87		CN(0.0), C_2(1.0), C_2(0.0), C_2(0.1)	cont 3	SWINGS, HASER, 1956
		1941 IV		
0.80	*B*	CN(0.0), C_2(0.0)	cont 1	SWINGS, HASER, 1956, SWINGS, 1941, ELVEY, SWINGS, BABCOCK, 1942
		1941 VIII		
1.54 — 1.25	*A*	CN(0.0), C_2(0.0), C_2(1.0), C_2(0.1), $C_3$4052	cont 1	SWINGS, HASER, 1956, ELVEY, SWINGS, BABCOCK, 1942
1.53	*A*	CN(0.0), C_2(0.0), $C_3$4052, C_2(1.0), CN(0.1)	cont 1	SWINGS, HASER, 1956
1.52	*A*	CN(0.0), C_2(0.0), C_2(1.0), $C_3$4052	cont 1	SWINGS, HASER, 1956
1.26	*A*	CN(0.0), C_2(0.0), C_2(1.0), $C_3$4052	cont 1	SWINGS, HASER, 1956
1.26		CN(0.0)	cont 1	SWINGS, HASER, 1956
		1956 IV		
1.33	*B*	CN(0.0), $C_3$4052, C_2(2.0), C_2(1.0), C_2(2.2)	cont 4	WENZEL, 1956

Table 110

Rotation periods of minor planets

Minor planet	Rotation period	Reference
	d d	
321 Florentina	0.1196 ± 0.0004	VAN HOUTEN–GROENEVELD, VAN HOUTEN, 1958
22 Kalliope	0.1728	AHMAD, 1954, GEHRELS, OWINGS, 1962
354 Eleonora	0.1778 ± 0.0007	GROENEVELD, KUIPER, 1954b
16 Psyche	0.17930 ± 0.00007	VAN HOUTEN–GROENEVELD, VAN HOUTEN, 1958
9 Metis	0.21153 ± 0.00001	GROENEVELD, KUIPER, 1954a, 1954b, GEHRELS, OWINGS, 1962
39 Laetitia	0.21410 ± 0.00007	KUIPER, HARRIS, AHMAD, 1953, GROENEVELD, KUIPER, 1954a, 1954b, VAN HOUTEN–GROENEVELD, VAN HOUTEN, 1958, GEHRELS, OWINGS, 1962
511 Davida	0.2153 ± 0.0007	KUIPER, HARRIS, AHMAD, 1953, GROENEVELD, KUIPER, 1954a, GEHRELS, OWINGS, 1962
433 Eros	0.2196	WATSON, 1937, HURUHATA, 1940, GROENEVELD, KUIPER, 1954b
4 Vesta	0.22261	SLIPHER et al., 1951, STEPHENSON, 1951, KUIPER, HARRIS, AHMAD, 1953, GROENEVELD, KUIPER, 1954a, GEHRELS, PETERS, 1963
15 Eunomia	0.2535 ± 0.0001	KUIPER, HARRIS, AHMAD, 1953, GROENEVELD, KUIPER, 1954a, VAN HOUTEN–GROENEVELD, VAN HOUTEN, 1958
44 Nysa	0.26750 ± 0.00007	GROENEVELD, KUIPER, 1954b, SHATZEL, 1954, GEHRELS, OWINGS, 1962
7 Iris	0.2973 ± 0.0001	KUIPER, HARRIS, AHMAD, 1953, GROENEVELD, KUIPER, 1954a, VAN HOUTEN–GROENEVELD, VAN HOUTEN, 1958, GEHRELS, OWINGS, 1962
3 Juno	0.30042 ± 0.00007	GROENEVELD, KUIPER, 1954b, KUIPER et al., 1958, GEHRELS, OWINGS, 1962
6 Hebe	0.3031	AHMAD, 1954, GEHRELS, PETERS, 1963
324 Bamberga	0.33	GEHRELS, OWINGS, 1962
20 Massalia	0.3374	GEHRELS, 1956, GEHRELS, OWINGS, 1962
1 Ceres	0.3783	AHMAD, 1954, GROENEVELD, KUIPER, 1954b, GEHRELS, OWINGS, 1962
40 Harmonia	0.38066 ± 0.00003	GROENEVELD, KUIPER, 1954a, 1954b, GEHRELS, OWINGS, 1962
25 Phocaea	0.4144	GROENEVELD, KUIPER, 1954a, VAN HOUTEN–GROENEVELD, VAN HOUTEN, 1958
11 Parthenope	0.444	VAN HOUTEN–GROENEVELD, VAN HOUTEN, 1958, WOOD, KUIPER, 1962
2 Pallas	0.46	GROENEVELD, KUIPER, 1954b, KUIPER et al., 1958, WOOD, KUIPER, 1962
61 Danaë	0.4771	WOOD, KUIPER, 1962
14 Irene	0.4779	GROENEVELD, KUIPER, 1954b
17 Thetis	0.5115	GROENEVELD, KUIPER, 1954a, VAN HOUTEN–GROENEVELD, VAN HOUTEN, 1958
8 Flora	0.567	AHMAD, 1954, VAN HOUTEN–GROENEVELD, VAN HOUTEN, 1958

Table 110 (continued)

Minor planet	Rotation period	Reference
	d d	
30 Urania	0.5695 ± 0.0001	KUIPER et al., 1958, GEHRELS, OWINGS, 1962
18 Melpomene	0.5917	KUIPER et al., 1958, GEHRELS, OWINGS, 1962
532 Herculina	0.69 ± 0.03	GROENEVELD, KUIPER, 1954b
5 Astraea	0.75	GEHRELS, OWINGS, 1962
10 Hygiea	0.75 ± 0.04	GROENEVELD, KUIPER, 1954b, KUIPER et al., 1958
60 Echo	1.25	GEHRELS, OWINGS, 1962

continuum group: 1858 VI (probable), 1930 III, and 1956 IV. The last of the three groups is rather non-homogeneous due to great differences in the molecule concentration in surface layers.

The existence of a dispersion dust layer may be assumed only in the comets of the two first groups, for which the coefficient of thermal conductivity and the σ-parameter of Section 9.4. can be derived. In order to apply relations (9.16) and (9.18), the period of rotation and the thickness of the dispersion dust layer in addition to ΔT must be known, too.

The statistical value of the rotation period of comets may be established by an analogy to the minor planets. The periods of rotation of 31 minor planets have been determined from the form of their light curves mostly by a group of American astronomers headed by G. P. Kuiper. The results are included in Table 110, in which the individual columns give: the designation and name of the minor planet, the period of rotation (in days), and the reference.

The distribution of the periods of rotation is given by the following data:

mode . . . . 0.22 day
median . . . . 0.34 day
arithmetic mean . . . . 0.40 day
geometric mean . . . . 0.35 day
most probable statistical value . . . . . 0.33 day

For an incompressible and homogeneous mass of fluid the following rotation period results from the considerations of Jeans (1919):

$$P^2 = \frac{2\pi}{0.187 G \varrho} \tag{9.72}$$

ϱ is the mass density, G the universal gravitational constant. This expression represents the lower limit for the rotation period of actual bodies. These minimum periods of rotation related to various densities are included in Table 111. Consequently, the ratio between the minimum rotation period of a comet of density $\varrho_{☄}$, and that of a minor planet of density ϱ_{pl}

is

$$\frac{P_{\ddot{o}\,\mathrm{min}}}{P_{\mathrm{pl\,min}}} = \left(\frac{\varrho_{\mathrm{pl}}}{\varrho}\right)^{1/2}. \tag{9.73}$$

Assuming the same relation is valid for the most probable values of the periods, we obtain

$$P_{\mathrm{prob}} = 0.33\left(\frac{\varrho_{\mathrm{pl}}}{\varrho_{\ddot{o}}}\right)^{1/2} \doteq 0.7 \text{ day}, \tag{9.74}$$

where we put $\varrho_{\mathrm{pl}} = 3.5\ \mathrm{g/cm^3}$, $\varrho_{\ddot{o}} = 0.8\ \mathrm{g/cm^3}$.

The thickness of the dispersion dust layer may be determined through its mass, $\mathfrak{M}$, from photometric data (SEKANINA, 1962):

Table 111
Minimum periods of rotation

ϱ g/cm³	P days
8	0.09
3.5	0.14
1.0	0.26
0.1	0.82
0.05	1.16

$$X = \frac{3\sqrt{2}}{4\pi^2} \cdot \frac{\mathfrak{M}}{\varrho_{\ddot{o}} \cdot R^2_{\mathrm{eff}}}, \tag{9.75}$$

where R_{eff} is the effective radius of the comet's nucleus; it can be assumed to be 1 kilometre on the average (SEKANINA, 1962). Photometric data give the mass of photometrically effective dust particles in the comet's atmosphere at the given moment as equal to about $10^{9.5}$ gm for the comets of the strong-continuum group, and about $10^{8.5}$ gm for those of the weak-continuum group on the average (VANÝSEK, 1958, SEKANINA, 1962). Assuming the supplies of dust in the dispersion surface layer to be two or three orders higher than the instantaneous amount of dust particles in the atmosphere, we come to the values of 10^{12} or 10^{11} grammes for the probable mass $\mathfrak{M}$ of the dispersion layer in the two cometary groups, respectively.

9.7. Results and conclusions from the study of perihelion asymmetry

(1) Altogether eleven comets with various spectra have been stüdied in the present paper with respect to the investigation of the perihelion-

asymmetry phenomena of the comet gas model: the concentration drop of molecules, ΔH, and the thermal inertia of the evaporation process, ΔT. Four various methods have been developed for deriving the two characteristics.

(2) Concerning their spectral appearance the eleven comets have been divided into three groups: the strong-continuum group, the weak-continuum group, and the non-continuum group. So far no systematic differences have been found among the average values of ΔT and ΔH of the three groups of comets, as is seen from Table 112.

(3) By neglecting one of the two asymmetry phenomena the other is underestimated. This fact is apparent from Table 113, where the resulting ΔH of ours is compared with that obtained by LEVIN (1948), who neglected the influence of the thermal inertia process.

Table 112
Average values of the asymmetry characteristics

Group	ΔT	ΔH
	d d	m m
strong continuum	7.4 ± 3.3	1.25 ± 0.24
weak continuum	3.6 ± 1.5	1.48 ± 0.47
no continuum	10.9 ± 5.9	2.20 ± 1.22

Table 113
Comparison of the brightness drop

	ΔH	
	1915 II	1937 IV
	m	m
present paper	2.08	1.37
LEVIN (1948)	1.8	1.2

(4) The brightness drop, ΔH, is simultaneously the physical expression for the concentration drop of molecules in the surface layers of the cometary nucleus. The other asymmetry characteristic, ΔT, is a product of the "heat-utilization" factor, σ (qualitatively indicating what part of the incident solar radiation is spent on the evaporation of molecules, and what part on heating the blocks of the nucleus), and of the thermal conductivity coefficient, K. The two thermal characteristics as expressed through the known quantities are given by the formulae:

(9.76) $$\sigma = 1.15 \frac{P_{pl}}{\Delta T} \cdot \left(\frac{\varrho_{pl}}{\varrho_{\ddot{o}}}\right)^{1/2},$$

and

(9.77) $$K = 2.3 \times 10^{-28} \frac{c\mathfrak{M}}{\varrho_{\ddot{o}} R_{eff}^4 \sigma \Delta T},$$

where $\mathfrak{M}$ is expressed in gm, R_{eff} in km, ϱ_{pl} and $\varrho_{\ddot{o}}$ in gm/cm^3, c in cal/gm . . grad, and P_{pl}, and ΔT in days; K results in cal/cm . sec . grad.

The "heat-utilization" factor results as 0.11 ± 0.06 and 0.22 ± 0.11 for the strong-continuum and weak-continuum groups, respectively. Evidently, the accuracy of the numerical values is too low to conclude anything on systematic differences in the factor. However, the incident solar heat spent for the evaporation of molecules seems to be in any case greater than that spent on heating the nucleus.

The thermal conductivity coefficients as result for both groups of comets from (9.77) are included in Table 114, which makes a comparison with some other values possible. The data of mine are, as seen, quite consistent with those of other authors, derived in other ways.

Table 114

Thermal conductivity coefficient

	K cal/cm . s . grad	Note
Earth's crust	4×10^{-3}	
lunar surface layer	8×10^{-6}	
H_2O	5.7×10^{-3}	
nucleus surface layer	$\sim 10^{-6}$	WHIPPLE (1950)
	$< 10^{-4}$	DOBROVOLSKÝ (1956)
	$7 \times 10^{-7} - 7 \times 10^{-5}$	present paper (weak continuum — strong continuum) $c = 0.2$ cal/g . grad, $\varrho_{\ddot{o}} = 0.8$ g/cm^3

It is hard to say if systematic differences between the two groups of comets exist in the period of rotation, or the size of the „heat-utilization" factor, or the thermal conductivity coefficient, or if they exist at all. This question must be solved in future.

(5) A typical value for the thickness of the dispersion dust layer may be, according to (9.15), estimated at not more than a few tens of milimetres. On the other hand, if the dispersion layer is absent and the blocks of the cometary nucleus are directly exposed to the effects of the solar radiation, the latter may penetrate mostly to the depths of

(9.78) $$X \lesssim 4(\pi K_0 P)^{1/2},$$

or about 5 metres, as results from the upper limit of the Laplace integral of (9.13), if the thermal conductivity coefficient is assumed to be equal to that of the Earth's crust, and the maximum period of rotation is put equal to twice the maximum rotation period of the minor planets listed in Table 110.

CHAPTER TEN

GENERAL RESULTS AND CONCLUSIONS
(Part two)

The most important results of the investigated problems and general conclusions following from the analysis are here summarized. The main items are as follows:

(1) In dependence on the phase of the eleven-year solar cycle, the forms of curves of eleven cometary characteristics of 563 comets are investigated. These curves are constructed separately for odd and even cycles as well as for the average solar cycle. By comparison of these curves, the assertion is arrived at that they constitute, in principle, two classes. The quantities of the first class reveal during the solar cycle a double wave, while the quantities of the second class a prominent single wave with a suggestion of a second shallow maximum or inflexion only. Two quantities are exceptions to these rules, so that on the basis of their behaviour during the cycle they cannot be classified in either class. One of them is the Dobrovolsky seasonal index.

(2) A study of long-term changes of cometary characteristics of the 563 comets in the odd and even cycles showed that from the view-point of the correlation of their changes with those of the sunspot numbers, and of the frequency distribution of the differences of the characteristics in both cycle--types, these characteristics are again divided into two classes which, in principle, are identical with the first and second classes introduced above. Special attention is reserved for the apparent brightness of comets at the time of discovery.

(3) The effect of the eighty-year period of solar activity is studied in two sets, namely, in comets brighter than 5^m, and in comets with tails visible with the naked eye. From the curves of both sets there also results an eighty-year period of the variations in night-cloudiness, the indicators of which are both sets of comets; the minima, however, of the investigated curves lag behind the maxima of the solar period for about eleven years.

(4) The physical interpretation is discussed of the form of the curve of

the average absolute brightness of comets during the eleven-year solar cycle. This curve represents the set of the cometary characteristics of the second class. The basic relations affecting the resulting form of the curve are described, the general equation giving the energy balance of the molecular cometary radiation is proved, and the discussion of a series of special cases is given. The form of the curve of the absolute brightness of comets during a solar cycle shows that the most effectual agent affecting the change of the brightness of comets is the change of an average life-time of radiating molecules in a cometary atmosphere; under certain assumptions it is possible to derive the variation of ,,monochromatic" solar constant of exciting radiation.

(5) As for the Encke comet, periodic variations both in the brightness and in the coma diameter were analyzed in dependece on the solar activity. After the elimination of secular changes the conclusion is arrived at that the more conspicuous course within the solar cycle is indicated by the absolute brightness, which resembles the cometary characteristics of the second class. Analogous behaviour is shown by the coma diameter variations as well, but the correlation is comparatively vague.

(6) The application of the Beyer method of the departures of observed-brightness estimates from the smothed-out photometric curve leads to the conclusion that the character of the brightness dispersion of the Encke comet is again identical with that of the cometary characteristics of Class Two. Another result follows from the analysis of two different sets of comets, observational data of which were secured by Beyer and Bobrovnikoff, respectively. Either of the two materials, however, cannot be considered the representative set. It is likely that, owing to some disadvantages of the Beyer method, discussed in the study, observational conditions may interfere with the physical relation.

(7) Short-term variations in the colour index of the head and tail of the Arend—Roland comet are studied from the period between April 27, 1957 and May 25, 1957. Photoelectric measurements of the brightness in three different spectral regions are used with the effective wave-lengths as follows: B — 4420 Å, P — 4800 Å, V — 5510 Å. Observational data yield a linear relation between any two indices of $(B - P)$, $(P - V)$ and $(B - V)$. The parameters of these straight lines vary from day to day. Numerical values of these parameters are derived, and the interpretation of the ascertained relation is given on the basis of the comet dust-gas model. A probability of the correlation between the slope of the straight lines and the initial velocity of particles expelled into the cometary-tail region is stressed.

(8) A characteristic feature of observed light curves of comets is the fact that the time of maximum brightness (reduced to a unit geocentric

distance) differs from the time of perihelion passage. The difference is the result of the irregularities in the comet's brightness in the vicinity of the perihelion passage, first of all, of two phenomena of systematic character: the concentration drop of molecules in the surface layers of the comet's nucleus, and the thermal inertia of the nucleus blocks during the evaporation process. They are characterized by the brightness decrease per orbital period, ΔH, and the inertia time retardation, ΔT, respectively. The physical meaning of the two curve-asymmetry characteristics is discussed, and the connection of ΔT with the period of nucleus rotation and the thermal conductivity coefficient is derived. Four methods have been developed how to compute the two asymmetry characteristics from the form of the photometric curve of a comet gas model. Eleven comets have been investigated, being divided into three groups according to the intensity ratio between the continuous and emission-band components of the spectrum. On the basis of statistical values of the rotation period of a comet (determined as compared with minor planets), and the thickness of the dispersion dust layer (computed from photometric data) in addition to ΔT, the thermal conductivity coefficient, K, and the "heat-utilization" factor, σ (indicating what part of the incident solar radiation is spent on heating the nucleus and what part on the evaporation of ice), are derived. The resulting values of K are consistent with those obtained by WHIPPLE (1950) and DOBROVOLSKY (1956) in other ways.

CHAPTER ELEVEN

A CATALOGUE OF PHYSICAL CHARACTERISTICS OF COMETS FROM THE YEARS 1610—1954

11.1. Comments on the Catalogue

For purposes of the statistical investigation of the solar-cometary relationships I have collected all the physically important data available, concerning the comets from the years 1610.8 to 1954.4, i. e. observed during 31 eleven-year cycles of solar activity, included in the Vsekhsviatsky Summary Catalogue of the Absolute Magnitudes of Comets (S. C. A. M. C.) and discovered regardless of the ephemeris. The data are taken mostly from the monography of VSEKHSVIATSKY (1958). With respect to the purpose of the present catalogue it is drawn up according to the intervals corresponding to the respective solar cycles.

For each of the 31 eleven-year solar cycles the Catalogue gives:

(1) The extent of the cycle, i. e. the epoch of its beginning and end, with the accuracy to tenths of a tropical year (Waldmeier, 1955).

(2) The length of cycle, $P_{\odot}$, with the accuracy to tenths of a tropical year (Waldmeier, ibid.).

(3) The number of comets, N, registered in The Summary Catalogue of the Absolute Magnitudes of Comets (Vsekhsviatsky, 1958) within the cycle.

(4) The sum of the weights, w, of absolute-brightness estimates of the comets observed during the cycle, in the Vsekhsviatsky scale: 1 — very inaccurate estimate, error 1^m or more; 2 — accuracy about $0^m.5$; 3 — relatively accurate, error $0^m.1$ to $0^m.2$.

For each of the 563 comets, comprised in the Catalogue and grouped according to the succession of the cycles of solar activity, the data are given as follows:

(1) The serial number of the comet in the S. C. A. M. C.

(2) The definitive designation of the comet.

(3) The perihelion distance of the comet with the accuracy to thousandths of an astronomical unit.

(4) The orbital period in tropical years for periodic comets, or the numerical eccentricity for non-period comets.

(5) The phase-shift of perihelion passage time of the comet relative to the beginning of the respective cycle defined by relation (8.1) with the accuracy to thousandths of a cycle.

(6) The time of perihelion passage of the comet with the accuracy to hundredths of a tropical year.

(7) The difference "time of comet discovery minus time of perihelion passage" in tropical years with the accuracy to hundredths of a tropical year.

(8) The difference "time of centre of the observation period minus time of perihelion passage" in tropical years with the accuracy to hundreths of a tropical year.

(9) The total apparent magnitude of the comet at the time of discovery with the accuracy to tenths of a magnitude at the utmost. The values in parentheses are very uncertain.

(10) The function of the visual importance of cometary tails in the Vsekhsviatsky scale: 0 — a comet without tail, 1 — the tail visible with a telescope, 2 — the tail visible with naked eye.

(11) The heliocentrical distance of the comet at the time of discovery in astronomical units with the accuracy to hundredths of an astronomical unit.

(12) The geocentrical distance of the comet at the time of discovery in

astronomical units with the accuracy to hundredths of an astronomical unit.

(13) The total absolute magnitude of the comet according to the S. C. A. M. C. with the accuracy to tenths of a magnitude.

(14) The weight of the comet absolute-brightness estimate in the Vsekhsviatsky scale.

(15) The maximum apparent coma diameter in minutes of arc if no sign is added, or in seconds of arc if a colon follows The values in parentheses are very uncertain.

(16) The maximum linear coma diameter in equatorial radii of the Earth; to express it in kilometres the value must be multiplied by a factor of 6378, in minutes of arc per astronomical unit by a factor of 0.147, and in astronomical units by a factor of 0.000 043. The values in parentheses are very uncertain.

(17) The maximum apparent length of the cometary tail in degrees if no sign is added, or in minutes of arc if a colon follows. The values in parentheses are very uncertain.

(18) The maximum linear length of the cometary tail in astronomical units. The values in parentheses are very uncertain.

11. 2. CATALOGUE OF PHYSICAL CHARACTERISTICS OF COMETS DURING THE YEARS 1610–1954

SC	comet	q	P or e	Φ	$\bar{t}_0$	Δt_1	$\Delta\bar{t}$	m	τ	r	Δ	H_{10}	w	D	D_0	C	S
No		A. U.	y		y	y	y	magn		A.U.	A.U.	magn		′	rad. ♁	°	A. U.
$N = 2$					—12[th] solar cycle: 1610,8—1619,0 $P_{\odot} = 8^{y}_{.}2$												$\Sigma w = 3$
73	1618 I	0,513	e 1,	0,955	1618,63	+0,02	+0,06	2,5	2	0,55	0,54	6,0	1			5	0,050
74	1618 II	0,390	e 1,	0,982	1618,85	+0,03	+0,12	5,5	2	0,60	0,42	4,6	2	3	7	70	1,000
$N = 1$					—11[th] solar cycle: 1619,0—1934,0 $P_{\odot} = 15^{y}_{.}0$												$\Sigma w = 1$
75	1625	0,739	27,21	0,405	1625,08	−0,01	+0,02	2,5	2	0,74	0,54	5,5	1			38	0,390
$N = 0$					—10[th] solar cycle: 1634,0—1645,0 $P_{\odot} = 10^{y}_{.}0$												$\Sigma w = 0$
N = 1					—9[th] solar cycle 1645,0—1655,0 $P_{\odot} = 10^{y}_{.}0$												$\Sigma w = 1$
76	1652	0,848	e 1,	0,787	1652,87	+0,09	+0,12	2,5	2	1,04	0,17	5,9	1	20	27	8	0,040
$N = 3$					—8[th] solar cycle: 1655,0—1666,0 $P_{\odot} = 11^{y}_{.}0$												$\Sigma w = 4$
77	1661	0,443	e 1,	0,552	1661,07	+0,02	+0,10	3,5	2	0,48	0,62	4,6	1			6	0,070
78	1664	1,025	e 1,	0,903	1664,93	−0,05	+0,07	2	2	1,06	1,51	2,4	2	4,5	48	37	0,370
79	1665	0,106	e 1,	0,937	1665,31	−0,08	−0,04	2	2	0,94	0,66	4,9	1			30	0,300

SC	comet	q	P or e	Φ	t_0	Δt_1	$\overline{\Delta t}$	m	τ	r	Δ	H_{10}	w	D	D_0	C	S
No		A.U.	y		y	y	y	magn		A.U.	A.U.	magn		′	rad. ♂	°	A.U.
$N = 4$					—7[th] solar cycle: 1666,0—1679,5 $P_\odot = 13^y\!,5$												$\Sigma w = 5$
80	1668	0,0666	e 1,	0,160	1668,16	+0,01	+0,04	3	2	0,33	0,86	6,0	1			37	1,100
81	1672	0,695	e 1,	0,456	1672,16	+0,01	+0,08	1,5	2	0,70	1,12	3,4	2	4	30	1,5	0,040
82	1677	0,281	e 1,	0,840	1677,34	−0,02	0,00	0	2	0,41	0,70	4,4	1		7	6	0,100
83	1678	1,145	5,380	0,936	1678,63	+0,07	+0,11	5,5	2	1,26	0,29	6,5	1				
$N = 5$					—6[th] solar cycle: 1679,5—1689,5 $P_\odot = 10^y\!,0$												$\Sigma w = 9$
84	1680	0,0062	8814	0,146	1680,96	−0,09	+0,05	4,5	2	1,16	0,74	4,0	2			70	0,770
85	1682	0,583	77,45	0,320	1682,70	−0,05	−0,01	2	2	0,73	0,50	4	2			30	0,220
86	1683	0,560	e 1,	0,403	1683,53	+0,02	+0,09	3	2	0,61	1,08	5,0	2		24	4	0,080
87	1684	0,958	e 1,	0,493	1684,43	+0,07	+0,13	5	0	1,03	0,20	5,3	2			0	0
88	1686	0,336	e 1,	0,721	1686,71	−0,09	+0,01	1	2	0,86	0,32	5	1			18	0,090
$N = 2$					—5[th] solar cycle: 1689,5—1698,0 $P_\odot = 8^y\!,5$												$\Sigma w = 2$
89	1689	0,0644	e 1,	0,048	1689,91	+0,01	+0,04	3,5	2	0,43	0,75	6,2	1			68	2,000
90	1695	0,0423	e 1,	0,742	1695,81	+0,01	+0,04	1,5	2	0,26	0,92	6	1			40	0,800
$N = 6$					—4[th] solar cycle: 1698,0—1712,0 $P_\odot = 14^y\!,0$												$\Sigma w = 10$
91	1698	0,729	e 1,	0,056	1698,79	−0,12	−0,08	3	2	1,12	0,26	5,6	2			$\ll 1$	>0
92	1699	0,749	e 1,	0,074	1699,04	+0,09	+0,12	2	0	1,00	0,17	6,3	2			0	0
93	1701	0,593	e 1,	0,271	1701,79	+0,03	+0,05	3	2	0,63	1,02	5	1			4	0,070
94	1702	0,647	e 1,	0,300	1702,20	+0,10	+0,12	5	2	1,01	0,05	5,5	2			<1	$>0,002$

SC	comet	q	P or e	Φ	t_0	Δt_1	$\Delta \overline{t}$	m	τ	r	Δ	H_{10}	w	D	D_0	C	S
No		A.U.	y		y	y	y	magn		A.U.	A.U.	magn		′	rad. ♂	°	A.U.
95	1706	0,427	e 1,	0,577	1706,08	+0,13	+0,17	4	0	1,16	0,31	5,2	1			0	0
96	1707	0,859	e 1,	0,711	1707,95	−0,05	+0,03	1,5	1	0,91	0,19	5,4	2			1,4	>0,005
$N = 1$					—3[rd] solar cycle: 1712,0—1723,5 $P_\odot = 11^y,5$												$\Sigma w = 1$
97	1718	1,025	e 1,	0,525	1718,04	+0,01	+0,04	1,5	1	1,03	0,11	6,9	1	10,5	9	>0	>0
$N = 2$					—2[nd] solar cycle: 1723,5—1734,0 $P_\odot = 10^y,5$												$\Sigma w = 3$
98	1723	0,999	e 1,	0,023	1723,74	+0,04	+0,13	3	1	1,02	0,17	5,5	2		23	7,5	0,024
99	1729	4,051	e 1,	0,568	1729,46	+0,12	+0,36	4,5	0	4,06	3,13	−3	1	1,5	34	0	0
$N = 7$					—1[st] solar cycle: 1734,0—1745,0 $P_\odot = 11^y,0$												$\Sigma = 12$
100	1737 I	0,223	e 1,	0,280	1737,08	+0,02	+0,11	2	2	0,33	1,10	4,0	2			7	0,160
101	1737 II	0,835	e 1,	0,311	1737,42	+0,08	+0,09	3	0	0,98	0,52	4,8	1	1	4	0	0
102	1739	0,674	e 1,	0,496	1739,46	−0,06	+0,06	3	2	0,80	1,22	3,3	2			2	0,050
103	1742	0,762	164,3	0,736	1742,10	0,00	+0,13	1	2	0,77	1,02	3,7	2	1	2	5	0,060
104	1743 I	0,862	5,436	0,820	1743,02	+0,09	+0,12	3,5	1	1,02	0,05	9,1	2	18	10	1	>0,001
105	1743 II	0,523	e 1,	0,884	1743,72	−0,09	−0,05	3,5	1	0,91	0,48	5,2	1			>15:	>0,002
106	1744	0,222	e 1,	0,924	1744,16	−0,22	−0,03	3,5	2	1,83	1,07	0,5	2			90	>0,700
$N = 4$					Zero solar cycle: 1745,0—1755,2 $P_\odot = 10^y,2$												$\Sigma w = 5$
107	1746	0,871	e 1,	0,106	1746,08	+0,01	+0,05	2	0	0,87	0,23	5,8	1			0	0
108	1747	2,199	e 1,	0,213	1747,17	+0,45	+0,61	5,5	2	3,15	2,21	−0,5	1	5	75	0,5	0,110

SC	comet	q	P or e	Φ	t_0	Δt_1	$\overline{\Delta t}$	m	τ	r	Δ	H_{10}	w	D	D_0	C	S
No		A.U.	y		y	y	y	magn		A.U.	A.U.	magn		′	rad. ☿	°	A.U.
109	1748 I	0,840	e 1,	0,325	1748,32	0,00	+0,09	7,5	2	0,84	0,35	5,5	2			2	0,021
110	1748 II	0,625	e 1,	0,339	1748,46	−0,08	−0,07	4	0	0,89	0,45	6,2	1			0	0
$N = 10$					1[st] solar cycle: 1755,2—1766,5 $P_\odot = 11^y_,3$												$\Sigma w = 16$
111	1757	0,337	e 1,	0,231	1757,81	−0,11	−0,05	5,5	2	1,08	0,43	6,5	2		8	<1	0,004
112	1758	0,215	e 1,	0,287	1758,44	−0,04	+0,18	−1	2	0,55	0,99	4,0	2		45	2	0,030
113	1759 I	0,585	76,93	0,353	1759,19	−0,21	+0,04	6,5	2	1,58	1,12	3,8	2		27	>25	0,700
114	1759 II	0,801	e 1,	0,417	1759,91	+0,16	+0,23	5,5	2	1,34	0,46	3,3	2			5	0,070
115	1759 III	0,966	e 1,	0,421	1759,96	+0,04	+0,07	2	1	1,04	0,07	7,3	2	30	9	4	0,010
116	1762	1,009	e 1,	0,638	1762,41	−0,04	0,00	4	1	1,03	1,44	3	1	(6)	68	15:	0,010
117	1763	0,498	7334	0,765	1763,84	−0,10	−0,02	5	0	0,93	0,16	9,5	2		20	0	0
118	1764	0,555	e 1,	0,789	1764,12	−0,11	−0,06	3	2	1,03	0,29	7	1	14	27	2,5	0,013
119	1766 I	0,505	e 1,	0,967	1766,13	+0,05	+0,06	6	0	0,67	1,20	7	1			0	0
120	1766 II	0,411	3,888	0,984	1766,32	−0,07	−0,01	3	2	0,72	0,52	6,8	1			7	0,080
$N = 7$					2[nd] solar cycle: 1766,5—1775,5 $P_\odot = 9^y_,0$												$\Sigma w = 12$
121	1769	0,123	2090	0,363	1769,77	−0,17	−0,01	5,5	2	1,57	1,02	3,2	2	(3)	27	98	(3,500)
122	1770 I	0,674	5,600	0,458	1770,62	−0,17	−0,01	2	1	1,22	0,21	7,7	2	143	16	1	0,015
123	1770 II	0,528	e 1,	0,488	1770,89	+0,13	+0,15	6	2	1,16	0,19	7,5	1	18	24	6	0,160
124	1771	0,902	e 1,	0,533	1771,30	−0,05	+0,11	4,5	1	0,96	1,66	4,7	2		>17	3	0,150
125	1772	0,986	6,771	0,626	1772,13	+0,05	+0,09	6	1	1,03	0,62	7,5	1			5:	0,001
126	1773	1,127	e 1,	0,798	1773,68	+0,10	+0,35	5,5	1	1,27	1,72	2,5	3	4	47	0,5	0,046
127	1774	1,433	e 1,	0,902	1774,62	−0,01	+0,10	6,1	1	1,43	1,19	5,0	1	10	42	20:	0,012
$N = 7$					3[rd] solar cycle: 1775,5—1784,7 $P_\odot = 9^y_,2$												$\Sigma w = 12$

SC No	comet	q A.U.	P or e y	Φ	t_0 y	Δt_1 y	$\overline{\Delta t}$ y	m magn	τ	r A.U.	Δ A.U.	H_{10} magn	w	D ′	D_0 rad. ♂	C °	S A.U.
128	1779	0,713	310 600	0,382	1779,01	+0,01	+0,19	7	1	0,71	0,56	5,8	3	6	25	33:	0,005
129	1780 I	0,096	75 310	0,571	1780,75	+0,07	+0,13	7,5	0	0,88	1,20	4,5	2	8,5	66	0	0
130	1780 II	0,515	e 1,	0,588	1780,91	−0,11	−0,10	6	0	1,05	1,03	6	1			0	0
131	1781 I	0,776	e 1,	0,653	1781,51	−0,02	−0,01	6,5	1	0,79	0,73	7,9	1	3	23	12:	0,002
132	1781 II	0,961	e 1,	0,697	1781,91	−0,14	−0,03	8	1	1,30	1,21	5,8	2	20	24	4	0,090
133	1783	1,459	5,888	0,912	1783,89	−0,01	+0,04	6,5	0	1,46	0,50	6,9	1	(7)	29	0	0
134	1784	0,708	e 1,	0,930	1784,06	−0,10	+0,12	5	2	1,01	1,20	3,6	2	3	50	6	0,090

$N = 18$

4th solar cycle: 1784,7—1798,3
$P_{\odot} = 13^{y}{,}6$

$\Sigma w = 27$

SC No	comet	q A.U.	P or e y	Φ	t_0 y	Δt_1 y	$\overline{\Delta t}$ y	m magn	τ	r A.U.	Δ A.U.	H_{10} magn	w	D ′	D_0 rad. ♂	C °	S A.U.
135	1785 I	1,143	e 1,	0,027	1785,07	−0,05	0,00	7	0	1,19	0,48	7,5	1			0	0
136	1785 II	0,427	1326	0,042	1785,27	−0,08	−0,03	7	2	0,81	1,43	5,6	2	5	25	8	0,180
137	1786 I	0,335	3,281	0,101	1786,08	−0,03	−0,03	5	1	0,45	0,63	9,0	1				
138	1786 II	0,411	9373	0,134	1786,52	+0,06	+0,18	7,3	1	0,74	1,16	4,2	2			1,5	0,050
139	1787	0,349	e 1,	0,196	1787,36	−0,09	+0,06	5,5	1	0,86	1,35	5	1				
140	1788 I	1,063	e 1,	0,306	1788,86	+0,04	+0,09	5,5	1	1,09	0,38	7,5	2			3	0,020
141	1788 II	0,757	151	0,308	1788,89	+0,08	+0,14	7,5	0,5	0,97	1,05	7,1	1	6	41		
142	1790 I	0,747	e 1,	0,393	1790,04	−0,02	0,00	6,5	0	0,77	0,65	7,5	1	4	27	0	0
143	1790 II	1,044	13,90	0,396	1790,08	−0,06	−0,02	5,5	0	1,10	0,38	7,7	1			0	0
144	1790 III	0,798	e 1,	0,418	1790,39	−0,10	0,00	7	1	1,02	1,74	5,8	2	(5)	29	4	0,060
145	1792 I	1,293	e 1,	0,540	1792,04	−0,08	−0,02	6,0	1	1,36	0,89	6,0	1	6	36	15:	0,005
146	1792 II	0,966	e 1,	0,610	1792,99	+0,03	+0,09	2	1	0,99	0,21	6,1	2	30	33	3,5	0,013
147	1793 I	0,403	e 1,	0,672	1793,84	−0,10	+0,04	5	0	1,01	1,01	6,3	2			0	0
148	1793 II	1,494	389,7	0,676	1793,89	−0,16	−0,05	7,5	0	1,69	0,97	5,5	1			0	0
149	1795	0,334	3,292	0,829	1795,97	−0,12	−0,09	5,5	0	1,03	0,26	8,4	2	5	10	0	0
150	1796	0,580	e 1,	0,849	1796,25	0,00	+0,02	8	0,5	1,58	0,59	7	1	1	5	>0	>0
151	1797	0,527	e 1,	0,943	1797,52	+0,10	+0,13	3	0	0,96	0,10	8,8	2	10	6,5	0	0
152	1798 I	0,485	e 1,	0,997	1798,26	+0,02	+0,08	6	0	0,52	0,97	8,4	2			0	0

$N = 12$

5th solar cycle: 1798,3—1810,6
$P_{\odot} = 12^{y}{,}3$

$\Sigma w = 17$

SC	comet	q	P or e	Φ	t_0	Δt_1	$\overline{\Delta t}$	m	τ	r	Δ	H_{10}	w	D	D_0	C	S
No		A.U.	y		y	y	y	magn		A.U.	A.U.	magn		′	rad. ☿	°	A.U.
153	1798 II	0,780	e 1,	0,057	1799,00	−0,07	−0,06	7	0,5	0,92	0,14	11,0	1	5	4,1	>0	>0
154	1799 I	0,840	e 1,	0,112	1799,68	−0,08	−0,01	6,5	1	1,02	1,65	5,5	2	10	55	4	0,060
155	1799 II	0,626	e 1,	0,137	1799,98	+0,01	+0,03	4,5	2	0,62	0,98	6,2	2			7	0,100
156	1801	0,256	e 1,	0,268	1801,60	−0,07	−0,05	6	0	0,81	0,37	9	1			0	0
157	1802	1,094	e 1,	0,357	1802,69	−0,04	+0,02	6,8	0,5	1,12	0,39	8,3	1	3	9,5	>0	>0
158	1804	1,071	e 1,	0,473	1804,12	+0,06	+0,10	6,7	0	1,14	0,22	8,0	2	3	10,2	0	0
159	1805	0,340	3,292	0,617	1805,89	−0,09	−0,04	5	1	0,84	0,44	8,0	2	5	15	3	0,0032
160	1806 I	0,906	6,729	0,627	1806,01	−0,15	−0,10	4,5	0	1,11	0,18	7,5	1	6	8,2	0	0
161	1806 II	1,082	e 1,01018	0,707	1806,99	−0,13	0,00	6,5	1	1,34	1,82	6	1	7	66	3:	0,001
162	1807	0,646	1714	0,766	1807,72	−0,03	+0,25	2	2	0,68	1,20	1,6	2	6	49	10	0,300
163	1808 I	0,390	e 1,	0,818	1808,36	−0,13	−0,12	6	0	1,19	0,58	6,8	1	3	12	0	0
164	1808 II	0,608	e 1,	0,832	1808,53	−0,05	−0,04	7	0	0,72	0,74	9,0	1			0	0
$N = 19$					6[th] solar cycles 1810,6→1823,3 $P_{\odot} = 12{,}^{y}7$												$\Sigma w = 33$
165	1810	0,970	e 1,	0,013	1810,76	−0,12	−0,05	6,5	1	1,24	1,01	6,3	2				
166	1811 I	1,035	3094.	0,087	1811,70	−0,47	+0,23	2	2	2,72	2,16	0,0	3	28	273	70	1,300
167	1811 II	1,582	754,5	0,093	1811,78	+0,10	+0,23	6,5	1	1,60	0,74	5,2	1			10:	0,007
168	1812	0,777	73,19	0,166	1812,71	−0,16	−0,06	6,5	2	1,28	1,78	4,2	2	1	11	3	0,090
169	1813 I	0,699	e 1,	0,202	1813,17	−0,07	−0,02	6,5	0	0,89	0,39	9,0	1			0	0
170	1813 II	1,215	e 1,	0,219	1813,38	−0,13	−0,07	5,5	1	1,41	0,87	4,4	2			8:	0,002
171	1815	1,213	73,93	0,372	1815,32	−0,14	+0,10	7,5	1	1,43	1,41	4,3	2	(4)	(41)	1	0,040
172	1816	0,0485	e 1,	0,438	1816,16	−0,10	−0,08	7,5	0	1,22	0,48	8	1			0	0
173	1818 I	0,747	27,74	0,591	1818,10	+0,05	+0,06	7,5	0	0,80	0,53	9	1			0	0
174	1818 II	1,198	e 1,	0,594	1818,15	−0,16	+0,01	7	1	1,53	1,34	5,7	1				
175	1818 III	0,855	e 1,	0,656	1818,93	−0,02	+0,07	7	0	0,87	0,66	8	1			0	0
176	1819 I	0,335	3,296	0,667	1819,07	−0,16	−0,10	8	1	1,29	0,80	8	2				
177	1819 II	0,342	e 1,	0,700	1819,49	+0,01	+0,17	1,5	2	0,36	0,77	4,0	3	2	26	8	0,180
178	1819 III	0,774	5,618	0,705	1819,55	−0,10	−0,05	8	0	0,97	0,78	8,8	2			0	0
179	1819 IV	0,892	5,098	0,731	1819,89	+0,02	+0,10	6,5	0	0,89	0,30	8,5	2	6	14	0	0
180	1821	0,092	e 1,	0,836	1821,22	−0,16	−0,02	6,5	2	1,59	1,68	3,4	2	4	46	7	0,360
181	1822 I	0,504	e 1,	0,924	1822,34	+0,02	+0,08	4,5	1	0,53	0,88	6,7	1			>0	>0
183	1822 III	0,847	e 1,	0,940	1822,54	−0,13	−0,09	5,5	0	1,19	0,68	7,0	2			0	0
184	1822 IV	1,145	5449	0,961	1822,81	−0,28	−0,11	6,5	1	1,88	1,70	3,0	2	2,5	18	4	0,130

SC	comet	q	P or e	Φ	t_0	Δt_1	$\overline{\Delta t}$	m	τ	r	Δ	H_{10}	w	D	D_0	C	S
No		A.U.	y		y	y	y	magn		A.U.	A.U.	magn		′	rad. ♁	°	A.U.
$N = 20$					7[th] solar cycle: 1823,3—1833,9 $P_{\odot} = 10^y{,}6$												$\Sigma w = 32$
185	1823	0,227	e 1,	0,060	1823,94	+0,04	+0,18	3	2	0,66	0,82	4,2	2	(2)	18	5	0,070
186	1824 I	0,592	e 1,	0,116	1824,53	0,00	+0,04	5	0	0,60	0,76	7	1			0	0
187	1824 II	1,050	e 1,	0,136	1824,74	−0,18	+0,03	7	1	1,52	0,64	6,5	2				
188	1825 I	0,889	3551	0,199	1825,41	−0,03	+0,05	6,5	1	0,92	1,15	5,6	2	7	41	1,5	0,010
189	1825 II	0,883	e 1,	0,220	1825,63	−0,03	0,00	6	0	0,90	0,95	6,5	1			0	0
191	1825 IV	1,241	4472	0,249	1825,94	−0,41	+0,09	6	2	2,35	2,76	2,2	2	(6)	30	14	1,150
192	1826 I	0,902	6,721	0,275	1826,21	−0,05	+0,26	6,5	1	0,98	0,15	7,5	1	1,5	16	>0	>0
193	1826 II	2,007	1 385 200	0,284	1826,31	−0,46	−0,24	8,5	0	2,75	1,88	2,4	1	3	34	0	0
194	1826 III	0,188	e 1,	0,285	1826,32	−0,08	−0,07	7,5	0	0,93	0,45	9,5	2			0	0
195	1826 IV	0,853	6278	0,327	1826,77	−0,17	0,00	7,5	1	1,39	0,82	6,5	2	4	16	6:	0,001
196	1826 V	0,269	e 1,	0,338	1826,88	−0,07	+0,04	6,5	2	0,96	1,04	7,0	2			8	0,280
197	1827 I	0,506	e 1,	0,358	1827,10	−0,12	−0,07	6,5	1	1,01	1,26	6,3	1			6:	0,002
198	1827 II	0,807	63,83	0,390	1827,43	+0,04	+0,08	5,5	0	0,84	0,54	7,0	1			0	0
199	1827 III	0,138	2611	0,415	1827,70	−0,11	−0,05	4,5	1	1,17	1,31	7,3	1	7	34	15:	0,003
200	1829	0,345	3,316	0,541	1829,03	−0,32	−0,18	11	1	1,49	0,58	8,5	2	9	32	18:	0,0035
201	1830 I	0,921	e 1,	0,658	1830,27	−0,07	+0,16	3	2	1,01	0,19	5,2	3	2	7,5	8	0,024
202	1830 II	0,126	e 1,	0,725	1830,99	+0,03	+0,13	2	2	0,42	1,02	6,2	2	4	16	3	0,044
203	1832 I	0,343	3,312	0,853	1832,34	+0,08	+0,12	8,5	0	0,74	0,36	9,5	1			0	0
204	1832 II	1,183	e 1,	0,890	1832,73	−0,18	−0,13	7,5	0	1,58	0,85	6,0	1			0	0
206	1833	0,464	e 1,	0,980	1833,69	+0,06	+0,08	6	1	0,67	1,06	7,0	2	1,0	7,5	>0	0,0007
$N = 13$					8[th] solar cycle: 1833,9—1843,5 $P_{\odot} = 9^y{,}6$												$\Sigma w = 19$
207	1834	0,513	e 1,	0,036	1834,25	−0,07	−0,02	3,5	0	0,76	0,62	7,0	2	5	19	0	0
208	1835 I	2,040	e 1,	0,139	1835,23	+0,07	+0,12	8,3	1	2,06	1,12	4,2	1	4	31	5:	0,005
209	1835 II	0,344	3,314	0,182	1835,65	−0,09	−0,07	6,5	0	0,86	1,52	7,7	1			0	0
210	1835 III	0,587	76,30	0,206	1835,88	−0,29	+0,11	10	2	2,07	2,59	4,4	3			20	0,130
211	1838	0,344	3,313	0,528	1838,97	−0,35	−0,18	11,5	0	2,07	1,59	9,4	2	10	20	0	0

SC	comet	q	P or e	Φ	t_0	Δt_1	$\overline{\Delta t}$	m	τ	r	Δ	H_{10}	w	D	D_0	C	S
No		A.U.	y		y	y	y	magn		A.U.	A.U.	magn			rad. ♂	°	A.U.
212	1840 I	0,618	599 300	0,636	1840,01	−0,09	+0,01	4,5	1	0,92	0,82	6,3	1				
213	1840 II	1,221	3 789	0,656	1840,20	−0,13	−0,05	7,5	0	1,43	1,17	6,1	1	>1	>9	0	0
214	1840 III	0,749	e 1,	0,661	1840,25	−0,07	−0,04	6,5	1	0,91	1,34	6,0	1			5	0,170
215	1840 IV	1,481	367,2	0,726	1840,87	−0,05	+0,14	9	0	1,50	1,11	6,5	1	1,3	8	0	0
216	1842 I	0,345	3,331	0,873	1842,28	−0,17	−0,03	12,5	0	1,33	1,90	9,5	1	1	7	0	0
217	1842 II	0,504	e 1,	0,944	1842,96	−0,14	−0,09	2,5	1	1,18	0,68	8,8	2			15:	0,002
218	1843 I	0,0055	512,4	0,965	1843,16	−0,06	+0,04	3,5	2	0,88	0,69	4,9	1	45:	4,5	64	1,250
219	1843 II	1,616	e 1,000 183	0,983	1843,34	−0,01	+0,18	7,5	0	1,62	1,69	4,2	2	3	34	0	0

$N = 48$ — 9[th] solar cycle: 1843,5—1856,0 — $P_{\odot} = 12^{y}5$ — $\Sigma w = 85$

SC No	comet	q	P or e	Φ	t_0	Δt_1	$\overline{\Delta t}$	m	τ	r	Δ	H_{10}	w	D	D_0	C	S
220	1843 III	1,692	7,436	0,023	1843,79	+0,10	+0,29	5,8	1	1,72	0,79	4,2	2	2	28	20:	0,021
221	1844 I	1,186	5,459	0,094	1844,67	−0,03	+0,15	7	1	1,19	0,20	8,0	2			>0	>0
222	1844 II	0,855	98 030	0,103	1844,79	−0,28	+0,06	6,5	1	1,92	1,42	4,9	2	3	28	10:	0,007
223	1844 III	0,252	e 1,000 353	0,116	1844,95	+0,01	+0,13	2	2	0,27	0,90	4,9	2	3	29	10	0,190
224	1845 I	0,905	e 1,000 25	0,122	1845,02	−0,03	+0,10	6,5	0	0,92	0,79	9,2	2			0	0
225	1845 II	1,255	e 1,	0,144	1845,30	−0,15	−0,06	7	0	1,50	0,66	7,5	1			0	0
226	1845 III	0,401	308 000	0,154	1845,43	−0,01	+0,03	3	2	0,41	0,77	4,0	2			5	0,040
228	1846 I	1,482	e 1,	0,205	1846,06	+0,01	+0,14	7	1	1,48	1,00	6	1	8	62	>0	>0
229	1846 II	0,856	6,601	0,209	1846,11	−0,21	0,00	10,5	1	1,44	0,96	8,0	2			45:	0,006
230	1846 III	0,650	5,569	0,212	1846,15	+0,01	+0,08	7,5	0	0,65	0,66	7,7	1	10	13	0	0
231	1846 IV	0,664	75,71	0,214	1846,18	−0,04	+0,08	6	1	0,72	1,08	7,2	2	3	48	17:	0,007
232	1846 V	1,376	e 1,	0,232	1846,40	+0,16	+0,28	9	0	1,64	1,72	6,2	2			0	0
233	1846 VI	1,529	13,38	0,234	1846,42	+0,06	+0,10	9	1	1,56	0,68	8,0	1			>0	>0
234	1846 VII	0,634	499,9	0,234	1846,43	−0,10	−0,04	7,7	1	0,96	0,35	8,1	2			20:	0,0047
235	1846 VIII	0,831	e 1,	0,266	1846,83	−0,10	−0,05	8,5	0	1,15	0,87	8	1			0	0
236	1847 I	0,0426	10 219	0,299	1847,24	−0,14	−0,03	7,5	2	1,49	1,17	6,8	2	6	33	4,3	0,060
237	1847 II	2,115	e 1,	0,314	1847,43	−0,08	+0,25	9,5	0	2,13	1,98	4,8	2			0	0
238	1847 III	1,766	44 229	0,329	1847,61	−0,10	+0,30	6,5	1	1,80	1,77	5,3	2	5,2	61	8:	0,010
239	1847 IV	1,485	e 1,	0,328	1847,60	+0,06	+0,19	8	0	1,52	1,04	5,7	2	3	23	0	0
240	1847 V	0,488	68,07	0,335	1847,69	−0,14	−0,06	9,5	1	1,21	0,80	9,6	1			15:	0,003
241	1847 VI	0,329	e 1,000 17	0,350	1847,87	−0,12	−0,01	5	2	1,15	0,40	7,3	2	30	89	2	0,016

SC	comet	q	P or e	Φ	t_0	Δt_1	$\overline{\Delta t}$	m	τ	r	Δ	H_{10}	w	D	D_0	C	S
No		A.U.	y		y	y	y	magn		A.U.	A.U.	magn		′	rad. ♁	°	A.U.
242	1848 I	0,320	e 1,	0,415	1848,69	−0,09	−0,06	7	0	0,82	0,84	8,1	1	1,5	11	0	0
243	1848 II	0,337	3,296	0,432	1848,90	−0,25	−0,12	11,5	1	1,58	1,04	8,5	3	8	35	2	0,020
244	1849 I	0,960	e 1,	0,444	1849,05	−0,23	−0,10	7,5	1	1,70	1,20	4,6	2	3	25	2	0,010
245	1849 II	1,159	e 1,000 71	0,472	1849,40	−0,11	+0,11	6,5	1	1,35	0,75	7,2	2	6	23	5:	>0,001
246	1849 III	0,894	8 375	0,474	1849,43	−0,15	+0,04	7,5	0	1,36	0,45	7,4	2	(2)	(3,5)	0	0
247	1850 I	1,081	28 909	0,565	1850,56	−0,23	0,00	8,5	1	1,72	1,52	6,0	2	10	31	5	0,050
248	1850 II	0,565	e 1,	0,584	1850,80	−0,14	−0,03	8,5	1	1,20	0,63	7,9	2	3	8,9	(>0)	(>0)
250	1851 II	1,173	6,390	0,642	1851,52	−0,03	+0,11	10	0	1,18	0,71	9,5	2	3	15	0	0
251	1851 III	0,985	e 1,	0,652	1851,65	−0,07	+0,02	7,5	0	1,07	0,83	7,6	2			0	0
252	1851 IV	0,142	e 1,	0,660	1851,75	+0,06	+0,10	4	1	0,75	0,98	6,0	2	2,1	20	1	0,018
253	1852 I	0,338	3,296	0,696	1852,20	−0,18	−0,09	11,5	1	1,35	1,55	9,8	1	2	25	1	0,015
254	1852 II	0,905	e 1,	0,704	1852,30	+0,07	+0,11	9,5	0	1,02	0,81	9,8	2			0	0
255	1852 III	0,861	6,621	0,738	1852,73	−0,08	−0,03	9,5	1	0,96	1,44	8,1	2	1,7	10,2		
256	1852 IV	1,250	61,56	0,742	1852,78	−0,22	+0,06	7,5	1	1,70	1,13	5,0	2	5	27	5:	0,010
257	1853 I	1,092	e ,1	0,772	1853,15	+0,03	+0,09	6,5	0	1,11	0,59	7,3	1	2	34	0	0
258	1853 II	0,909	782,1	0,788	1853,35	−0,09	0,00	7,5	2	1,11	0,92	6	1			10	0,020
259	1853 III	0,307	e 1,000 25	0,814	1853,67	−0,23	+0,06	7,5	2	1,84	2,21	4,8	2	1,6	16	12,5	0,180
260	1853 IV	0,173	e 1,001 23	0,823	1853,79	−0,09	+0,03	7,5	2	1,05	1,0	7,0	2	5	24	4	0,050
261	1854 I	2,045	e 1,	0,841	1854,01	−0,11	+0,02	8,5	1	2,12	1,12	4,3	2			4:	0,004
262	1854 II	0,277	e 1,	0,858	1854,23	−0,01	+0,04	2	2	0,28	0,96	7,0	2			5	0,080
263	1854 III	0,648	e 1,	0,878	1854,47	−0,05	+0,03	5,5	1	0,76	1,06	6,4	2	5	16	1,5	0,030
264	1854 IV	0,799	1 089	0,906	1854,82	−0,12	−0,01	7,5	0	1,16	0,85	8,3	2	6	34	0	0
265	1854 V	1,357	944,2	0,917	1854,96	+0,08	+0,22	9,5	0	1,37	1,60	6,9	1	5	53	0	0
266	1855 I	2,194	500,1	0,928	1855,10	+0,18	+0,26	8,5	0	2,34	1,32	4,0	1	0,7	8,2	0	0
267	1855 II	0,568	252,2	0,953	1855,41	+0,01	+0,09	9,7	0	0,59	0,62	11,3	2	1	4,1	0	0
268	1855 III	0,337	3,296	0,960	1855,50	+0,03	+0,08	6	0	0,46	0,95	9,4	2	1,5	13	0	0
269	1855 IV	1,231	e 1,	0,992	1855,90	−0,03	+0,04	6	0	1,25	1,04	8,1	2	15	27	0	0

$N = 38$ — 10[th] solar cycle: 1856,0—1867,2, $P_{\odot} = 11^{y}2$ — $\Sigma w = 70$

SC	comet	q	P or e	Φ	t_0	Δt_1	$\overline{\Delta t}$	m	τ	r	Δ	H_{10}	w	D	D_0	C	S
270	1857 I	0,772	e 1,	0,109	1857,22	−0,08	+0,02	7,5	1	0,95	1,56	7,1	2	3	34	5:	0,002
271	1857 II	0,620	5,538	0,111	1857,24	−0,03	+0,10	5,5	1	0,65	1,10	7,7	2	2,8	17	11:	0,003
272	1857-III	1,368	e 1,	0,137	1857,54	−0,07	−0,03	5	1	0,79	1,25	9,0	1	2	14	30:	0,010

SC	comet	q	P or e	Φ	t_0	Δt_1	$\overline{\Delta t}$	m	τ	r	Δ	H_{10}	w	D	D_0	C	S
No		A.U.	y		y	y	y	magn		A.U.	A.U.	magn		′	rad. ☉	°	A.U.
273	1857 IV	0,747	234,7	0,147	1857,65	−0,09	+0,03	8,5	1	0,95	0,67	9,1	1	3	14	5:	>0,001
274	1857 V	0,563	2 463	0,156	1857,75	−0,11	−0,05	7,5	2	1,04	0,67	4,9	3	6	20	4	0,100
275	1857 VI	1,009	6 143	0,168	1857,88	−0,02	+0,04	9	0	1,02	0,63	9,9	1	3	14	0	0
277	1858 I	1,026	13,74	0,192	1858,15	−0,14	−0,03	8	1	1,28	0,81	7,8	2	6	34		
278	1858 II	0,769	5,620	0,208	1858,33	−0,15	0,00	7,5	1	1,16	0,60	9,0	2	3	12		
279	1858 III	1,146	6,018	0,209	1858,34	−0,01	+0,04	9,5	0	1,15	0,36	11,3	1			0	0
280	1858 IV	0,544	e 1,	0,217	1858,43	−0,04	+0,04	7	1	0,63	1,08	8,6	2	4	20	30:	0,010
281	1858 V	1,694	7,448	0,241	1858,70	−0,02	+0,09	11,5	0	1,70	1,51	8,1	2			0	0
282	1858 VI	0,578	1 950	0,246	1858,75	−0,33	+0,05	7,5	2	2,23	2,39	3,3	3			41	0,550
283	1858 VII	1,427	6 002	0,248	1858,78	−0,10	−0,01	5,5	0	1,53	1,16	5,9	1	6	20	0	0
285	1859	0,201	e 1,000 03	0,304	1859,41	−0,16	−0,03	7,5	1	1,41	0,80	7,0	2	4	25	20:	0,004
286	1860 I	1,199	e 1,	0,369	1860,13	+0,03	+0,03	9	0	1,21	0,59	7	1	30:	2	0	0
287	1860 II	1,307	e 1,	0,373	1860,18	+0,11	+0,19	8,5	0	1,44	2,05	5,2	1	2	29	0	0
288	1860 III	0,293	e 1,	0,398	1860,46	0,00	+0,17	3	2	0,30	0,97	5,8	3			20	0,280
289	1860 IV	0,683	e 1,	0,422	1860,73	+0,08	+0,09	7,5	0	0,93	0,58	9,5	1	4	14	0	0
290	1861 I	0,921	415,4	0,484	1861,42	−0,16	+0,05	(5)	1	1,30	0,69	5,5	2	20	49	3,5	0,020
291	1861 II	0,822	409,4	0,486	1861,44	−0,08	+0,41	4,5	2	1,12	0,90	3,9	3	13	31	118	0,340
292	1861 III	0,839	e 1,	0,529	1861,93	+0,06	+0,11	7	0	0,93	0,71	9,3	2	4	9,6	0	0
294	1862 II	0,981	e 1,	0,578	1862,47	+0,03	+0,07	4,5	1	1,00	0,11	9,4	2	34	20	30:	0,001
295	1862 III	0,963	119,6	0,593	1862,64	−0,10	+0,04	6	2	1,20	1,55	4,0	3	4	44	25	0,220
296	1862 IV	0,803	e 1,	0,624	1862,99	−0,08	+0,04	7	0	0,95	1,63	6,5	1	3	32	0	0
297	1863 I	0,795	e 1,000 07	0,633	1863,09	−0,18	−0,04	9	0,5	1,42	0,60	8,4	2	1	10		
298	1863 II	1,068	e 1,	0,648	1863,26	+0,02	+0,32	5,5	1	1,09	0,70	5	2	30:	10,2	1	0,050
299	1863 III	0,629	17 740	0,652	1863,30	−0,02	+0,05	6	2	0,65	0,85	6,8	1			3	0,050
300	1863 IV	0,707	18 368	0,702	1863,86	−0,02	+0,12	4	1	0,71	0,80	5,7	3	3	34	10,6	0,150
301	1863 V	0,771	e 1,	0,713	1863,99	0,00	+0,09	6,5	1	0,77	0,80	8,2	1	4	17	1,5	0,100
302	1863 VI	1,313	e 1,000 65	0,713	1863,99	−0,22	+0,04	7,5	1	1,76	2,05	4,2	2	5	44	8:	0,010
303	1864 I	0,626	e 1,	0,765	1864,57	+0,12	+0,16	8,5	0	1,09	1,82	7,0	1	1,5	18	0	0
304	1864 II	0,909	3 934	0,770	1864,62	−0,11	+0,02	6,5	1	1,18	1,26	6,2	3	32	26	40	0,090
305	1864 III	0,931	55 242	0,784	1864,78	−0,22	+0,08	9,5	1	1,70	1,74	5,2	2	2	28	20:	0,008
306	1864 IV	0,771	e 1,	0,802	1864,98	−0,02	+0,08	7	0,5	0,80	1,38	5,2	2	1	9,6		
307	1864 V	1,115	e 1,000 06	0,803	1864,99	+0,01	+0,05	9	0	1,12	0,99	9,5	1	2	14	0	0
308	1865 I	0,025	e 1,	0,807	1865,04	+0,01	+0,15	3	2	0,56	1,15	3,8	2	10	61	25	0,450
310	1866 I	0,977	33,18	0,896	1866,03	−0,06	+0,01	6	1	1,05	1,21	8,0	1	12	12	>0	≥0
312	1867 I	1,577	40,09	0,987	1867,05	+0,01	+0,12	7	1	1,58	1,21	7,2	2	3	25	>0	>0

SC	comet	q	P or e	Φ	t_0	Δt_1	$\overline{\Delta t}$	m	τ	r	Δ	H_{10}	w	D	D_0	C	S
No		A.U.	y		y	y	y	magn		A.U.	A.U.	magn		′	rad. ♉	°	A.U.
$N = 29$					11[th] solar cycle: 1867,2–1878,9 $P_\odot = 11^y\!,7$												$\Sigma w = 56$
313	1867 II	1,564	5,696	0,016	1867,39	−0,14	+0,06	9	1	1,62	0,71	8,4	2	5	25		
314	1867 III	0,330	e 1,	0,056	1867,85	−0,11	−0,06	6,5	1	1,01	1,01	7,9	2	4	27	15:	0,004
316	1868 II	0,579	e 1,	0,109	1868,48	−0,03	+0,02	5	1			7,6	2	5	24	>3	0,020
317	1868 III	0,334	3,288	0,128	1868,70	−0,16	−0,09	11,5	1	1,27	1,70	9,0	2	1,6	32	>0	>0
319	1869 II	1,231	e 1,	0,220	1869,77	+0,01	+0,06	9	0	1,24	1,97	5,1	2	1,5	14	0	0
320	1869 III	1,064	5,484	0,229	1869,88	+0,03	+0,08	8,5	0	1,08	0,26	11,4	1	6	10	0	0
321	1870 I	1,009	e 1,	0,285	1870,53	−0,12	−0,06	7	1	1,23	1,62	6,2	2	3	27	20:	0,008
322	1870 II	1,817	2 056	0,297	1870,67	−0,01	+0,15	7,5	0	1,82	1,23	3,8	3	5,5	46	0	0
324	1870 IV	0,389	e 1,	0,322	1870,97	−0,07	−0,06	8,5	0	0,78	0,50	11,1	1	4	12	0	0
325	1871 I	0,654	5 188	0,362	1871,44	−0,17	−0,01	(8)	1	1,44	1,87	5,3	2	2,2	26	10:	0,010
326	1871 II	1,083	e 1,	0,374	1871,57	−0,12	+0,02	8	0,5	1,29	1,42	6,5	2	4	33		
328	1871 IV	0,691	e 1,	0,408	1871,97	−0,13	+0,02	8,5	0	1,12	1,27	8,0	2	3	27	0	0
330	1872	0,064	e 1,	0,492	1872,96	−0,04	−0,04	6,3	1	0,54	0,83	6,3	1	75:	6,8	8:	0,003
332	1873 II	1,344	5,207	0,537	1873,48	+0,02	+0,16	9,5	0	1,35	0,70	8,5	2	5	23	0	0
334	1873 IV	0,794	3 375	0,555	1873,69	−0,05	−0,01	6,5	0,5	0,91	1,24	6,7	2	3	25		
335	1873 V	0,385	53 900	0,560	1873,75	−0,11	+0,03	6,5	1	1,05	1,13	6,4	3	10	61	>3	0,040
337	1873 VII	0,747	28,03	0,574	1873,92	−0,06	−0,05	8,5	0	0,82	0,25	11,6	1	6	10	0	0
338	1874 I	0,045	e 1,	0,597	1874,19	−0,05	−0,04	8,5	1	0,62	0,68	11,0	1	3	10	2:	0,0006
339	1874 II	0,886	e 1,	0,598	1874,20	+0,08	+0,17	6,5	0	1,02	0,93	5,0	2	7	45	0	0
340	1874 III	0,676	13 708	0,626	1874,52	−0,23	+0,03	7,5	2	1,68	1,72	5,7	3	4	41	48	0,290
341	1874 IV	1,688	306,0	0,627	1874,54	+0,09	+0,21	8,5	0	1,69	1,38	6,2	2	3	27	0	0
342	1874 V	0,983	24 368	0,637	1874,65	−0,09	+0,03	7,5	0	1,12	0,60	9,4	2	5	21	0	0
343	1874 VI	0,508	e 1,	0,650	1874,80	+0,13	+0,18	8,5	0	1,18	1,30	7,6	1	4	17	0	0
346	1877 I	0,807	e 1,	0,842	1877,05	+0,06	+0,13	6,5	0	0,90	0,46	8,2	2	20	53	0	0
347	1877 II	0,950	19 765	0,862	1877,29	−0,03	+0,11	10,5	1	0,96	1,40	5,7	3	10	61	2:	0,050
348	1877 III	1,009	10 717	0,865	1877,32	−0,05	+0,03	7,5	0	1,07	1,45	6,7	2	1,5	18	0	0
350	1877 V	1,070	e 1,	0,879	1877,49	+0,26	+0,30	7,5	1	1,82	0,90	6,0	2	3	18	5:	0,002
351	1877 VI	1,576	e 1,	0,897	1877,70	0,00	+0,12	9,5	0	1,58	1,86	6,3	2	2	14	0	0
352	1878 I	1,392	e 1,	0,970	1878,55	−0,04	−0,01	7,5	0	1,41	0,49	8,5	2			0	0

SC	comet	q	P or e	Φ	t_0	Δt_1	$\overline{\Delta t}$	m	τ	r	Δ	H_{10}	w	D	D_0	C	S
No		A.U.	y		y	y	y	magn		A.U.	A.U.	magn		′	rad. ♁	°	A.U.
$N = 47$					12th solar cycle: 1878,9 — 1889,6; $P_\odot = 10^y\!,7$												$\Sigma w = 101$
356	1879 II	0,897	e 1,	0,039	1879,32	+0,14	+0,23	6,5	1	1,20	1,60	4,5	2	3	33	0,4:	0,0003
358	1879 IV	0,991	e 1,	0,071	1879,66	−0,01	+0,02	10,5	0	0,99	1,10	10,5	1	4	40	0	0
359	1879 V	0,990	e 1,	0,080	1879,76	−0,12	−0,03	7,5	0	1,23	1,85	5,5	2	2,7	48	0	0
360	1880 I	0,005	e 1,	0,109	1880,07	+0,01	+0,04	5	2	0,22	0,85	8,0	2	3	14	>50	1,000
361	1880 II	1,814	e 1,000 84	0,150	1880,50	−0,24	+0,06	7,5	1	2,10	1,98	2,7	2	2	25	6:	0,0004
362	1880 III	0,355	e 1,	0,166	1880,68	+0,06	+0,15	5,5	1	0,71	0,50	7,7	3	8,7	75	4	0,030
363	1880 IV	1,067	5,492	0,182	1880,85	+0,01	+0,11	8	0	1,21	0,29	12,2	2	14,5	20	0	0
364	1880 V	0,387	e 1,	0,183	1880,86	+0,11	+0,12	4,8	0	1,09	0,26	7,4	1			0	0
365	1880 VI	0,660	e 1,	0,183	1880,86	+0,10	+0,25	7,5	1	0,99	1,58	5,8	3	7	78	>0,8:	>0,001
366	1881 I	1,738	7,566	0,202	1881,06	−0,47	−0,14	10,5	0	2,18	1,19	7,4	3	1,7	18	0	0
367	1881 II	0,591	e 1,	0,232	1881,38	−0,05	−0,03	7	0	0,74	1,11	8,4	1	2	15	0	0
368	1881 III	0,734	2429	0,239	1881,46	−0,07	+0,38	3	2	0,75	0,31	4,1	3	20	66	15	0,150
369	1881 IV	0,634	e 1,	0,256	1881,64	−0,11	+0,03	6	2	1,00	1,77	6,3	2	7	55	10	0,240
370	1881 V	0,735	8,687	0,262	1881,70	+0,14	+0,17	7,5	0	0,85	0,83	9,0	1	2	14	0	0
371	1881 VI	0,449	e 1,	0,262	1881,70	+0,01	+0,07	6,5	1	0,47	1,11	8,3	1	2	15	8:	0,002
373	1881 VIII	1,923	612,2	0,279	1881,89	−0,01	+0,07	8,5	0	1,93	1,10	4,3	1	4,2	32	0	0
374	1882 I	0,061	1 174 000	0,331	1882,44	−0,23	−0,02	7	2	2,04	1,74	4,1	3	3,5	48	> 5	0,940
375	1882 II	0,008	760,9	0,356	1882,71	−0,04	+0,34	0	2	0,68	1,35	0,8	3	3	89	30	1,250
376	1882 III	0,956	e 1,000 07	0,371	1882,87	−0,17	−0,05	10	1	1,39	1,50	7,4	2	2	20	9:	0,004
377	1883 I	0,760	e 1,	0,396	1883,14	+0,01	+0,08	6,5	1	0,77	1,16	7,0	3	10,5	126	70:	0,023
378	1883 II	0,309	64,63	0,475	1883,98	+0,04	+0,10	6	1	0,49	0,85	7,0	2	2	14	105:	0,030
379	1884 I	0,776	71,56	0,483	1884,07	−0,40	−0,02	10	1	2,43	2,37	4,9	3	11,5	79	20	0,250
380	1884 II	1,280	5,400	0,535	1884,62	−0,08	+0,10	9,5	1	1,33	0,44	8,9	2	3	20	> 0	>0
381	1884 III	1,572	6,774	0,559	1884,88	−0,17	+0,11	9,5	1	1,69	0,82	6,2	3	3	11,6	> 0	>0
383	1885 II	2,508	e 1,002 85	0,625	1885,59	−0,08	0,00	11	1	2,52	1,53	5,8	2	1,7	16	4:	0,002
384	1885 III	0,749	274,5	0,627	1885,61	+0,06	+0,11	8,5	0	0,86	1,12	9,5	2	3	29	0	0
386	1885 V	1,080	e 1,	0,654	1885,90	+0,09	+0,18	7,5	1	1,20	1,85	6,3	1	3	38	> 0	>0
387	1886 I	0,642	e 1,000 45	0,688	1886,26	−0,34	−0,01	11	2	1,90	1,73	5,2	3	15	41	>10	0,070
388	1886 II	0,479	e 1,000 23	0,695	1886,34	−0,42	−0,09	9,5	2	2,57	1,78	6,6	3	3	34	3	0,030
389	1886 III	0,843	e 1,013 0	0,695	1886,34	−0,01	+0,03	4,5	1	0,85	0,98	4,9	2	1,5	10	1	0,020
390	1886 IV	1,328	5,595	0,704	1886,43	−0,04	+0,02	8,5	0	1,35	0,56	8,9	2	2	10	0	0
391	1886 V	0,270	745	0,704	1886,43	−0,11	+0,02	8	0	1,28	1,40	7,5	2	2,3	18	0	0

SC	comet	q	P or e	Φ	t_0	Δt_1	$\overline{\Delta t}$	m	τ	r	Δ	H_{10}	w	D	D_0	C	S
No		A.U.	y		y	y	y	magn		A.U.	A.U.	magn		′	rad. ♁	°	A.U.
393	1886 VII	0,998	6,648	0,747	1886,89	−0,15	+0,13	11	0	1,33	1,13	9,2	3	3	17	0	0
394	1886 VIII	1,481	e 1,	0,749	1886,91	−0,15	+0,32	10	0	1,68	2,12	4,8	2	1	27	0	0
395	1886 IX	0,663	e 1,000 38	0,753	1886,96	−0,20	+0,15	8	1	1,41	1,93	4,9	3	10	92	7	0,190
396	1887 I	0,010	e 1,	0,760	1887,03	+0,02	+0,04	1,5	2	0,43	0,66	6,3	2	7	27	41	0,350
397	1887 II	1,630	999,4	0,777	1887,21	−0,15	−0,02	9,5	0	1,78	1,45	5,4	2	3	31	0	0
398	1887 III	1,007	e 1,000 42	0,779	1887,24	−0,11	−0,04	10,5	0	1,26	0,25	10,9	2	4	15	0	0
399	1887 IV	1,394	6 725	0,800	1887,46	−0,10	+0,02	9,5	1	1,49	0,48	9,6	3	2	6	3,6:	0,001
400	1887 V	1,199	72,40	0,829	1887,77	−0,12	+0,31	9	1	1,34	2,10	5,0	2	2	29	10:	0,049
401	1888 I	0,699	2 182	0,870	1888,21	−0,08	+0,20	3	2	0,91	0,87	4,7	3	5,5	<70	8	0,130
403	1888 III	0,902	969 600	0,905	1888,58	+0,02	+0,14	9,5	1	0,93	1,48	7,6	1	30:	5,5	7:	0,003
405	1888 V	1,528	2 367	0,916	1888,70	+0,13	+0,41	9,5	1	1,59	1,82	5,8	2	4	38	3:	0,002
406	1889 I	1,815	e 1,001 26	0,951	1889,08	−0,41	+0,92	11	0,5	2,55	2,82	3,6	3	6	48	(1)	(0,030)
407	1889 II	2,256	579 200	0,978	1889,36	−0,11	+0,67	13	1	2,40	2,76	5,3	1	1,5	14	15:	0,012
408	1889 III	1,102	128,3	0,988	1889,47	+0,01	+0,07	9,5	0	1,12	1,12	9,4	1	2	14	0	0
409	1889 IV	1,040	9 739	0,995	1889,55	0,00	+0,17	3,5	1	1,04	0,37	6,5	3	5	25	30:	0,003

$N = 48$ — 13th solar cycle: 1889,6 – 1901,7; $P_{\odot} = 12^{y}{,}1$ — $\Sigma w = 103$

SC No	comet	q	P or e	Φ	t_0	Δt_1	$\overline{\Delta t}$	m	τ	r	Δ	H_{10}	w	D	D_0	C	S
410	1889 V	1,950	7,074	0,012	1889,75	−0,24	+0,53	11	1	2,09	1,45	7,2	3	5	44	15:	0,014
411	1889 VI	1,356	8,917	0,019	1889,83	+0,05	+0,14	10,5	0	1,38	0,66	11,5	2	4	17	0	0
412	1890 I	0,270	e 1,	0,039	1890,07	−0,12	−0,07	9,5	1	1,18	1,02	8,8	2	3	20	45:	0,011
413	1890 II	1,908	e 1,000 41	0,068	1890,42	−0,21	+0,74	7,5	1	2,11	2,60	3,3	1	1	41	15:	0,008
414	1890 III	0,764	e 1,	0,076	1890,52	+0,02	+0,06	8	0,5	0,78	1,55	8,6	1	1,8	21		
415	1890 IV	2,047	11 040	0,083	1890,60	+0,27	+0,36	8	0	2,35	1,46	5,2	1	1	27	0	0
416	1890 V	1,324	6,691	0,092	1890,71	+0,05	+0,15	11	0	1,34	0,86	9,7	2	2	16	0	0
417	1890 VI	1,260	57 510	0,093	1890,73	−0,17	−0,02	9	0	1,61	1,57	8,1	2	1	10	0	0
418	1890 VII	1,817	6,373	0,101	1890,82	+0,06	+0,17	11,5	0	1,83	0,95	9,0	2	30:	3	0	0
419	1891 I	0,398	e 1,	0,142	1891,32	−0,08	+0,03	8,5	1	0,85	1,26	8,8	2	4	34	30:	0,009
421	1891 III	0,340	3,303	0,182	1891,80	−0,22	−0,12	16,8	1	1,53	1,60	9,1	3	2	14	15:	0,005
422	1891 IV	0,971	e 1,	0,188	1891,87	−0,13	−0,03	12	0	1,18	0,92	9,4	1	1	7	0	0
424	1892 I	1,027	24 480	0,220	1892,26	−0,08	+0,40	4	2	1,16	1,13	3,2	3	10	68	15	0,560
425	1892 II	1,971	e 1,000 34	0,228	1892,36	−0,15	+0,26	11	0	2,06	2,37	5,4	2	3	48	0	0

SC	comet	q	P or e	Φ	t_0	Δt_1	$\overline{\Delta t}$	m	τ	r	Δ	H_{10}	w	D	D_0	C	S
No		A.U.	y		y	y	y	magn		A.U.	A.U.	magn		′	rad. ♁	°	A.U.
426	1892 III	2,142	6,902	0,236	1892,45	+0,40	+0,57	5,5	1	2,38	1,48	3,0	2	30	340	30:	0,130
428	1892 V	1,434	6,634	0,276	1892,94	−0,16	−0,08	11,5	1	1,51	1,10	9,8	2	1	7,5	>0	>0
429	1892 VI	0,976	e 1,	0,280	1892,99	−0,33	+0,08	10,5	1	2,14	2,30	5,3	3	3	25	5	0,094
430	1893 I	1,195	e 1,001 59	0,283	1893,02	−0,14	+0,02	10	1	1,41	1,75	7,8	3	5	22	>0	>0
431	1893 II	0,675	44 410	0,323	1893,51	+0,01	+0,24	3,5	2	0,78	0,50	6,6	3	8	27	18	0,250
432	1893 III	0,989	6,622	0,325	1893,53	−0,16	0,00	11	0	1,28	1,31	10,1	3	1,5	9	0	0
433	1893 IV	0,812	2 516	0,340	1893,72	+0,07	+0,21	7	1	0,97	1,74	6,6	3	2	23	10	0,820
434	1894 I	1,147	7,418	0,373	1894,11	+0,12	+0,22	10	1	1,22	0,48	10,4	2	5	17	3:	0,001
435	1894 II	0,983	959	0,387	1894,28	−0,03	+0,17	6,5	1	1,02	0,94	6,5	1	25	89	6	0,035
437	1894 IV	1,392	5,855	0,428	1894,78	+0,11	+0,21	13	1	1,45	1,02	10,2	2	1	9,5	>0	>0
439	1895 II	1,298	7,219	0,499	1895,64	0,00	+0,23	10,5	1	1,30	0,35	11,4	2	<9	48	5:	0,0004
440	1895 III	0,843	e 1,	0,512	1895,80	+0,09	+0,13	7,5	0	1,03	0,40	10,5	1	10	23	0	0
441	1895 IV	0,192	e 1,	0,526	1895,96	−0,08	+0,12	6,5	2	0,95	1,61	5,2	3	3	45	30	0,133
442	1896 I	0,585	e 1,000 65	0,536	1896,09	+0,03	+0,12	7,5	1	0,66	0,56	8,8	2	6	26	2	0,020
444	1896 III	0,566	e 1,000 48	0,553	1896,29	−0,01	+0,09	6,5	1	0,59	0,55	10,3	3	2,8	12	8:	0,0016
445	1896 IV	1,143	e 1,	0,572	1896,52	+0,15	+0,20	10	0	1,43	1,71	8,4	1	2	20	0	0
446	1896 V	1,455	6,646	0,597	1896,82	−0,14	+0,03	12,5	0	1,56	1,01	9,9	2	1	7	0	0
448	1896 VII	1,110	6,441	0,603	1896,90	+0,04	+0,16	8	1	1,12	0,27	9,9	1	6	10	30:	0,003
449	1897 I	1,063	e 1,000 94	0,621	1897,11	−0,27	+0,10	11	0	1,84	1,49	7,7	2	4	34	0	0
451	1897 III	1,357	e 1,	0,689	1897,94	−0,15	−0,09	8	1	1,56	0,81	8,0	2	>5	16	10:	0,003
452	1898 I	1,095	402,8	0,712	1898,21	0,00	+0,33	7	1	1,10	1,58	5,7	3	2,6	27	2	0,070
456	1898 V	1,501	e 1,	0,740	1898,56	−0,10	−0,02	11,5	0	1,59	0,63	9,6	2	2	8,2	0	0
457	1898 VI	0,626	e 1,	0,745	1898,62	−0,17	−0,09	10	0	1,30	1,96	7,6	2	9	27	0	0
458	1898 VII	1,702	e 1,001 03	0,752	1898,70	−0,26	+0,11	8	1	2,22	1,10	5,0	3	5	34	>0	>0
459	1898 VIII	2,285	210 800	0,754	1898,72	+0,17	+0,27	12	0	2,38	2,19	5,8	1	1,5	19	0	0
460	1898 IX	0,420	e 1,	0,760	1898,80	−0,10	−0,06	8	1	1,04	1,56	7,0	3	5	41	30:	0,006
461	1898 X	0,756	158 700	0,768	1898,89	−0,09	−0,04	7	1	0,99	0,62	9,2	3	5	34	10:	0,003
462	1899 I	0,327	e 1,000 34	0,800	1899,28	−0,11	+0,11	7	1	1,08	0,79	5,4	3	12	41	10	0,120
464	1899 III	1,019	13,67	0,805	1899,34	−0,17	0,00	11,5	0	1,34	1,80	8,5	2	2	24	0	0
466	1899 V	1,786	e 1,	0,836	1899,71	+0,03	+0,15	11	0	1,81	2,03	6,5	2	1	16	0	0
467	1900 I	1,332	e 1,	0,886	1900,32	−0,24	+0,04	11	1	1,85	1,59	8,0	2	1,5	20	>0	>0
468	1900 II	1,015	e 1,000 33	0,908	1900,59	−0,03	+0,18	6,5	1	1,03	0,46	8,2	2	25	82	6	0,052
469	1900 III	0,932	6,524	0,935	1900,91	+0,06	+0,17	10,5	1	1,02	0,93	10,8	1	3	7,5	3:	0,001
470	1901 I	0,245	39 080	0,968	1901,31	−0,03	+0,06	1,5	2	0,46	0,75	5,9	2	1,5	12	30	0,540

SC	comet	q	P or e	Φ	t_0	Δt_1	$\overline{\Delta t}$	m	τ	r	Δ	H_{10}	w	D	D_0	C	S
No		A.U.	y		y	y	y	magn		A.U.	A.U.	magn		′	rad. ☉	°	A.U.
$N = 40$					14[th] solar cycle: 1901,7–1913,6 $P_\odot = 11^y\!,9$												
472	1902 I	0,451	e 1,	0,055	1902,35	−0,07	−0,06	7,5	1	0,73	0,51	11,7	2	3	10	30:	0,004
473	1902 II	0,746	4,826	0,067	1902,50	+0,06	+0.03	9	0	0,90	1,10	9 5	1	1,5	11	0	0
474	1902 III	0,401	1 403 000	0,101	1902,90	−0,23	+0,06	9	1	1,78	1,29	6,0	3	5	20	10	0,120
475	1903 I	0,411	43 100	0,126	1903,20	−0,16	−0,01	10	1	1,40	1,78	8,6	2	4	27	4	0,080
476	1903 II	2,774	e 1,	0,128	1903,22	−0,30	−0,02	11	0	2,97	2,01	5,0	2	2	48	0	0
477	1903 III	0,499	e 1,	0,129	1903,23	+0,06	+0.12	8,5	0	0,80	1,50	9,0	1			0	0
478	1903 IV	0,330	e 1,	0,165	1903,66	−0,19	−0,02	8	1	1,52	0,75	6,3	3	15	31	17	0,080
480	1904 I	2,708	e 1,001 36	0,208	1904,18	+0,11	+0,68	9	1	2,74	2,23	2,8	2	2,1	34	20:	0,051
481	1904 II	1,882	e 1,	0,264	1904,84	+0,12	+0,31	11	0,5	1,95	2,32	6,6	2	30:	7,5	(>0,6:)	(>0,001)
484	1905 II	1,395	6,906	0,281	1905,04	−0,05	+0,15	10	0	1,41	0,93	9,0	2	3	16	0	0
485	1905 III	1,115	226,2	0,313	1905,42	−0,19	−0,07	11,5	1	1,14	0,73	11,5	3	3	14		
486	1905 IV	3,339	e 1,001 05	0,345	1905,80	+0,37	+0,04	10,5	1	3,57	2,60	3,7	2	1,2	41	30:	0,040
487	1905 V	1,052	e 1,000 19	0,346	1905,82	+0,06	+0,12	7	1	1,12	0,26	9,5	3	15	27	30:	0,0097
488	1905 VI	1,296	e 1,000 18	0,359	1905,97	+0,10	+0,22	9	1	1,41	1,04	7,7	1	12	89	>0	>0
489	1906 I	0,215	e 1,	0,366	1906,06	−0,13	+0,02	8	1	1,29	1,42	8,3	3	5	31	>10	0,240
490	1906 II	0,723	e 1,	0,373	1906,14	+0,07	+0,09	8	0	1,83	1,50	10,2	2	3	31	0	0
492	1906 IV	1,698	6,584	0,389	1906.33	+0,31	+0,47	11,5	0	1,97	1,02	8,4	2	2,5	17	0	0
494	1906 VI	1,632	7,773	0,426	1906,77	+0,10	+0,19	11	0	2,67	0,72	9,5	2	4,5	9,6	0	0
495	1906 VII	1,213	583,1	0,431	1906,83	+0,03	+0,13	8,5	0	0,22	0,68	8,1	2	7	28	0	0
496	1907 I	2,052	e 1,001 02	0,463	1907,21	−0,02	+0,47	11	0	1,06	1,45	6,5	1	1,4	14	0	0
497	1907 II	0,923	164,3	0,465	1907,23	+0,04	+0,09	6,5	1	0,95	0,30	10,4	2	15	31	8:	0,0009
498	1907 III	1,147	4,129	0,479	1907,40	+0,02	+0,04	13	0	1,15	0,98	12,3	2	2	14	0	0
499	1907 IV	0,512	8,741	0,503	1907,68	−0,24	+0,69	9,5	2	1,82	1,64	4,0	3	9	25	17	0,220
500	1907 V	0,983	e 1,	0,504	1907,70	+0,08	+0,22	8,5	0	1,10	0,96	9,8	2	>10	15	0	0
503	1908 III	0,945	e 1,000 69	0,612	1908,98	−0,31	+0,03	9	1	2,00	1,65	4,2	3	5	26	11	0,190
504	1909 I	0,843	2 038	0,650	1909,43	+0,02	+0,11	9,3	0	0,86	0,90	10,9	3	2	13	0	0
507	1909 IV	1,382	6,481	0,690	1910,05	+0,02	+0,19	9	1	1,39	0,42	9,5	1	3	11,6	>0	>0
~~508~~	~~1910 I~~	~~0,129~~	3 906 000	0,702	1910,71	−0,02	+0,23	1	2	0,13	1,12	5,0	3	5	17	40	0,500
510	1910 III	1,948	1 056 000	0,757	1910,84	−0,11	+0,33	8,5	1	1,98	1,54	4,8	2	>6	25	45:	0,042
512	1910 V	1,655	— 7,438	0,788	1911,58	+0,01	+0,29	9,5	1	1,66	0,68	9,1	3	3	8	10	0,003
514	1911 II	0,684	1 898	0,835	1909,91	−0,07	+0,03	4	1	0,70	1,32	7,4	3	16	19	1	0,050

SC	comet	q	P or e	Φ	t_0	Δt_1	$\overline{\Delta t}$	m	τ	r	Δ	H_{10}	w	D	D_0	C	S
No		A.U.	y		y	y	y	magn		A.U.	A.U.	magn		′	rad. ♂	°	A.U.
516	1911 IV	0,303	e 1,000 17	0,839	1911,69	+0,05	+0,25	3	2	0,38	0,96	5,9	2	4	68	15	0,260
517	1911 V	0,489	2 126	0,850	1911,82	−0,27	0,00	10	2	1,99	1,21	5,1	3	12	41	30	0,400
518	1911 VI	0,788	9 246	0,855	1911,87	−0,14	+0,03	7,5	1	1,22	0,90	6,5	3	9	44	4	0,070
519	1911 VII	1,226	8,071	0,855	1911,87	+0,04	+0,15	12	0	1,25	1,68	10,3	2	3,5	42	0	0
522	1912 II	0,716	56 650	0,929	1912,76	−0,07	+0,28	5	2	0,88	0,91	6,2	3	4	36	5,5	0,120
523	1912 III	1,107	e 1,	0,934	1912,81	+0,03	+0,09	10	0	1,13	0,97	8,0	2	5	34	0	0
524	1912 IV	1,030	13,52	0,934	1912,82	−0,02	+0,11	10,8	1	1,06	1,23	8,6	2	3	23	>0	>0
525	1913 I	0,407	e 1,	0,958	1913,10	−0,10	−0,09	8	1	0,98	0,83	9	1				
526	1913 II	1,457	5 419	0,981	1913,37	−0,03	+0,12	9,5	1	1,47	1,13	7,7	3	4	27	>3:	>0,0015
$N = 35$					15[th] solar cycle: 1913,6—1923,6 $P_{\odot} = 10^{y}{,}0$												$\Sigma w = 66$
527	1913 III	1,529	17,77	0,002	1913,62	+0,05	+0,22	10,0	1	1,55	0,54	10,3	2	12:	3,5	4:	0,007
528	1913 IV	1,356	13 180	0,010	1913,70	−0,03	+0,09	8,5	1	1,37	1,44	8,0	2	10	48	6:	>0,002
529	1913 V	0,976	6,511	0,024	1913,84	−0,03	+0,06	10	1	0,98	0,60	10,5	2	3	12	30:	0,005
530	1913 VI	1,254	61,73	0,030	1913,90	−0,16	−0,08	7,8	1	1,53	0,59	8,8	2	20	40	3,5	0,104
531	1914 I	0,543	e 1,	0,075	1914,35	+0,02	+0,05	4	2	0,58	0,62	8,3	2	3	13	12	0,126
532	1914 II	1,198	e 1,	0,082	1914,42	−0,01	+0,26	9,5	1	1,57	0,77	9,4	3	10	38	40:	0,017
533	1914 III	3,747	e 1,003 67	0,098	1914,58	−0,10	+0,15	14,0	1	3,79	2,82	4,7	2	0,7	14	0,1:	>0,001
534	1914 IV	0,713	e 1,	0,099	1914,59	+0,12	+0,33	3,5	1	1,13	0,28	6,5	2	4	44	14:	>0,003
535	1914 V	1,104	e 1,000 16	0,122	1914,82	−0,86	0,00	10,5	2	4,25	3,50	1,1	3	5	51	10	0,680
538	1915 II	1,005	e 1,000 24	0,194	1915,54	−0,43	+0,38	8,5	2	2,57	2,70	3,7	2	7	44	6	0,043
539	1915 III	0,972	5,871	0,207	1915,67	−0,41	−0,11	16	0	2,14	1,41	9,2	2	2	15	0	0
540	1915 IV	0,443	e 1,	0,218	1915,78	−0,08	−0,06	9,5	0	0,84	1,51	10	1			0	0
541	1916 I	1,558	6,362	0,248	1916,08	−0,18	+0,07	9,5	1	1,70	0,72	8,8	2	2,5	11	2:	>0,001
542	1916 II	1,340	5,434	0,259	1916,19	−0,04	+0,10	10	0	1,36	0,39	10,7	2	5	15	0	0
543	1916 III	0,471	16,34	0,285	1916,45	−0,11	−0,11	1	0	1,02	0,05	6	1			0	0
544	1916 IV	0,753	e 1,	0,311	1916,71	+0,18	+0,19	11	0	1,40	0,41	12	1			0	0
545	1917 I	0,190	145,3	0,367	1917,27	−0,06	+0,08	6,5	2	0,71	1,41	7,8	3	2	22	5	0,110
546	1917 II	0,764	e 1,	0,378	1917,38	−0,07	0,00	9,5	1	0,89	1,56	10,5	2	7	15		
547	1917 III	1,686	193 100	0,386	1917,46	−1,21	+0,21	12,8	1	5,11	4,10	6,1	2	2,5	20	2:	>0,001
549	1918 II	1,101	e 1,	0,483	1918,43	+0,02	+0,03	10,5	0	1,11	1,04	10	1			0	0

SC	comet	q	P or e	Φ	t_0	Δt_1	$\overline{\Delta t}$	m	τ	r	Δ	H_{10}	w	D	D_0	C	S
No		A.U.	y		y	y	y	magn		A.U.	A.U.	magn		′	rad. ♁	°	A.U.
550	1918 III	1,882	6,707	0,514	1918,74	+0,16	+0,21	14	0	1,94	0,95	11,0	2	1	7	0	0
554	1919 II	1,013	6,697	0,619	1919,79	+0,03	+0,19	9	0	1,02	1,06	10,9	3	5	12	0	0
555	1919 III	0,485	69,06	0,619	1919,79	−0,15	−0,01	8	1	1,36	0,36	9,6	3	25	33	1,5	>0,007
557	1919 V	1,115	e 1,000 22	0,633	1919,93	−0,29	−0,06	8	0	2,07	2,41	4,7	1	4	61	0	0
558	1920 I	0,298	e 1,	0,641	1920,01	−0,05	−0,04	8,5	0	0,51	0,63	12,4	1			0	0
559	1920 II	1,321	5,161	0,684	1920,44	−0,04	+0,20	11	1	1,42	0,72	10,1	2	2	11	1:	0,0004
560	1920 III	1,149	e 1,	0,734	1920,94	0,00	+0,13	10	0	1,15	0,27	11,9	2	8	16	0	0
561	1921 I	1,116	67,01	0,774	1921,34	−0,03	+0,04	10	0	1,14	0,70	10,5	2	3	18	0	0
562	1921 II	1,008	13 850	0,775	1921,35	−0,15	+0,20	9	1	1,40	1,95	6,4	3	6	27	17	0,034
563	1921 III	1,041	6,048	0,785	1921,45	−0,18	+0,03	12	1	1,34	0,44	12,4	1	10	12	30:	0,0017
564	1921 IV	0,340	3,303	0,793	1921,53	+0,04	+0,07	8,8	0	0,50	0,99	11,2	2			0	0
565	1921 V	1,629	1 398	0,822	1921,82	+0,23	+0,36	9,5	0	2,14	1,47	6,3	1			0	0
566	1922 I	0,889	4,980	0,877	1922,37	0,00	+0,13	11,8	0	0,89	0,29	13,1	1	5	6	0	0
567	1922 II	2,259	e 1,000 86	0,922	1922,82	−0,02	+0,62	10,5	1	2,26	1,86	5,3	2	2,1	25	17:	0,0016
568	1923 I	0,924	1 790	0,941	1923,01	−0,11	+0,02	7	0	1,20	0,96	7,5	1	3,5	24	0	0

$N = 44$

16[th] solar cycle: 1923,6—1933,8

$P_{\odot} = 10^{y}\!,2$

$\Sigma w = 93$

570	1923 III	0,778	e 1,	0,027	1923,88	−0,02	+0,02	8	1	1,05	0,46	10	1	>8	22	35:	0,005
571	1924 I	1,756	26,370	0,059	1924,20	+0,03	+0,30	10	1	1,76	2,19	6,4	1	12:	5	>0	>0
572	1924 II	0,406	e 1,001 317	0,106	1924,68	+0,03	+0,08	4	1	0,50	0,91	7,5	2	6	41	4	0,110
574	1924 IV	2,442	7,658	0,137	1925,00	−0,03	+0,02	16	0	2,44	1,47	11,4	2	45:	7,5	0	0
575	1925 I	1,110	e 1,000 629	0,162	1925,25	0,00	+0,56	9,0	1	1,11	1,76	5,4	3	18	170	1	0,026
576	1925 II	5,547	16,29	0,154	1925,17	+2,73	+3,18	13,5	0	(6,0)	(5,3)	2,5	2	2	70	0	0
577	1925 III	1,633	6,910	0,194	1925,58	−0,35	+0,54	8	1	2,34	1,37	4,6	2	5	42	10:	0,0158
580	1925 VI	4,181	e 1,001 941	0,204	1925,68	−0,46	+0,52	11	1	4,42	3,42	2,5	2	2	49	>0	>0
581	1925 VII	1,566	e 1,000 428	0,211	1925,75	+0,13	+0,41	8	1	1,63	1,60	5,5	3	3	20	2,1	0,054
583	1925 IX	1,862	6,919	0,220	1925,84	−0,15	+0,02	13,1	1	1,95	0,94	10,4	2	0,7	5	1:	0,0005
585	1925 XI	0,764	e 1,000 505	0,228	1925,93	−0,06	+0,01	8	1	0,89	0,62	9,6	3	3	20	1	0,018
586	1926 I	1,347	e 1,	0,236	1926,01	+0,03	+0,15	9,5	0	1,37	0,72	10,6	2	6	31	0	0
588	1926 III	0,323	e 1,000 514	0,247	1926,12	−0,17	0,00	8	1	1,27	0,47	10,4	2	10	27	1	0,030
592	1926 VII	0,755	e 1,	0,333	1927,00	+0,07	+0,17	8	0	0,91	1,37	11,3	2			0	0

SC	comet	q	P or e	Φ	t_0	Δt_1	$\Delta \bar{t}$	m	τ	r	Δ	H_{10}	w	D	D_0	C	S
No		A.U.	y		y	y	y	magn		A.U.	A.U.	magn		′	rad. ♁	°	A.U.
594	1927 II	1,036	e 1,	0,345	1927,12	−0,09	+0,02	9	0	1,19	1,53	8,3	1			0	0
595	1927 III	1,772	8,516	0,355	1927,22	−0,37	−0,09	12	1	2,20	1,39	9,0	3	3	21	3,5:	0,012
596	1927 IV	3,684	83 060	0,357	1927,24	−0,05	+1,92	10	1	3,68	2,70	1,9	2	2,5	48	10:	0,030
598	1927 VI	1,213	11,188	0,377	1927,45	−0,02	+0,10	8,0	0	1,23	0,36	9,5	2	3	7,5	0	0
599	1927 VII	4,040	6,011	0,379	1927,47	−0,30	+0,13	16	1	1,77	0,99	10,7	2	60	23	1	0,001
601	1927 IX	0,176	e 1,	0,427	1927,96	−0,05	+0,15	1	2	0,74	1,01	5,2	2	1	6	35	0,610
602	1928 I	1,860	7,238	0,439	1928,08	+0,06	+0,22	12,5	0	1,86	0,87	10,0	3	1	7	2:	0,0006
604	1928 III	0,995	6,354	0,454	1928,23	−0,02	0,00	11	0	1,0	0,2	14,2	1			0	0
605	1928 IV	0,745	27,91	0,514	1928,84	+0,04	+0,09	6	1	0,79	0,92	9,5	2	1	8	>0	>0
606	1929 I	2,091	6,416	0,551	1929,22	−0,17	−0,01	11,0	1	2,23	1,25	8,0	2	4	37	2:	0,001
607	1929 II	1,528	6,383	0,576	1929,48	+0,10	+0,26	10	1	1,56	0,57	10,5	2	1,2	5	1:	0,0003
608	1929 III	2,042	10,90	0,577	1929,49	+0,10	+0,15	13,5	1	2,06	1,05	10,8	2	0,7	5,5	>0	>0
609	1930 I	1,087	176 900	0,631	1930,04	+0,09	+0,13	10,0	1	1,22	0,24	12,5	2	6	10	2:	0,0002
610	1930 II	0,672	18 180	0,633	1930,06	−0,09	+0,01	7	1	1,01	0,88	8,4	3	3	18	2	0,030
611	1930 III	0,482	486,9	0,651	1930,24	−0,02	+0,12	7,0	1	0,53	1,07	8,8	3	10	48	4	0,066
612	1930 IV	2,079	e 1,000 379	0,656	1930,29	−0,10	+0,32	10,5	1	3,03	3,0	6,8	2	>2	61	>0,1:	>0,0002
613	1930 V	1,153	e 1,	0,662	1930,35	+0,06	+0,13	9,0	1	1,22	0,62	8,9	3	12	32	>3:	>0,0008
614	1930 VI	1,011	5,427	0,672	1930,45	−0,12	+0,04	9,5	1	1,22	0,36	11,7	2	3,1	3	>30:	0,0017
615	1930 VII	0,408	e 1,	0,692	1930,66	+0,21	+0,34	13,5	0	1,93	0,99	10,9	2			>0	0
619	1931 III	1,047	356,4	0,769	1931,44	+0,10	+0,48	7	1	1,18	1,63	4,7	2	5	61	30:	0,041
620	1931 IV	0,074	e 1,002 065	0,789	1931,65	−0,04	+0,14	5	2	0,40	1,20	6,6	2	>5	48	2	0,056
621	1931 V	2,331	e 1,002 217	0,815	1931,91	+0,40	+0,52	12	1	2,72	1,75	6,0	3	2	27	2,6:	0,006
622	1932 I	1,254	302,0	0,839	1932,16	+0,09	+0,18	9	0	1,34	0,55	9,3	2	10	40	0	0
626	1932 V	1,037	281,8	0,889	1932,67	−0,07	+0,09	8	1	1,18	0,86	8,1	2	5	20	2	0,031
627	1932 VI	2,314	e 1,001 376	0,894	1932,72	−0,25	−0,21	8,5	1	2,53	1,99	3,5	3	3	30	30:	0,022
628	1932 VII	1,647	e 1,	0,895	1932,73	−0,31	+0,01	13	1	2,23	1,33	9,2	2	2	24	2:	0,0014
631	1932 X	1,131	262,0	0,922	1933,00	−0,04	+0,14	8	1	1,15	0,89	8,7	2	7	20	1	0,016
632	1933 I	1,001	e 1,	0,931	1933,10	+0,03	+0,11	8	1	1,01	0,59	10,2	2	3	11	>1,1:	>0,0002
635	1933 IV	1,014	e 1,	0,975	1933,55	−0,01	−0,01	10	0	1,02	0,25	13	1			0	0
636	1933 V	2,495	7,486	0,978	1933,58	+0,21	+0,94	13	1	2,54	1,64	8,0	2	18:	11	>3:	0,0020

$N = 34$

17th solar cycle: 1933,8−1944,2

$P_{\odot} = 10^{y}4$

$\Sigma w = 79$

SC	comet	q	P or e	Φ	t_0	Δt_1	$\overline{\Delta t}$	m	τ	r	Δ	H_{10}	w	D	D_0	C	S
No		A.U.	y		y	y	y	magn		A.U.	A.U.	magn		′	rad. ♂	°	A.U.
638	1934 II	3,486	e 1,	0,085	1934,68	+0,74	+0,83	13	0	4,24	3,90	4,4	2	12:	5,5	0	0
640	1935 I	0,811	900,3	0,130	1935,15	−0,13	+0,07	10	1	1,25	1,15	10,0	2	4	19	35:	0,010
644	1936 I	4,043	e 1,001 97	0,246	1936,36	−0,72	+0,50	14	1	4,83	3,93	5,4	3	30:	8	30:	0,0005
645	1936 II	1,100	1 642	0,262	1936,52	−0,15	+0,07	9	2	1,38	1,55	6,9	3	9	41	6	0,056
646	1936 III	0,518	887,3	0,263	1936,54	0,00	+0,18	5,5	2	0,52	1,01	8,4	2	48:	6,8	2	0,044
647	1936 IV	1,462	8,532	0,284	1936,75	−0,03	+0,04	12	0	1,50	1,52	13,3	2	36:	2,7	0	0
648	1936 V	1,953	593,3	0,295	1936,87	+0,72	+0,84	13	0	3,41	2,63	5,5	2	30:	8,9	0	0
650	1937 II	0,621	e 1,	0,321	1937,14	+0,02	+0,12	7	1	0,63	0,86	10,4	3	>6	27	>1	0,014
652	1937 IV	1,734	e 1,000 160	0,353	1937,47	−0,37	−0,04	12	1	2,40	1,70	6,0	3	5	27	20:	0,008
653	1937 V	0,863	160 600	0,367	1937,62	−0,11	0,00	7	1	1,16	1,52	6,1	3	18	137	20	0,195
656	1939 I	0,717	1 651	0,510	1939,10	−0,05	+0,08	8	1	0,81	0,71	9,2	2	3	12	8	0,078
658	1939 III	0,528	7 889	0,526	1939,27	+0,02	+0,08	3	2	0,54	0,67	7,1	3	5	24	20	0,240
659	1939 IV	1,762	10,58	0,531	1939,32	−0,21	−0,04	15	1	1,91	0,91	12,2	2	30:	2	1:	0,0004
661	1939 VI	0,748	156,0	0,558	1939,60	−0,03	+0,21	8,0	1	0,78	0,82	8,5	2	3	18,5	>1	0,0008
662	1939 VII	1,871	6,949	0,568	1939,71	−0,25	+0,03	17	0,5	2,03	1,93	11,2	2	20:	2,0	(2:)	(0,0006)
663	1939 VIII	1,749	5,637	0,572	1939,75	+0,27	+0,33	15,8	0	2,0	1,2	12,4	1			0	0
664	1939 IX	0,945	e 1,	0,582	1939,85	−0,01	+0,08	9,2	0	0,95	0,95	10,7	3	6	22	0	0
668	1940 III	1,064	e 1,	0,656	1940,62	+0,13	+0,26	9	0,5	1,31	1,92	10,9	2	1	10		
669	1940 IV	1,086	e 1,	0,670	1940,77	−0,17	+0,05	10,5	0	1,57	0,65	10,3	2			0	0
670	1941 I	0,368	e 1,	0,696	1941,04	−0,36	+0,03	13	2	2,75	2,17	6,3	3	19	82	20	0,200
671	1941 II	0,942	371,8	0,697	1941,05	−0,01	+0,11	10	1	1,00	0,55	12,0	3	12	12,3	3:	0,0003
673	1941 IV	0,790	18 110	0,699	1941,07	−0,03	+0,31	6	2	0,82	0,36	5,9	3	12	61	>20	0,100
675	1941 VI	5,523	16,47	0,742	1941,52	−1,01	−0,27	13	1	5,55	6,44	3,5	1	3	11		
676	1941 VII	1,305	5,538	0,745	1941,55	−0,01	+0,12	10	1	1,30	0,30	11,7	3	24:	1,4	3:	0,0003
677	1941 VIII	0,875	e 1,000 968	0,757	1941,67	−0,27	+0,14	11	1	1,88	0,90	7,1	3	5	29	1,7	0,018
679	1942 II	1,287	85,52	0,801	1942,13	+0,06	+0,08	13	0	1,34	0,36	13,2	2	20:	>1,0	0	0
681	1942 IV	1,445	e 1,000 893	0,820	1942,33	−0,24	+0,01	8	1	1,97	1,33	6,0	3	13	57	42:	0,014
684	1942 VII	3,390	7,886	0,850	1942,64	+0,61	+1,15	15	1	3,49	2,51	7,8	2	15:	4,1	2:	0,002
685	1942 VIII	4,113	e 1,	0,860	1942,74	−0,62	−0,53	15	0	4,5	3,6	5,7	1			0	0
686	1942 IX	1,596	38,96	0,881	1942,96	−0,11	+0,09	13	1	1,77	0,79	9,5	3	3	14,5	12:	0,008
687	1943 I	1,354	2 274	0,894	1943,10	−0,16	+0,16	10	2	1,61	0,66	4,6	3	29	102	17	0,290
688	1943 II	0,758	e 1,	0,946	1943,64	+0,03	+0,10	8	0	0,85	0,60	11	1			0	0
690	1943 IV	1,527	6,800	0,970	1943,89	+0,01	+0,14	15	0	1,53	0,66	13,7	2	1,3	5,5	0	0
692	1944 I	0,872	e 1,	0,984	1944,03	−0,12	−0,05	9	1	1,20	0,54	10,7	2	6	25	>1	0,010

SC	comet	q	P or e	Φ	t_0	Δt_1	$\overline{\Delta t}$	m	τ	r	Δ	H_{10}	w	D	D_0	C	S
No		A.U.	y		y	y	y	magn		A.U.	A.U.	magn		′	rad. ☿	°	A.U.
$N = 56$					18[th] solar cycle: 1944,2—1954,4 $P_{\odot} = 10^{y}\!,2$												$\Sigma w = 118$
694	1944 III	1,277	14,87	0,025	1944,46	−0,09	+0,17	10	0	1,36	0,52	11,6	1	12:	1,4	0	0
695	1944 IV	2,226	e 1,001 995	0,033	1944,54	−0,15	+0,46	12	0	2,31	2,41	8	2	10:	4,0	0	0
696	1945 I	2,400	e 1,	0,079	1945,01	−0,71	+0,14	14,5	0	3,60	4,40	7,4	1	30:	6	0	0
697	1945 II	1,235	4,555	0,108	1945,30	−0,03	+0,05	10	0	1,24	0,27	11,9	1			0	0
698	1945 III	0,998	e 1,	0,115	1945,37	+0,07	+0,10	10	0	1,06	0,73	10,4	1			0	0
701	1945 VI	0,194	e 1,	0,173	1945,96	−0,07	−0,06	7	1	0,80	0,73	9,6	2			<1	>0,002
702	1945 VII	0,006	e 1,	0,175	1945,99	−0,05	−0,05	7	0	0,83	0,62	10,8	1			0	0
703	1946 I	1,724	e 1,001 201	0,204	1946,28	−0,19	+0,57	9	1	1,90	0,94	6,1	3	2,5	29	35:	0,011
704	1946 II	1,018	e 1,	0,212	1946,36	+0,05	+0,13	8	1	1,07	0,17	9,5	3	9	10	7	0,0065
708	1946 VI	1,136	e 1,	0,257	1946,82	−0,22	+0,93	9	1	1,69	2,42	4,8	3	1,5	8,9	2:	0,0040
709	1946 VII	1,754	3,580	0,266	1946,91	−0,27	−0,22	12	0	1,84	0,7	9,8	2			0	0
710	1947 I	2,407	e 1,	0,284	1947,10	−0,26	+0,70	10,0	1	2,64	2,05	5,2	3	1,1	12	6:	0,0036
712	1947 III	0,962	e 1,	0,308	1947,34	−0,10	−0,04	9	0	1,16	0,67	11,2	2	4	18,5	0	0
713	1947 IV	0,560	3 357	0,312	1947,38	−0,15	+0,10	10,5	1	1,28	0,47	9,2	3	5	18,5	50:	>0,067
714	1947 V	1,408	e 1,	0,315	1947,41	−0,03	+0,08	11,3	1	1,42	0,47	11,5	2	3	18,5	2:	0,0004
715	1947 VI	2,828	e 1,001 045	0,327	1947,54	0,00	+0,61	12	1	2,83	1,88	8,0	2	12:	2	3:	0,003
716	1947 VII	1,867	6,591	0,336	1947,63	+0,06	+0,26	12,5	0	1,87	0,87	10,9	2			0	0
717	1947 VIII	3,267	e 1,	0,340	1947,67	+1,10	+2,06	14	1	4,89	4,19	4,1	2	24:	14,5	1:	0,0088
719	1947 X	0,744	e 1,	0,361	1947,88	−0,01	+0,13	8,0	0	0,75	1,06	9,9	1			0	0
721	1947 XII	0,110	e 1,000 032	0,365	1947,92	+0,02	+0,08	3	2	0,27	0,85	6,0	3	10	80	25	0,360
722	1947 XIII	1,648	7,253	0,365	1947,92	+0,13	+0,24	16	1	1,70	0,85	14,4	2	15:	1,4	1,4:	0,0003
723	1948 I	0,748	e 1,	0,385	1948,13	−0,40	+0,34	11	1	2,50	2,01	6,5	3	4	20	5	0,090
724	1948 II	1,499	e 1,	0,385	1948,13	−0,08	+0,44	10	1	1,55	1,84	7,7	2	1	11,5	<1	0,0053
725	1948 III	4,709	e 1,	0,399	1948,27	+0,40	+0,56	13	1	4,85	3,95	4,3	1	(1)	(27)	10:	0,035
726	1948 IV	0,208	e 1,	0,409	1948,37	+0,05	+0,29	3,5	2	0,64	0,52	8,4	2	8	27	3	0,046
727	1948 V	2,107	e 1,	0,409	1948,37	−0,17	+0,79	10	1	2,23	2,19	5,3	3	3	38	30:	0,039
731	1948 IX	2,311	7,475	0,447	1948,76	−0,11	+0,48	11	1	2,34	1,34	7,8	3	2	15	10:	0,010
732	1948 X	1,274	e 1,	0,452	1948,81	+0,09	+0,26	7,8	1	1,36	1,15	8,5	2	1	16	3:	0,0012
733	1948 XI	0,135	171 000	0,453	1948,82	+0,01	+0,22	−3	2	0,40	0,66	5,5	3	30	143	30	0,310
734	1948 XII	0,557	4,996	0,459	1948,88	+0,04	+0,12	9	1	0,65	0,56	12	2	2	8	>3,3:	>0,0006
736	1949 I	2,518	e 1,	0,503	1949,33	−0,79	+0,53	15,5	1	3,94	3,09	6,2	3	42:	27	2:	>0,007
737	1949 II	2,248	6,854	0,540	1949,71	−0,06	+0,06	13,7	0	2,26	1,27	9,6	2	6:	1,5	0	0

SC	comet	q	P or e	Φ	t_0	Δt_1	$\overline{\Delta t}$	m	τ	r	Δ	H_{10}	w	D	D_0	C	S
No		A.U.	y		y	y	y	magn		A.U.	A.U.	magn		′	rad. ♁	°	A.U.
738	1949 III	1,028	2,311	0,547	1949,78	+0,10	+0,11	16	1	1,10	0,16	16	1	>12:	0,3	>1,6:	>0,0001
739	1949 IV	2,059	e 1,	0,551	1949,82	−0,32	+0,52	13	1	2,47	1,84	7,5	3	2,9	42	5:	0,007
741	1949 VI	2,234	7,273	0,559	1949,90	−0,27	−0,07	12,8	1	2,38	1,38	7,6	2	2,7	19	30:	0,023
742	1950 I	2,553	e 1,000 671	0,574	1950,05	−0,67	+0,56	13,0	1	3,60	2,66	6,8	2	2,5	34	6:	0,010
748	1950 VII	1,386	6,691	0,663	1950,96	+0,14	+0,26	11,5	1	1,50	0,61	12,1	1	2,8	11,5	0,7:	>0,0002
749	1951 I	2,572	e 1,000 855	0,671	1951,04	−0,66	+0,31	8	1	3,59	2,85	4,0	2	2,4	32	13:	0,025
750	1951 II	0,719	e 1,003 119	0,675	1951,08	+0,02	+0,39	8,5	1	0,73	1,31	9,8	3	1	7,5	1	0,005
751	1951 III	0,338	3,298	0,686	1951,20	−0,65	−0,12	21	1	3,01	2,70	11	2	7,0	34	15:	0,004
752	1951 IV	1,117	5,493	0,701	1951,35	−0,04	+0,11	10,5	0,5	1,13	0,51	12,0	2	9	82		
757	1951 IX	1,665	6,381	0,748	1951,83	+0,25	+0,36	15	1	1,88	1,12	12,1	1	(1)	(7)	>0,3:	>0,0002
758	1951 X	1,821	7,765	0,754	1951,89	−0,13	+0,15	14	1	1,86	0,89	11,9	2	1,5	9	2:	0,0008
759	1952 I	0,740	262 700	0,768	1952,03	−0,43	−0,12	10	1	2,69	1,94	9,0	3	3	9,5	>1	0,010
760	1952 II	1,599	6,510	0,775	1952,10	−0,34	−0,14	16	1	2,01	1,09	10,8	2	1	6	2:	0,0026
763	1952 V	1,288	462	0,808	1952,44	−0,07	+0,07	10	0	1,36	1,74	8,5	2	10	25	0	0
764	1952 VI	1,202	e 1,	0,817	1952,53	−0,06	+0,15	10	0	1,27	0,91	9,1	3	8	41	0	0
766	1953 I	1,665	1 445	0,864	1953,01	−0,38	+0,05	15	0	2,44	2,03	6,6	3	10	130	0	0
767	1953 II	0,778	e 1,	0,870	1953,07	−0,13	+0,24	10	1	1,17	1,5	9,5	2	4	34		
768	1953 III	1,022	6 974	0,902	1953,40	−0,12	+0,24	9	0	1,26	2,0	8,2	3	6,5	68	0	0
769	1953 IV	1,449	7,01	0,906	1953,44	+0,67	+0,70	18	1	2,77	2,12	12,5	1			>0	>0
771	1953 VI	1,691	6,90	0,933	1953,72	−0,10	+0,06	15	1	1,73	0,78	14,8	2	20:	1,7	3:	0,0026
773	1954 I	2,113	e 1,	0,966	1954,05	+0,01	+0,31	19	1	2,77	2,1	11,7	2	12:	3,5	<1	<0,125
774	1954 II	0,072	e 1,	0,967	1954,06	−0,14	−0,09	11	1	1,46	0,63	11,1	3			30:	0,0044
775	1954 III	0,556	5,21	0,971	1954,10	−0,02	+0,07	8,8	1	0,58	1,04	12,1	2	1	7	12:	0,0039
776	1954 IV	2,353	14,1	0,972	1954,11	+0,56	+1,15	15	1	2,90	2,3	9,0	1	3:	1		

Appendix

The two parts of "Some Problems of Cometary Physics Investigated on the Basis of Photometric Data" are a version of the author's thesis to obtain the scientific degree of "Kandidát fyzikálně matematických věd" (Candidate of Sciences in Physics and Mathematics, CSc.). The original version of the thesis, which has been defended in November, 1963, and the heading of which is "An Analysis of Some Problems of Cometary Physics Based on Photometric Data", will not be published. It differs mostly only in details from this version in a few chapters. The only essential difference consists in including two more sections to the chapter dealing with the brightness variations connected with the comet's interior structure in the original version. The sections analyze the secular variations in the absolute brightness of short-period comets on the basis of the conception of desorption of gases from the solid nucleus. The study is published separately in the Bulletin of the Astronomical Institutes of Czechoslovakia Vol. 15 (1964), 1.

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Table 107

The asymmetry and photometric characteristics of eleven comets

Method of expanding in a series

Comet	t_0	per.	N	int r	r	$t-t_0$	B_0	B_1	B_2	B_3	L	H_0	ΔH	ΔT	Ref.
	U. T.			A. U.	A. U.	d	m m				cal/Mol	m	m	d	
1941 VIII	1941 Sept. 3.20	B	22	0.93−1.94	1.196	+45.7	7.60±0.02	+1.928±0.055	+0.074±0.072	−0.322±0.106	3770	6.49	0.62	3.9	BEYER, 1942
1956 IV	1956 June 19.18	A	35	1.19−1.94	1.484	−60.3	7.36±0.03	−2.289±0.052	+0.291±0.063	+0.585±0.075	6930	4.64	0.84	19.2	BEYER, 1959

Method of symmetric positions

Comet	t_0	$N_A(r)$	$N_B(r)$	int r_A	int r_B	r	$t-t_0$	$n_A(r)$	$H_{\Delta A}(r)$	$n_B(r)$	$H_{\Delta B}(r)$	L	H_0	ΔH	ΔT	Ref.
	U. T.			A. U.	A. U.	A. U.	d		m m		m m	cal/Mol	m	m	d	
1858 VI	1858 Sept. 30.46	4n	5n	0.61−0.81	0.58−0.85	0.686	∓ 16.5	3.89±0.17	1.91±0.02	4.54±0.12	2.49±0.02	6510	3.61	0.97	0.1	BOBROVNIKOFF, 1942
1915 II	1915 July 17.66	5n	6n	1.38−2.46	1.43−2.60	2.051	∓113.8	2.40±0.22	7.24±0.06	2.98±0.36	8.52±0.08	2400	5.08	2.08	6.9	BOBROVNIKOFF, 1942
1937 IV	1937 June 20.08	7n	5n	1.74−2.12	1.74−2.02	1.881	∓ 56.6	2.53±0.31	8.25±0.03	3.41±0.21	8.78±0.01	2800	6.06	1.37	8.9	BOBROVNIKOFF, 1942
1956 IV	1956 June 19.18	19	13	1.30−1.87	1.41−1.87	1.555	∓ 68.3	5.41±0.41	6.85±0.04	5.85±0.15	7.70±0.02	6110	4.10	1.15	21.3	BEYER, 1959

Method of four points

Comet	t_0	$N_A(r)$	$N_B(r)$	$N(q)$	$N(t_m)$	r	$t-t_0$	$H_{\Delta A}(r)$	$H_{\Delta B}(r)$	$H_\Delta(q)$	t_m-t_0	L	H_0	ΔH	ΔT	Ref.
	U. T.					A. U.	d	m m	m m	m m	d d	cal/Mol	m	m	d	
1862 III	1862 Aug. 23.41	7n	5n	5n	5n	1.013	∓18.1	5.48±0.03	5.91±0.05	4.80±0.01	−2.8±0.5	18 930	5.22	0.53	0.0	BOBROVNIKOFF, 1942
1921 II	1921 May 10.42	2n	1n	4n	8n	1.080	∓22.6	7.15±0.04	7.73±0.05	6.99±0.05	−4.7±4.0	7 670	6.23	1.40	2.6	BOBROVNIKOFF, 1942
1932 V	1932 Sept. 1.85	5n	5n	3n	10n	1.086	∓20.6	8.18±0.01	8.73±0.01	7.60±0.02	−1.0±0.3	21 550	5.75	2.24	5.0	BOBROVNIKOFF, 1942
1937 V	1937 Aug. 15.66	5n	3n	5n	3n	0.938	∓20.0	6.17±0.02	6.28±0.01	6.13±0.02	−6.8±0.2	(450)	6.16	0.18	0	BOBROVNIKOFF, 1942
1941 VIII	1941 Sept. 3.20	9	18	7	8	1.233	∓48.9	7.52±0.02	7.70±0.01	6.63±0.02	−2.1±2.0	3 340	6.61	0.75	4.2	BEYER, 1942

Generalized method of photometric exponent

Comet	t_0	per.	$N(r)$	$N(q)$	$N(t_m)$	int r	r	$t-t_0$	$n(r)$	$H_\Delta(r)$	$H_\Delta(q)$	t_m-t_0	L	H_0	ΔH	ΔT	Ref.
	U. T.					A. U.	A. U.	d		m m	m m	d d	cal/Mol	m	m	d	
1862 III	1862 Aug. 23.41	B	5n	5n	5n	0.96−1.02	1.013	+18.1	14.66±1.26	5.91±0.05	4.80±0.01	−2.8±0.5	18 100	4.53	1.66	2.2	BOBROVNIKOFF, 1942
1930 III	1930 March 28.79	B	11n	2n	3n	0.50−1.19	0.780	+26.1	4.93±0.04	7.29±0.01	5.37±0.02	+5.7±0.2	6 630	5.84	4.64	12.3	BOBROVNIKOFF, 1942
1941 IV	1941 Jan. 27.65	B	7n	6n	6n	0.83−1.25	0.989	+31.4	3.20±0.11	5.87±0.02	5.28±0.01	+8.0±0.3	5 210	5.07	1.84	15.7	BEYER, 1942

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NĚKTERÉ PROBLÉMY FYZIKY KOMET ŘEŠENÉ NA PODKLADĚ FOTOMETRICKÝCH ÚDAJŮ

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Souhrn

Tato práce je pokračováním analýzy řady otázek kometární fyziky, jejíž první část byla publikována v tomto časopise v minulém roce (Sekanina, 1962). Nyní věnuje autor nejvíce místa vlivů změn sluneční činnosti na změny v jasu komet. Tato otázka je zkoumána jednak na obsáhlém materiálu 563 komet, jednak na pozorováních komety Encke v jejích 44 návratech k Slunci a jednak na fluktuacích pozorovaných jasností dvou souborů komet (Beyerova metoda). Dále jsou v práci zkoumány krátkodobé fluktuace v barevném indexu hlavy a chvostu komety Arend—Rolandovy a velmi častý jev — asymetrie světelné křivky komet (po redukci na jednotkovou geocentrickou vzdálenost) vůči perihelu.

Pokud to nevede k příliš složitým a na pozorovací materiál těžko aplikovatelným vzorcům, jsou tyto problémy řešeny na podkladě pracho-plynového modelu komety. Jinak je použito zjednodušení ve formě plynového modelu, z jehož platnosti ovšem máme právo usuzovat i na aplikabilitu modelu pracho-plynového. Poslední kapitola obsahuje katalog fyzikálních charakteristik 563 komet z let 1610—1954, jež byly objeveny nezávisle na efemeridě. Může sloužit jako materiál pro řadu statistických úvah.

НЕКОТОРЫЕ ПРОБЛЕМЫ КОМЕТНОЙ ФИЗИКИ РАССМАТРИВАЕМЫЕ НА ОСНОВАНИИ ФОТОМЕТРИЧЕСКИХ ДАННЫХ

Часть вторая

З. Секанина

Резюме

Настоящая работа является продолжением анализа ряда вопросов кометной физики, первая часть которого была опубликована в этом журнале на прошлом году (Секанина, 1962). Здесь наиболее места посвящено влиянию изменений солнечной активности на изменения блеска комет. Этот вопрос исследуется отчасти на многочисленном статистическом материале 563 комет, отчасти на наблюдениях кометы Энке в 44 ее возвращениях к Солнцу, и отчасти на флюктуациях наблюдаемых яркостей двух коллекций комет

(метод Бейера). Далее в работе исследованы коротковременные изменения показателя цвета в голове и хвосте кометы Аренда-Ролана, и очень частое явление — асимметрия кривой блеска комет (приведенного к единице геоцентрического расстояния) по отношению к перигелию.

Пока не приходится к очень сложным, к наблюдательному материалу плохо применительным формулам, эти проблемы решаются на основании пыле-газовой модели кометы. В противном случае используется упрощение в виде газовой модели, по действию которой мы имеем право судить даже на применимость пыле-газовой модели. Последняя глава содержит каталог физических характеристик 563 комет периода 1610—1954, которые были открыты независимо от эфемериды. Каталог может служить материалом для ряда статистических исследований.