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SOME PROBLEMS OF COMETARY PHYSICS INVESTIGATED ON THE BASIS OF PHOTOMETRIC DATA

PART ONE

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PREFACE

Photometrical data of any physical body are collected, besides other reasons, for studying the stability of its brightness. Comets belong to those cosmic objects, the apparent as well as real brightness of which changes considerably. If the terrestrial influences are not considered, which introduce a number of errors into obtained brightness estimations, the objective agents accounting for changes of the comet brightness are as follows:

1. geocentric distance of a comet;
2. heliocentric distance of a comet, or the surface temperature of a comet nucleus;
3. state of the Sun at the moment;
4. structure and physical properties of a comet nucleus under the conditions existing;
5. period during which the comet nucleus is exposed to the effects of the solar activity (secular influences).

The first and partly the second items are the problems of celestial mechanics and they make no difficulty at present. A part of the second point, viz. the dependence of the surface temperature on the heliocentric distance, as well as each of the other belong to the problems of cometary physics. With respect to their character the changes of comet brightness may be divided into two groups:

1. brightness changes with the heliocentric distance;
2. brightness changes with time:
   a) periodical changes;
   b) irregular changes:
      ba) short-term changes,
      bb) secular changes.

An analysis of photometrical data may be performed in two different ways:

1. by starting from the form of the dependence found from the gathered observational material and looking for pure empirical correlations;
2. by postulating a certain physical comet model (simplified, of course) and discuss the agreement between theory and observations.

In the present study I will follow, as far as possible, the latter way. A comet dust-gas model is applied as the physical theory. Hence, one of the main targets of the study is to prove the ability of a dust-gas model to interpret the quantitative relations found.

In Chapter One I am trying to give a brief synopsis of the cometary photometry successes reached so far. Observational effects on brightness estimates are discussed a little more, because this important problem, being outside the interests of the study, is not further analyzed in detail.

A fundamental problem of photometry of comets and cometary physics altogether is the character of the dependence of the comet brightness (more exactly comet-head brightness) on the heliocentrical distance; an analysis of the photometrical curve of the comet is the method of investigation. This problem is therefore discussed most thoroughly, especially in Chapter One and several other chapters. At the same time the importance of the photometrical efficiency of dust in the cometary atmosphere is stressed. Up to now only small attention has been paid to this question in world literature.

In connection with the release of dust into the cometary atmosphere an interaction between a comet and interplanetary matter is of great importancy, which is discussed in a separate chapter. The Stanyukovich theory of microscopic explosions (when cosmic velocities occur) is applied to the conditions in interplanetary space.

The next chapter deals with the study of cometary-head dimensions, which is closely connected with photometrical parameters and check on each other. In this field the results are incomplete at present and a number of difficulties arise in treating observational material.

Part Two of this investigation, which will follow in the course of next year, analyzes the influence on comets of the periodicity of the solar activity, short-term fluctuations in the colour-index of comets, and the perihelion asymmetry of the cometary light curves.

On principle I try to give the observational material used, and if it is not possible because of the extent of the study I consistently indicate the reference. The synopsis of physical characteristics of comets distributed according to the solar cycles will be appended to Part Two. It may serve for cotingent further investigations.

CHAPTER ONE

HISTORY AND EVOLUTION OF PHOTOMETRY OF COMETS

The history of photometry of comets in the best sense of the term, i. e. that of determining their brightness and empirical study of observed photometrical curves, is connected with the evolution of knowledge of kinematic and dynamic properties of comets. The brightness of comets changes in many greater limits and substantially faster than that of any other cosmic objects (excepting supernovae), and hence, they have always attracted observers' attention. The performed brightness estimates, however, were not in any way contributing to science as long as no information on the comet position in space was obtained. Finding the law of universal gravitation was of fundamental importance for photometry of comets as a part of comet astronomy. Methods of computing comet orbits, based on the law, made it possible to determine the heliocentric and geocentric distances of a comet for any moment and to construct in such a way its photometrical curve.

It was NEWTON (1687) that first investigated the problem of integrated-brightness variation of comets and its utilization for determining "photometrical parallax" of comets. He assumed the radiation of comets was due only to reflected solar light; hence, the following expression was used by him for the apparent comet brightness:

$$I = I_0 . \Delta^{-2} . r^{-2}, \tag{1.1}$$

I_0 was a constant. This formula was used many times, and even verified in practice (SCHMIDT, 1863, MÜLLER, 1897). At the same time another reduction formula was suggested, which assumed that comets hold their luminosity constant (analogously to stars), so that their apparent brightness varied with Δ^{-2} (e. g. HERSCHEL, 1912).

At the end of the 19th century the first critical remarks appeared regarding the validity of formula (1.1) (BERBERICH, 1888, HOLETSCHEK, 1896, 1905, 1913, 1916, 1917). HOLETSCHEK (1893) suggested the following formula for expressing the changes in the comet brightness:

$$I = I_0 . \Delta^{-2} . r^{-n}, \tag{1.2}$$

where quantity n, called the **photometrical exponent** at present, was put by him equal to 4. If we denote I_Δ the comet brightness reduced to a unit of geocentrical distance,

$$I_\Delta = I . \Delta^2,$$

we get the **general definition** of the photometrical exponent from (1.2):

$$n(r) = -\frac{r}{I_\Delta} \cdot \frac{dI_\Delta}{dr}. \tag{1.3}$$

HOLETSCHEK himself considered the photometrical exponent to be a constant magnitude. Let us here stress that relation (1.3) results from (1.2) only provided n = const. The generalization to an arbitrary, variable n was definitorically introduced later. Then the mathematical form of (1.2) met with changes too, as we shall see in the next chapter.

HOLETSCHEK having suggested relation (1.2) in 1893, applied in his later studies the old law (1.1) and contented himself with referring to systematic departures of observations from it. The present current method of determining the photometrical exponent from the form of the photometrical curve was introduced by ORLOV (1911a, 1911b) and the variability of the photometrical exponent was first pointed out by VSEKHSVIATSKY (1925). This fact led VSEKHSVIATSKY (1936a, 1936b) and some other authors (FILIPPOV, 1929, DEICH, 1932) to look for photometrical formulae more suitable than that of HOLETSCHEK. However, their attempts had a purely empirical character and the suggested formulae are not used in practical computations. Meanwhile, formula (1.2) has been applied to a great extent without any change up to now.

Since the beginning of this century a number of astronomers have dealt with deriving numerical values of the so-called photometrical parameters, i. e. exponent n and absolute magnitude $H_0 = -2.5 \log I_0$, for many comets. ORLOV (1911a, 1911b) and KRITZINGER (1914) were the first to do so, BOBROVNIKOFF (1941, 1942) and BEYER (1933, 1937a, 1937b, 1937c, 1942, 1947, 1950a, 1950b, 1955, 1958, 1959) dealt with the problem most thoroughly, and in this country mainly Bouška (1949a, 1949b, 1949c, 1949d, 1950a, 1950b, 1951a, 1951b, 1951c, 1953), Vanýsek (1949, 1952), Bouška and Vanýsek (1949), Vanýsek and Hřebík (1954), and Hruška (1957a) took part in this investigation. Moreover, Hruška and Vanýsek (1958) have recently made up the catalogue of photometrical parameters of 120 comets from the years 1853 to 1955.

Vsekhsviatsky (1925, 1928, 1933, 1935, 1937, 1948a, 1948b) and his collabolators, Konopleva (1950) and Vodopianova (1954a, 1954b, 1954c, 1956), proceeded in another way. Vsekhsviatsky statistically studied the distribution of observed photometrical exponents and found that the mean value was close to $\bar{n} = 4$. This value was held by him as "a characteristic of the mean law of the comet-brightness variation, when the ice evaporation from the comet takes place in an average homogeneous force field of the Sun (i. e. corpuscular radiation field — Z. S.)" (Vsekhsviatsky, 1958). Although Vsekhsviatsky's interpretation may be hardly accepted, nevertheless, the method has some justification for purely statistical purposes because of its applicability to a considerably greater number of comets than that of the general form of formula (1.2). Besides, for the comets being observed within a small interval of heliocentric distances the photometrical exponent results, as a rule, in such a great error, that the approximation $n = 4$ is quite sufficient and often closer to facts. In this way Vsekhsviatsky obtained the H_{10}-quantity called by him the absolute magnitude and defined by the relation

$$H_{10} = H - 5 \log \Delta - 10 \log r, \tag{1.4}$$

H is the apparent magnitude. By this method the General Catalogue of Absolute Magnitudes of Comets has been made up, including H_{10} for 803 apparitions of comets at present (Vsekhsviatsky, 1958).

A history of photometry of comets in a narrow sense, i. e. that of the study of physical processes occurring in the cometary head on the basis of photometrical data, began less than twenty years ago. An impulse to investigations of this kind was given by Levin's paper (1943), who applied the ideas of the processes of sorption (i. e. absorption, adsorption, chemosorption) and desorption to the mechanism of release of gas from the comet nucleus. He indicated that the comet brightness depended among others on the surface temperature of the nucleus, sort and concentration of gas. The resulting photometrical exponent is in close connection with the heat necessary to evaporate a certain amount of the gas and it is a function of the heliocentric distance. Levin (1947) presumes the character of the sorption of gas in the comet nucleus is an interstage between adsorption on the one hand and absorption and chemosorption on the other. It is likely that over the whole life-time of the comet nucleus a high degree of the gas diffusion has taken place in its crystal structure as well as inside individual crystals. On the other hand it is hard to accept an idea of the uniform distribution of gas within the crystal; it is clear the maximum concentration of gas is near the crystal surface. During the approach of a comet to the Sun the heating of the nucleus occurs; the evaporation of gas comes first of all in the surface layer. However, there exists a rather great inertia of the process so that the heat attains deeper layers of nucleus blocks only after the perihelion passage. It produces the transfer of molecules sorbed in deep parts towards the nucleus surface and sometimes their evaporation, too. The brightness of the comet then increases with its receding from the Sun. Such a case is relatively frequent and appears as the retardation of the maximum brightness regarding the perihelion passage.

Besides it an opposite effect is always present. The continuous evaporation of gas during an approach of a comet to the Sun leads to the fall of the sorbed-gas concentration in surface layers of the nucleus and, hence, to the decrease of the comet brightness after the perihelion passage. In this case the time of the perihelion passage is preceded by the moment of the maximum brightness. Both the mentioned processes act simultaneously and the character of the resulting effect depends on the respective powers.

Here are some fundamental ideas of the physical conditions connected with the mechanism of release of gas from the comet nucleus. As it is the gas that produces the observed effect, the discussed physical model is often called the comet gas model.

The gas describes the cometary atmosphere as a homogeneous physical medium. However, various physical investigations indicate the cometary atmosphere must be considered as a heterogeneous medium. The reasons are both logical and experimental.

The assumption of the release of gas from the stony blocks of the comet nucleus (whether close to the monolith or more divided) leads to considerations about the interaction between gas molecules and "parent" bodies, in which the gas was sorbed, and especially about the process of ejection of dust particles together with gas molecules. The acceptance of this process conditions the acceptance of the fact that there are two constituents which take part in the comet radiation: the gas and the dust.A number of observed disruptions of comets prove in a telling way that the forces securing the stability of the comet nucleus are relatively ~~weak,~~ not to say those uniting individual particles of the nucleus. Otherwise, there exists some evidence indicating a porous structure of the nucleus material.

Experimental reasons for physical non-homogeneity of the cometary atmosphere are in close connection with finding the differences in physical properties of the so-called "new" and "old" comets and based on photometrical investigations, spectral analysis and polarization measurements of the comet light.

The differences between "new" and "old" comets are not a cosmogonic problem to all intents and purposes, even when there is no doubt that they contribute to the solution of evolution problems of the solar system. The criterion of "age" of comets is not represented by the duration of the period, during which a comet stays in the same state as observed, but by a number of its revolutions round the Sun. Expressed in units of time, the scale of ageing is specific for each comet, according to its orbital period. The "new" comets are those approaching the Sun for the first time or several times extremely, while the "old" comets are those known for many passages through the perihelion.

A statistical analysis of photometrical exponents of a great number of comets having been performed by many authors indicates that there exists a systematic difference between long-period and non-period comets on the one hand and short-period comets on the other. The former have a far smaller average photometrical exponent. The difference is explained both by the absence of the dust and by the higher values of the evaporation heat of gas molecules in atmospheres of short-period comets. Analogously there exist systematic differences in the absolute magnitude of both groups of comets. These differences indicate the various amount and different type of sorbed gases in the nucleus for the two groups. Long-period and non-period comets have vast supplies of gas placed in surface layers of the nucleus, while in the same regions of the nucleus of short-period comets the supplies of gas are exhausted.

A statistical analysis of photometrical exponents also indicates that the n-value decreases both in the vicinity of the Sun and in rather large heliocentric distances (HRUŠKA, 1957a), so that curve $n = n(r)$ attains its maximum as a rule in the range between 1 — 2 A. U., sometimes even farther from the Sun.

A photometrical and spectral investigation of the "new" and "old" comets was thoroughly carried out by OORT and SCHMIDT (1951). They divided comets into four groups according to the semi-major axis of the primitive orbit, a, i. e. of that undisturbed by gravitational influence of Jupiter or other planets:

I. new comets, $\frac{1}{a}$ less than 0.0001 $(A.U.)^{-1}$;

II. fairly new comets, $\frac{1}{a}$ between 0.0001 and 0.0020 $(A.U.)^{-1}$;

III. old comets, $\frac{1}{a}$ between 0.0020 and 0.0400 $(A.U.)^{-1}$;

IV. periodical comets, $\frac{1}{a}$ greater than 0.0400 $(A.U.)^{-1}$.

As to spectral characteristics of comets, OORT and SCHMIDT gave a few interesting data following from BALDET's catalogue of cometary spectra, containing comets from the years 1864 to 1925. They found that seven out of eight comets with intense continuous spectra belonged to the groups of new and fairly new comets, and that six out of eleven comets, for which primitive orbits of $\frac{1}{a}$ less than 0.00025 $(A.U.)^{-1}$ were established, had intense continuous spectra, five of the six having the perihelion distance larger than 1 A.U. The authors concluded that an intense continuous spectrum was a feature of new and fairly new comets. From the above mentioned, however, the conclusion could be drawn that new comets own continuous spectra not because they are new but because they are observed in large heliocentric distances, in other words, an intense continuous spectrum is a general property of comets in large heliocentric distances. OORT and SCHMIDT contradict such a conclusion, referring to the fact that BALDET's list includes at least 9 old comets (without an intense continuous spectrum or even a continuous spectrum at all) out of 17 comets with perihelion distances larger than 1 A.U.

Fluorescence, a process exciting gas molecules for radiation, produces simultaneously a partial polarization of the molecular light (LEVIN, 1947), to 7.6 per cent in transitions $\Sigma \rightarrow \Sigma$ as well as $\Pi \rightarrow \Pi$ and to 19 per cent in transitions $\Sigma \rightarrow \Pi$. Owing to a very low gas density collisions of molecules may be left out of account, the unpolarized constituent of excited radiation is negligible and the observed degree of polarization must be ranged between the two extreme values, while the polarization degree of the solar light reflected on dust particles is considerably higher, sometimes exceeding 30 per cent. The latter fact has recently been experimentally verified by RICHTER (1959). Earlier ÖHMAN (1941a, 1941b) found a 10 per cent polarization in the comet 1941 I and a 24 per cent polarization in the comet 1941 IV. The latter was characterized by an intense continuous spectrum (SWINGS, 1941, ELVEY, SWINGS, BABCOCK, 1943). The connection of the polarization degree with the appearance of spectrum was later proved by many other explorers.

The differences between "old" and "new" comets are reflected even in a value of the heliocentric distance $\bar{r}$ of the maximum growth of coma dimensions (see Chapter Six).

The average values of the heat of evaporation, L, photometrical exponent, n, and heliocentrical distance $\bar{r}$ for the Oort-Schmidt distribution of comets are given in Table 1.

Table 1

Average photometrical parameters of investigated groups of comets

Group of comets	L cal/mol	n	$\bar{r}$ A. U.
I. new	2700 ± 500	2.8	3.3 ± 0.4
II. fairly new	3400 ± 500	3.7	2.5 ± 0.3
III. old	4900 ± 400	3.8	1.6 ± 0.2
IV. periodical	6500 ± 1000	4.2	1.7 ± 0.3

The differences in spectrum, i.e. in the intensity ratio between the emission band spectrum and the continuous spectrum, as well as those in the parameters given in the table are produced by the same effect. Any comet is a conglomerate of gas and dust; both gas and dust are characterized by the specific (and different from each other) values of L, n and the specific appearance of spectrum. The reason why values L, n and spectra of respective groups of comets differ from each other is the different ratio between the abundance of the two constituents of the conglomerate: the influence of dust in the two first groups is much greater than in the two others.

The following correlation clearly appears on the basis of the results of investigations reached so far between the appearance of spectrum, photometrical exponent (or heat of evaporation) and degree of polarization:

(a) continuous spectrum ↔ low photometrical exponent ↔ high degree of polarization;

(b) emission molecular band spectrum ↔ high photometrical exponent ↔ low degree of polarization.

A "pure" gaseous model could explain neither ascertained differences between "old" and "new" comets nor the form of the photometrical exponent curve.

Hence, the only logical solution of the disagreement between the gaseous model and observations was originating and working up a new physical hypothesis, a comet dust-gas model. Some considerations of this character, even when vague and in some points incorrect, were pronounced by BOBROVNIKOFF (1942) twenty years ago. Nine years later the same problem was discussed in the already mentioned paper of OORT and SCHMIDT (1951), while the simplest mathematical analysis of the new model was first performed by VANÝSEK (1952). The present mathematical methods of the comet dust-gas model and its next development are dealt with in the substantial part of the study.

Simultaneously with improving observational methods and accumulating material of sufficient abundancy some other fields of investigation have started to develop, mainly the solar-cometary relationships and study of the secular variation of comet-radiation parameters.

A correlation between the solar activity and the brightness of the Encke comet was first found by BERBERICH (1888). Later this relation was confirmed by BOSLER (1909), but also authors appeared, negating it (LINK, 1948, KONOPLEVA, 1954, DOBROVOLSKY, 1957) and finding another way of interpretation (HOLETSCHEK, 1916). Analogous studies were lately extended to other comets (RICHTER, 1939, 1941, 1949, 1954b, BEYER, 1950a, KONOPLEVA, 1954, DOBROVOLSKY 1955, 1958). Conclusions of various authors differ from each other and till now there are many obscurities in this question, as to both short-term fluctuations and long-term periodical changes.

Secular variations of the absolute brightness of comets were thoroughly dealt with mainly by two investigators, VSEKHSVIATSKY (1927, 1930, 1950, 1958) in the U.S.S.R. and BOBROVNIKOFF (1942, 1948) in the U.S.A. An influence of observational methods and instrument used, however, made itself felt here even in a higher degree than in studying comet-brightness changes. Results are therefore unreliable.

A detailed discussion of the accuracy of observational methods is not the subject of this paper. Regarding the fundamental importance, however, of photometrical observations for our conclusions, it is necessary for at least some aspects of this problem to be briefly mentioned. Whether HOLETSCHEK's old method or BEYER's new method or the most extended extrafocal method is used, the obtained values are always affected by a number of systematic errors (in addition to accidental errors). The agents are (besides the method used) as follows:

a) comet, viz. head dimensions and the brightness distribution over the disc;

b) state of the sky, viz. transparency, high cloudiness, twilight, the Moon etc.;

c) observational instrument, viz. its dimensions and magnification used;

d) observer, viz. properties of his subjective percept.

The first, second and fourth effects are irremovable in practice. The dissensions in view on the way how to reduce them are in connection with this fact. A number of investigators suggest a reduction of the heterogeneous set of brightness estimations to the system of only observer (BOBROVNIKOFF, 1941, 1942, 1948, GADOMSKI, 1947). On the other hand VSEKHSVIATSKY (1928) proved that no observer estimated the brightness always in the same photometrical system; he believes the determination of the average value from several observers' estimates is more advantageous.

No universal view exists among astronomers on the influence of the instrument used. BOBROVNIKOFF (1941) found that the difference between the observed magnitude and that of the photovisual system depended linearly on the aperture of the instrument, D, i. e.

$$\Delta H = - a . \Delta D, \; a > 0. \qquad (1.5)$$

This reduction formula was objected to by LEVIN (1947), and strongly criticized by VSEKHSVIATSKY (1958). LEVIN points out the method of comparison between focal comet images and extrafocal star images is not suitable. A comparison of surface brightness of a diffuse object with that of the extrafocal image of a star-like object yields an estimate independent of instrument dimensions and magnification used only in such a case, when the former object has sharp contours. However, this requirement is not fulfilled in comets. This is just the way in which systematic differences arise consisting in downward bias of comet brightness in big telescopes when great magnifications are used. Hence, correction ΔH should be dependent first of all on the magnification used. According to VSEKHSVIATSKY the light-gathering power of the telescope and the magnification used are the magnitudes affecting ΔH.

In 1943, BOBROVNIKOFF (1943) obtained more than 700 estimates of the brightness of the Whipple-Fedtke comet through various instruments to verify the character of the dependence of ΔH on instrument dimensions. He observed the comet with the naked eye and through a few different telescopes. BOBROVNIKOFF's brightness estimates as well as the parameters of the instrument used, viz. the diameter of the entrance pupil, D, the reciprocal value of the light-gathering power, S, and the magnification used, M, are included in Table 2.

Table 2

Visual brightness estimates as related to the instrument used

D mm	$S = \frac{F}{D}$	M	ΔH
			m
6	3.8	1	0.00
28	5.2	8	+0.21
32	4.0	3.5	+0.18
54	3.5	4	+0.18
63	10.8	20	+0.76
240	15.7	60	+1.61

If the error of individual measurement is assumed to be $\pm 0^m.2$, the ΔH-data are of an accuracy of about $\pm 0^m.026$. Further, an analysis is carried out of the influence of respective instrument characteristics on the observed comet brightness. The complication of the problem will appear to the full.

We denote I the observed comet brightness (through a given instrument) and assume two different types of its dependence on the instrument characteristics; firstly, a power form

$$I(D, S, M) \sim D^{\alpha}.S^{\beta}.M^{\gamma}, \tag{1.6}$$

and secondly, an exponential form:

$$I(D, S, M) \sim \exp\,[\varkappa D + \lambda S + \mu M], \tag{1.7}$$

$\alpha, \beta, \gamma, \varkappa, \lambda, \mu$ are constants which may be found by the method of least squares. Let us point out that BOBROVNIKOFF's formula (1.5) is identical with (1.7) for $\lambda \equiv \mu = 0$. Table 3 includes the constants as well as residuals O-C between ΔH of Table 2 and that from (1.6) or (1.7). The computation has been performed for various forms of the dependence.

Table 3

Parameters of the reduction formula $I = I(D, S, M)$

Case	α	β	γ	O-C	$\varkappa$	λ	μ	O-C
				m				m
general	−0.36 ± 0.12	−1.02 ± 0.24	+0.33 ± 0.18	± 0.107	−0.106 ± 0.023	−0.088 ± 0.011	+0.010 ± 0.005	± 0.024
independent of M	−0.16 ± 0.06	−0.62 ± 0.13	—	± 0.144	−0.068 ± 0.007	−0.069 ± 0.005	—	± 0.031
independent of S	−0.04 ± 0.18	—	−0.32 ± 0.15	± 0.240	+0.027 ± 0.045	—	−0.028 ± 0.007	± 0.090
independent of D	—	−0.59 ± 0.27	−0.12 ± 0.12	± 0.183	—	−0.047 ± 0.015	−0.015 ± 0.003	± 0.058
independent of $M+S$	−0.40 ± 0.08	—	—	± 0.312	−0.159 ± 0.016	—	—	± 0.172
independent of $M+D$	—	−0.85 ± 0.10	—	± 0.197	—	−0.110 ± 0.007	—	± 0.116
independent of $S+D$	—	—	−0.36 ± 0.05	± 0.241	—	—	−0.024 ± 0.001	± 0.092

The following conclusions may be drawn from the data of Table 3:

a) exponential form satisfies observations much better;

b) general form of the dependence gives always the best agreement with the material;

c) BOBROVNIKOFF's formula represents the worse, agreement with observations (residual $\pm 0^m.17$) than any other exponential case under consideration.

The general form of the exponential dependence is the only satisfying observation with a higher accuracy than that of observed ΔH. In spite of it, even this form must be considered as a merely formal expression of the sought-for dependence because of the sign of the coefficient at M. It indicates that the observed comet brightness increases when using greater magnification, which is impossible. Since the values of M-coefficient are in every case small, it seems the magnification is of little importance for estimations of the comet brightness. The situation is complicated also by the fact, that the magnification of two instruments used is smaller than the corresponding normal magnification. It probably produces some change in respective ΔH, too. It seems that the effect of the observational instrument itself cannot satisfactorily explain the course of ΔH found empirically assuming the dependence may be in general expressed in the form of (1.6) or (1.7). Then the three other effects that cannot be abolished must be of the same order. Most authors incline to the opinion that the error of good visual observations is about $\pm 0^m.2$ to $\pm 0^m.3$ (VSEKHSVIATSKY, 1928, BOBROVNIKOFF, 1942, BEYER, 1952, HRUŠKA, VANÝSEK, 1958). Any correction may hardly reduce it markedly.

At present more and more photometrical measurements are made by photoelectrical methods. This is one of the ways of improving the observational material of physical data and giving precision to our conception of physical processes taking place in comets.

With a view of application to photoelectrical measurements of comet brightness a series of methods are being worked out in this study that cannot be fully exploited for treating present visual observations.

CHAPTER TWO

A COMET DUST-GAS MODEL. FUNDAMENTAL METHODS OF DETERMINING ITS PHYSICAL PARAMETERS

2.1. INTRODUCTION

Under physical parameters we shall generally understand such quantities which are necessary and sufficient for the computation of the brightness of a comet provided its distance from the Sun, r, and from the Earth, Δ, are given, and which by their nature characterize simultaneously certain physical conditions in the comet. In this respect they differ from the photometrical parameters. The physical parameters are with a sufficient accuracy constant for the given comet for a long enough time-interval.

Let us accept a dust-gas model of the comet and find the relation between the physical and photometrical parameters.

The connection between the surface temperature of the comet nucleus and the number of molecules, n_0, released by the process of free evaporation from a unit surface per unit of time, is given by the relation which was first used by LEVIN (1943):

$$n_0 = N_0 \cdot \left(\frac{\varkappa T}{2\pi m}\right)^{1/2} . e^{-\frac{L}{R_0 T}} \tag{2.1}$$

where N_0 is the concentration of molecules in the surface layer of the nucleus, m the mass of the average molecule, $\varkappa$ the Boltzmann constant, R_0 the gas constant, T the absolute temperature of the comet-nucleus surface and L the heat necessary for evaporation of a certain amount of gas. Expressions similar to (2.1) result even if some other release mechanism, e. g. evaporation through the isolative disperse surface layer of dust, effusion of gas etc., is considered instead of free evaporation. As the gas-coma brightness may be assumed to be proportional to the number of released molecules, which is LEVIN's way of doing it, formula (2.1) gives the dependence of the brightness of the gas constituent of a cometary atmosphere on the heliocentrical distance. The form of relation $T = T(r)$ only must be known.

There is no doubt that the surface temperature of the cometary nucleus increases with an approach of the comet to the Sun. However, the exact mathematical form of the dependence has not been found so far.

If a body of tiny dimensions in a thermal equilibrium state is the question, its absolute temperature T is

$$T(r) = T_0 . r^{-1/2}, \tag{2.2}$$

where T_0 is its absolute temperature at a unit heliocentrical distance; accepted T_0-values are ranged within 300° — 350° K.

The comet nucleus is neither in a thermal equilibrium state nor of tiny dimensions, since its diameter is probably ranged within 1 to 10 kilometers. Moreover, formula (2.2) does not take into account the rotation effect of the nucleus. MARKOVICH (1959) showed that for these reasons the formula could not correctly express changes of the surface temperature of the comet nucleus.

The correct expression results from the partial differential equation for conduction of heat, applied to the physical conditions that are — according to our conceptions — in the comet atmosphere. Such an analysis was performed by MARKOVICH (1959); he indicated that the surface temperature of the comet nucleus changed in a different way from that given by formula (2.2). Its magnitude as well as variation depend to a great extent on the structure of the comet nucleus and on what gases are released from it. We should not forget that the thermal solar radiation acts in two directions: both increases the comet-nucleus temperature and produces release of frozen gases and tiny dust particles from the nucleus. By means of numerical quadrature of the equation of heat conduction for some special cases MARKOVICH showed the dependence of the surface temperature of the comet nucleus on the heliocentrical distance might be satisfactorily written in a formally analogous form to (2.2):

$$T(r) = T_0 . r^{-\alpha}, \tag{2.3}$$

where, however, $\alpha < 0.5$ and has somewhat smaller value prior to the perihelion passage than after it. Nevertheless, so far formula (2.2) has been often applied in statistical investigations and therefore we cannot avoid it throughout this study.

The problem of the presence of dust in cometary atmospheres will be discussed in detail in the next chapter.

2.2. THE ANALYTICAL FORM OF THE PHOTOMETRICAL CURVE

Let us denote by symbols without index the quantities concerning the whole coma; index d will be used for the same quantities concerning the dust coma and index g for those related to the gas coma. Let us introduce the following denotations:

- $I_\Delta(r)$ — the brightness of the coma in the heliocentric distance r and geocentric distance $\Delta = 1$ A.U.;
- $H_\Delta(r)$ — the magnitude of the comet corresponding to $I_\Delta(r)$;
- $n(r)$ — the photometrical exponent defined by the well-known formula;
- $\eta(r)$ — the function giving the dependence of I_Δ on the heliocentric distance (physical exponent);
- I_0 — the absolute brightness of the comet;
- H_0 — the absolute magnitude of the comet;
- B — the quantity resulting from the average heat of evaporation of gases L, the gas constant R_0, and the absolute temperature of the nucleus surface in $r = 1$ A.U., $T_0 : B = L/R_0T_0$;
- k — the ratio of the absolute brightness of the dust- and gas coma;
- $\Psi(r)$ — the ratio of the brightness of the dust- and gas coma in a given heliocentrical distance.

If the measurements of the brightness are free of the phase-effect the following relations are applicable:

$$I_\Delta = I_{\Delta d} + I_{\Delta g}, \tag{2.4}$$

$$I_{\Delta d} = I_{0d} . r^{-\eta_d}, \tag{2.5}$$

$$I_{\Delta g} = I_{0g} . r^{-\eta_g}, \tag{2.6}$$

$$I_\Delta = I_0 . r^{-\eta}, \tag{2.7}$$

so that

$$I_\Delta = I_\bullet . \frac{kr^{-\eta_d} + r^{-\eta_g}}{1 + k}, \tag{2.8}$$

$$n(r) = \frac{n_d k r^{-\eta_d} + n_g r^{-\eta_g}}{k r^{-\eta_d} + r^{-\eta_g}}, \tag{2.9}$$

or

$$n(r) = \frac{n_d k \mathrm{e}^{-\int_1^r \frac{n_d}{r} dr} + n_g \mathrm{e}^{-\int_1^r \frac{n_g}{r} dr}}{k \mathrm{e}^{-\int_1^r \frac{n_d}{r} dr} + \mathrm{e}^{-\int_1^r \frac{n_g}{r} dr}}. \tag{2.10}$$

These relations apply to arbitrary forms of the functions $\eta_d = \eta_d(r)$ and $\eta_g = \eta_g(r)$ which are related to the corresponding photometrical exponents by differential equations of the form ($i = d,g$):

$$\ln r . \frac{\mathrm{d}\eta_i(r)}{\mathrm{d} \ln r} + \eta_i(r) = n_i(r). \tag{2.11}$$

Now we shall consider quite a general form of the functions n_d and η_d, and if for n_g and η_g we insert the expressions following from (2.1) and (2.3),

$$\left. \begin{aligned} n_g &= \frac{\alpha}{2} + \alpha B r^\alpha, \\ \eta_g &= \frac{\alpha}{2} + B \frac{r^\alpha - 1}{\ln r}, \end{aligned} \right\} \tag{2.12}$$

we obtain the resulting expression for the magnitude of the comet in the heliocentric distance r:

$$H_\Delta(r) = H_0 + 2.5 \log \frac{1 + k}{k r^{-\eta_d} + r^{-\frac{\alpha}{2}} \exp\left[B(1 - r^\alpha)\right]}. \tag{2.13}$$

Hence, the photometrical curve of the comet is characterized by three parameters, H_0, B, k, called the physical parameters.

2.3. DERIVATION OF THE GENERAL EXPRESSIONS FOR THE PHYSICAL PARAMETERS

2.3.1. Method of expanding in a series

Expanding (2.13) in a series of the form:

$$H_\Delta(r) = \sum_{i=0}^{p} a_i \left(\log \frac{r}{r_0}\right)^i \tag{2.14}$$

(r_0 is the geometrical mean of the heliocentric distances for which measurements of the comet brightness were carried out) and neglecting the terms with $p > 2$ we obtain, with respect to

$$\frac{\mathrm{d}}{\mathrm{d}\log r}(r^{-n_d}) = -r^{-n_d}.\frac{n_d(r)}{\mathrm{mod}},$$

the following expressions for the coefficients a_i:

$$\left.\begin{aligned} a_0 &= H_\Delta(r_0), \\ a_1 &= \frac{5}{4(1+\Psi)}[\alpha + n_d\Psi + 2\alpha B r_0^\alpha], \\ a_2 &= \frac{5}{16\,\mathrm{mod}(1+\Psi)^2}\Big[-\frac{\alpha^2}{4}\Psi + \Psi n_d(\alpha - n_d) + \\ &+ \Psi\,\mathrm{mod}\,(1+\Psi)\frac{\mathrm{d}\,n_d}{\mathrm{d}\log r} + (\alpha + 2\Psi n_d)\,\alpha B r_0^\alpha - \Psi\alpha^2 B^2 r_0^{2\alpha}\Big]. \end{aligned}\right\} \quad (2.15)$$

Quantities Ψ, n_d and $\frac{\mathrm{d}\,n_d}{\mathrm{d}\log r}$ must be taken in r_0. Eliminating $\Psi(r_0)$ from the second and third equations of (2.15) we obtain the quadratic equation for B, when $n(r_0) \neq n_d(r_0)$:

$$\alpha^2 B^2 r_0^{2\alpha}\Delta\nu + \alpha B r_0^\alpha \Delta\mu + \mu_d\nu - \mu\nu_d + \frac{1}{4}\alpha^2\Delta\nu = 0; \quad (2.16)$$

here

$$\left.\begin{aligned} \mu &= \frac{4}{25}a_1^2 - \frac{4}{5}\,\mathrm{mod}\,a_2, \\ \mu_d &= n_d^2 - \mathrm{mod}\frac{\mathrm{d}n_d}{\mathrm{d}\log r}, \\ \nu &= \frac{\alpha}{2} - \frac{2}{5}a_1, \\ \nu_d &= \frac{\alpha}{2} - n_d, \\ \Delta\mu &= \mu_d - \mu, \\ \Delta\nu &= \nu_d - \nu. \end{aligned}\right\} \quad (2.17)$$

Then the sought-for root of (2.16) is equal to:

$$B = \frac{1}{2\alpha r_0^\alpha.\Delta\nu}\left[\Delta\mu + \left\{(\Delta\mu)^2 - 4\Delta\nu\left(\mu_d\nu - \mu\nu_d + \frac{\alpha^2}{4}\Delta\nu\right)\right\}^{\frac{1}{2}}\right]. \quad (2.18)$$

For the ratio Ψ we obtain the expression

$$\Psi(r_0) = \frac{\frac{\alpha}{2} + \alpha B r_0^\alpha - \frac{2}{5}a_1}{\frac{2}{5}a_1 - n_d}, \quad (2.19)$$

and

$$k = \Psi(r_0) . r_0^{\eta_d - \frac{\alpha}{2}} . \exp\left[B(1 - r_0^{\alpha})\right]. \tag{2.20}$$

The fundamental equation (2.13) together with (2.15) and the other equations gives the expression for the absolute brightness H_0.

Thus, equations (2.18), (2.20) and (2.13) together with the other equations make it possible to determine the physical parameters of the comet designed according to the dust-gas model for an arbitrary form of the dependence of the photometrical esponent of the dust coma on the heliocentric distance.

In the special case, when

$$n(r_0) = n_d(r_0), \tag{2.21}$$

equation (2.18) loses its validity. The heat of evaporation is now given by the requirement of a finite solution of equation (2.19), so that

$$B = \frac{1}{r_0^{\alpha}}\left(\frac{2a_1}{5\alpha} - \frac{1}{2}\right), \tag{2.22}$$

and by inserting (2.21) and (2.22) into the last equation of (2.15) we determine:

$$\Psi(r_0) = \frac{\dfrac{\alpha}{2\,\mathrm{mod}}\left(a_1 - \dfrac{5}{4}\,\alpha\right) - a_2}{a_2 - \dfrac{5}{4}\,\dfrac{\mathrm{d}\, n_d}{\mathrm{d}\log r}}. \tag{2.23}$$

The parameters k and H_0 will be derived in the same way as before. Thus, from the material, a_0, a_1, a_2 have to be determined.

2.3.2. Method of the photomaterical exponent

Our considerations will be based on two equations. From equations (2.7) and (2.8) follows:

$$\eta(r_0) = -\frac{\log \dfrac{k r_0^{-\eta_d} + r_0^{-\eta_g}}{1 + k}}{\log r_0} \tag{2.24}$$

and according to (2.9)

$$n(r_0) = \frac{n_d k r_0^{-\eta_d} + n_g r_0^{-\eta_g}}{k r_0^{-\eta_d} + r_0^{-\eta_g}}; \tag{2.25}$$

let r_0 be again the geometrical mean of the heliocentric distances, for which we know H_Δ. By eliminating parameter k from the two equations we find the transcendental equation for B, of the form of:

$$\beta\, e^{\gamma B} + \delta B + \varepsilon = 0, \tag{2.26}$$

where

$$\left.\begin{aligned} \beta &= (n - n_d)\, r_0^{\alpha/2}, \\ \gamma &= r^{\alpha} - 1, \\ \delta &= \alpha r_0^{\alpha}\,(r_0^{\eta_d} - r_0^{\eta}), \\ \varepsilon &= \left(\frac{\alpha}{2} - n\right) r_0^{\eta_d} + \left(n_d - \frac{\alpha}{2}\right) r_0^{\eta} . \end{aligned}\right\} \tag{2.27}$$

The expression for k follows directly from (2.24):

$$k = \frac{r_0^{-\eta} - r_0^{-\eta_g}}{r_0^{-\eta_d} - r_0^{-\eta}}. \tag{2.28}$$

In this case, H_0, $n(r_0)$ and $\eta(r_0)$ have to be determined from the material. This method fails for $r_0 \to 1$ A.U.

2.4. TREATMENT OF THE MATERIAL

The coefficients $a_0, \ldots, a_p$ in equation (2.14) will be computed from the following system of linear equations

$$\sum_{i=0}^{p} \sum_{l=1}^{N} a_i \left(\log \frac{r_l}{r_0}\right)^{i+j} = \sum_{l=1}^{N} H_{\Delta l} \left(\log \frac{r_l}{r_0}\right)^j, \; j = 0, \ldots, p, \tag{2.29}$$

where N is the number of observations. The coefficients for a higher p result usually in large errors so that it is advisable to take only $p = 2$ into consideration. This enables us to determine a_1 and a_2 from the graph of the function $\dfrac{H_\Delta - a_0}{\log \dfrac{r}{r_0}}$, which we plot in dependence on $\log \dfrac{r}{r_0}$, as the ordinate on the axis y and the slope of the relation, respectively.

The method of the photometrical exponent requires, first of all, the determination of the absolute magnitude of the comet H_0. This may be done either directly from the diagram or by a suitable extrapolation (if $q > 1$ A.U. or if no observations from the neighbourhood of $r = 1$ A.U. are available).

If we know H_0, the expression for η follows from equation

$$H_\Delta(r_0) = H_0 + 2.5\eta(r_0) \log r_0. \tag{2.30}$$

2.5. COMPARISON OF THE TWO METHODS. PHYSICAL PARAMETERS OF THE COMET 1943 I

The comparison of the results following from the derived methods is carried out on the comet 1943 I, Whipple-Fedtke. The relation (2.2) is here accepted to be possible for comparing the numerical results with those obtained in another way earlier by various authors. As according to spectroscopical data (McKellar, 1943) there was no substantial influence of the dust on the comet brightness it is possible to put $n_d = \eta_d = 2$.

The computation of the physical parameters of the comet 1943 I was based on the material collected and grouped into daily means by Gadomski (1947). It turned out that the observations prior to the passage of the comet through the perihelion are not numerous and at the same time homogeneous enough, so that they were unsuitable for treatment. The comet passed through its perihelion on February 6, 1943. In its vicinity the comet revealed numerous anomalies in the run of its brightness, as shown in Fig. 1. In the anomalous region, the observations (daily means) are plotted by circles, in the normal region, registered in the diagram, by discs. The smoothed out curve is given in a full line and its extrapolated part in a dashed line. In the perihelion, the comet was by almost 1^m fainter than would correspond to the most probable extrapolation. The maximum of the light curve was retarded in respect to the perihelion by about 18 days and the comet attained in it the brightness of $H_\Delta = 4.74^m$, which is by about 0.5^m brighter than it should have been. The average photometrical exponent from the period 4th till 26th February results in $n = -53$, which sufficiently illustrates the tumultuous development

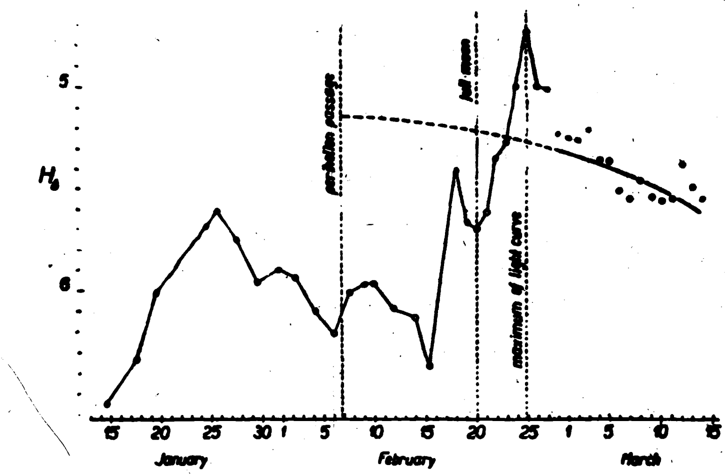


Fig. 1. Light curve of the Whipple-Fedtke comet in the vicinity of the perihelion.

of the comet in this period. That cannot be substantially influenced even by the fact that on February 20th the Moon attained her full, so that the brightness of the comet in the short period round that date was somewhat underestimated.

Observations made between February 27th and July 3rd were taken into consideration, with the exception of those of the periods from March 19th till March 22nd, April 16th till April 22nd, when the brightness was underestimated owing to the Full Moon, and those of April 12th, as these observations distinctly deviate from the given dependence. Thus, there remained 58 daily means that comprised 512 observations, all of which were made after the passage of the comet through the perihelion. The weight of each of the daily means was put equal to the number of the individual observations within it. The run of H_Δ with the logarithm of the heliocentric distance is shown in Fig. 2. The values of the weight ≥ 20 are plotted by crosses, those of the weight < 20 but ≥ 10 by discs and the values of the weight < 10 by circles.

The elimination of the two periods round the Full Moon subdivided the applied material into three groups corresponding to the time-intervals from February 27th till March 18th, from March 23rd till April 15th and from April 23rd till July 3rd. The photometrical parameters were then computed for six time-intervals Δt and the intervals of the heliocentric distance Δr, corresponding to them, with the mean value of $\log r_0$ and the total number of observations N given in Table 4. These photometrical parameters were then treated by both mentioned methods. It turned out that for shorter intervals Δr, the value a_2 was very unreliable and liable to cause a complete misrepresentation of its physical

Table 4
Distribution of measurements into groups

No	Δt	Δr	$\log r_0$	N
I	27. II. — 3. VII.	1.39 — 2.44	0.1805	512
II	27. II. — 15. IV.	1.39 — 1.67	0.1670	446
III	23. III. — 3. VII.	1.51 — 2.44	0.2211	204
IV	27. II. — 18. III.	1.39 — 1.48	0.1537	308
V	23. III. — 15. IV.	1.51 — 1.67	0.1968	138
VI	23. IV. — 3. VII.	1.75 — 2.44	0.2719	66

meaning. Therefore no use at all was made of the values a_2 obtained by the method of least squares for group IV and VI, as can be seen from Table 5. Since it is evident from Fig. 2 that the deviations from the straight-line cannot be considerable, the method of expansion in a series has been applied, too, for the case of putting $a_2 = 0$ by definition. The method of the photometrical exponent has also been applied in two ways. First of all, the absolute magnitude of the comet H_0 has been determined in advance by expanding the function H_Δ in the point $\log r = 0$, which yielded

$$H_\Delta = \underset{\pm 0.051}{3.073} + \underset{\pm 0.50}{15.66} \log r - \underset{\pm 1.1}{1.6} (\log r)^2, \qquad (2.31)$$

whereupon from H_0 the physical exponent $\eta(r_0)$ has been computed according to (2.30). In the second case, $\eta(r_0)$ has been put equal to $n(r_0)$ by definition, since $\eta(r_0)$ differs only very slightly from $n(r_0)$. Table 5 gives a summary of the individual values of the physical parameters determined for the given group Δr by means of the given method; the individual columns show:

No — the serial number;

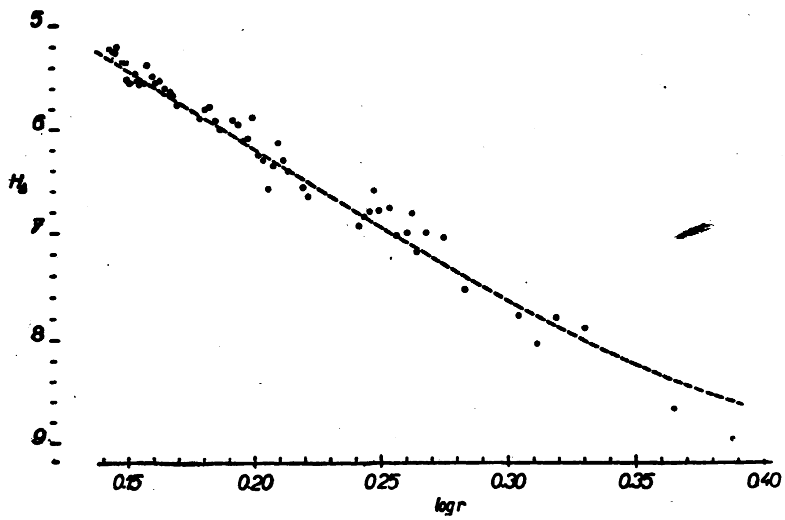


Fig. 2. The post-perihelion photometrical curve of the Whipple-Fedtke comet.

Table 5

Physical parameters of the Whipple — Fedtke comet in the Δt-groups

No	Method	Δt	$H_{\Delta}(r_0)$	$a_1(r_0)$ / $n(r_0)$	$a_2(r_0)$ / $\eta(r_0)$	$\left(\frac{d}{dr} n(r)\right)_{r_0}$ / $H_n(r_0)$	B	$w(B)$	k	$w(k)$	H_0	$w(H_0)$
			m m								m m	
1	A1	I	5.850 ± 0.004	15.10 ± 0.11	—1.56 ± 1.14	—0.36 ± 0.26	10.73 ± 0.17	6.92	0.035 ± 0.004	12.50	3.21 ± 0.04	[illegible]2.50
2	A1	II	5.642 ± 0.004	15.40 ± 0.17	+13.90 ± 6.20	+3.29 ± 1.47	9.05 ± 0.82	0.30	<0.029	—	3.37 ± 0.24	0.17
3	A1	III	6.453 ± 0.007	14.53 ± 0.20	+4.49 ± 2.63	+0.94 ± 0.55	9.10 ± 0.38	1.39	0.014 ± 0.010	2.00	3.52 ± 0.12	0.69
4	A1	IV	5.451 ± 0.003	16.60 ± 0.37	(—157.7 ± 27.8)	(—38.5 ± 6.8)	—	—	—	—	—	—
5	A1	V	6.108 ± 0.006	17.17 ± 0.49	+8.21 ± 7.56	+1.81 ± 1.67	10.69 ± 0.90	0.25	<0.028	—	3.05 ± 0.27	0.14
6	A1	VI	7.151 ± 0.014	13.12 ± 0.54	(+44.16 ± 8.58)	(+8.20 ± 1.59)	—	—	—	—	—	—
7	A2	I	5.848 ± 0.003	14.99 ± 0.07	0.00 def	0.00 def	10.47 ± 0.04	62.50	0.032 ± 0.002	25.00	3.25 ± 0.01	50.00
8	A2	II	5.648 ± 0.003	15.68 ± 0.12	0.00 def	0.00 def	11.05 ± 0.08	15.63	0.030 ± 0.003	11.11	3.13 ± 0.02	12.50
9	A2	III	6.460 ± 0.006	14.77 ± 0.14	0.00 def	0.00 def	9.85 ± 0.08	15.63	0.025 ± 0.006	2.78	3.37 ± 0.04	3.13
10	A2	IV	5.442 ± 0.003	16.20 ± 0.37	0.00 def	0.00 def	11.56 ± 0.24	1.74	0.030 ± 0.009	1.23	3.04 ± 0.05	2.00
11	A2	V	6.109 ± 0.006	17.18 ± 0.49	0.00 def	0.00 def	11.61 ± 0.30	1.11	0.016 ± 0.006	2.78	2.90 ± 0.08	0.78
12	A2	VI	7.194 ± 0.012	15.21 ± 0.39	0.00 def	0.00 def	9.53 ± 0.22	2.07	0.015 ± 0.005	4.00	3.37 ± 0.09	0.62
13	B1	I	5.848 ± 0.003	6.00 ± 0.03	6.15 ± 0.11	3.14 ± 0.01	11.78 ± 0.09	24.69	0.051 ± 0.011	1.65	3.07 ± 0.05	4.00
14	B1	II	5.648 ± 0.003	6.27 ± 0.05	6.17 ± 0.13	3.03 ± 0.02	11.52 ± 0.13	11.83	0.038 ± 0.014	1.02	3.07 ± 0.06	2.78
15	B1	III	6.460 ± 0.006	5.91 ± 0.06	6.13 ± 0.09	3.19 ± 0.03	11.46 ± 0.12	13.89	0.039 ± 0.007	4.08	3.07 ± 0.06	2.78
16	B1	IV	5.442 ± 0.003	6.48 ± 0.15	6.17 ± 0.13	2.95 ± 0.06	11.22 ± 0.27	2.74	0.022 ± 0.020	0.50	3.07 ± 0.09	1.23
17	B1	V	6.109 ± 0.006	6.87 ± 0.19	6.17 ± 0.10	2.72 ± 0.09	10.46 ± 0.30	2.22	<0.017	—	3.07 ± 0.13	0.59
18	B1	VI	7.194 ± 0.012	6.09 ± 0.16	6.06 ± 0.08	3.05 ± 0.11	10.67 ± 0.29	2.38	0.021 ± 0.011	1.65	3.07 ± 0.16	0.39
19	B2	I	5.848 ± 0.003	6.00 ± 0.03	$n(r_0)$ def	3.14 ± 0.01	11.29 ± 0.06	27.78	0.045 ± 0.004	6.25	3.14 ± 0.02	12.50
20	B2	II	5.648 ± 0.003	6.27 ± 0.05	$n(r_0)$ def	3.03 ± 0.02	11.86 ± 0.10	10.00	0.044 ± 0.006	2.78	3.03 ± 0.03	5.56
21	B2	III	6.460 ± 0.006	5.91 ± 0.06	$n(r_0)$ def	3.19 ± 0.03	10.82 ± 0.09	12.35	0.036 ± 0.005	4.00	3.19 ± 0.04	3.13
22	B2	IV	5.442 ± 0.003	6.48 ± 0.15	$n(r_0)$ def	2.95 ± 0.06	12.34 ± 0.30	1.11	0.043 ± 0.019	0.28	2.95 ± 0.09	0.62
23	B2	V	6.109 ± 0.006	6.87 ± 0.19	$n(r_0)$ def	2.72 ± 0.09	12.59 ± 0.36	0.77	0.023 ± 0.014	0.51	2.72 ± 0.14	0.26
24	B2	VI	7.194 ± 0.012	6.09 ± 0.16	$n(r_0)$ def	3.05 ± 0.11	10.74 ± 0.29	1.19	0.021 ± 0.011	0.83	3.05 ± 0.16	0.20

method — the applied method:

$A1$ — method of expansion in a series with the computed coefficient a_2;

$A2$ — method of expansion in a series with the coefficient $a_2 = 0$ by definition;

$B1$ — method of exponent with the quantity $\eta(r_0)$ computed from (2.30) under application of H_0 from (2.31);

$B2$ — method of exponent with the quantity $\eta(r_0) = n(r_0)$ by definition;

Δt — the given Roman numerals correspond with the data of Table 4;

$H_\Delta(r_0)$ — the magnitude of the comet in the heliocentric distance r_0 (see Table 4);

$a_1(r_0)$, or $n(r_0)$ — in the case of the method of expansion in a series, $\left(\frac{\mathrm{d}H_\Delta}{\mathrm{d}(\log r)}\right)_{r_0}$, in the case of the method of exponent, the photometrical exponent in r_0;

$a_2(r_0)$, or $\eta(r_0)$ — in the case of the method of expansion in a series, $\frac{1}{2}\left(\frac{\mathrm{d}^2H_\Delta}{\mathrm{d}(\log r)^2}\right)_{r_0}$, in the case of the method of exponent, the physical exponent in r_0;

$\left[\frac{\mathrm{d}}{\mathrm{d}r}\, n(r)\right]_{r_0}$, or $H_n(r_0)$ — in the case of the method of expansion in a series, the change of the photometrical exponent in r_0, in the event of the method of exponent, the photometrical parameter defined by the relation $H_n(r_0) = H_\Delta(r_0) - 2.5\, n(r_0) \log r_0$;

B — the function of the desorption heat (see above) and its probable error μ_B;

$w(B)$ — the weight of quantity B;

k — the ratio of the absolute brightness of both parts of the coma (see above) and its probable error μ_k;

$w(k)$ — the weight of the quantity k;

H_0 — the absolute magnitude of the comet and its probable error μ_{H_0};

$w(H_0)$ — the weight of the quantity H_0.

Each of these values of the physical parameters have then been treated in Table 6 which shows their weighted values both for the individual method and summarily.

The columns indicate;

method — the applied method (see above);

B — the resulting value B and its probable error;

(p. p. e.)$_B$ — the probable error of the quantity B expressed in per cent;

N'_B — the number of the individual quantities B applied from Table 5;

k — the resulting value k and its probable error;

(p. p. e.)$_k$ — the probable error of the quantity k expressed in per cent;

N'_k — the number of the individual values k which have been used;

H_0 — the resulting value H_0 and its probable error;

(p. p. e.)$_{H_0}$ — the probable error of the quantity H_0 in per cent;

N'_{H_0} — the number of individual values H_0 applied.

In this Table, H_0 is in method $B1$ substituted by the value from formula (2.31), with which the computation was made.

From the summary of the individual values of the physical parameters it can be seen that their dispersion is relatively very low, which surprises particularly in the case of quantity k. The material concerning the comet under consideration is very rich indeed, so that it was feasible to eliminate all apparently

Method	B	$(p.p.e.)_B$	N'_B	k	$(p.p.e.)_k$	N'_k	H_0	$(p.p.e.)_{H_0}$	N'_{H_0}
		%			%		m m	%	
$A1 + A2 + B1 + B2$	10.955 ± 0.093	0.85	22	0.0321 ± 0.0013	4.0	19	3.194 ± 0.011	0.34	22
$A1 + A2$	10.471 ± 0.098	0.94	10	0.0295 ± 0.0016	5.4	8	3.219 ± 0.013	0.40	10
$B1 + B2$	11.426 ± 0.078	0.68	12	0.0389 ± 0.0018	4.6	11	3.096 ± 0.013	0.42	12
$A1$	10.416 ± 0.251	2.41	4	0.0321 ± 0.0049	15.3	2	3.213 ± 0.013	0.40	4
$A2$	10.476 ± 0.125	1.19	6	0.0287 ± 0.0017	5.9	6	3.225 ± 0.022	0.68	6
$B1$	11.527 ± 0.099	0.86	6	0.0368 ± 0.0033	9.0	5	3.073 ± 0.051	1.66	—
$B2$	11.316 ± 0.123	1.09	6	0.0402 ± 0.0021	5.2	6	3.110 ± 0.022	0.71	6

Table 6

Resulting physical parameters of the Whipple — Fedtke comet

erroneous estimates of the brightness without more pronounced prejudice of its ampleness. The error of the exponent is only about 0.5 per cent, which is ten times less than in the case of the exponents of other comets, whereat these exponents are not yet considered as unreliable. This also explains the small errors in the physical parameters.

After inserting the resulting values of the physical parameters into formula (2.13) and rearrangement of the latter, we obtain, in accordance with the accepted assumptions, the following mathematical form of the photometric curve:

$$H_\Delta(r) = \underset{\pm 0.04}{6.96} + 5 \log r - \qquad (2.32)$$

$$-2.5 \log \left\{ 1 + \underset{\pm 0.18}{1.78} . 10^6 \, r^{7/4} \exp \left[-\underset{\pm 0.09}{10.96} \sqrt{r} \right] \right\},$$

while the ratio of the brightness of both parts of the coma varies with the heliocentric distance as follows:

$$\Psi(r) = \underset{\pm 0.57}{5.61} . 10^{-7} \, r^{-7/4} \exp \left[\underset{\pm 0.09}{10.96} \sqrt{r} \right]. \qquad (2.33)$$

A comparison of the two methods used indicates the method of the photometrical exponent leads to somewhat higher values of L, ratio k and the absolute brightness. However, these differences are not considerable and may be explained by simplifying assumptions made in the beginning of this section. Generally, the method of expanding in a series is more suitable when brightness measurements from a relatively wide interval of heliocentric distances are at our disposal, while the method of the photometrical exponent in the opposite case.

Let us now compare the obtained resulting values of L with those gained by some other authors. For $T_0 = 350°$ K we get

$$L = 7280 \pm 70 \text{ cal/mol}$$

from the A-method, and

$$L = 7940 \pm 50 \text{ cal/mol}$$

from the B-method.

On the basis of the assumption of the pure gaseous model, i. e. for $k = 0$, Vorontsov-Velyaminov (1943) found $L = 7090 \pm 450$

cal/mol and Martynov (1944a, 1944b) from a more abundant material $L = 6650 \pm 513$ cal/mol.

The former and the latter values do not contradict one another, as L must increase with increasing k.

CHAPTER THREE

INFLUENCE OF THE DUST ON THE PHOTOMETRICAL PROPERTIES OF COMETS. SOLUTION OF THE DUST-GAS MODEL IN DETAIL

3.1. PROBLEM

As given in the preceding chapter the existence of the dust in cometary atmospheres was in outline first introduced into photometrical calculations by Vanýsek (1952). The comet dust-gas model became an important generalization of the Levin gas model. Also Vanýsek (1958) dealt with the determination of the momentary amount of dust in atmospheres of a few comets with intense continuous spectra. His calculation is based on the following assumptions:

(a) dust and gas constituents take an equal part in the total coma radiation;
(b) dust particles in the atmosphere have a certain frequency distribution;
(c) the intensity of the solar light reflected by the dust particle cloud falls with the square of heliocentrical distance;
(d) the phase-effect may be replaced by a factor of $0^m.5$;
(e) the albedo of reflecting material in coma is 0.1.

The target of the present chapter is to give a photometrical model of the dust coma. It is easy to show that the two last assumptions are a matter of convention, the analysis of the photometrical curve of a comet dust-gas model is able to abolish assumption (a), and assumption (b) may be replaced by a simpler one of the mean dimension of dust particles.

The main problem is assumption (c). The determination of the photometrical exponent of the dust comet head is the question. It may be divided into two items:

(1) relations between the exponent and the basic physical characteristics of the dust (or solid) radiation constituent;
(2) dependence of the exponent on the heliocentric distance.

3.2. FUNDAMENTAL EQUATIONS OF THE MODEL AND DISCUSSION

The basic physical characteristics of the dust radiation constituent describe the state of the dust (or solid matter) in a cometary head from the photometrical point of view. They are as follows:

(a) effective radius of the nucleus, i. e. the radius of a monolithic spherical nucleus;
(b) mean dust particle radius;
(c) equivalent thickness of the layer of photometrically effective dust, i. e. the thickness of the layer of a theoretically maximum concentration of particles at the surface of a monolithic nucleus;

(d) entire number of photometrically effective dust particles in a cometary atmosphere and the corresponding mass;

(e) other physical properties of dust particles and the nucleus (such as albedo, phase-effect, mass density etc.).

The magnitude of a body reflecting the solar light at a geocentric distance of 1 A.U. is given by the formula:

$$h_\Delta = \sigma - 2.5 \log A_0 . F(\varphi) - 5 \log R + 5 \log r, \qquad (3.1)$$

where A_0 is the albedo of a body, $F(\varphi)$ the function of the phase angle, i. e. the intensity ratio between the light reflected in angle φ to that reflected in the angle of 0°, R the radius of a body (in cm) and r the heliocentric distance of a body (in A. U.). Constant σ depends on the photometrical system used. The visual region of the spectrum will be further considered. The Moon as a calibration object then gives

$$\sigma = 38^m.8 \pm 0^m.05,$$

while the four brightest minor planets give

$$\sigma = 38^m.34 \pm 0^m.05.$$

The initial values of brightness, albedo and dimensions were taken from the papers of KUIPER (1954) and WATSON (1942).

Let us denote the effective radius of the comet nucleus R and its phase-effect $F_n(\varphi)$. The reduced nucleus brightness is expressed by the formula:

$$H_{\Delta d}(N) = \sigma - 2.5 \log A_0 . F_n(\varphi) - 5 \log R + 5 \log r. \qquad (3.2)$$

Let us further denote the radii of individual dust particles in the comet atmosphere as $\varrho_1, \varrho_2, \ldots, \varrho_\nu$ respectively. The total number of particles is ν and the summary surface exposed to the solar radiation proportional to $\sum_{(\nu)} \varrho_i^2$. Since tiny particles are the question (comparable in dimension with the wave-length of the visible radiation) diffraction of light must be taken into account. Owing to it the effective cross-section of a particle differs from its real cross-section. The differences are expressed by the Debye function $\Phi(\varrho)$. Moreover, the phase-effect of the dust particle cloud, $F_p(\varphi)$, is different from that of the comet nucleus. The reduced magnitude of the dust particle cloud in the cometary atmosphere may be got by adding (3.1) from 1 to ν. As the screening of particles by each other or by the nucleus is of no consequence we get

$$H_{\Delta d}(P) = \sigma - 2.5 \log [A_0 . F_p(\varphi) . \Phi(\varrho)] - 2.5 \log \sum_{(\nu)} \varrho_i^2 + 5 \log r. \qquad (3.3)$$

Inside a unit of volume there is on an average the following number of dust particles (of spherical shape and radius ϱ) when the maximum particle concentration takes place:

$$N_d = \frac{\sqrt{2}}{8\varrho^3}. \qquad (3.4)$$

The relation between the equivalent thickness of a dust layer, D, and the entire number of dust particles included in it, ν, is, according to the definition of the former, given by the formula:

$$\nu = 4\pi R^2 . D . N_d \qquad (3.5)$$

and the summary exposed surface of particles

$$\sum_{(\nu)} \varrho_i^2 = \pi R^2 D . \sqrt[3]{2 N_d} . \tag{3.6}$$

After inserting (3.6) into (3.3) and adding the brightness of the nucleus and that of the dust cloud the expression for the reduced brightness of the dust constituent of comet radiation results in

$$H_{\Delta d} = \sigma - 2.5 \log A_0 - 5 \log R - 2.5 \log E(r) + 5 \log r ; \tag{3.7}$$

here

$$E(r) = F_n(\varphi) + \pi D . F_\nu(\varphi) . \Phi(\varrho) . \sqrt[3]{2 N_d} \tag{3.8}$$

is the so-called function of dust. It is a special combination of the basic physical characteristics of the dust radiation constituent. Comparing (3.8) with (3.4) we get the equivalent thickness of the dust layer:

$$D(r) = \frac{\varrho \sqrt{2}}{\pi} \cdot \frac{E(r) - F_n(\varphi)}{F_\nu(\varphi) . \Phi(\varphi)} , \tag{3.9}$$

and from (3.5) the entire number of photometrically effective dust particles in the coma:

$$\nu(r) = \left(\frac{R}{\varrho} \right)^2 \cdot \frac{E(r) - F_n(\varphi)}{F_\nu(\varphi) . \Phi(\varrho)} . \tag{3.10}$$

The entire mass of the dust cloud is then simply

$$\mathfrak{M}_\nu(r) = \frac{4}{3} \pi \varrho^3 . s . \nu(r) , \tag{3.11}$$

where s is the particle mass density.

In the mentioned relations there are present a series of magnitudes that may be considered to be constant from the statistical point of view. First of all it is a question of the mean radius of dust particles. This problem has lately been studied by VANÝSEK (1960a). He has photometrically derived a colour excess of the light reflected by dust particles for a few comets of recent years. It depends on the character of reflecting solid particles in a cometary atmosphere, namely on their form, size and conductivity. The Mie classic theory has been used with VAN DE HULST's applications (VAN DE HULST, 1957) and the most effective dimension seemed to be about $\varrho \approx 2.10^{-5}$ cm when assuming the solar light being scattered by dielectric spherical particles.

VANÝSEK (1961) has also solved the same problem in another way. He derived the dependence of the polarization degree on the phase angle for a few suitable combinations of particle dimensions and refractive index, and compared the theoretical curves with those obtained from measurements. The comparison was performed for the Arend-Roland comet of 1957, polarization measurements of which had been get by BLAHA, HRUŠKA, ŠVESTKA and VANÝSEK (1958), and for the Mrkos comet of the same year on the basis of the observational material secured by MARTEL (1960). In the former comet the most effective dust particle radius resulted in $1.6.10^{-5}$ cm, in the latter about $1.9.10^{-5}$ cm. A dielectric character of particles was again postulated. Otherwise, LILLER (1957) drew the conclusion that metal particles may have had a dimension of about 6.10^{-5} cm, i. e. that of the same order as dielectric particles.

In relation (3.3) and next, symbol ϱ has been used for two not quite identical magnitudes. The "mean size" of dust particles in equation (3.3) means their mean quadratic radius, $\varrho^{(2)}$, while in equation (3.4) their mean cubic radius, $\varrho^{(3)}$. If $U(\varrho)$ is a frequency distributiom of the dust particle size, the ratio between the two magnitudes is

$$\frac{\varrho^{(3)}}{\varrho^{(2)}} = \left[\int_0^\infty U(\varrho)\, d\varrho\right]^{1/6} \cdot \left[\int_0^\infty \varrho^2 U(\varrho)\, d\varrho\right]^{-1/2} \cdot \left[\int_0^\infty \varrho^3 U(\varrho)\, d\varrho\right]^{1/3} \qquad (3.12)$$

However, current forms of the distribution function $U(\varrho)$ lead to rather small differences between $\varrho^{(2)}$ and $\varrho^{(3)}$, far smaller than the inaccuracy is in the particle size itself. For instance, let us assume the form

$$U(\varrho) = \text{const.}\exp\left[-h^2(\varrho - \varrho_0)^2\right]$$

and two distinct, almost extreme values of h, viz $h = \frac{1}{\varrho_0}$ and $h = \frac{1}{10\varrho_0}$ corresponding to relative frequencies $\frac{U(2\varrho_0)}{U(\varrho_0)} = 0.37$ and $\frac{U(10\varrho_0)}{U(\varrho_0)} = 0.45$ respectively (ϱ_0 is the "mean linear size" of particles). Ratio $\varrho^{(3)}/\varrho^{(2)}$ in the former case is 1.05 and in the latter 1.16. Hence, the differences do not exceed 20 per cent and both the magnitudes may be put equal to each other.

Another quasi-constant quantity is the albedo of reflecting stuff. Assuming the albedo of dust particles to be the same as that of other bodies without atmosphere scattering the solar light the following investigations may serve for determining its average value:

(a) albedo data of the main minor planets (WATSON, 1942);

(b) albedo of the Moon (ibid.);

(c) indirect determining albedo of the set of 17 comets (RICHTER, 1948);

(d) monochromatic albedo of 8 stony meteorites experimentally investigated by RICHTER (1959).

As a result we get 0.128 ± 0.046, 0.073, 0.086, 0.125 ± 0.023 (λ4200 Å) and 0.142 ± 0.024 (λ5250 Å) respectively. All the values are ranged round 0.10 ± ± 0.015.

As to the comet dust particle mass density we may use an analogy between their physical properties and those of the cosmic bodies met with the Earth. There exist several direct as well as indirect methods for determining the density of the latter bodies. However, the existence of at least three quite different groups of these bodies was proved, as for the mass density. Besides iron meteorites the mean mass density of which was established as 7.72 gm cm^{-3} by KRINOV (1947), normal stony meteorites and porous stony meteorites are the question. The former have according to KRINOV (ibid.) the mean mass density 3.54 gm cm^{-3}, while the latter are of much smaller density and so far no meteorite of such properties has been found. There is no doubt the differences are of high importance. In connection with the Bowen correlation theory of meteor streams and rainfalls the problem was dealt with by Kvíz (1960). He divided meteors into two groups: cometary meteors, products of comet disintegration, and asteroidal meteors, products of collisions of asteroids. The former are, according to Harvard Observatory investigations (JACCHIA, 1955, MCCROSKY, 1955, WHIPPLE, 1955a, 1955b), of the mass density of about 0.1 gm cm^{-3}, while that of the latter is ranged between 3 and 8 gm cm^{-3}. All stream-meteors and the greater part of bright sporadic meteors belong to the former group, while meteorites, the minor part of bright sporadic meteors, probably the greater part of faint sporadic meteors and obviously almost all micrometeorites (ibid.) to the latter. Other authors, however, consider for porous meteors mass density about 1 gm cm^{-3} (LEVIN, 1956, CEPLECHA, 1958, VANÝSEK, 1958, CEPLECHA, PADEVĚT, 1961). Hence, the dust particle mass density has been in a certain degree a question of convention up to now.

When deriving formulae (3.7) to (3.11) we have introduced functions $\Phi(\varrho)$, $F_n(\varphi)$, and $F_p(\varphi)$. The first of them is dependent on the particle dimensions only, so that the accuracy of its determination is given by that of ϱ. Numerical values of $\Phi(\varrho)$ were computed by DEBYE (1908).

The problem of phase-curve form of particles of various dimensions has recently been discussed in detail by RICHTER. We omit his phase-effect investigation of nuclei of 14 comets (RICHTER, 1948) that cannot be considered to be reliable. RICHTER did not take into account the variable concentration of dust particles in the nucleus region, but this effect is at least of the same order as the phase-effect. The mutual comparison of similar statistical methods proves their unreliability. While RICHTER in the paper in question found an extraordinarily quick decrease of the comet-nucleus brightness even at the phase angle of 10° (about $1^m.5$), BOBROVNIKOFF (1942) and some other explorers ascertained in this way no phase effect at all.

The results of two other papers of RICHTER (1956, 1959) are of considerably higher importance, where laboratory measurements of artificial dust clouds are summarized. The former of the two papers dealt with the phase curve of 0.01-cm particles of a number of terrestrial materials, among others river sand, glass, corundum and quartz. The curve form turns out qualitatively the same in every case; first the cloud brightness decreases with the increasing phase angle, as a rule rather slightly, so that the minimum is flat and not too deep, after which a markedly greater increase of the brightness takes place. Between 80° and 170° of phase angle (according to the sort of material) the phase effect is zero; the next increase occurs owing to diffraction of light. As to the quantitative considerations, the form of the phase curve depends on the sort of material as well as on the particle concentration inside the cloud.

In the latter of the two papers the analogous analysis of samples of 14 stony- and 3 iron meteorites has been carried out. The smoothed-out phase curves are given for three groups of particles according to the dimensions: 1 to 10 cm particles, 10^{-2}-cm particles and 10^{-4}-cm particles. In addition, metal particles have been treated separately from dielectrical particles. The form of the dispersion indicatrix depends in a high degree both on the properties of material and on the particle size and form. Generally, the Lambert photometrical law (LAMBERT, 1760) is not in any case quite consistent with experiments. Some phase curves obtained by RICHTER are given in Table 7. For comparison the Lambert photometric law, and the mean phase curve of the Moon (STEBBINS, BROWN, 1907, KING, 1909, ROUGIER, 1933, 1937) are listed.

For our purposes the data about dielectric particles of 10^{-4} cm and partly those of 10^{-2} cm are important. It is interesting that the photometrical efficiency of 10^{-4}-cm particles is least in the phase angle of 0°. The increase, however, is relatively slight and reaches 1^m as far as in angle of 120°, where only few co-

Table 7
Phase effect

φ	artificial cloud						Lambert's law	Moon
	dielectric particles			metal particles				
	$a=10^{-4}$ cm	$a=10^{-3}$ cm	$a=10^{0}$-10^{1} cm	$a=10^{-4}$ cm	$a=10^{-3}$ cm	$a=10^{0}$-10^{1} cm		
	m	m	m	m	m	m	m	m
0°	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	—0.05	+0.01	+0.03	—0.30	+0.01	+0.10	+0.01	+0.22
20	—0.18	+0.05	+0.14	—1.72	+0.09	+0.21	+0.05	+0.46
30	—0.32	+0.09	+0.31	—2.20	+0.19	+0.30	+0.11	+0.71
40	—0.45	+0.12	+0.48	—2.04	+0.28	+0.35	+0.20	+0.97
50	—0.57	+0.15	+0.67	—1.63	+0.40	+0.39	+0.32	+1.22
60	—0.67	+0.19	+0.90	—1.18	+0.58	+0.44	+0.47	+1.48
70	—0.76	+0.22	+1.14	—0.65	+0.77	+0.51	+0.65	+1.76
80	—0.84	+0.23	+1.40	—0.21	+0.94	+0.59	+0.87	+2.08
90	—0,89	+0.22	+1.67	—0.06	+1.09	+0.67	+1.13	+2.43
100	—0.96	+0.20	+1.95	—0.23	+1.22	+0.78	+1.44	+2.82
110	—0.99	+0.11	+2.24	—0.48	+1.32	+0.91	+1.81	+3.25
120	—1.00	0.00	+2.54	—0.83	+1.39	+1.08	+2.26	+3.76
130	—1.10	—0.22	+2.86	—1.18	+1.39	+1.29	+2.81	+4.35
140	—1.25	—0.56	+3.18	—1.61	+1.23	+1.54	+3.50	+5.06
150	—1.63	—0.96	+3.41	—2.04	+0.90	+1.84	+4.41	+5.99
160	—2.39	—1.50	+3.37	—2.58	+0.22	+2.35	+5.70	
170	—4.60	—2.12	+2.82	—3.25		+2.82	+7.95	

mets have been observed. Particles of 10^{-3} cm show, on the contrary, only a little positive phase effect within the range of angles between 0° and 120°, not more than $0^{m}.25$.

The largest metal particles studied by RICHTER show an agreement with the Lambert law within $\pm\ 0^{m}.5$ to about 90°, dielectric particles even a bit further on. The phase curve of the latter lies in the 0° to 130° region between the phase curve of LAMBERT and that of the Moon.

3.3. PHOTOMETRICAL EXPONENT OF THE SOLID PART OF THE COMET HEAD. GENERAL CONSIDERATIONS

Applying definition (1.3) to equation (3.7) we get the general expression for the photometrical exponent of the solid component of the comet head as follows:

$$n_d(r) = 2 - \frac{r}{E(r)} \cdot \frac{dE(r)}{dr}, \qquad (3.13)$$

or

$$E(r) = E_0 . \exp \left[\int_0^{\ln r} (2 - n_d)\, dy \right], \qquad (3.14)$$

E_0 is the function of dust in a unit of heliocentric distance. The relation between the number of particles, $\nu(r)$, and the photometrical exponent, n_d, results from (3.13) after inserting $\nu(r)$ from (3.10):

$$\left.\begin{aligned} n_d(r) &= 2 - \frac{r}{\nu(r) + \left(\frac{R}{\varrho}\right)^2 \cdot \frac{F_n(\varphi)}{F_p(\varphi) \cdot \Phi(\varrho)}} \times \\ &\times \left\{ \frac{\mathrm{d}\nu(r)}{\mathrm{d}r} + \frac{1}{F_p(\varphi)} \left[\nu(r) \cdot \frac{\mathrm{d}F_p(\varphi)}{\mathrm{d}\varphi} + \left(\frac{R}{\varrho}\right)^2 \cdot \frac{1}{\Phi(\varrho)} \cdot \frac{\mathrm{d}F_n(\varphi)}{\mathrm{d}\varphi} \right] \cdot \frac{\mathrm{d}\varphi}{\mathrm{d}r} \right\}. \end{aligned}\right\} \quad (3.15)$$

An analogous expression may be derived for the relation between exponent n_d and equivalent thickness D.

Photometrical exponent n_d, of course, is only an auxiliary magnitude, which makes it possible in our considerations and calculations to apply photometrical measurements. From the physical point of view relation (3.15) represents a differential equation of the sought for dependence $\nu = \nu(r)$. Its solution in the form of (3.15) is very difficult because of unknown analytical expressions of functions $F_p(\varphi)$ and $F_n(\varphi)$. In fact they are the functions of three variables, viz. heliocentrical distance of a comet, r, geocentrical distance of a comet, Δ, and heliocentrical distance of the Earth, $R_♁$, as

$$\varphi = \arccos \frac{r^2 + \Delta^2 - R_♁^2}{2r\Delta}. \quad (3.16)$$

Owing to (3.16), moreover, there is in (3.15)

$$\frac{\mathrm{d}\varphi}{\mathrm{d}r} = \frac{\partial\varphi}{\partial r} + \frac{\partial\varphi}{\partial\Delta} \cdot \frac{\mathrm{d}\Delta}{\mathrm{d}r} + \frac{\partial\varphi}{\partial R_♁} \cdot \frac{\mathrm{d}R_♁}{\mathrm{d}r}. \quad (3.17)$$

Functions $F_p(\varphi)$ and $F_n(\varphi)$ change from case to case. If the course of the number of photometrically effective dust particles is studied for a certain comet and if the course of exponent $n_d(r)$ is known, integral relation (3.14) may preferably be used. If we succeed in determining ν in a certain distance, r_ν, we derive $E(r_\nu)$ from relation

$$E(r_\nu) = F_n(\varphi_\nu) + F_p(\varphi_\nu) \cdot \Phi(\varrho) \cdot \left(\frac{\varrho}{R}\right)^2 \cdot \nu, \quad (3.18)$$

and insert into (3.14) for establishing E_0. Especially for "new" comets the first term on the right side of (3.18) is by several orders lower than the second, and therefore negligible. Further procedure is nothing but a problem of numerical quadrature of (3.14) and inserting into (3.10).

If the course of the number of dust particles of a statistical set of comets is investigated, the average magnitudes must be introduced according to the rules of the compensation computation.

Before we derive methods for determining n_d we are discussing some special solutions of differential equation (3.15). We shall assume $F_p(\varphi) = \text{const}$ and $F_n(\varphi) = \text{const}$ in a considered region of investigation. Then the expression within the square brackets on the right side of (3.15) is zero. If we denote

$$\alpha(r) = \frac{1}{\nu(r)} \cdot \left(\frac{R}{\varrho}\right)^2 \cdot \frac{F_n(\varphi)}{F_p(\varphi) \cdot \Phi(\varrho)}, \quad (3.19)$$

we can write

$$C = \alpha(r) \cdot \nu(r) = \text{const}.$$

Table 8

Percentage change of the number of dust particles in the cometary atmosphere

n_d	$r = 0.5$		$r = 1.0$		$r = 1.5$		$r = 2.0$		$r = 2.5$		$r = 3.0$	
	$\frac{1}{\nu}\|\Delta_r\nu\|$	$\frac{1}{\nu}\|\Delta_t\nu\|$	$\frac{1}{\nu}\|\Delta_r\nu\|$	$\frac{1}{\nu}\|\Delta_t\nu\|$	$\frac{1}{\nu}\|\Delta_r\nu\|$	$\frac{1}{\nu}\|\Delta_t\nu\|$	$\frac{1}{\nu}\|\Delta_r\nu\|$	$\frac{1}{\nu}\|\Delta_t\nu\|$	$\frac{1}{\nu}\|\Delta_r\nu\|$	$\frac{1}{\nu}\|\Delta_t\nu\|$	$\frac{1}{\nu}\|\Delta_r\nu\|$	$\frac{1}{\nu}\|\Delta_t\nu\|$
	%	%	%	%	%	%	%	%	%	%	%	%
2.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2.2	4.00	1.37	2.00	0.49	1.33	0.26	1.00	0.17	0.80	0.12	0.67	0.09
2.4	8.00	2.75	4.00	0.97	2.67	0.53	2.00	0.34	1.60	0.25	1.33	0.19
2.6	12.00	4.12	6.00	1.46	4.00	0.79	3.00	0.52	2.40	0.37	2.00	0.28
2.8	16.00	5.49	8.00	1.94	5.33	1.06	4.00	0.69	3.20	0.49	2.67	0.37
3.0	20.00	6.86	10.00	2.43	6.67	1.32	5.00	0.86	4.00	0.61	3.33	0.47
3.2	24.00	8.24	12.00	2.92	8.00	1.59	6.00	1.03	4.80	0.74	4.00	0.56
3.4	28.00	9.61	14.00	3.40	9.33	1.85	7.00	1.20	5.60	0.86	4.67	0.65
3.6	32.00	10.98	16.00	3.89	10.67	2.12	8.00	1.37	6.40	0.98	5.33	0.75

Some of the four following cases may take place:

a) The simplest case, $n_d = 2$, which leads to a constant number of particles $\nu = \nu_o$, is physically unjustified;

b) if $n_d = \text{const} \neq 2$, then, provided that $\alpha(r) \to 0$,

$$\nu = \nu_0 . r^{2-n_d}, \tag{3.20}$$

ν_0 is the constant of integration;

c) if $n_d = \text{const}$, and if $\alpha(r) > 0$, then

$$\nu = \nu_0 . \exp\left[(2 - n_d)\int_1^r \frac{1+\alpha}{r}\,\mathrm{d}r\right] =$$

$$= [\nu_0 + C] . r^{2-n_d} - C\,; \tag{3.21}$$

d) if, finally, $n_d = n_d(r)$,

$$\nu = [\nu_o + C] . r^2 . \exp\left[-\int_1^r \frac{n_d(r)}{r}\,\mathrm{d}r\right] - C. \tag{3.22}$$

Table 8 indicates for the given heliocentrical distance r and the given exponent n_d the change of the number of particles within 0.1 A.U., $|\Delta_r\nu|$, as part of the already ejected particles, provided that $\alpha(r) \to 0$ for the given r. In this case the quantity $|\Delta_r\nu|$ will be obtained from the formula

$$|\Delta_r\nu| = 0.10\,(n_d - 2) . \frac{\nu}{r} . \tag{3.23}$$

In addition to this quantity, the table indicates also the change of the number of particles during 1 day, $|\Delta_t\nu|$, under the same assumption as above, and for the case of a parabolical orbit of the comet. We have

$$|\Delta_t\nu| = 2.43 . 10^{-2}\,(n_d - 2) . \frac{\nu}{r^{3/2}} . \tag{3.24}$$

3.4. STATISTICAL METHOD FOR DERIVING DEPENDENCE $n_d = n_d(r)$

For a certain comet, the real form of the function $n_d = n_d(r)$ can be derived only from observations of the brightness in some spectral range unaffected by any emission-band of molecular radiation. Technically, this is difficult to achieve. An approximate form of this curve may also be derived from the total radiation, provided that molecular radiation is negligible in comparison with continuous radiation. This requirement can be fulfilled in some of the so-called "new" comets, however, mostly only within a certain interval of the heliocentrical distances.

Another possible solution offers itself in the theoretical approach on the basis of the physical processes in the comet. In this way LEVIN'S formula for the gas coma has been derived. In the case of the dust coma, however, the problem is much more complicated. This approach has not yet been used. An attempt of such character will be sketched in Section 3.7.

Excluding this, the only possibility consists in a statistical analysis of the material, which is being carried out in this and two following sections.

The basic idea of the statistical method of determining photometrical exponent of the dust cometary atmosphere consists in applying fundamental relations of the comet dust-gas model, i. e. formulae (2.9), and (2.8) in connection with (2.7). Provided we know the photometrical and physical exponents of the coma as a whole, those of its gas constituent and the ratio between contributions of the two radiation constituents, the mentioned formulae may be considered as two equations for the photometrical and physical exponent of the dust coma. The solution makes no difficulty.

In the fundamental equations of the dust-gas model given at the beginning of Chapter Two the brightness is corrected for the phase effect. From the material, of course, we get values affected by this effect, so that the derived relations must be modified. Let us denote $F(\varphi)$ the function of the phase angle for the coma as a whole and $F_d(\varphi)$ that for the dust constituent (for the gas one $F_g(\varphi) \equiv 1$), we get the following expression for the total photometrical exponent instead of (2.9):

$$n(r) = \frac{kn_dF_d.r^{-\eta_d} + n_g.r^{-\eta_g}}{kF_d.r^{-\eta_d} + r^{-\eta_g}} - \frac{k.r^{1-\eta_d}}{kF_d.r^{-\eta_d} + r^{-\eta_g}}\cdot\frac{\mathrm{d}F_d}{\mathrm{d}r} + \frac{r}{F}\frac{\mathrm{d}F}{\mathrm{d}r}\cdot \quad (3.25)$$

The corresponding physical exponent is given by

$$\eta(r) = \frac{\log\dfrac{(1+k).F}{kF_d.r^{-\eta_d} + r^{-\eta_g}}}{\log r}. \quad (3.26)$$

These relations, however, cannot be used for determining n_d and η_d. We must realize that the phase effect also influences values of the total photometrical and physical exponents derived from the measured (or estimated) brightnesses of a comet. Therefore fictional exponents $n'(r)$ and $\eta'(r)$ follow from the material instead of $n(r)$ and $\eta(r)$ given by (3.25) and (3.26) respectively. The relations between the two are as follows

$$n' = n - \frac{r}{F}\cdot\frac{\mathrm{d}F}{\mathrm{d}r}, \qquad \eta' = \eta - \frac{\log F}{\log r}. \quad (3.27)$$

Just after inserting (3.27) into (3.25) and (3.26), and solving the two last mentioned we get the resulting expressions for n_d and η_d, applicable to the observational material:

$$n_d = n' + \frac{r}{F_d}\cdot\frac{\mathrm{d}F_d}{\mathrm{d}r} + \frac{n' - n_g}{(1+k).r^{\eta_g - \eta'} - 1}, \quad (3.28)$$

$$\eta_d = \eta_g + \frac{\log\dfrac{kF_d}{(1+k).r^{\eta_g - \eta'} - 1}}{\log r}. \quad (3.29)$$

The treatment of the material and discussion of known magnitudes are simultaneously with the results of this method included in the two following sections, where, for simplicity, we shall write n instead of n' and η instead of η'.

Let us point out here that the selection of the material is discussed very attentively for getting results as reliable as possible, even at the cost of not having general validity.

3.5. MATERIAL AND ITS TREATMENT

The statistical method is applied to comets with a period $P > 1000$ years. The material has been taken from HRUŠKA's and VANÝSEK's new list of photometrical parameters (HRUŠKA, VANÝSEK, 1958), which has been submitted to a further analysis. First of all, all listed exponents, of which there were 144 values for 84 comets, have been classified according to their size.

Figure 3 shows the distribution of n into the intervals $\left(\frac{k}{2}, \frac{k+1}{2}\right)$, where $k = 0, 1, \ldots, 27$. Figure 4 gives the corresponding cumulative distribution function.

Figures 3 and 4 show clearly that the maximum frequency appears in $n = 3$, so that $n = 3.0$ is the mode. However, the dispersion n is considerable, so that this material cannot be directly used in the sense mentioned in the previous section. Here, evidently several different groups of "new" comets are concerned. Now the question arises as to how to select a homogeneous working set from the bulk of material. It is obvious that the exponent $n \approx 10$, for instance, is not characteristic for the group of comets under consideration, so that it would misrepresent the results in a high degree. The material may be made more accurate by the requirement of a mode to be equal simultaneously to the

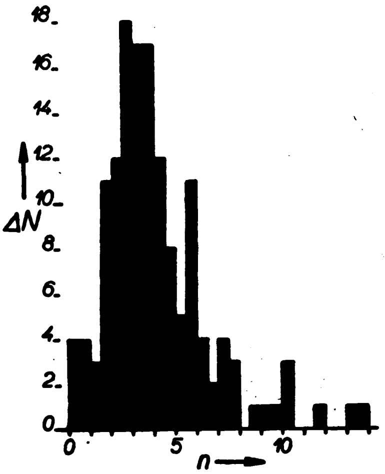


Fig. 3. Distribution of the photometrical exponents according to their size.

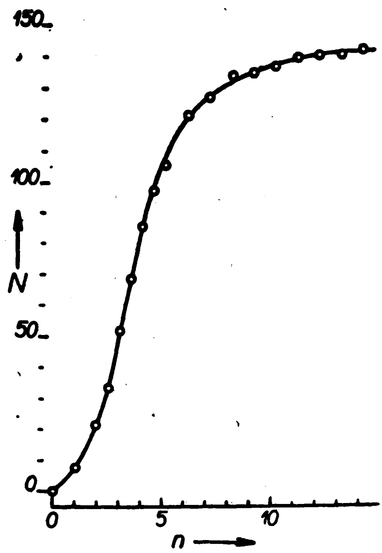
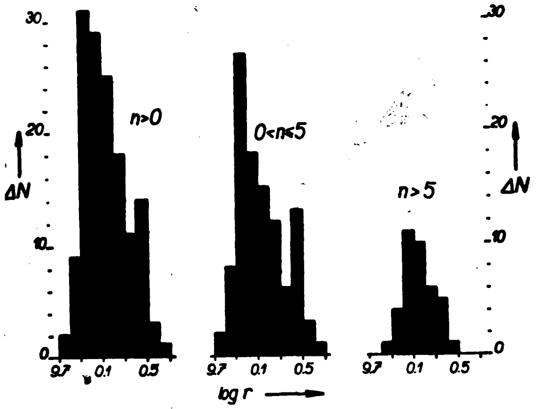


Fig. 4. Cumulative distribution function of the photometrical exponents.

median as well as to the arithmetical mean. If all positive exponents are taken into consideration, then the requirement of a median equal to 3.0 makes it necessary to take all $n \leq 4.8$, and the requirement of the arithmetical mean to be equal to 3.0 leads to the condition of $n \leq 5.0$. The latter value has been chosen for the upper limit of the individual exponents. This resulted in the division of the original material into two groups, namely, that with an $n \leq 5$, which will be analysed in the following, and that with an $n > 5$.

The distribution of the exponents according to the heliocentrical distance is given in Fig 5, for the whole material as well as for both groups which shows that the distribution in these three cases is rather analogous.



◀ *Fig 5.* Distribution of the photometrical exponents according to the heliocentrical distance.

After omission of one measurement in the heliocentrical distance $r = 6.5$ A.U. and supplementation by the photometrical exponent of the comet 1957d (Mrkos), computed by the author on the basis of 15 ELIAS observations (VINTER HANSEN, 1957b), the whole material consisted of 105 values of exponents for 69 comets. A synopsis of this material may be found in Tab. 9. The first column gives the serial number of the exponent, then follow the comet's denotation, the distance of the perihelion q, the orbital period P (for ellipses) or the eccentricity e of the orbit (in the case of parabolae and hyperbolae), the Briggs logarithm of the heliocentrical distance log r for which the value of the exponent is valid, the photometrical exponent $n(r)$, the corresponding mean phase angle φ (if the pre- as well as post-perihelion observations are present, the average of the two is given), the Briggs logarithm of the respective geocentric distance log Δ, and the position of the comet with regard to the perihelion at the time of observation (*per*): *A* — observations prior to the perihelion passage, *B* — following the perihelion passage, *AB* — the pre- as well as post-perihelion observations are approximately of the same frequency, *Ab* — the number of observations prior to the perihelion is of a higher order than after the perihelion, *aB* — the same of a lower order; the last column gives the quantity H_{10} according to VSEKHSVIATSKY (1956).

Owing to its little extent, the used material was unsuitable for a separate study prior and following the passage of perihelion. Therefore, at least, the following system of classification for the position of the measurement with regard to the perihelion has been introduced:

A	+ 1
B	— 1
AB	0
Ab	+ 0.5
aB	— 0.5

Table 9

Synopsis of the photometrical exponents ⩽ 5.0 of comets with an orbital period > 1000 years

No	Comet	*q*	*P*;*e*	log *r*	*n*(*r*)	*φ*	log *Δ*	*per*	H_{10}
1	1947 XII	0.110	*e*1.000032	9.791	2.0	63°	0.048	*B*	6.0
2	1941 I	0.368	*e*1.	9.792	1.1	93	9.875	*A*	6.3
3	1948 XI	0.135	171000	9.836	2.0	101	9.785	*B*	5.5
4	1858 VI	0.578	1950	9.840	3.47	63	0.041	*A*	3.3
5	1858 VI	0.578	1950	9.844	3.76	76	9.950	*AB*	3.3
6	1858 VI	0.578	1950	9.844	4.50	90	9.851	*B*	3.3
7	1903 IV	0.330	*e*1.	9.874	2.38	100	9.740	*A*	6.3
8	1939 III	0.528	7889	9.888	3.08	63	0.030	*B*	7.1
9	1886 I	0.642	*e*1.00045	9.892	0.72	49	0.120	*A*	5.2
10	1930 II	0.672	18180	9.897	4.27	68	9.987	*Ab*	8.4
11	1893 II	0.675	44410	9.910	2.24	66	9.999	*B*	6.6
12	1874 III	0.676	13708	9.916	4.78	65	0.003	*Ab*	5.7
13	1874 III	0.676	13708	9.924	3.8	64	0.010	*A*	5.7
14	1910 I	0.129	3906000	9.932	3.8	29	0.221	*B*	5.0
15	1911 V	0.489	2126	9.935	2.9	88	9.733	*A*	5.1
16	1898 X	0.756	158700	9.936	2.8	78	9.856	*Ab*	9.2
17	1886 IX	0.663	*e*1.00038	9.938	2.63	58	0.058	*Ab*	4.9
18	1896 III	0.566	*e*1.00048	9.950	4.8	70	9.930	*B*	10.3
19	1955 III	0.537	*e*1.	9.951	3.21	56	0.073	*B*	7.0
20	1911 V	0.489	2126	9.954	3.2	45	0.146	*B*	5.1
21	1911 II	0.684	1898	9.958	4.34	76	9.839	*B*	7.4
22	1937 V	0.863	160600	9.959	0.56	77	9.820	*B*	6.1
23	1957d	0.355	*e*1.	9.963	3.40	47	0.137	*B*	4.5
24	1886 IX	0.663	*e*1.00038	9.968	2.8	52	0.099	*A*	4.9
25	1886 II	0.479	*e*1.00023	9.970	2.05	37	0.199	*A*	6.6
26	1899 I	0.327	*e*1.00034	9.979	2.73	70	9.887	*AB*	5.4
27	1895 IV	0.192	*e*1.	9.982	3.4	29	0.233	*AB*	5.2
28	1911 V	0.489	2126	9.984	3.43	61	9.940	*AB*	5.1
29	1911 VI	0.788	9246	9.988	3.55	58	0.038	*A*	6.5
30	1937 V	0.863	160600	9.988	1.52	57	0.032	*AB*	6.1
31	1906 I	0.215	*e*1.	9.991	4.2	52	0.094	*A*	8.3
32	1937 V	0.863	160600	9.996	0.72	54	0.058	*AB*	6.1
33	1941 IV	0.790	18110	9.999	2	50	0.108	*B*	5.9
34	1937 II	0.621	*e*1.	0.000	3.74	73	9.766	*B*	10.4
35	1937 V	0.863	160600	0.000	0.8	60	0.000	*A*	6.1
36	1941 I	0.368	*e*1.	0.000	1.99	59	0.008	*A*	6.3
37	1902 III	0.401	1403000	0.002	2.63	74	9.752	*A*	6.0
38	1948 I	0.748	*e*1.	0.004	4.4	62	9.967	*B*	6.5
39	1937 II	0.621	*e*1.	0.013	4.47	70	9.772	*B*	10.4
40	1877 II	0.950	19765	0.021	3.8	54	0.057	*AB*	5.7
41	1947 XII	0.110	*e*1.000032	0.021	1.8	34	0.225	*B*	6.0
42	1947 III	0.962	*e*1.	0.024	2.33	65	9.876	*A*	11.2
43	1899 I	0.327	*e*1.00034	0.025	2.3	67	9.802	*B*	5.4
44	1882 I	0.061	1174000	0.033	2.8	59	9.975	*A*	4.1
45	1948 I	0.748	*e*1.	0.045	4.4	61	9.894	*B*	6.5
46	1899 I	0.327	*e*1.00034	0.054	3.77	61	9.829	*AB*	5.4
47	1952 I	0.740	262700	0.059	4.33	51	0.072	*B*	9.4
48	1946 II	1.018	*e*1.	0.060	2.62	60	9.645	*B*	9.5
49	1933 I	1.001	*e*1.	0.068	3.39	58	9.896	*B*	10.2
50	1951 II	0.719	*e*1.003119	0.077	3.47	52	0.037	*B*	9.8
51	1908 III	0.945	*e*1.00069	0.083	3.14	46	0.120	*A*	4.2
52	1912 II	0.716	56650	0.083	3.21	46	0.121	*aB*	6.2
53	1936 II	1.100	1642	0.083	4.62	37	9.738	*AB*	6.9

(continued)

Table 9 (continued)

No	Comet	q	P;e	log r	n(r)	φ	log Δ	per	H_{10}
54	1941 I	0.368	el.	0.083	1.9	50	0.057	*A*	6.3
55	1941 VIII	0.875	el.000968	0.093	3.0	53	9.930	*A*	7.1
56	1881 III	0.734	2429	0.114	2.40	47	0.073	*aB*	4.1
57	1914 II	1.198	el.	0.116	0.10	46	9.775	*AB*	9.4
58	1948 I	0.748	el.	0.118	4.8	49	9.932	*B*	6.5
59	1948 XI	0.135	171000	0.126	3.66	46	9.810	*B*	5.5
60	1914 V	1.104	el.00016	0.127	1.5	34	0.246	*A*	1.1
61	1941 VIII	0.875	el.000968	0.128	3.62	47	9.937	*AB*	7.1
62	1908 III	0.945	el.00069	0.134	5.00	46	0.051	*A*	4.2
63	1886 I	0.642	el.00045	0.146	4.9	35	0.240	*B*	5.2
64	1860 III	0.293	el.	0.152	2.8	41	9.855	*B*	5.8
65	1948 I	0.748	el.	0.154	2.97	44	9.979	*B*	6.5
66	1860 III	0.293	el.	0.155	3.0	41	9.856	*B*	5.8
67	1925 I	1.110	el.000629	0.159	3.28	37	0.220	*B*	5.4
68	1853 III	0.307	el.00025	0.164	4.2	34	0.231	*AB*	4.8
69	1913 II	1.457	5419	0.173	4.27	37	9.878	*aB*	7.7
70	1915 II	1.005	el.00024	0.210	3.84	36	0.022	*AB*	3.7
71	1943 I	1.354	2274	0.232	2.93	20	9.905	*AB*	4.6
72	1917 III	1.686	193100	0.235	2.8	34	0.059	*B*	6.1
73	1865 I	0.025	el.	0.250	3.8	28	0.330	*B*	3.8
74	1948 I	0.748	el.	0.254	3.6	33	0.118	*A*	6.5
75	1914 V	1.104	el.00016	0.258	3.1	24	0.367	*B*	1.1
76	1915 II	1.005	el.00024	0.265	1.66	32	0.117	*A*	3.7
77	1941 VIII	0.875	el.000968	0.267	2.5	20	9.983	*B*	7.1
78	1937 IV	1.734	el.000160	0.274	3.3	32	0.170	*B*	6.0
79	1915 II	1.005	el.00024	0.285	2.99	26	0.078	*B*	3.7
80	1937 IV	1.734	el.000160	0.288	3.88	30	0.165	*AB*	6.0
81	1917 III	1.686	193100	0.290	1.97	18	0.025	*aB*	6.1
82	1946 I	1.724	el.001201	0.294	3.8	25	0.169	*AB*	6.1
83	1937 IV	1.734	el.000160	0.303	3.25	28	0.186	*AB*	6.0
84	1914 V	1.104	el.00016	0.307	3.50	20	0.408	*AB*	1.1
85	1892 I	1.027	24480	0.320	3.9	28	0.255	*B*	3.2
86	1946 VI	1.136	el.	0.328	3.81	19	0.435	*AB*	4.8
87	1925 VII	1.566	el.000428	0.365	1.5	14	0.150	*B*	5.5
88	1925 VII	1.566	el.000428	0.365	2.5	14	0.150	*B*	5.5
89	1890 II	1.908	el.00041	0.400	2.55	16	0.493	*aB*	3.3
90	1930 IV	2.079	el.000379	0.411	0.5	18	0.489	*B*	6.8
91	1914 V	1.104	el.00016	0.449	3.7	11	0.555	*A*	1.1
92	1948 V	2.107	el.	0.454	4.44	20	0.454	*AB*	5.3
93	1907 I	2.052	el.00102	0.471	2.5	19	0.432	*AB*	6.5
94	1898 VIII	2.285	210800	0.472	4.8	11	0.319	*B*	5.6
95	1922 II	2.259	el.00086	0.472	0.5	12	0.565	*AB*	5.3
96	1951 I	2.572	el.000855	0.473	2.1	18	0.494	*AB*	4.0
97	1951 I	2.572	el.000855	0.473	1.54	18	0.504	*A*	4.0
98	1950 I	2.553	el.000671	0.480	0.4	18	0.512	*AB*	6.8
99	1932 VI	2.314	el.001376	0.487	4.03	13	0.354	*B*	3.5
100	1932 VI	2.314	el.001376	0.488	2.46	13	0.358	*B*	3.5
101	1949 I	2.518	el.	0.498	5.0	16	0.540	*A*	6.2
102	1904 I	2.708	el.00136	0.515	3.45	12	0.597	*B*	2.8
103	1898 VII	1.702	el.00103	0.521	2.7	12	0.599	*AB*	5.0
104	1889 I	1.815	el.00126	0.590	1.6	14	0.617	*AB*	3.6
105	1905 IV	3.339	el.00105	0.659	2.2	12	0.635	*B*	3.7

This system has been adhered to in the further tables in the column *per*, where the mean value for the given group of exponents is indicated.

In Tab. 9 the material is already arranged according to the heliocentrical distances. The treatment has been carried out by intervals in log *r*, as shown in Tab. 10. The first column gives the interval in log *r*, the second the mean value of log *r*, followed by the mean value of *n*; the next columns give the mean phase angle, the mean geocentric distance, the mean value of the position of measurement with regard to the perihelion, and the number of the measured values, *N*.

Table 10

Dependence of the values *n* under observation on log *r*

int log *r*	log *r*	*n*(*r*)	φ	log Δ	*per*	*N*
			°			
9.701 — 9.800	9.792	1.55	78.0	9.962	0.00	2
9.801 — 9.900	9.864	3.02	76.3	9.938	0.06	8
9.901 — 0.000	9.964	2.90	58.9	0.011	—0.02	26
0.001 — 0.100	0.049	3.28	55.8	9.935	—0.18	19
0.101 — 0.200	0.140	3.32	41.8	0.006	—0.43	14
0.201 — 0.300	0.262	3.09	27.5	0.116	—0.35	13
0.301 — 0.400	0.341	3.00	20.0	0.297	—0.50	7
0.401 — 0.500	0.469	2.66	15.7	0.465	—0.08	12
0.501 — 0.600	0.542	2.58	12.7	0.604	—0.33	3
0.601 — 0.700	0.659	2.20	12.0	0.635	—1.00	1

The statistical smooth-out of these values is given in Table 11. The columns of this table indicate: the interval in log *r*, the mean value of log *r*, the mean value and probable error of the exponent *n*, of the phase angle φ, of the geocentric distance Δ and of the position of measurement as referred to the perihelion; the last column shows the number of the measured values, *N*. The last line gives the values of these quantities for the whole extent of the heliocentrical distances.

Table 11

Dependence of the statistically smoothed-out values *n* on log *r*

int log *r*	log *r*	*n*(*r*)	φ	log Δ	*per*	*N*
			° °			
9.601 — 9.900	9.850	2.73 ± 0.28	76.6 ± 3.9	9.943 ± 0.027	+0.05	10
9.701 — 0.000	9.932	2.84 ± 0.13	63.8 ± 1.9	9.992 ± 0.015	0.00	36
9.801 — 0.100	9.979	3.05 ± 0.10	60.4 ± 1.4	9.973 ± 0.013	—0.07	53
9.901 — 0.200	0.033	3.12 ± 0.10	53.8 ± 1.2	9.985 ± 0.013	—0.17	59
0.001 — 0.300	0.138	3.24 ± 0.10	43.6 ± 1.4	0.008 ± 0.017	—0.30	46
0.101 — 0.400	0.228	3.17 ± 0.12	31.9 ± 1.2	0.108 ± 0.021	—0.41	34
0.201 — 0.500	0.357	2.91 ± 0.15	21.4 ± 0.9	0.286 ± 0.023	—0.28	32
0.301 — 0.600	0.438	2.76 ± 0.20	16.6 ± 0.7	0.430 ± 0.021	—0.25	22
0.401 — 0.700	0.495	2.62 ± 0.25	14.9 ± 0.5	0.502 ± 0.016	—0.19	16
0.501 — 0.800	0.596	2.49 ± 0.26	12.5 ± 0.3	0.612 ± 0.006	—0.50	4
			° °			
9.601 — 0.800	0.135	3.00 ± 0.08	44.6 ± 1.5	0.097 ± 0.016	—0.20	105

The dependence $n = n(r)$ is shown in Fig. 6. The vertical abscissae in the mean values n indicate their probable errors.

There still remains the unsolved problem, whether this material gives us the real picture of the statistical validity of the dependence $n = n(r)$, or whether that material is influenced by a certain selection-effect, owing to which certain values n for a certain r are for various reasons unaccessible to our observation. In principle, the problem turns round the question, whether the continuous decrease of n from a certain r with increasing heliocentrical distance is real or not. If we assume, for instance, that the average absolute magnitude of the comets of our group (with $P > 1000$ years) is about $H_0 \doteq 6^m.0$, then we obtain for $r = 5$ A.U., and $n(r) = 5$ the apparent magnitude of about $16^m.5$, which is a value lying under the limiting magnitude at the discovery of the comet. However, on the other hand, many comets may be followed much farther while receding from the Sun. Consequently the following effect ought to produce itself: the mean photometrical exponent should for a greater r be — in the case of the presence of selection — smaller prior to than following the perihelion. Consequently, the total photometrical exponent $n(r)$ should be smaller than the photometrical exponent after the perihelion (for the given r). The insufficiency of the material, however, makes a thorough investigation of this phenomenon impossible. There may be obtained only certain orientation-values, which are given in Table 12, and which attain only $r = 3$ A. U. For the given r, the table contains the mean photometrical exponent n (total exponent) and the mean post-perihelion exponent n_b, both with probable errors. In the mentioned table a certain trace of the above analyzed effect can be seen which, however, is rather uncovincing. It may be said that if there exists a selection-effect of the size of the photometrical exponent, then it will have no essential influence on the course of dependence $n = n(r)$, at least up to $r = 3$ A.U. On the other hand, the theory requires a continuous decrease of n with r from a certain r on.

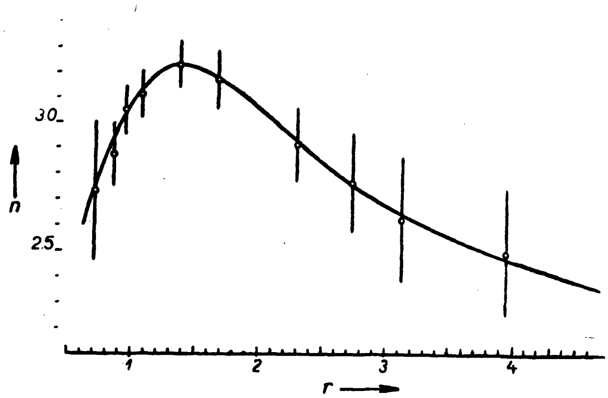


Fig. 6. Relation $n = n(r)$ for the comets of the first group with $P > 1000$ years.

Table 12

Difference between the total and post-perihelion values of the photometrical exponents

r	n	n_b
1.49	3.22 ± 0.11	3.14 ± 0.13
2.06	3.03 ± 0.14	2.93 ± 0.15
2.80	2.73 ± 0.20	2.79 ± 0.27

From Table 9 can be seen, moreover, that in the comets under observation "the mean absolute brightness" increases with an increasing heliocentrical distance, as follows from the trend of quantity H_{10}. This effect cannot be explained physically. On its existence partake partly the form of the definition of the quantity H_{10}, partly a certain selection-effect. From the definition of H_{10} it follows that for $r > 1$ A.U. H_{10} is in the case of $\eta < 4$ systematically smaller than H_0, where H_0 is the real absolute magnitude. This may be considered as an explanation of a certain part of the mentioned phenomenon. For the sake of simplicity, let us furthermore assume that any comet may be discovered only when it has attained the limiting apparent magnitude M. Then, if we put approximately $\Delta \doteq r$, the absolute (real) magnitude is equal to

$$H_0 = M - 5\left(1 + \frac{1}{2}\eta\right).\log r, \qquad (3.30)$$

so that the "limiting" absolute brightness increases with increasing heliocentri cal distance for the given η. Even if in reality M changes from case to case, similarly as η, this does not affect the statistical validity (3.30). Thus, with an increasing distance from the Sun we see a lower percentage of comets which, of course, has nothing in common with the problem of the selection-effect in the photometrical exponents in so far as the mentioned dependence is not selective with regard to n.

The dependence of the phase angle on the heliocentric distance for the investigated set of comets is given in Fig. 7, where individual points represent data arising from the material. The dash-dot line indicates the maximum phase angle possible, which may be expressed as:

$$\cos \varphi_{\max} = \left(1 - \frac{1}{r^2}\right)^{1/2} \qquad (3.31a)$$

for $r > 1$ A.U. and

$$\cos \varphi_{\max} = -1 \qquad (3.31b)$$

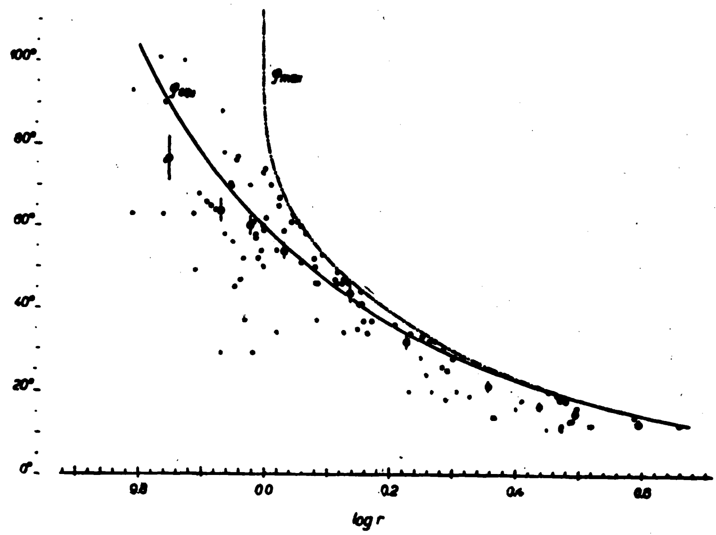


Fig. 7. Phase angle as related to the heliocentric distance.

for $r < 1$ A.U. The full line corresponds to the phase angle of heliocentric and geocentric distances being equal,

$$\cos \varphi_{equ} = 1 - \frac{1}{2r^2}. \tag{3.32}$$

Full circles give the average course of the relation, as follows from the studied material. Abscissae represent the mean errors. Mathematically the relation may be expressed in a generalized form of (3.32)

$$\cos \varphi_{comp} = 1 - \frac{1}{2r^2}[1 + \mu_1(r)], \tag{3.33}$$

where $\mu_1(r)$ is an empirical function of the form:

$$\mu_1(r) = 0.32 \frac{\sin \frac{1}{3}\pi[5\sqrt{r} - 4] - 1}{\sqrt{r}}. \tag{3.34}$$

When $r \to \infty$ formulae (3.31a), (3.32) as well as (3.33) converge to $1 - \frac{1}{2r^2}$.

For a number of heliocentrical distances Table 13 includes the mean phase angle observed, φ_{obs}, that for $\Delta = r$, φ_{equ}, maximum angle φ_{max} and mean angle computed from (3.33), φ_{comp}; further, the differences between the respective magnitudes, the mean error of the result and the root-mean-square deviation are given.

Table 13

Phase angle as related to the heliocentric distance

r	φ_{obs}	φ_{equ}	φ_{max}	φ_{comp}	o-e	o-m	o-c	m. e. of φ_{obs}	$\delta\varphi_{obs}$
	°	°	°	°	°	°	°	°	°
0.708	76.6	89.9	180.0	72.6	—13.3	—103.4	+4.0	±5.8	17.3
0.855	63.8	71.6	180.0	65.8	— 7.8	—116.2	—2.0	±2.9	17.0
0.953	60.4	63.3	180.0	60.9	— 2.9	—119.6	—0.5	±2.1	15.1
1.079	53.8	55.2	68.0	54.7	— 1.4	— 14.2	—0.9	±1.7	13.3
1.374	43.6	42.7	46.7	42.3	+ 0.9	— 3.1	+1.3	±2.1	13.9
1.690	31.9	34.4	36.3	32.2	— 2.5	— 4.4	—0.3	±1.8	10.3
2.275	21.4	25.4	26.1	20.8	— 4.0	— 4.7	+0.6	±1.3	7.3
2.742	16.6	21.0	21.4	16.5	— 4.4	— 4.8	+0.1	±1.1	4.9
3.126	14.9	18.4	18.7	14.8	— 3.5	— 3.8	+0.1	±0.8	3.1
3.945	12.5	14.5	14.7	13.2	— 2.0	— 2.2	—0.7	±0.5	0.9

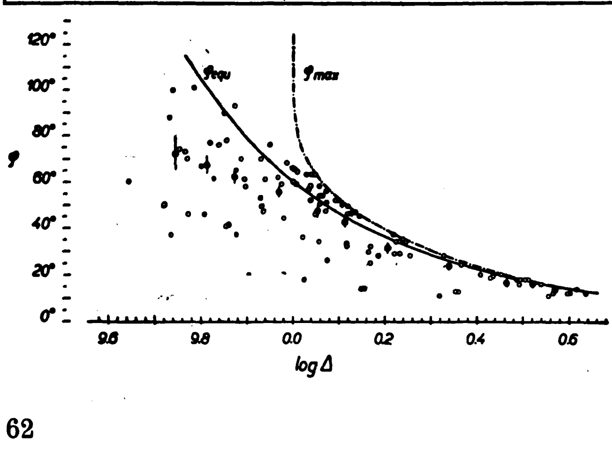


Fig. 8. Phase angle as related to the geocentric distance.

In the same way the dependence of the phase angle on the geocentric distance was studied. The result is in Fig. 8. The same comments are valid as before. The relation may be again expressed in the form of

$$\cos \varphi_{comp} = 1 - \frac{1}{2\Delta^2} [1 + \mu_2(\Delta)] , \tag{3.35}$$

where we must put

$$\mu_2(\Delta) = \frac{0.18 \sin 2\pi (\sqrt{\Delta} - 0.85) - 0.34}{\sqrt{\Delta}} . \tag{3.36}$$

The comparison of the material with the computations is given in Table 14.

Table 14
Phase angle as related to the geocentric distance

Δ	φ_{obs}	φ_{equ}	φ_{max}	φ_{comp}	o-e	o-m	o-c	m. e. of φ_{obs}	$\delta\varphi_{obs}$
	°	°	°	°	°	°	°	°	°
0.556	72.1	128.2	180.0	69.1	—56.1	—107.9	+3.0	±7.4	20.9
0.652	67.1	100.2	180.0	67.1	—33.1	—112.9	0.0	±3.9	18.8
0.746	62.3	84.1	180.0	64.0	—21.8	—117.7	—1.7	±3.2	19.3
0.931	55.7	65.0	180.0	56.3	— 9.3	—124.3	—0.6	±2.2	15.6
1.127	47.3	52.7	62.5	48.1	— 5.4	— 15.2	—0.8	±2.2	15.4
1.294	42.7	45.5	50.6	41.9	— 2.8	— 7.9	+0.8	±2.0	13.2
1.607	31.3	36.3	38.5	32.4	— 5.0	— 7.2	—1.1	±1.9	9.8
2.188	24.0	26.4	27.2	21.8	— 2.4	— 3.2	+2.2	±1.1	5.0
2.904	16.7	19.8	20.1	16.8	— 3.1	— 3.4	—0.1	±1.1	4.5
3.319	16.0	17.5	17.3	15.5	— 1.3	— 1.5	+0.5	±0.8	3.0
3.707	13.9	15.5	15.7	14.4	— 1.6	— 1.8	—0.5	±0.8	2.6

3.6. THE FORM OF DEPENDENCE $n_d = n_d(r)$ FOR THE LONG-PERIOD AND NON-PERIOD COMETS DERIVED BY A STATISTICAL METHOD

Here are three questions that must be solved before we come to the calculations themselves:

1) course of the total photometrical and physical exponents;

2) form of the F_d-curve as given in equations (3.28) and (3.29);

3) typical values of physical parameters k, B and α of the comet dust-gas model.

As a matter of fact the first question has been solved already. The course of the photometrical exponent is given in Fig. 6 or Table 11, and the physical exponent may be obtained its numerical or graphical quadrature; modifying (2.8) we find

$$\eta(r) = \frac{1}{\ln r} \int_1^r \frac{n(r)}{r} \, dr . \tag{3.37}$$

The phase effect will be established from RICHTER's investigations discussed in Section 3.2. Since long-period and non-period comets are the question, in which the dust takes a substantial part in the total radiation, the nucleus-effect is quite negligible and hence $F_d(\varphi) \equiv F_p(\varphi)$ may be put according to the denotation introduced in Section 3.2. Regarding what is known about the dimensions and character of photometrically effective particles the values given in the second and third columns of Table 7 may be considered to be representative. If the phase effect is expressed in a magnitude scale as difference $H(\varphi)$, the function of the phase angle is

$$F_d(\varphi) = 10^{-0.4H(\varphi)}. \tag{3.38}$$

This is just the quantity appearing in relation (3.29). If we realize that

$$\frac{1}{F_d} \cdot \frac{dF_d}{dH} = -0.921$$

and neglect the last term on the right side of (3.17) which is practically zero, the second term of relation (3.28) results in

$$\frac{r}{F_d} \cdot \frac{dF_d}{dr} = -52.8r \left[\frac{\partial\varphi}{\partial r} + \frac{\partial\varphi}{\partial\Delta} \cdot \frac{d\Delta}{dr} \right] \cdot \frac{dH}{d\varphi^\circ} \cdot \tag{3.39}$$

Partial derivates $\frac{\partial\varphi}{\partial r}$ and $\frac{\partial\varphi}{\partial\Delta}$ cannot evidently be computed from (3.33) and (3.35) respectively. Equation (3.16) must be used for this purpose from which the following expression may be derived when the general form of function $\Delta = \Delta(r)$ is assumed:

$$\frac{r}{F_d} \cdot \frac{dF_d}{dr} = 52.8 \sin^{-1}\varphi . \left[1 + \frac{r}{\Delta} \cdot \frac{d\Delta}{dr} \right] \cdot \left[\frac{r}{\Delta} - \cos\varphi \right] \cdot \frac{dH}{d\varphi^\circ} \cdot \tag{3.40}$$

The result is in a high degree dependent on ratio Δ/r and derivative $\frac{d\Delta}{dr}$. The statistical dependence between both the distances is represented in Fig. 9 and indicates an extraordinarily high dispersion. Values of $\frac{d\Delta}{dr}$ of the smoothed curve are ranged in a wide interval from $+0.14$ to $+1.53$; they are probably quite fictional. It is obvious that the analytical relation established from the statistical dependence must be of a rather low weight. It is likely that we will be closer to facts when we put $\Delta = r$. Let us compute correlation ratio, η^2 ($\log \Delta$, $\log r$), for both the relation of Fig. 9 and that of $\Delta = r$. We get $\eta_1^2 = 0.617 \pm 0.062$ and $\eta_2^2 = 0.529 \pm \pm 0.070$ respectively. The difference between both the values can serve as a criterion of reliability of the $\Delta = r$ approximation; it is $0.088 \pm \pm 0.094$ and linear regression is proved. If we put $\Delta = r$ in (3.40) we get

$$\left(\frac{r}{F_d} \cdot \frac{dF_d}{dr} \right)_{\Delta = r} = 105.54 \frac{dH}{d\varphi^\circ} . (4r^2 - 1)^{-1/2}. \tag{3.41}$$

This is the sought for expression.

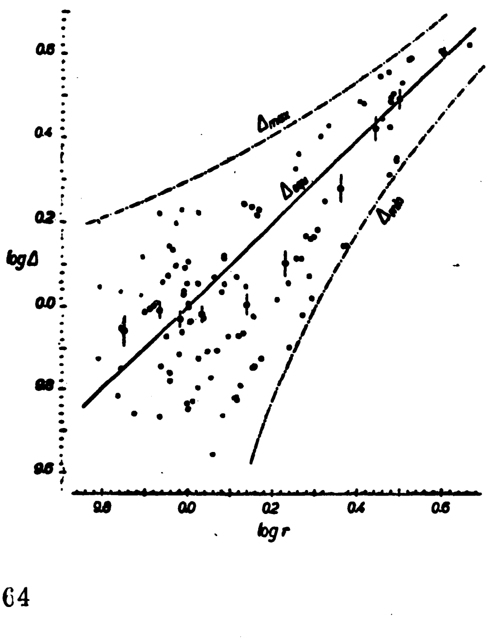


Fig. 9. Geocentric distance as related to the heliocentric distance.

Physical parameter B may be determined on the basis of the results of statistical investigations performed so far. OORT and SCHMIDT (1951) secure a list of 53 new, fairly new and old comets. A pure gaseous model was applied, so that the resulting mean parameter, $\overline{B} = 5.8$ is probably somewhat underestimated. An analysis of 15 comets with intense continuous spectra and $n < 5$ (see Chapter Four) gives $\overline{B} = 6.7$. In both the cases an assumption of T being proportional $r^{-1/2}$ was accepted. So far no material exists for determining the average value of B corresponding to another α. In the only synopsis of comets, where a general form of relation $T = T(r)$ is used (MARKOVICH, 1959), there is only one comet, 1881 III, with a sufficiently high k to be held as belonging to the group of comets under consideration. Its parameters are $B = 16.0$ and $\alpha = 0.2$. Assuming for $\alpha = \frac{1}{2}$ temperature T_0 equal to 330° K the respective values of the heat of evaporation are included in Table 15.

Table 15

Probable combinations of mean physical parameters

Comb.	B	α	L cal/mol	T,°K
I	5.8	0.5	3800	330
II	6.7	0.5	4400	330
III	16.0	0.2	3800	120
			4400	140

Ratio k has a considerable dispersion. It is likely that it is not lower than 1 on the average. The n_d-curves have been computed for the three given combinations of parameters B and α, and in each case for $k = 1, 2$ and 3. The results are included in Table 16. The individual columns give the heliocentrical distance, total photometrical and physical exponent, function of the phase angle, phase effect $\frac{r}{F_d} \cdot \frac{dF_d}{dr}$ and resulting values of the photometrical exponent of the dust coma. The dependence of the resulting form of n_d on B, α and k accepted is conspicuously expressed.

The dependence of n_d on the heliocentrical distance is shown in Fig. 10, 11, 12. The numbers in the indivi-

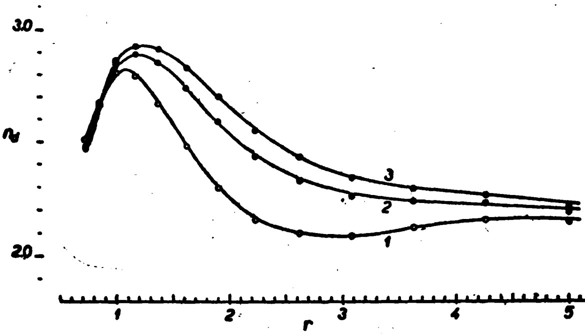


Fig. 10. Dependence of n_d on the heliocentric distance for $B = 5.8$, $\alpha = 0.5$ (Case I).

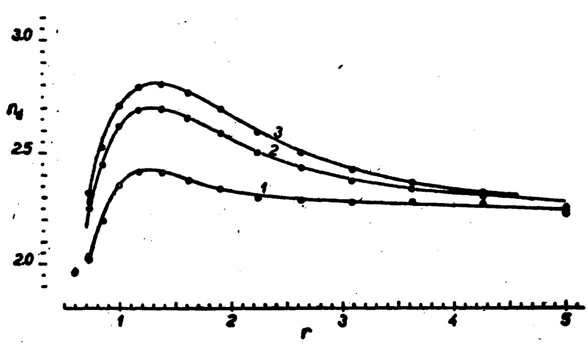


Fig. 11. Dependence of n_d on the heliocentric distance for $B = 6.7$, $\alpha = 0.5$ (Case II).

Table 16

Dependence of $n_d(r)$ on the heliocentric distance

$\log r$	$n(r)$	$\eta(r)$	F_d	$\frac{r}{F_d} \cdot \frac{dF_d}{dr}$	$n_d(r)$ Case I $k = 1$	Case I $k = 2$	Case I $k = 3$	Case II $k = 1$	Case II $k = 2$	Case II $k = 3$	Case III $k = 1$	Case III $k = 2$	Case III $k = 3$
9.86	2.78	2.93	1.351	−0.332	2.510	2.479	2.469	2.020	2.250	2.320	2.065	2.265	2.328
9.93	2.93	3.00	1.292	−0.253	2.681	2.679	2.678	2.196	2.446	2.525	2.385	2.534	2.582
0.00	3.07	3.07	1.247	−0.183	2.807	2.847	2.860	2.357	2.622	2.710	2.657	2.772	2.810
0.07	3.18	3.13	1.215	−0.184	2.792	2.893	2.927	2.415	2.692	2.791	2.787	2.889	2.925
0.14	3.23	3.17	1.177	−0.189	2.670	2.849	2.912	2.411	2.697	2.804	2.797	2.914	2.956
0.21	3.20	3.19	1.143	−0.171	2.482	2.736	2.829	2.376	2.656	2.768	2.696	2.853	2.909
0.28	3.10	3.18	1.115	−0.143	2.297	2.587	2.700	2.338	2.588	2.694	2.525	2.722	2.796
0.35	2.95	3.15	1.091	−0.121	2.156	2.434	2.550	2.300	2.501	2.592	2.327	2.546	2.632
0.42	2.79	3.10	1.072	−0.102	2.100	2.329	2.429	2.286	2.432	2.500	2.179	2.391	2.478
0.49	2.64	3.05	1.054	−0.087	2.093	2.263	2.341	2.276	2.373	2.420	2.081	2.269	2.350
0.56	2.52	2.99	1.041	−0.073	2.126	2.240	2.294	2.277	2.335	2.364	2.050	2.201	2.269
0.63	2.42	2.93	1.030	−0.056	2.161	2.231	2.262	2.270	2.302	2.318	2.048	2.164	2.218
0.70	2.31	2.87	1.020	−0.040	2.152	2.192	2.212	2.223	2.239	2.247	2.028	2.114	2.155

dual curves indicate the value k. Each curve consists, in principle, of three parts. For great r the course of n_d is almost constant. Any attempt to find a certain correlation is futile, since the values n_d are rather uncertain in this domain od r. It may be said that this is a domain where the physical properties of the dust coma are stable, where the total number of dust particles in it changes only very slowly. The second part of the curve, which occupies the domain from about $r_2 \approx 1.5$ A.U. to $r_1 \approx 4$ A.U., shows a clear trend of n_d with the heliocentrical distance. Mathematically it can be expressed in the form

$$n_d = 2 + a . r^{-c}, \qquad (3.42)$$

where a, c are coefficients depending on accepted B, α and k. In this period the dust coma begins rapidly to grow. As soon as a certain rate of growth of the number of dust particles is attained, the smoothing out of the exponent n_d, and finally its decrease as well, set in. This, however, is already the third part of the curve. But the decrease of n_d does in no way mean a reversal in the extent of the dust coma. In this domain the increase of the number of dust particles in the coma is approximately constant (provided that $\alpha(r) \to 0$).

In the interval of r, where the formula (3.42) is valid, the number of particles in the coma may be expressed by the relation:

$$\nu = \left[\nu_0 + \left(\frac{R}{\varrho}\right)^2\right] . \exp\left[\frac{a}{c} . r^{-c}\right] - \left(\frac{R}{\varrho}\right)^2 . \qquad (3.43)$$

As for the numerical values of the coefficients a, c from equations (3.42) and (3.43), they are for the given B, α and k shown in Table 17 together with the limits of their validity, r_2 and r_1.

From the study of the photometrical exponents of long-period and non-period comets we thus arrive at the conclusion that there exists a statistical

Table 17
Coefficients of empirical dependence $n_d = n_d(r)$

	I			II			III		
	$k = 1$	$k = 2$	$k = 3$	$k = 1$	$k = 2$	$k = 3$	$k = 1$	$k = 2$	$k = 3$
a	2.63	1.59	1.76	0.50	1.05	1.39	6.92	2.49	2.48
c	3.47	1.60	1.45	0.60	0.92	1.08	3.84	1.91	1.70
r_2	1.5	1.5	1.7	1.3	1.6	1.5	1.9	1.8	1.8
r_1	2.6	3.1	3.4	2.8	5.0	5.0	3.6	5.0	5.0

dependence of the photometrical exponent of the dust coma n_d on the heliocentrical distance in the following sense:

a) for $r > r_1$, there is $n_d \doteq$ const, $2 < n_d < n_1$, n_1 depends in a certain degree on B, α and k. Approximately we can put $n_1 = 2.2 \pm 0.1$;

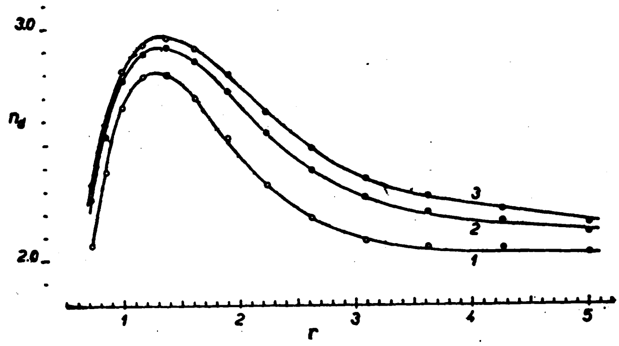


Fig. 12. Dependence of n_d on the heliocentric distance for $B = 16$, $\alpha = 0.2$ (Case III).

b) for $r_2 < r < r_1$ we have approximately the relation (3.42), in which case the coefficients a, c depend on B, α and k according to Table 17;

c) for $r < r_2$ exponent n_d changes considerably, attains its maximum and lies within the interval $2 < n_d < n_2$, n_2 depends again on B, α and k. Approximately $n_2 = 2.7 \pm 0.3$.

3.7. AN ATTEMPT TO SKETCH THE PHYSICAL DERIVATION OF RELATION $n_d = n_d(r)$

The introduction of the dust as a photometrical agent into quantitative considerations slightly complicated the calculations (if compared with the pure gaseous model). Mathematical controlling of the process of release of the dust in any detail is very difficult. The following procedure must be considered as an outline of a semi-analytical solution of the problem because of a number of simplifying assumptions and approximations.

Let us point out that the dependence of n_d on the heliocentric distance is important when the dust contributes to the total comet radiation in a high degree, i. e. in comets with intense continuous spectra.

Let us consider such a comet. Statistical studies indicate (e. g. Oort, Schmidt, 1951) that an intense continuous spectrum is a feature of the so-called "new" comets with a semi-major axis $\frac{1}{a} < 0.002$. In Chapter Four we will see that

$$\left(\frac{R}{\varrho}\right)^2 \approx 10^{-3 \pm 1} \nu$$

may be written in such comets, so that the $\left(\frac{R}{\varrho}\right)^2$-terms are negligible when compared with those of ν. In such a way we get

$$n_d(r) = 2 - \frac{r}{\nu(r)} \left[\frac{d\nu(r)}{dr} + \frac{\nu(r)}{F_d(\varphi)} \cdot \frac{dF_d(\varphi)}{d\varphi} \cdot \frac{d\varphi}{dr}\right] \tag{3.44}$$

and after correcting for the phase effect:

$$n_d(r) = 2 - \frac{r}{\nu(r)} \cdot \frac{d\nu(r)}{dr}. \tag{3.45}$$

Let us assume in accordance with a number of explorers (WHIPPLE, 1950, VANÝSEK, 1952, DOBROVOLSKY, 1953c, MARKOVICH, 1958) the disperse surface layer of the conglomerate forming the comet nucleus represents the source of dust. The next considerations of this section will concern the pre-perihelion period.

The main mechanism, which gets the dust into a cometary atmosphere, is connected with the process of evaporation of frozen gas masses from surface layers of the nucleus, for the dust is released with the gas. We assume the amount of the released dust depends on the impulse magnitude of the gas. As the initial molecule velocity changes with the heliocentrical distance quite slightly, a direct proportionality between the increase in the number of dust particles in the cometary atmosphere per unit of time, $\frac{d\nu}{dt}$, and the number of evaporated molecules during the same time must be valid. Besides it, the volume of the disperse layer also affects the increment of photometrically effective dust particles in the cometary atmosphere.

Collisions with cosmic velocities of a comet with micrometeorites in interplanetary space contribute to pulverization of the nucleus surface and release of dust particles from it. The number of particles expelled per unit of time is roughly proportional to the collision frequency (see Chapter Five).

In addition, it is necessary to take into account the fact that, owing to chaotic motions of dust particles in the atmosphere, a certain number of them come into collisions with the nucleus again soon after ejection. In this way an additional fragmentation of the nucleus surface occurs and a further increase of the number of dust particles takes place in the cometary atmosphere. Hence, increment $d\nu$ is proportional to the momentary number of photometrically effective dust particles in the comet head.

Finally, the observed variation of the number of dust particles is also proportional to the interval of time, during which particles remain inside the comet atmosphere region, i. e. to their effective "life-time".

If we denote the number of gas molecules evaporated from the nucleus surface per unit of time as n_m, the number of photometrically effective dust particles in the atmosphere at a given moment as ν, the frequency of collisions with micrometeorites as f_M, the number of dust particles included in a dust layer near the nucleus surface at the same moment as N, and the "life-time" of dust particles as τ, we get the following expression for increment $d\nu$:

$$d\nu = \text{const}.n_m.\nu.f_M.N.\tau.dt. \tag{3.46}$$

In the case of molecular flow taking place in comets, n_M may be written in the form (DOBROVOLSKY, 1953c, MARKOVICH, 1959):

$$n_m \sim \frac{P}{\sqrt{2\pi.m.k_0.T}}, \tag{3.47}$$

P is the pressure of saturated vapour above the gas-ice, T the surface temperature of the nucleus, m the mass of a molecule and k_0 BOLTZMANN's constant. If all heat is spent for evaporation of the ice, DOBROVOLSKY (1953c) applied an approximate relation between pressure P and temperature T as follows:

$$P = A.T^\beta,$$

where A and β are constants characterizing a gas. With regard to (2.3) relation (3.47) has the form:

$$n_m \sim T^{\beta-1/2} \sim r^{\alpha(1/2-\beta)}. \tag{3.48}$$

The frequency of collisions of a comet with micrometeorites may be easily expressed if we know the cross-section of the comet nucleus, S, micrometeorite space concentration, c_M, and collision velocity, V:

$$f_M = S.c_M.V. \tag{3.49}$$

In Chapter Five we will find that $c_M \sim r^{-\gamma}$, $\gamma \rightarrow 0_+$; especially for comets with extended orbits, the dominant component of the average collision velocity is very close to the radial orbital velocity of the comet, so that

$$f_M \sim r^{-\gamma}.\left|\frac{dr}{dt}\right|. \tag{3.50}$$

If the interaction between a dust particle and surrounding gas is weak the particle life-time may be expressed as a function of its initial velocity, v, and effective acceleration on it, g:

$$\tau \approx \frac{v}{g}. \tag{3.51}$$

Dynamical effects of gas on the motion of a dust particle in the cometary atmosphere is characterized by the time of relaxation (HRUŠKA, 1959):

$$T_E = \frac{\dfrac{M}{v_0}.\exp\left[\left(\dfrac{j}{v_0}\right)^2\right]}{4\pi\varrho^2 n_0 m \sum\limits_{n=0}^{\infty}\left(\dfrac{j}{v_0}\right)^{2n}.\dfrac{\binom{n+2}{2}}{\Gamma\left(n+\dfrac{5}{2}\right)}}, \tag{3.52}$$

where v_0 is the most probable "thermal" velocity of molecules, n_0 their concentration, m the mass of a molecule, j the velocity of a dust particle relative to the gas, M and ϱ its mass and radius. Using the most probable values, $v_0 = 10^5$ cm.s^{-1}, $n_0 = 3.10^4$ cm^{-3} (Dobrovolsky, 1953b), $m = 4.10^{-23}$ gm, $j = 10^5$ cm.s^{-1}, $M = 10^{-13}$ gm, $\varrho = 2.10^{-5}$ cm (Vanýsek, 1960a), we get the time of relaxation

$$T_E \approx 6 \text{ years},$$

i. e. by three orders higher than the life-time of a dust particle; the form of equation (3.51) is therefore quite satisfying. As (see Chapter Five)

$$v \sim r^{-1} . v_0^{1/2},$$

$$g \sim r^{-2},$$

the life-time of a dust particle is proportional to

$$\tau \sim r^{1-\frac{\alpha}{4}} . \tag{3.53}$$

Let us study now the drop of gas supplies, N_m, in the surface layer of the comet nucleus:

$$N_m(t) = N_m^{(0)} - \text{const} \int_{t_0 - \frac{1}{2}P}^{t} T^{\beta - 1/2} \, \mathrm{d}t . \tag{3.54}$$

Here $N_m^{(0)}$ is the number of molecules initially (i. e. in aphelium) included in the surface layer, t_0 is the time of perihelion passage of a comet, P the orbital period. On Dobrovolsky's assumptions Markovich (1958) found that exponent α of equation (2.3) is connected with exponent β by the relation

$$\alpha = \frac{4}{2\beta - 1} . \tag{3.55}$$

Assuming the heliocentrical distances under consideration are small relative to the semi-major axis, the integral on the right side of (3.54) may be replaced by expression

$$\int_{\infty}^{r} r^{-\mu} \left(1 - \frac{q}{r}\right)^{-1/2} \mathrm{d}r , \quad \mu = \alpha\beta - \frac{1}{2}(1 + \alpha) ,$$

q is the perihelion distance; with respect to (3.55) we get

$$N_m(r) = N_m^{(0)} \left[1 - \frac{2}{\pi} \arcsin \left(\frac{q}{r}\right)^{1/2}\right],$$

where, moreover, the boundary condition $N_m(q) = 0$ is introduced.

The solution of the equation of heat conduction, performed by Markovich (1959), leads always to exponents α higher than those of relation (3.55). The same effect follows from observations. Physical reasons for it are quite obvious: only a part of the incident solar heat is spent for evaporation of frozen gases, the other part increases the temperature of the nucleus directly; the entire increase of the temperature is therefore faster than that given by (3.55). The most probable value is close to $\mu = 3$ ($\alpha = 0.2$, $\beta = 18$) instead of $\mu = \frac{3}{2}$ and the variation of the number of gas molecules may be expressed in the form

$$N_m(r) = N_m^{(0)} . \left[1 - \xi\left\{1 - \left(1 - \frac{q}{r}\right)^{1/2} . \left(1 + \frac{1}{2} . \frac{q}{r}\right)\right\}\right].$$

Constant ξ will be determined from the boundary condition $N_m(q) = 0$ and then

$$N_m(r) = N_m^{(0)} . \left(1 - \frac{q}{r}\right)^{1/2} . \left(1 + \frac{1}{2} \cdot \frac{q}{r}\right). \tag{3.56}$$

In large heliocentrical distances mostly the gas as the volatile component of the mixture escapes from the surface layer. The dust, on the contrary, is expelled together with the gas only in smaller distances from the Sun; we may assume that here the dust ejection runs with a higher power of the gas-concentration drop. Dust supplies in surface layers are not, as a rule, renewed in a sufficient degree, so that their gradual exhaustion occurs. Inevitability of a rapid disintegration of the dust layer in a comet was pointed out by VANÝSEK (1952). He asserts that such a layer may exist for not more than a few revolutions round the Sun. This fact is experimentally corroborated by the absence of a continuous spectrum in the majority of short-period comets.

According to what has just been said, in heliocentrical distances $r > 1.6\, q$ the drop of the dust concentration may be approximately expressed

$$\ln \frac{N}{N^{(0)}} \sim -\left(\frac{q}{r}\right)^2, \tag{3.57}$$

where $N^{(0)}$ is the initial dust-particle concentration in the layer. With an increasing heliocentric distance ratio $N/N^{(0)} \rightarrow 1$ rapidly and is of no importance in distances larger than 2 A.U. In small heliocentric distances, on the contrary, function $N = N(r)$ prevails over that of $n_m = n_m(r)$ in relation (3.46).

An approximate validity of (3.57) may be proved still in another way; from the considerations of this section the following relation holds good in large heliocentrical distances

$$\mathrm{d}N \approx -\mathrm{d}\nu$$

or, with respect to (3.46) and (3.45) or (3.22)

$$\ln \frac{N(r)}{N^{(0)}} = \mathrm{const} \int_{\infty}^{r} r^{3-\alpha\beta+\frac{1}{4}\alpha-\gamma} . \exp\left[-\int_{1}^{r} \frac{n_d(r)}{r}\, \mathrm{d}r\right] \mathrm{d}r\,.$$

After integrating it we get an approximate relation

$$\frac{N(r)}{N^{(0)}} \sim \exp\,[-\,\mathrm{const}\,.\,r^s]$$

with the exponent

$$s(r) = 4 - \alpha\beta + \frac{1}{4}\alpha - \gamma - \frac{1}{\ln r}\int_{1}^{r} \frac{n_d(r)}{r}\, \mathrm{d}r\,; \tag{3.58}$$

the last term on the right side changes only quite slightly with r, the others are constants.

If in the expansion of function (3.56) higher-order terms are taken into account than those of the second order, exponent $s(r)$ is given by the formula sufficiently accurate for every $r \gg 1.3\,q$:

$$s(r) = -\left\{2 + \frac{\frac{1}{3}\cdot\frac{r}{q} + \frac{3}{4}}{\left(\frac{r}{q}\right)^2 + \frac{1}{3}\cdot\frac{r}{q} + \frac{3}{8}}\right\}. \tag{3.59}$$

According to our consideration both the expressions, (3.58) as well as (3.59), should converge to the resulting common s_0 in large heliocentrical distances:

$$s_0 = \lim_{r \to \infty} s(r).$$

In small distances from the Sun (3.58) and (3.59) may differ from each other rather more. A comparison is performed in Table 18. The statistical method of Section 3.6. has been applied to relation (3.58) and ratios $\overline{\left(\frac{r}{q}\right)}$ from

Table 18

Comparison of the course of exponent $s(r)$ derived in different ways

r	$\overline{\left(\frac{r}{q}\right)}$	$s(r)$		
		from (3. 59)	from (3. 58); Case III	
			$k = 1$	$k = 3$
0.71	1.57	—2.38	—1.91	—2.12
0.86	1.56	—2.38	—2.09	—2.26
0.96	1.64	—2.36	—2.18	—2.34
1.10	1.73	—2.34	—2.25	—2.38
1.40	1.96	—2.29	—2.31	—2.46
1.71	2.22	—2.25	—2.30	—2.46
2.35	2.49	—2.21	—2.19	—2.39
2.78	2.85	—2.18	—2.12	—2.34
3.15	3.15	—2.15	—2.07	—2.29
3.76	3.76	—2.13	—2.02	—2.23

Table 9 to that of (3.59). The results of Table 18 corroborate our statement: exponents $s(r)$ derived in different ways converge to the value of —2. The dispersion of $s(r)$ in smaller heliocentrical distances is consistent with our considerations, too.

By inserting (3.48), (3.50), 3.53) and (3.57) into the balance equation (3.46) we find

$$d\nu = -\,\mathrm{const}.\nu.r^{1-\alpha\beta+\frac{1}{4}\alpha-\gamma}.\exp\left[-\frac{\varepsilon}{r^2}\right]dr, \tag{3.60}$$

where ε is a positive constant, and, finally, after inserting (3.60) into (3.45):

$$n_d(r) = 2 + a.\exp\left[-\frac{b}{r^2}\right].r^{-c}, \tag{3.61}$$

where coefficients a, b, c vary from case to case; the numerical value a depends on the initial volume of the dust layer, heat conductivity, specific heat, density, micro- and macroscopic structure of the meteoric material and a series of other physical magnitudes, while the other two coefficients are given by

$$\left.\begin{aligned} b &= \varepsilon > 0 \\ c &= \alpha\beta + \gamma + \frac{1}{4}\alpha - 2 \,. \end{aligned}\right\} \quad (3.62)$$

From the physical point of view formula (3.61) is nothing but a hypothesis. However, let us pay attention to the following fact: for rather large r (and relatively low b) term $\exp\left[-\frac{b}{r^2}\right]$ is close to a unit and formula (3.61) is then identical with formula (3.42) found empirically. The drop of n_d in small heliocentric distances following from (3.61) is also well expressed in empirical curves (Fig. 10, 11, 12).

Let us study in what degree formula (3.61) is consistent with the statistical n_d-curves in detail. When applying the method of least squares we find coefficients a, b, c from the system of normal equations:

$$\left.\begin{aligned} \ln a.N - b.\left[\frac{1}{r^2}\right] - c.[\ln r] - [\ln(n_d - 2)] &= 0, \\ \ln a.\left[\frac{1}{r^2}\right] - b.\left[\frac{1}{r^4}\right] - c\left[\frac{1}{r^2}\ln r\right] - \left[\frac{1}{r^2}\ln(n_d - 2)\right] &= 0, \\ \ln a.[\ln r] - b\left[\frac{1}{r^2}\ln r\right] - c\,[(\ln r)^2] - [\ln r.\ln(n_d - 2)] &= 0, \end{aligned}\right\} (3.63)$$

where N is the number of measurements and square brackets denote the summation.

In such a way, formula (3.61) has been compared with all the curves from Fig. 10, 11, 12 and the results are included in Table 19; the coefficients are given with their probable errors. Moreover, the maximum n_d-exponent, $n_{d\,\max}$, as well as the corresponding heliocentrical distance, $r_{\max}$, are indicated as computed from

$$n_{d\,\max} = 2 + a.\left(\frac{c}{2be}\right)^{c/2}, \quad r_{\max} = \left(\frac{2b}{c}\right)^{1/2}.$$

Finally, the mean residual, ε, is given between the empirical curve and that computed from (3.61).

The conclusions can be drawn as follows:

1) the formula satisfies best of all Case III (mean residual $\pm$ 0.018), worst Case I;

2) the formula satisfies better curves with higher k;

3) maxima of curves are ranged close to 1.3 A.U. within $\pm$ 0.25 A.U. and there may be hardly found any correlation with accepted B, α and k;

4) the highest maximum is in Case III, the lowest in Case II; its value always increases with increasing k;

5) a comparison of c from Tables 17 and 19 indicates good agreement in fact: the former gives $\bar{c} = 1.84$, the latter $\bar{c} = 1.94$. For $\alpha = 0.2$, $\beta = 18.0$ and $\gamma = 0$ we get from (3.62) $c = 1.55$;

6) values of all the three coefficients with increasing k increase in Case II, conspicuously decrease in Case III and no trend is apparent in Case I.

In all the studied cases the residuals are less than proper errors of observed photometrical exponent. In this sense the general form of (3.61) may be considered sufficient for mathematically expressing the photometrical exponent n_d for non-period and long-period comets. Formula (3.61) itself may be found semiempirical as to its character. Since the form of function $n_d = n_d(r)$ is of little importance for short-period comets and other comets in which the gas has a dominant influence on photometrical parameters, formula (3.61) will be further applied as a general approximate expression for the photometrical exponent n_d.

Table 19

Coefficients of semi-empirical dependence $n_d = n_d(r)$

	Case I			Case II			Case III		
	$k = 1$	$k = 2$	$k = 3$	$k = 1$	$k = 2$	$k = 3$	$k = 1$	$k = 2$	$k = 3$
a	2.68 ± 0.63	2.17 ± 0.25	2.67 ± 0.17	0.88 ± 0.21	2.04 ± 0.12	2.45 ± 0.08	11.95 ± 2.56	5.84 ± 0.28	4.73 ± 0.14
b	1.25 ± 0.21	1.02 ± 0.10	1.17 ± 0.06	1.06 ± 0.23	1.28 ± 0.05	1.29 ± 0.03	2.90 ± 0.21	2.00 ± 0.04	1.76 ± 0.03
c	2.32 ± 0.20	1.60 ± 0.10	1.61 ± 0.05	0.90 ± 0.20	1.33 ± 0.05	1.40 ± 0.03	3.83 ± 0.17	2.41 ± 0.04	2.09 ± 0.02
r_{max}	1.04	1.13	1.21	1.53	1.39	1.35	1.23	1.29	1.30
$n_{d\,max}$	2.77	2.80	2.89	2.38	2.68	2.80	2.79	2.92	2.97
ε	± 0.059	± 0.039	± 0.024	± 0.037	± 0.019	± 0.012	± 0.026	± 0.015	± 0.011

3.8. NUMERICAL ANALYSIS OF THE PHOTOMETRICAL EXPONENT CURVE OF THE COMET DUST-GAS MODEL. COMET 1957 III

In the last chapter we dealt with the solution of the comet dust-gas model based on two physical parameters (besides the absolute magnitude), viz. the heat of evaporation of molecules and the ratio of brightness of the two physical components.

Now, on the basis of the discussion of the form of the n_d-curve, the opportunity arises of an important generalization of the methods derived in the last chapter.

We shall assume that we have a series of numerous estimates (or measurements) of comet brightness at our disposal sufficient for establishing the course of the total photometrical exponent. The obtained values of the exponent satisfy the basic relation of the dust-gas model:

$$n(r) = \frac{n_d k . \exp\left[-\int_1^r \frac{n_d}{r}\, dr\right] + n_g . \exp\left[-\int_1^r \frac{n_g}{r}\, dr\right]}{k . \exp\left[-\int_1^r \frac{n_d}{r}\, dr\right] + \exp\left[-\int_1^r \frac{n_g}{r}\, dr\right]}, \quad (3.64)$$

where the partial photometrical exponents are

$$
\left.\begin{aligned}
n_g(r) &= \frac{\alpha}{2} + \alpha B r^{\alpha}, \\
n_d(r) &= 2 + a.\exp\left[-\frac{b}{r^2}\right].r^{-c}.
\end{aligned}\right\} \quad (3.65)
$$

In such a case relation (3.64) includes six unknown parameters — k, B, α, a, b, c. The problem is to find their numerical values. There exists no exact analytical solution of equation (3.64) and approximate methods must be applied. Ratio k turns out the magnitude of the greatest importance when deriving them. Since exponent $n_d(r)$ given by the second equation (3.65) has not its integral in a close form, function $\eta_d(r)$ must be expressed as a function of η and η_g according to (2.8).

For "new" comets, which are characterized by a high k, it is acceptable to transcribe relation (3.64) into the form

$$
n = n_d + \frac{n_g - n_d}{1 + k} . r^{\eta - \eta_g} \quad (3.66)
$$

and expand functions $n_d = n_d(r)$, $n_g = n_g(r)$ and $\eta_g = \eta_g(r)$ in a series at the point $r = 1$ A.U. The course of exponent n is known from the material and that of function $\eta = \eta(r)$ may be simply determined by means of graphical integration of n. We get the series of the form

$$
n(r) = \sum_{i=0}^{m} A_{2i} (\ln r)^i + r^{\eta} \sum_{i=0}^{m} A_{2i+1} (\ln r)^i, \quad (3.67)
$$

where

$$
\left.\begin{aligned}
A_0 &= 2 + a.e^{-b}, \\
A_1 &= (1 + k)^{-1}.\left[\alpha\left(\frac{1}{2} + B\right) - A_0\right], \\
A_2 &= ae^{-b}.[2b - c], \\
A_3 &= (1 + k)^{-1}.[\alpha^2 B - A_2] - \alpha^2 A_1\left(\frac{1}{2} + B\right), \\
A_4 &= a.e^{-b}\left[2b(b-1) - 2bc + \frac{1}{2}c^2\right], \\
A_5 &= \frac{1}{2}A_1\alpha^2 B^2 + (1 + k)^{-1}.\left[\alpha A_2\left(\frac{1}{2} + B\right) - A_4 - \alpha^3 B^2\right].
\end{aligned}\right\} \quad (3.68)
$$

The proper computation must start from the form of dependence $n = n(r)$ in the neighbourhood of $r = 1$ A.U. For the series to converge it is necessary to compute with $m > 2$. The six parameters of relation (3.64), of course, will be derived from coefficients $A_0, \ldots, A_5$ only.

For short-period comets which low values of ratio k are typical for, basic relation (3.64) must be transcribed into another form:

$$
n - 2 = n_g (1 + k)^{-1}.r^{\eta - \eta_g}.\left[1 + \frac{1}{n_g}\left\{(1 + k)(n_d - 2).r^{\eta_g - \eta} - n_d\right\}\right]. \quad (3.69)
$$

By expanding functions $n_d = n_d(r)$, $n_g = n_g(r)$ and $\eta_g = \eta_g(r)$ in a series at the point $r = 1$ A.U. again we find expression (3.69) in the form:

$$\ln\,[n(r) - 2] - \eta(r).\ln r = \sum_{i=0}^{m} A'_{2i}\,(\ln r)^i + r^{-\eta} \sum_{i=0}^{m} A'_{2i+1}\,(\ln r)^i, \quad (3.70)$$

where it is now

$$\left.\begin{aligned}
A'_0 &= \ln \alpha B - \ln(1 + k) + \frac{1}{B}\left(\frac{1}{2} - \frac{2}{\alpha}\right) - \frac{a}{\alpha B}\,e^{-b},\\
A'_1 &= -\frac{1 + k}{\alpha B}\,.ae^{-b},\\
A'_2 &= \alpha B + \frac{\alpha}{2} + \frac{1}{B}\left(2 - \frac{\alpha}{2}\right) - \frac{a}{\alpha B}\,(2b - c - \alpha)\,e^{-b},\\
A'_3 &= \frac{1 + k}{\alpha B}\,a.e^{-b}\left(\alpha B + 2b - c - \frac{\alpha}{2}\right),\\
A'_4 &= \frac{1}{2}\alpha^2 B + \frac{\alpha}{B}\left(\frac{1}{4}\alpha - 1\right) + \frac{a}{\alpha B}\,e^{-b}\cdot\left\{2b\,(1 - b + c + \alpha) - \frac{(c + \alpha)^2}{2}\right\},\\
A'_5 &= \frac{1 + k}{\alpha B}\,ae^{-b}\cdot\left\{\alpha B\left(\frac{1}{2}\alpha B + 2b - c\right) - 2b\left(1 - b + c + \frac{\alpha}{2}\right) + \frac{1}{2}\left(c + \frac{\alpha}{2}\right)^2\right\}.
\end{aligned}\right\} \quad (3.71)$$

The length of the interval of heliocentric distances and the size of index m must be approximately the same as before.

The present state in methods of determining comet brightness makes it possible to derive nothing more but the average photometrical exponent for most comets. Comet Arend-Roland is, however, one of a few comets for which — owing to the numerous and homogeneous observational material — it is possible to watch the photometrical-exponent variations with the heliocentrical distance and construct the photometrical-exponent curve within a wide range of heliocentric distances.

The photometrical curve of comet Arend-Roland has been constructed on the basis of BEYER's 51 visual observations (BEYER, 1959) and it is represented in Fig. 13. There is no difference between the pre-perihelion and post-perihelion form of the curve, which was independently pointed out by BOUŠKA (VINTER HANSEN, 1957a). First, let us find, whether the departures from the straight-line (giving the course of the photometrical curve) are real or represent observational errors. BEYER (1952) states that the accuracy of his method is about $\pm\,0^m.30$. If the constant photometrical exponent is acceptable for expressing relation

$$I = I_0.r^{-n},$$

the maximum departure from the straight-line must be less than the proper errors of the method. The maximum departure is defined by the normal law of errors as a departure the probability of which is equal to $1/N$, where N is the number of observations. If the i-th departure is denoted as ε_i then the constant photometrical exponent is acceptable when the following condition is fulfilled:

$$\gamma\left(\frac{\sum\limits_{i=1}^{N}\varepsilon_i^2}{N - 1}\right)^{\frac{1}{2}} \leq 0^m.3\,. \quad (3.72)$$

Here γ is the numerical factor between the maximum departure, ε_m, and the mean departure, $\bar{\varepsilon}$,

$$\varepsilon_m = \gamma\bar{\varepsilon};$$

γ fulfils the relation

$$\frac{2}{\sqrt{\pi}} \int_0^{\gamma/\sqrt{2}} e^{-t^2} dt = 1 - \frac{1}{N}.$$

The maximum departure of the observational material used is $\pm 0^m.73$ and considerably exceeds the limit permitted by (3.72). The reality of curvature of the observed photometrical curve is, hence, proved beyond any doubt.

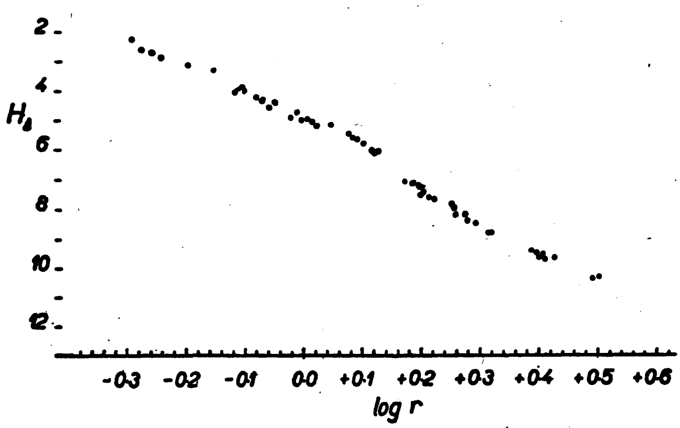


Fig. 13. Photometrical curve of the Arend-Roland comet.

Then the curve was divided into a series of intervals and within each of them the mean photometrical exponent was computed by means of the method of east squares. The results are included in Table 20, where the individual columns contain: the serial number of the interval of heliocentric distances, its range,

Table 20
Empirical dependence $n = n(r)$ of the Arend-Roland comet

No	int. r	N	$\bar{r}$	n	ε_m
1	0.51 — 0.71	6	0.58	2.77 ± 0.29	±0.16
2	0.53 — 0.79	8	0.66	2.87 ± 0.20	±0.21
3	0.53 — 0.98	14	0.75	3.15 ± 0.11	±0.24
4	0.78 — 1.06	12	0.92	3.51 ± 0.18	±0.17
5	0.97 — 1.06	5	1.02	3.93 ± 0.41	±0.07
6	1.11 — 1.35	8	1.25	4.17 ± 0.18	±0.17
7	0.99 — 1.81	23	1.37	4.37 ± 0.12	±0.17
8	1.48 — 1.81	10	1.63	4.25 ± 0.28	±0.14
9	1.48 — 2.08	16	1.74	4.19 ± 0.12	±0.16
10	1.78 — 2.57	13	2.12	3.90 ± 0.11	±0.12
11	1.88 — 2.67	11	2.26	3.72 ± 0.12	±0.29
12	1.88 — 3.17	13	2.38	3.64 ± 0.10	±0.11
13	2.43 — 3.17	8	2.67	3.46 ± 0.19	±0.16

the number of measurements, the mean heliocentrical distance and the entire photometrical exponent within the interval. The linearity of the photometrical curve within individual intervals is maintained. It is proved by the maximum departures, ε_m, computed separately for each interval, as seen from the last column of Table 20.

The dependence of the photometrical exponent on the heliocentric distance is represented in Fig. 14, where also function $\eta = \eta(r)$, obtained by means of

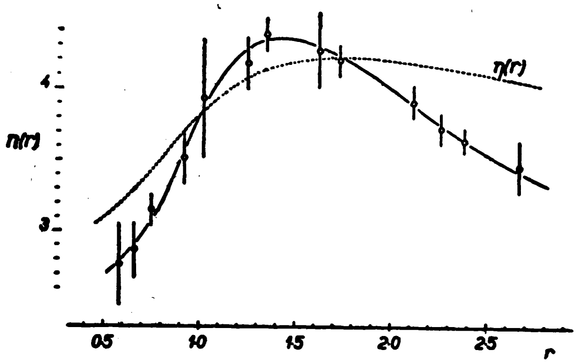


Fig. 14. Photometrical-exponent curve of the Arend-Roland comet.

graphical integration of $n(r)$, is plotted in a dashed curve. At $r = 1.4$ A.U. curve $n = n(r)$ has a maximum, which is characteristic for the comet dust-gas model and its existence follows directly from the basic equation of the model (Vanýsek, Hřebík, 1954). A series of other comets recently investigated by Beyer (1958, 1959) also have the point of inflexion on their photometrical curve, e. g. comet 1956a at $r = 2.2$ A.U., comet 1957d at $r = 1.2$ A.U. etc., and from comets observed earlier comet 1942g appeared the same effect at $r = 1.6$ A.U. as follows from the material collected by Gadomski (1947).

A lot of spectral and polarization measurements of comet Arend-Roland indicate a considerable amount of dust in the atmosphere. Therefore the series of form (3.67) must be used for the numerical analysis of the photometrical exponent. By means of the method of least squares we get the following coefficients $A_0, \ldots, A_5$ from the interval of heliocentric distances between 0.67 and 1.49 A.U. (for $m = 4$):

$$\left.\begin{aligned} A_0 &= +\,3.62 \pm 0.04,\\ A_1 &= +\,0.19 \pm 0.02,\\ A_2 &= +\,3.25 \pm 0.05,\\ A_3 &= -\,0.54 \pm 0.07,\\ A_4 &= -\,3.12 \pm 0.07,\\ A_5 &= +\,4.00 \pm 0.09. \end{aligned}\right\} \quad (3.73)$$

From the first, third and fifth equations of (3.68) we get immediately the expressions for the parameters of the dust part of the cometary atmosphere:

$$b = \frac{1}{2\,(2 - A_0)}\left[A_4 + \frac{1}{2}\cdot\frac{A_2^2}{2 - A_0}\right], \tag{3.74}$$

$$c = \frac{A_2}{2 - A_0} + 2b\,, \tag{3.75}$$

$$a = (A_0 - 2).e^b. \tag{3.76}$$

Eliminating $1 + k$ from the second and fourth equations we obtain the expression for B as a function of α:

$$\alpha^2 A_1 B^2 + \alpha B[\alpha A_1(\alpha - 1 - A_0) + A_3] + \frac{1}{2}\,\alpha^2 A_1\left(\frac{1}{2}\,\alpha - A_0\right) + \frac{1}{2}\,\alpha A_3 + A_1 A_2 - A_0 A_3 = 0 \tag{3.77}$$

and for ratio k:

$$k = \frac{1}{A_1}\left[\alpha\left(\frac{1}{2} + B\right) - A_0\right] - 1\,. \tag{3.78}$$

The computing procedure: we choose α and determine corresponding B and k; insert these values into the last equation of (3.68) and change as long as we find the agreement between it and (3.73). The approximations converge rapidly.

The resulting values of the six parameters of photometrical curve (3.64) found in this way are included in Table 21.

Table 21

Resulting parameters of the photometrical curve of the Arend-Roland comet

k	5.81 ± 0.75
α	0.28 ± 0.01
B	17.02 ± 0.51
a	11.51 ± 0.86
b	1.96 ± 0.07
c	1.92 ± 0.14

Table 22

Residuals between the empirical and theoretical curve of $n = n(r)$

No	O-C	No	O-C
1	—0.01	8	—0.04
2	—0.03	9	—0.02
3	+0.11	10	+0.01
4	—0.05	11	—0.03
5	+0.08	12	—0.01
6	—0.09	13	+0.04
7	+0.04		

Residuals O - C between photometrical exponents obtained from BEYER's material and those computed from (3.64) are given in Table 22. As seen, the agreement is very good even at $r > 2$ A.U.

The results of Table 21 are interesting in many directions. Intensity ratio k has an extraordinarily high value, which is consistent with a number of spectral measurements. A strong continuum in the head-spectrum of this comet was reported by PORTER (1957), FEHRENBACH, HASER, SWINGS and WOSZCZYK (1957), LILLER (1958), DOLIDZE and ARKHIPOVA (1957). Some authors even estimated

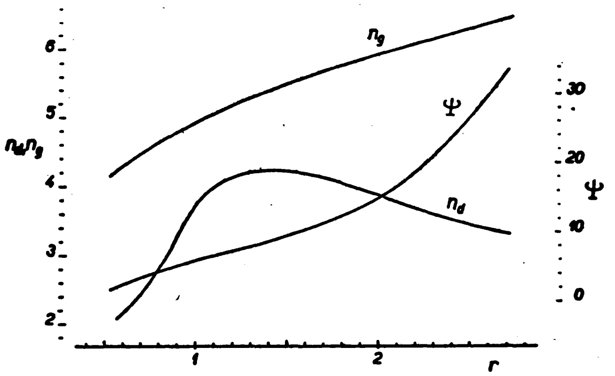


Fig. 15. Dependence of both the partial photometrical exponents of the Arend-Roland comet on the heliocentrical distance. Change of the intensity ratio with the heliocentrical distance.

from the spectrograms an approximate value of intensity ratio Ψ of both the physical components, e. g. RAJCHL (1958) found $\Psi \approx 2$ at $r = 0.75$ A.U., BOUŠKA and HERMANN-OTAVSKÝ (1958) estimated $\Psi \approx 3$ at $r = 0.66$ A.U. These values are within errors consistent with ratio Ψ computed from parameters of Table 21 according to the formula

$$\Psi(r) = \frac{I_{\Delta d}(r)}{I_{\Delta g}(r)} = k.\exp\left[\int_1^r \frac{n_g - n_d}{r}\,\mathrm{d}r\right]. \tag{3.79}$$

The computed course of Ψ with the heliocentric distance as well as that of $n_g = n_g(r)$ and $n_d = n_d(r)$ are in Fig. 15.

The correctness of the high value of ratio k is also fully supported by a number of polarization measurements. Polarization degree of the comet-head light reached extraordinarily high value. Thus, for instance, RICHTER (1958) reported the polarization being nearly 40 per cent, according to BLAHA, HRUŠKA, ŠVESTKA and VANÝSEK (1958) it was almost 30 per cent, BARBIER (1958) gave 41 $\pm$ 5 per cent, and even the lowest value, 26 per cent, published by HOPMANN, WIDORN and PURGATHOFER (1958) is still high enough.

The resulting value of exponent α agrees, on the whole, with both the investigation of MARKOVICH (1959) and earlier considerations of MINNAERT (1947). If we consider the nucleus surface temperature of the comet to be equal to about

$T_0 \approx 160°$ K — as follows from some theoretical considerations (DOBROVOLSKY, 1953c, MARKOVICH, 1959) — we get the heat of evaporation $L = 5400 \pm 160$ cal/mol, while the temperature determined from the 3883 Å CN-band by GLAGOLEVSKY (1959) leads to another result: $T_0 = 273°$ K and $L = 9200 \pm 280$ cal/mol.

The dust coma parameters indicate two characteristic features:

a) high exponent n_d, equal to 4 in maximum, gives evidence of a very violent development of the dust coma;

b) high coefficient b indicates a rapid drop of dust supplies within the surface layer.

In addition to this, coefficient β of gas may be derived from (3.62); assuming $\gamma = 0$ and inserting α and c from Table 21 we get $\beta = 14.3$. This value is consistent with the results of the laboratory experiments. The latter lie within the interval between 13.1 for CH_4 and 19.1 for C_2N_2 (DOBROVOLSKY, 1953c).

Thus we arrive at the conclusions as follows:

1) the comet dust-gas model quite complies with the empirical dependence of the photometrical exponent of the Arend-Roland comet on the heliocentric distance. The general form of this model — with six independent parameters — makes it possible to analyze both the physical components of the comet atmosphere, unless the inaccurate observational material prevent us from doing so.

2) Numerical results indicate that photometrically the dust considerably prevailed over the gas in the atmosphere of the Arend-Roland comet. The high photometrical exponent n_d disproves the assumption of the constant summary surface of dust particles, especially in comets with a substantial effect of the dust on the comet photometrical properties.

3) As to the gas coma parameters, the ascertained dependence of the nucleus surface temperature on the heliocentrical distance differs from that of tiny particles under conditions of thermal equilibrium, but is consistent with the mentioned considerations of MARKOVICH. Parameter B is close to the above value held as the "typical" one for comets with continuous spectra.

CHAPTER FOUR

SOME APPLICATIONS OF A COMET DUST-GAS MODEL. COMETS WITH STRONG CONTINUOUS SPECTRA

4.1. FUNCTION OF GAS

In Section 3.3 we dealt with general considerations concerning the photometrical efficiency of the dust in cometary atmospheres, especially the correlation between the number of dust particles, ν, and the corresponding photometrical exponent n_d. The latter determines the changes in the dust amount in the atmosphere, but in any way does not contribute to solving the problem of zero-point. The difficulty consists in the fact, that equation (3.7) contains two unknowns, namely effective radius of the nucleus, R, and the function of dust, E. The second equation has to be found.

In Section 2.1. the gas-part brightness of the cometary atmosphere is shown to be assumed proportional to the number of ejected molecules from a unit surface per unit of time, n_0, given by formula (2.1). Thus we can write

$$I_{\Delta g} = A \,.\, 4\pi R^2 . n_0 , \qquad (4.1)$$

where the photometrical efficiency of a radiating molecule, the ratio between the number of radiating and evaporated molecules, the mean life-time of the former, and the energy density of solar radiation are included in coefficient A. After inserting from (2.1) and modifying we obtain

$$H_{\Delta g} = -2.5 \log G - 5 \log R + \frac{5}{4} \alpha \log r + \frac{5}{2} \mathrm{mod}.B\,(r^{\alpha} - 1), \qquad (4.2)$$

where the following magnitude is introduced:

$$G = AN_0 . \left(\frac{8\pi r T_0}{m}\right)^{1/2} . \mathrm{e}^{-B}. \qquad (4.3)$$

It represents a certain combination of gas characteristics. We shall therefore call it the function of gas and investigate some of its properties.

First, the gas-component brightness may be generally expressed in the form

$$H_{\Delta g} = H_0 + 2.5 \log (1 + k) + 2.5 \eta_g(r) \log r. \qquad (4.4)$$

Analogously the dust-component brightness is described by the formula

$$H_{\Delta d} = H_0 + 2.5 \log \left(1 + \frac{1}{k}\right) + 2.5\, \eta_d(r) \log r. \qquad (4.5)$$

If we introduce (2.12) into (4.4), compare (4.4) with (4.2), and (4.5) with (3.7), we get the relation between the function of dust and the function of gas:

$$\frac{E}{G} = 10^{0.4\sigma} . \frac{k}{A_0} . r^{2-\eta_d}. \qquad (4.6)$$

In addition to this, the comparison of (4.4) with (4.2) gives the relation between the effective radius of the comet nucleus of the function of gas:

$$R = [G(1 + k)]^{-1/2} . \exp \left[- \frac{H_0}{5\, \mathrm{mod}}\right]. \qquad (4.7)$$

If the basic physical parameters of the comet photometrical curve as well as the effective radius of the nucleus are known, the function of gas can be computed from (4.7). The function of gas is indicated to depend mainly on the heat of evaporation of molecules, as seen from (4.3). If we succeed in determining R, k, B and H_0 for a number of comets, the calibration curve $G = G(B)$ may be established.

The problem of determining physical parameters of the photometrical curve was solved in the two preceding chapters, the problem of the comet nucleus model will be discussed in the following section.

4.2. THE STRUCTURE OF THE COMET NUCLEUS

There exist divergences in opinions on both the size and structure of the comet nucleus and its function in the comet life. With respect to these problems, comet models may be, in the main, divided into three groups:

a) comets without nucleus;
b) comets with compound nucleus;
c) comets with monolithic nucleus.

The first and third types are hypothetical extremes, in fact non-existing. The comet model without nucleus, suggested by LYTTLETON (1948), has been most objected to till now. A comet without any nucleus should have no interior stability and be destroyed under effects of differential gravitation. It could hold neither its roughly spherical form of the coma at all nor relatively small dimensions as observed. The assumption on the mechanism of producing gas radiation cannot be accepted either (evaporation of particles due to their collisions).

The most extended hypotheses are those concerning the compound comet nucleus. This intermediate group of views, however, is rather non-homogeneous and is represented by a string of conceptions from those of a low-concentration cluster of particles of various size to those of almost monolithic nucleus.

Among the former conceptions there belong DUBIAGO's considerations (DUBIAGO, 1942) concerning the stability of the cometary nucleus formed by a meteoric-particle shower. He proved the period of semi-desintegration of such a comet with the nucleus diameter of about 9000 kilometres and inner space density of 5.10^{-9} gm.cm^{-3} is roughly 100 years, if $q = 2.4$ A.U. and $P = 5.2$ years being considered, i. e. it is quite short. This model cannot be disproved dynamically, but it contradicts the spectroscopic and photometrical data. The summary surface of the particles should be so great that every comet would appear an intense continuous spectrum. The accepted diameter itself is too large. For instance, the apparent nuclei of two huge comets, 1858 VI and 1910 II, were in some periods of observation estimated to be not more than about 500 kilometres in diameter (CURTIS, 1910, BREDIKHIN, 1934). Moreover, as shown by ORLOV (1945), the measured apparent diameter of the nucleus is in close connection with the geocentric distance. It is evident that an observational effect takes place here. It is well-known that some unstable molecules, as CH, CH_2 or NH_2, have too short a life-time and are dissociated or ionized in a close vicinity of the comet nucleus (i. e. real nucleus) (SWINGS, 1943). The intensity discontinuity in the comet head, produced by the disintegration of such molecules, is probably often held as the nucleus boundary.

Another conception of the compound nucleus was suggested by VORONTSOV-VELYAMINOV (1945). In his study concerning the nucleus of the Halley comet he drew the conclusion that it consisted of about 10^7 blocks of dimensions about 100 metres, which were included inside the sphere of 60 kilometres in diameter. As the volume of the nucleus is filled up by the blocks to 15 per cent approximately, mutual collisions certainly occur, which lead to the fragmentation of the nucleus and in such a way to the loss of stability. DUBIAGO (1950) therefore believes the nucleus must be of considerably larger dimensions, about 500 kilometres in diameter and the number of blocks less to decrease the danger of disintegration by the process of fragmentation. On the other hand, RICHTER (1954a) finds a considerable dispersion in the nucleus dimensions, extended over three orders, from 10^0 to 10^3 kilometres. The differences in the size of individual comets are unlikely to be so conspicuous.

All the given data about the dimensions of nuclei were obtained by either photometrical methods (VORONTSOV-VELYAMINOV, RICHTER) or those of celestial mechanics (DUBIAGO).

An idea of the compound comet nucleus very close to the monolith (or even almost identical with it) has been held by BALDET (1931) on the basis of photometrical measurements, WHIPPLE (1950) by means of physical considerations and ORLOV (1960) by connecting the methods of celestial mechanics with photometrical relations. The three authors' results are consistent with each other within an order.

The determination of the comet-nucleus dimensions is most fully described by ORLOV. He proceeded from his theory of the cometary head (ORLOV, 1945) where he had studied parabolical envelopes of a few comets. He found there may have been up to four envelopes in a comet at the same time, and established that the ratios between top-distances of individual envelopes (in stationary state) from the nucleus were always the same.

ORLOV assumed the motion of any particle was controlled by three forces as follows:

a) attractive force of the Sun;

b) repulsive force of the Sun, i. e. radiation pressure;

c) repulsive force of the cometary nucleus, i. e. reflected solar radiation pressure.

The resulting motion arises under the effect of the vector sum of the three agents. If the motion is studied of a particle expelled in the very direction of the radius-vector to the Sun, the following differential equation holds good:

$$\frac{d^2\xi}{dt^2} = -\frac{k^2 (1 + \mu)}{(r - \xi)^2} + \frac{k^2\mu_1}{\xi^2 r^2}, \tag{4.8}$$

where ξ is the distance of a particle from the nucleus, k^2 the universal constant of gravitation, μ_1 the effective acceleration on a particle from the comet nucleus and $1 + \mu$ the repulsive acceleration from the Sun. The following expression is reached for the distance of the cometary-head top from the nucleus:

$$\xi_0 = (2r)^{1/2} \cdot \left(\frac{\mu_1}{1+\mu}\right)^{\frac{1}{4}}. \tag{4.9}$$

This formula is valid, in fact, for both parabolical envelopes as proved by ORLOV (1945), and cometary-head diameters (Chapter Six).

ORLOV justifiably assumes the repulsive forces due to the Sun and the comet nucleus are proportional to the illuminations from both bodies, respectively. Thus we can write:

$$\frac{10^{-0.4\,H_{0n}}}{10^{-0.4\,H_{\odot}}} = \frac{\mu_1}{1+\mu}, \tag{4.10}$$

where $H_{\odot}$ and H_{0n} are the absolute brightness of the Sun and the comet nucleus, respectively. Formula (4.9), being written at $r = 1$ A.U., characterizes the first envelope:

$$\xi_{\mathrm{I}} = \sqrt{2} \cdot \sqrt[4]{\frac{\mu_1}{1+\mu}}. \tag{4.11}$$

Comparing (4.10) with (4.11) we obtain

$$\xi_{\mathrm{I}}^4 = 4 \cdot 10^{0.4(H_{\odot} - H_{0n})}. \tag{4.12}$$

According to (3.2) we find

$$H_{0n} = \sigma - 2.5 \log A_0 - 5 \log R, \; [F_n(\varphi = 0) = 1], \tag{4.13}$$

so that the radius of the real cometary nucleus yields in

$$R = \frac{\xi_{\mathrm{I}}^2}{2\sqrt{A_\theta}} \cdot \exp\left[\frac{0.2}{\mathrm{mod}}(\sigma - H_{\odot})\right]. \tag{4.14}$$

Numerical values: from Section 3.2. we take $\sigma = 38^{\mathrm{m}}.8$, $A_\theta = 0.1$ and according to STEBBINS and KRON (1957) $H_{\odot} = -26^{\mathrm{m}}.73$. If ξ_{I} is expressed in A.U., the nucleus radius (in kilometres) is

$$\log R = 8.305 + 2 \log \xi_{\mathrm{I}}. \tag{4.15}$$

As a monolithic nucleus was considered in the computation, value R of (4.15) may be put equal to the so-called effective radius of the comet as introduced in Section 3.2. and applied in Section 4.1. Hence, ORLOV's conception of the real comet nucleus is quite close to that pronounced in the preceding sections of this study.

4.3. PROPERTIES OF THE FUNCTION OF GAS

Comparing (4.14) with (4.7) the function of gas may be expressed in dependence on the physical parameters of the comet dust-gas model and distance ξ_{I} of the first envelope from the nucleus:

$$G = \frac{4A_0}{(1+k)\,\xi_{\mathrm{I}}^4} \cdot \exp\left[\frac{0.4}{\mathrm{mod}}(H_{\odot} - H_0 - \sigma)\right]. \tag{4.16}$$

The writer succeeded in gathering both the photometrical data and the envelope-distance measurements for six comets of the 19th and 20th centuries, for which the numerical value of the function of gas may be computed directly from (4.16).

The re-treatment of ORLOV's data concerning the envelope distances gave the results included in Table 23.

Table 23

Envelope distances and nucleus diameters of calibration comets

Comet	ε_I, A. U.	$2R$, km
1811 I	0.000365 ± 0.0000135	53.7 ± 4.0
1858 VI	0.000115 ± 0.0000021	5.33 ± 0.19
1882 II	0.00020 ± 0.000019	16.1 ± 3.1
1908 III	0.000106 ± 0.0000048	4.53 ± 0.41
1910 I	0.000098 ± 0.0000031	3.87 ± 0.24
1910 II	0.000099 ± 0.0000014	3.95 ± 0.11

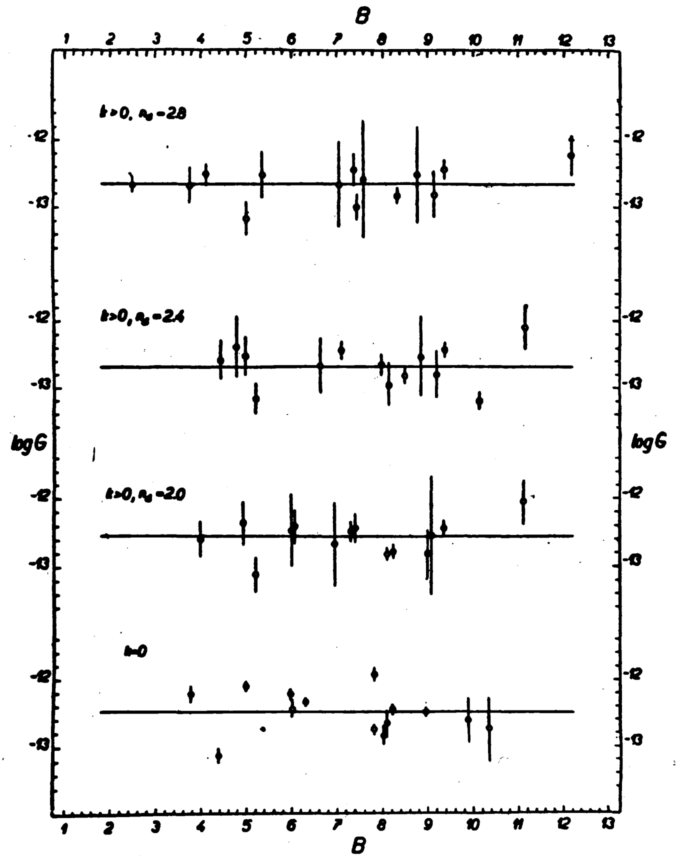


Fig. 16. **Function of gas of calibration comets as related to their evaporation heat.**

Table 24
Observational data of calibration comets

No	Comet	*per*	int r	r_0	a_0	a_1	a_2	Reference
					m m			
1	1607 (Halley)	*A*	0.59 — 0.86	0.71	+3.50 ± 0.09	+11.55 ± 1.20	+26.16 ± 22.13	Holetschek (1916)
2	1759 I (Halley)	*B*	0.71 — 1.58	1.16	+3.96 ± 0.06	+11.14 ± 0.36	— 7.58 ± 3.02	Holetschek (1916)
3	1811 I	*AB*	1.10 — 4.57	2.23	+3.22 ± 0.14	+ 8.79 ± 0.41	+ 1.39 ± 2.13	Kritzinger (1914)
4	1835 III (Halley)	*A*	0.60 — 1.82	0.97	+4.98 ± 0.11	+10.56 ± 0.60	+10.17 ± 4.03	Holetschek (1916), Vsekhsviatsky (1958)
5	1858 VI (Donati)	*B*	0.58 — 0.85	0.64	+2.09 ± 0.03	+10.49 ± 0.81	+ 8.26 ± 8.84	Bobrovnikoff (1942)
6	1882 II	*B*	0.01 — 4.47	0.61	—0.60 ± 0.14	+ 6.42 ± 0.16	+ 0.49 ± 0.12	Kritzinger (1914)
7	1882 II	*B*	0.32 — 3.98	0.99	+0.83 ± 0.08	+ 6.81 ± 0.20	+ 1.41 ± 0.56	Vsekhsviatsky (1958)
8	1882 II	*B*	0.55 — 4.47	1.80	+2.52 ± 0.21	+ 6.89 ± 0.38	+ 0.64 ± 1.38	Kritzinger (1914)
9	1908 III (Morehouse)	*A*	1.02 — 1.71	1.35	+5.41 ± 0.04	+ 9.29 ± 0.43	+16.43 ± 6.36	Bobrovnikoff (1942)
10	1910 I	*B*	0.29 — 3.29	1.05	+5.69 ± 0.08	+10.64 ± 0.18	+ 4.18 ± 0.76	Vsekhsviatsky (1958)
11	1910 II (Halley)	*A*	1.06 — 2.74	1.73	+8.48 ± 0.13	+13.81 ± 0.55	+ 0.42 ± 5.03	Bobrovnikoff (1942)
12	1910 II (Halley)	*A*	1.10 — 3.36	2.21	+9.44 ± 0.05	+17.02 ± 0.27	+ 5.13 ± 1.67	Holetschek (1916)
13	1910 II (Halley)	*B*	0.59 — 2.11	0.92	+4.03 ± 0.03	+ 8.18 ± 0.15	+ 1.92 ± 0.76	Holetschek (1916)
14	1910 II (Halley)	*AB*	0.59 — 1.95	0.85	+3.86 ± 0.04	+10.08 ± 0.21	+ 8.29 ± 1.00	Holetschek (1916)

Radius R of a vast majority of comets decreases slightly with time, according to HRUŠKA (1957b), for instance, it amounts to about 50 cm per year for short-period comets. Since distance ξ_I is of the same character as R, in relation (4.16) there are only two parameters, k and H_0, pre- and post-perihelion values of which may differ from each other, and in such a way can produce the variability of G. Moreover, a contingent dependence of G on the heat of evaporation, L, may be expected.

A method of expanding in a series has been applied for computing the basic physical parameters of the six comets. The results of treatment of observational material are included in Table 24. The individual columns give: the comet denotation and name, its position relative to the perihelion (symbols of Section 3.5. are used), the range of heliocentric distances, geometric mean of distances, coefficients a_i of series (2.14) and the reference to the observational material.

From a_i given in Table 24 the physical parameters k, B and H_0 have been computed for $n_d = 2.8$, 2.4 and 2.0. Parameters B and H_0, have also been determined for a pure gaseous model ($k = 0$). The resulting values of G for each of the mentioned cases are presented in Table 25.

The dependence of G on B is represented in Fig. 16. Full lines indicate mean values. They are also given in Table 26, where $\psi(B, G)$ are the correlation coefficients between the heat of evaporation and the function of gas.

Besides it the dependences of G on both the heliocentric distance and time have been studied in passing. Fig. 17 includes the function of gas as related to the heliocentric distance for the vast comet of 1882 (after its perihelion passage) and the Halley comet (before its perihelion passage). The data of the latter are corrected for the secular changes and concern the passage of

Table 25
Values of function of gas

No	Comet	log G			
		$k > 0, n_d = 2.8$	$k > 0, n_d = 2.4$	$k > 0, n_d = 2.0$	$k = 0$
1	1607	—12.56 ± 0.90	—12.36 ± 0.46	—12.45 ± 0.55	—12.74 ± 0.50
2	1759 I	—12.21 ± 0.32	—12.08 ± 0.35	—12.03 ± 0.35	—11.91 ± 0.12
3	1811 I	—13.15 ± 0.27	—13.14 ± 0.28	—13.09 ± 0.27	—13.12 ± 0.12
4	1835 III	—12.64 ± 0.67	—12.65 ± 0.43	—12.65 ± 0.63	—12.65 ± 0.22
5	1858 VI	—12.51 ± 0.74	—12.52 ± 0.62	—12.53 ± 0.89	—12.61 ± 0.36
6	1882 II	—12.64 ± 0.13	—13.18 ± 0.14	—12.80 ± 0.09	—12.21 ± 0.09
7	1882 II	—12.48 ± 0.17	—12.94 ± 0.33	—12.40 ± 0.25	—12.08 ± 0.08
8	1882 II	—12.65 ± 0.28	—12.56 ± 0.31	—12.35 ± 0.33	—12.21 ± 0.14
9	1908 III	—12.49 ± 0.38	—12.51 ± 0.30	—12.58 ± 0.29	—12.44 ± 0.13
10	1910 I	—12.82 ± 0.13	—12.79 ± 0.11	—12.77 ± 0.11	—12.74 ± 0.07
11	1910 II	—12.80 ± 0.37	—12.78 ± 0.37	—12.79 ± 0.36	—12.84 ± 0.16
12	1910 II	—12.42 ± 0.14	—12.42 ± 0.12	—12.43 ± 0.12	—12.48 ± 0.07
13	1910 II	—12.98 ± 0.21	—12.63 ± 0.17	—12.48 ± 0.16	—12.33 ± 0.05
14	1910 II	—12.42 ± 0.25	—12.41 ± 0.13	—12.42 ± 0.22	—12.44 ± 0.08

Table 26
Resulting mean values of G

Case	$\overline{\log G}$	$\psi(B, G)$
$k > 0,\ n_d = 2.8$	—12.625 ± 0.026	+0.329 ± 0.238
$k > 0,\ n_d = 2.4$	—12.640 ± 0.061	+0.037 ± 0.267
$k > 0,\ n_d = 2.0$	—12.553 ± 0.046	+0.293 ± 0.244
$k > 0$	—12.606 ± 0.027	+0.209 ± 0.148
$k = 0$	—12.486 ± 0.063	—0.228 ± 0.253
Σ	—12.576 ± 0.025	+0.089 ± 0.133

1910. Fig. 18, on the other hand, shows the secular drop of G for the Halley comet. The values are reduced to a unit of heliocentric distance. Full circles indicate the pre-perihelion data, open circles those after the perihelion passage.

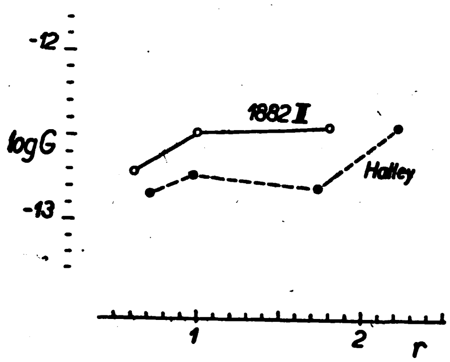


Fig. 17. Concentration drop in the comets 1882 II and Halley near the perihelion passage.

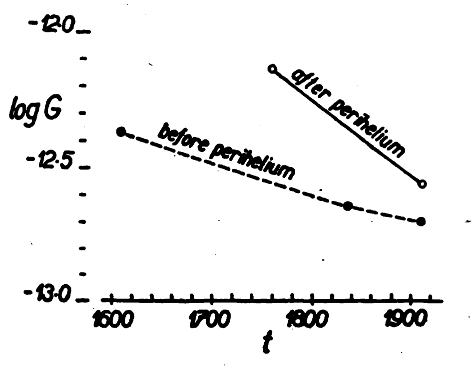


Fig. 18. Secular drop of the concentration of molecules in the Halley comet in a unit heliocentrical distance.

Analyzing the empirical dependences concerning the function of gas, we arrive at the following conclusions as to its properties:

1) The magnitude of the function of gas changes from case to case, but all the individual values of G, included in Table 25, lie within the order, from —12.0 to —13.0.

2) The obtained results show no marked trend of G with the heat of evaporation. Coefficients $\varphi(B, G)$ in Table 26 are too low to indicate any real correlation. No effect is seen in Fig. 16, either.

3) On the other hand, Fig. 17 shows a trend of increasing the function of gas with the comet's receding from the Sun and vice versa, for two comets. This effect can be explained as a drop of the concentration of molecules in the surface layer of the comet nucleus, as seen from (4.3). The change is given by a factor of about 1.5 per unit of heliocentric distance. It is likely that the drop is greater in the vicinity of the Sun and less in large heliocentric distances.

4) Another effect appears in the course of G with time. A well-expressed secular drop in the concentration of molecules in the Halley comet is seen in Fig. 18, of magnitude of about an order per 1000 years.

5) There appears no effect as for the dependence of the function of gas on the nucleus dimensions.

6) All the ascertained relations, both positive and negative, are of preliminary character and they call for verification. At present, the best approximation being allowed for statistical studies is

$$\log G = —12.6 \pm 0.1 \text{ p. e.},$$

which may serve as a calibration value for determining the dimensions of the cometary nucleus in a photometrical way.

4.4. COMETS WITH INTENSE CONTINUOUS SPECTRA

The introduction of the function of gas is also of great importancy for determining the total mass of photometrically effective dust particles in the cometary atmosphere, $\mathfrak{M}_p$, though it looks surprising at first sight.

According to (3.10) and (3.11) we can write

$$\mathfrak{M}_p(r) = \frac{4}{3}\pi\varrho s R^2 . E(r) . [F_p(\varphi) . \Phi(\varrho)]^{-1} . \left[1 - \frac{F_n(\varphi)}{E(r)}\right] . \tag{4.17}$$

Eliminating the function of gas from (4.6) and (4.7) the expression for the function of dust is as follows:

$$E(r) = \frac{k}{1+k} \cdot \frac{1}{A_0 R^2} . 10^{0.4(\sigma - H_0)} . r^{2-\eta_d} . \tag{4.18}$$

After inserting (4.18) into (4.17) we obtain the resulting formula

$$\mathfrak{M}_p(r) = \frac{4}{3}\pi s\varrho . \left[F_p(\varphi) . \Phi(\varrho) . A_0 . \left(1 + \frac{1}{k}\right)\right]^{-1} . 10^{0.4(\sigma - H_0)} . r^{2-\eta_d} (1 - Q) , \tag{4.19}$$

where

$$Q = 10^{-0.4\sigma} . F_n(\varphi) . \frac{A_0}{G.k} . r^{\eta_d - 2} < 0.01$$

for $k > 0.01$. This condition is fulfilled for any comet with a strong continuous spectrum, so that Q may be neglected. Then neither the function of gas, nor that of dust, nor the effective radius of the nucleus appear in formula (4.19), and mass $\mathfrak{M}_\nu$ depends on the physical parameters and functions $F_\nu(\varphi)$ and $\Phi(\varphi)$ only.

The mass of the dust included in the atmosphere of six comets was determined by VANÝSEK (1958) on several simplifying assumptions. He found the masses of about 10^{10} — 10^{11} gm, but the corrected values were to be about an order lower than those just given (VANÝSEK, 1960b).

In the present study the mass of the dust cloud in the cometary atmospheres is derived in another way. First of all the comets with an extraordinarily strong continuum (cont 1) and those with a relatively well pronounced continuum (cont 2) were selected from the Catalogue of Physical Characteristics of Comets (HRUŠKA, VANÝSEK, 1958). Photometrical data were taken over from a few authors (BEYER, 1942, 1950a, 1955, BOBROVNIKOFF, 1942, HOLETSCHEK, 1916, KRITZINGER, 1914, VSEKHSVIATSKY, 1958) and the basic physical parameters were determined by the method of expanding in a series. The mass of the photometrically effective dust was then computed from formula (4.19). In addition to this, the diameter and mass of the comet nucleus were also established. The results are included in Table 27. For each of the investigated comets the spectrum and the corresponding heliocentrical distance, r_{sp}, are given as well as the expression $\frac{1}{\nu}\left(\frac{R}{\varrho}\right)^2$ for purposes of Section 3.7. For comparison, a comet is included with a relatively weak continuous spectrum (cont 3).

Table 27

Nucleus diameter and dust-cloud mass of comets with continuous spectra

Comet	Spectrum	r_{sp} A. U.	$2R$ km	$\log \mathfrak{M}_n$ gm	$\log \mathfrak{M}_p$ gm	$\frac{1}{\nu}\left(\frac{R}{\varrho}\right)^2$
1882 I	cont		0.66 ± 0.30	14.73 ± 0.59	9.23 ± 0.15	0.0009
1882 II	cont strong		16.1 ± 3.1	18.89 ± 0.25	11.94 ± 0.13	0.0002
1904 I	cont		22 ± 11	19.30 ± 0.65	10.35 ± 0.37	0.0141
1907 IV	cont 1	0.6	5.6 ± 3.0	17.51 ± 0.70	10.34 ± 0.04	0.0010
1910 I	cont 1	0.5	3.87 ± 0.24	17.03 ± 0.08	9.57 ± 0.27	0.0027
1910 II	cont 1	0.65	3.95 ± 0.11	17.06 ± 0.04	10.62 ± 0.09	0.0002
1912 II	cont 2	0.8	2.28 ± 0.60	16.34 ± 0.34	9.13 ± 0.21	0.0026
1936 II	cont 1	1.1	1.59 ± 0.91	15.87 ± 0.74	9.04 ± 0.26	0.0015
1937 II	cont 2	0.65	0.22 ± 0.08	13.30 ± 0.47	8.26 ± 0.14	0.0002
1941 I	cont 2	1.1	2.8 ± 1.0	16.60 ± 0.47	9.63 ± 0.85	0.0012
1941 I	cont 1	2.0	2.8 ± 1.0	16.60 ± 0.47	9.51 ± 0.86	0.0016
1941 IV	cont 1	0.8	1.94 ± 0.63	16.13 ± 0.42	9.81 ± 0.19	0.0004
1941 VIII	cont 1	1.4	1.28 ± 0.38	15.59 ± 0.39	9.30 ± 0.11	0.0005
1946 II	cont 2	1.2	0.33 ± 0.20	14.72 ± 0.79	8.54 ± 0.36	0.0002
1948 I	cont 1	0.9	1.82 ± 0.47	16.95 ± 0.34	9.72 ± 0.05	0.0004
1948 I	cont 2	1.0	1.82 ± 0.47	16.95 ± 0.34	9.70 ± 0.05	0.0004
1948 I	cont 1	1.5	1.82 ± 0.47	16.95 ± 0.34	9.61 ± 0.05	0.0005
1948 IV	cont 3	0.8	1.17 ± 0.41	16.37 ± 0.46	8.47 ± 0.10	0.0031
1948 XI	cont 1	0.6	2.67 ± 0.79	17.45 ± 0.38	10.13 ± 0.15	0.0003
1948 XI	cont 1	2.2	2.67 ± 0.79	17.45 ± 0.38	9.88 ± 0.15	0.0006
1951 I	cont 1	2.6	5.1 ± 1.4	18.29 ± 0.36	9.94 ± 0.16	0.0020
1951 II	cont 1		0.41 ± 0.13	15.01 ± 0.41	8.25 ± 0.14	0.0006

Here are the main conclusions resulting from the investigation of comets with strong continuous spectra:

1) The mean values of physical parameters B and k are 8.0 and 0.9 respectively; if considering $n < 5.0$ only, $\overline{B} = 6.7$ and $\overline{k} = 1.1$. The latter magnitude has a rather high dispersion with $k_{max} \approx 10$. Some more detailed results are included in Table 28.

Table 28

Average physical parameters of comets with continuous spectra

n_d	all n		$n < 5$	
	B	k	B	k
2.0	8.1	0.5	7.0	0.6
2.4	8.2	1.1	6.9	1.3
2.8	7.6	1.1	6.2	1.3

2) The spectral classification introduced by HRUŠKA and VANÝSEK in their Catalogue may be considered a characteristic of the total mass of the dust cloud. The mean cloud mass (in gm) of the comets with the cont 1 spectrum is $10^{9.83 \pm 0.22\,m.e.}$, that of the comets with the cont 2 spectrum is $10^{9.06 \pm 0.28\,m.e.}$. The cloud mass of comets with the cont 3 spectrum probably does not exceed $10^{8.5}$ gm on an average.

3) The mean value of $\frac{1}{\nu}\left(\frac{R}{\varrho}\right)^2$ important for purposes of some theoretical considerations (Section 3.7.) is 0.0008 ± 0.00018 m. e., the dispersion does not exceed the order.

4) As to the dimensions of cometary nuclei, from Table 27 a value of diameter of 4.0 km results when two big comets, 1882 II and 1904 I are taken into account. Excluding them we get another result, 2.2 km. The corresponding mass of the nucleus lies within $2 \leftarrow 10.10^{16}$ gm. The distribution of nuclei according to their diameter is represented in Fig. 19 (the two great comets are not included).

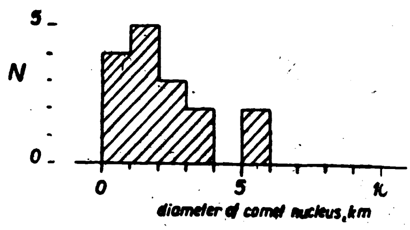


Fig. 19. Distribution of comets with continuous spectra according to their nucleus diameter.

CHAPTER FIVE

INTERACTION BETWEEN A COMET AND DUST CONSTITUENT OF INTERPLANETARY MATTER

5.1. INTRODUCTIÓN

In the collision of solid bodies with cosmic velocities there occur physical phenomena which may be mathematically described in the same way as an appearance of the explosion. STANYUKOVICH and FEDYNSKY (1947) pointed out that already for velocities of about 3 — 5 km/s the crystal structure of the meteoritical matter is broken up to such an extent that an explosion to all intents and purposes takes place. In fact, collisions with cosmic velocities in interior parts of the solar system lead not only to the total destruction of the crystal lattice but even to the evaporation of such small enough "projectiles". Generally, in order that the microscopic explosions may occur the projectile kinetic energy must be greater than the energy of the crystal lattice of a matter.

When studying collision of a meteorite, M, with a comet nucleus, N, we must introduce some special features into the general problem which may be expressed by the following suppositions:

1) M and N are bodies of incomparable mass and dimensions.

2) Physical characteristics of the materials forming M an N are roughly the same.

3) Influence of the N-gravitation field is so small that the escape velocity from N is negligible when compared with the velocity gained by the expelled material during the microscopic explosion process.

Under these conditions the process has roughly the following character: from the point of impact a shock wave characterized by a certain pressure starts to spread (STANYUKOVICH, 1955). The type of the running process (evaporation, fusion or pulverization of the stuff) now depends merely on the magnitude of the pressure. When the impact velocity reaches the order of 10^6 cm/s the complete evaporation of the meteorite and partial evaporation of the destroyed region of the surface of the comet nucleus take place; moreover, inside the microcrater the gas expands towards the sides. However, soon after being expelled the evaporated particles re-condense. The dimensions of the condensed particles are several orders greater than molecules (ZELDOVICH, RAISER, 1958). As was mentioned in Chapter Three, VANÝSEK (1960a) on the basis of colorimetrical measurements of a few comets had drawn the conclusion that photometrically effective dust particles in the cometary atmosphere are of the order of 10^{-5} cm.

5.2. BALANCE OF THE EXPELLED MATTER AND THE VELOCITY

The total mass expelled from a comet nucleus into space due to impact of a meteorite is equal to (STANYUKOVICH, 1960):

$$M_v = \frac{\bar{\eta}\eta M_0 V_{com}^2}{8\varepsilon_k^*}\left[\left(\frac{A}{\bar{\eta}}\right)^{3/8} - 1\right], \qquad (5.1)$$

where

$$(\bar{\eta})^{1/3} = A_0^{1/3} + \overline{\cos z}\,.\left(\frac{2\varepsilon_k^* \delta^2}{\eta \varrho_{com}^2 V_{com}^2}\right)^{1/3} \cdot \frac{8}{3c_x}\,.\ln \frac{V_{com}}{\sqrt{2\varepsilon_k^*}}\,, \tag{5.2}$$

A_0 is a coefficient depending on the characteristics of the matter, $\bar{z}$ the average angle in which the meteorite falls on the nucleus surface (measured from the perpendicular to the surface), δ the density of the meteorite, V_{com} the relative velocity between the nucleus and the meteorite, c_x a non-dimensional coefficient, ε_k^* the energy density necessary for breaking up the crystal lattice or the fine pulverization of the stuff, ϱ_{com} the comet nucleus density, λ the coefficient connecting the radius of the microcrater formed by the meteorite projectile, R_e, with the mass of the meteorite, M_0, $\lambda = R_e . M_0^{-1/3}$, and coefficient A is then defined by relation: $A = \frac{4}{3}\pi \varrho_{com} \lambda^3$; finally, η is the energy utilization coefficient ($\eta \leq 1$).

Let us further denote ϱ_M the space density of the meteorical matter and S the cross-section of the comet nucleus:

$$S = \bar{\alpha}^* \left(\frac{M_{com}}{\varrho_{com}}\right)^{2/3},$$

$\bar{\alpha}^*$ is a shape factor; a sphere has $\bar{\alpha}^* = \left(\frac{3\sqrt{\pi}}{4}\right)^{2/3}$. The total mass of meteorites which fall off on the comet nucleus surface per dt is

$$dm_M = S \varrho_M V_{com} dt \tag{5.3}$$

and the total mass expelled from the nucleus region where the crystal structure was destroyed owing to meteorite impacts during dt is

$$d(\Delta M_{com}) = \frac{M_p}{M_0} \cdot \frac{dm_M}{dt} \cdot dt = \frac{\bar{\eta}\eta V_{com}^3}{8\,\varepsilon_k^*}\left[\left(\frac{A}{\bar{\eta}}\right)^{2/3} - 1\right] \cdot \left(\frac{3\sqrt{\pi}}{4}\right)^{2/3} \cdot \left(\frac{M_{com}}{\varrho_{com}}\right)^{2/3} \cdot \varrho_M dt\,. \tag{5.4}$$

A velocity of the expelled stuff depends on angle φ measured from the perpendicular to the microcrater,

$$g = \left(\frac{2\varepsilon_k^*}{\bar{\eta}} \cos^3 \varphi\right)^{1/2}. \tag{5.5}$$

As the maximum opening of the microcrater is $\varphi_0 = \arccos\left(\frac{\bar{\eta}}{A}\right)^{\frac{1}{3}}$ and the mass distribution relative to φ is

$$dM_p = \frac{\bar{\eta}\eta M_0 V_{com}^2}{2\varepsilon_k^*} \cdot \frac{\sin \varphi}{2 \cos^3 \varphi}\, d\varphi, \tag{5.6}$$

the mean velocity of expelled particles results in;

$$\bar{g} = \frac{1}{M_p}\int_0^{M_p} g\, dM_p = \left(\frac{32\,\varepsilon_k^*}{\bar{\eta}}\right)^{1/2} \cdot \frac{\left(\frac{A}{\bar{\eta}}\right)^{1/6} - 1}{\left(\frac{A}{\bar{\eta}}\right)^{2/3} - 1}\,. \tag{5.7}$$

5.3. SPACE DENSITY OF THE METEORIC MATTER AND ITS VELOCITY RELATIVE TO THE COMET

To be able to integrate equation (5.4) it is necessary to know the concrete expressions for density ϱ_M and velocity V_{com}.

The problem of the space density of meteoric matter, especially in the vicinity of the Earth, has been solved by a number of authors; for a list see e. g. LEVIN (1956), MIRTOV (1960). There exist four rather different ways which may lead to some result:

a) Photometrical methods.
b) Estimates based on collections of meteoritic dust on the Earth's surface.
c) Observation of meteors.
d) Rocket and satellite research.

The most unreliable results are those obtained from the observations of meteors; the dispersion attains three orders. For our purposes they are omitted.

Further, there is a principle difference between the a)-method on the one hand and the b)-method on the other: photometrical methods (mostly zodiacal light and Fraunhoffer's corona) gather the information on those parts of the wide neighbourhood of the Earth which lie outside the sphere of activity of our planet, while collections of meteoritic dust give the data on the space density of dust particles inside the Earth's sphere of activity.

In the future the rocket and satellite probes appear to gather the best observational material. Nevertheless, at present we can arrive at the following conclusions:

1) Excepting VAN DE HULST's results (VAN DE HULST, 1947) we find 5.10^{-23} gm/cm^3 to be the most probable space density of minute dust particles (average diameter about 10^{-3} cm) at heliocentrical distance of 1 A.U. from photometrical measurements (ALLEN, 1947, BERH, SIEDENTOPF, 1953, ELSÄSSER, 1954, FESENKOV, 1947, SIEDENTOPF, 1954, SIEDENTOPF, 1955); the dispersion about one order.

2) Estimates of the fall of meteoritic dust from years 1950 to 1955 (MIRTOV, 1960) give the space density about two orders higher; some recent more reliable estimates (e. g. HANSA, ZACHAROV, 1958) are four or even five orders higher than those obtained by photometrical methods; the dispersion about two orders.

3) The densities resulting from impacts of micrometeorites on the surface of rockets and satellites (DUBIN, 1960, KOMISSAROV, NAZAROVA, NEUGODOV, POLOSKOV, RUSAKOV, 1958, LOVERING, 1959, MANRING, 1959) are about two or three orders higher than those from photometrical methods. Moreover, there are present short-term bursts in the impact frequency of intensity up to 10^4 times higher than the normal level (DUBIN, 1960, KOMISSAROV, NAZAROVA, NEUGODOV, POLOSKOV, RUSAKOV, 1958).

4) Recently WHIPPLE (1961) and HIBBS (1961) have shown that between heights of 100 and 100,000 kilometres the concentration of particles falls off roughly as the inverse 1.4 power of distance from the Earth surface. If the quantity of the dust fallen on the Earth surface corresponds to the densest parts of the dust cloud, the ratio between the density computed from the fall of meteoritic dust and that derived from photometrical data should be about 10,000 which is in good agreement with what has been said above. However, LEVIN (1961) states the effect found by WHIPPLE and HIBBS is not sustained by Soviet cosmic probes. Also SINGER (1961) pointed out that the existence of the

Whipple-Hibbs dust cloud was inconsistent with his theoretical conclusions. If all forces except gravitational forces are neglected the spatial density of dust, according to SINGER, is given by the formula

$$\varrho(\Delta)/\varrho_\infty = \left(u^2 + \frac{1}{\Delta}\right)^{1/2} . (2u)^{-1} \cdot \left[1 + \left(1 - \frac{1}{\Delta^2} \cdot \frac{u^2+1}{u^2+\frac{1}{\Delta}}\right)^{1/2}\right],$$

where Δ is the geocentric distance (in radii of the Earth) and u the geocentric velocity at infinity of the dust particle (in units of the escape velocity from the Earth's surface); ϱ_∞ is the spatial density of dust in interplanetary space. The resulting dependence of ϱ on the geocentric distance shows the existence of a modest dust shell around the Earth with maximum density at about 5,000 kilometres above sea level, not a layer with a uniformly decreasing density. SINGER also stressed that the concentration distribution of dust differed from the impact-rate distribution. The latter indicates another maximum in much lower altitudes, only a few hundred kilometres above the Earth's surface, and the rate of impacts drops more rapidly. The differences in the results obtained by various methods may, to some degree, be explained by this effect. Assuming $u = 0.1$ the interplanetary spatial density of dust of about 5×10^{-23} gm/cm³ corresponds to the accretion of some 10,000 tons per day.

5) There are some difficulties in computing the space density from both the collections of meteoritic dust and the impacts on the rocket surface. In the former case we do not know the mean velocity of the meteorite dust relative to the Earth, while in the latter case there are two unknown magnitudes, the mean velocity of the particles relative to the rocket and the mean mass of a particle. Therefore all results obtained in both the ways must be considered with reserve. SINGER (1961) emphasized that it was always the momentum which was measured by the rockets, satellites and cosmic probes. The differences in velocities of these vehicles cause the discrepancies among the observational data.

From what has just been said it could be concluded that a value of 5.10^{-23} is the best one of all. However, it is possible that photometrical methods underestimate the mass contribution of "heavy" particles, which are photometrically ineffective. Also rocket measurements probably underestimate the number of colliding particles because of inability to register those smaller than about 10^{-3} cm in diameter. Therefore we will further use 5.10^{-22} as the concrete value for $\varrho_M^{(0)}$ (see equation (5.8)).

Under conditions of the stationary distribution of interplanetary dust and the Poynting-Robertson effect the space density of the dust falls in inverse proportion to the heliocentric distance. Nevertheless, various authors put in the approximate formula

$$\varrho_M(r) = \varrho_M^{(0)} . r^{-\gamma} \qquad (5.8)$$

for the exponent numerical values from $\gamma = 0$ (BEHR, SIEDENTOPF, 1953, VAN DE HULST, 1947 SIEDENTOPF, 1954, 1955) up to $\gamma = 2$ (ALLEN, 1947, VAN DE HULST, 1947). It is likely that the real γ is not too far from zero.

Assuming the random distribution of dust-particle motions in interplanetary space velocity V_{com} may be put equal to the orbital velocity of the comet,

$$V_{com}^2 = \frac{GM_\odot}{R_\delta}\left(\frac{2}{r} - \frac{1}{a}\right), \qquad (5.9)$$

G is the universal gravitation constant, $M_\odot$ the mass of the Sun, a the semi-major axis of the cometary orbit, $R_♁ = 1.495 . 10^{13}$ cm; r and a are expressed in A. U.

5.4. THE TOTAL LOSS OF THE COMET MASS DUE TO METEORITE IMPACTS. THE SECONDARY EFFECT

From what was said about the balance between the projectile kinetic energy and the energy of the crystal lattice in Section 5.1 it is obvious that the condition for the origin of the pulverzation process can be written in the form

$$V_{com}(r) . \overline{\cos z} > \sqrt{2\varepsilon_k^*}, \tag{5.10}$$

so that according to (5.9) it takes place at heliocentrical distances less than

$$r_0 = \frac{GM_\odot \overline{\cos^2 z}}{R_♁ \varepsilon_k^* + \dfrac{GM_\odot \overline{\cos^2 z}}{2a}} . \tag{5.11}$$

For parabolical and near-parabolical comets ($1/a \leq 0.0001$) we can write simply

$$r_0^\infty = \frac{GM_\odot \overline{\cos^2 z}}{R_♁ \varepsilon_k^*}, \tag{5.12}$$

while for short-period comets the aphelion distance may be less than corresponding $r_0 = r_0(a)$; then the comet is throughout its orbit exposed to effects of the pulverization process and in such a case it must hold good

$$a < \frac{r_0^\infty}{2} \cdot \frac{1 - e}{1 + e}, \tag{5.13}$$

e is the numerical eccentricity of the orbit (see Section 5.7.).

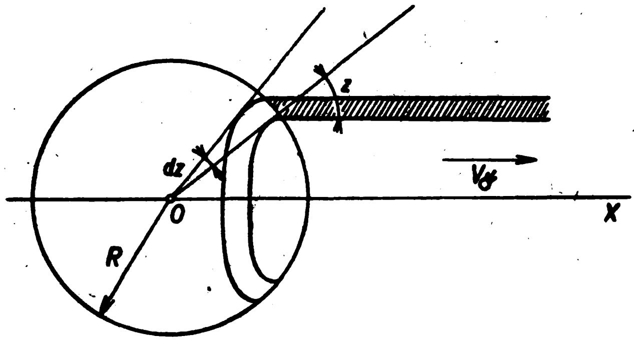
Magnitudes $\overline{\cos z}$ and $\overline{\cos^2 z}$ may be determined from the following consideration: Let us assume the spherical comet nucleus is moving inside a homogeneous cloud of dust particles with velocity V_{com} in direction OX (Fig. 20). If the number of particles in a unit volume of space is $\frac{\varrho_M}{M}$ (M is the mass of a particle), the rate of particle impacts on the belt of the sphere bounded by angles z, $z + dz$ is

$$f_r = \frac{\varrho_M}{M} . 2\pi R^2 V_{com} \sin z \cos z \, dz, \tag{5.14}$$

so that

$$\left. \begin{aligned} \overline{\cos z} &= \frac{\int_0^{\pi/2} f_r(z) \cos z \, dz}{\int_0^{\pi/2} f_r(z) \, dz} = \frac{2}{3}, \\ \overline{\cos^2 z} &= \frac{\int_0^{\pi/2} f_r(z) \cos^2 z \, dz}{\int_0^{\pi/2} f_r(z) \, dz} = \frac{1}{2} . \end{aligned} \right\} \tag{5.15}$$

Fig. 20. **A graph for deriving expressions $\overline{\cos z}$ and $\overline{\cos^2 z}$.**



After substituting

$$dt = -(2GM_{\odot})^{-1/2}.\left(1 - \frac{q}{r}\right)^{-1/2}.r^{1/2}.R_{\delta}^{3/2}\left[1 - \frac{r}{2a}\left(1 + \frac{q}{r}\right)\right]^{-1/2}.dr \tag{5.16}$$

and inserting (5.2), (5.8) and (5.9) into (5.4) we arrive at the expression, the integral of which from q to min (r_0, q') gives the total loss of the comet mass due to meteorite impacts along the orbit from the aphelion, q', to the perihelion, q:

$$\Delta M_{com} = \left(\frac{3\sqrt{\pi}}{4}\frac{M_{com}}{\varrho_{com}}A\right)^{2/3}.\frac{\eta\varrho_M^{(0)}GM_{\odot}}{4\varepsilon_k^*}\int_q^{\min(r_0,q')} r^{-(1+\gamma)}.\left(1 - \frac{r}{2a}\right)^{3/2}.\left(1 - \frac{q}{r}\right)^{-1/2} \times$$

$$\times\left[1 - \frac{r}{2a}\left(1 + \frac{q}{r}\right)\right]^{-1/2}.\left[A_0^{1/3} + \left(\frac{\varepsilon_k^*\delta^2}{\eta GM_{\odot}\varrho_{com}^2}\right)^{1/3}.\frac{4\,\overline{\cos z}}{3c_x}.r^{1/2}.R_{\delta}^{1/2}.\left(1 - \frac{r}{2a}\right)^{-1/2}\times\right.$$

$$\left.\times \ln\left\{\left(\frac{GM_{\odot}}{\varepsilon_k^*}\right).r^{-1}.R_{\delta}^{-1}.\left(1 - \frac{r}{2a}\right)\right\}\right]dr. \tag{5.17}$$

For periodical comets the solution of (5.17) must be carried out be means of numerical quadrature while the following expression results for near-parabolical comets:

$$\Delta M_{com} = X_0 q^{-\gamma}\sum_{k=0}^{\infty}\frac{(-1)^k}{k + \frac{1}{2}}\binom{\gamma-1}{k}(1-\varepsilon)^{k+\frac{1}{2}} + Y_0 q^{1/2-\gamma}\left\{\left(Z_0 + \ln\frac{\varepsilon}{q}\right)\times\right.$$

$$\times\sum_{k=0}^{\infty}\frac{(-1)^k}{k+\frac{1}{2}}\binom{\gamma-4/3}{k}(1-\varepsilon)^{k+\frac{1}{2}} + 2\ln\frac{1+\sqrt{1-\varepsilon}}{\sqrt{\varepsilon}}\sum_{k=0}^{\infty}\frac{(-1)^k}{k+\frac{1}{2}}\binom{\gamma-4/3}{k} -$$

$$\left. - \sum_{k=0}^{\infty}\sum_{l=0}^{k}\frac{(-1)^k}{k+\frac{1}{2}}\binom{\gamma-4/3}{k}\frac{(1-\varepsilon)^{l+1/2}}{l+\frac{1}{2}}\right\}, \tag{5.18}$$

where $\varepsilon = \frac{q}{r_0^{\infty}}$. The first and second progressions converge according to the d'Alembert criterion, the third according to the Cauchy criterion while the convergency of the fourth progression may be proved when combining both criteria.

If $\gamma > \frac{1}{3}$ formula (5.18) may be written in much simpler form by means of B-function:

$$\left.\begin{aligned} \Delta M_{com} &= X_0 q^{-\gamma} . B\left(\frac{1}{2}, \gamma\right) + Y_0 q^{1/3-\gamma} . B\left(\frac{1}{2}, \gamma - \frac{1}{3}\right) \times \\ &\times \left[Z_0 - \ln q - \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{\left(n + \gamma - \frac{5}{6}\right)\left(n + \gamma - \frac{4}{3}\right)} \right]. \end{aligned}\right\} \quad (5.19)$$

In the two formulae there is

$$\left.\begin{aligned} X_0 &= \frac{\varrho_M^{(0)}}{4\varepsilon_k^*} \eta G M_{\odot} . \left(\frac{3\sqrt{\pi}}{4} \frac{M_{com}}{\varrho_{com}} . A \sqrt{A_0} \right)^{2/3}, \\ Y_0 &= \frac{\varrho_M^{(0)} \overline{\cos z}}{3 c_x} . \left[\frac{3\sqrt{\pi}}{4} R_\delta^{1/2} A \frac{M_{com}}{\varrho_{com}} \frac{\eta \delta G M_{\odot}}{\varepsilon_k^*} \right]^{2/3}, \\ Z_0 &= \ln \frac{G M_{\odot}}{R_\delta \varepsilon_k^*} . \end{aligned}\right\} \quad (5.20)$$

A part of the expelled particles falls, after finishing the trajectories in the comet atmosphere, back on the nucleus surface. It may be proved that the condition for the origin of an explosion

$$\bar{g} . \overline{\cos z} > (2\varepsilon_k^*)^{1/2} \quad (5.21)$$

is on assumption (5.2) of Section 5.1. never fulfilled, regardless of the crystal-lattice energy. It would have to be $\delta \gtrsim 3\varrho_{com}$ in order that explosions might occur over a small section of the orbit. However, it is likely that such dust particles are immediately after their impact on the nucleus surface pulled up with the escaping gas into the comentary atmosphere again. Let us determine the ratio between the number of particles colliding with the nucleus and the entire number of particles running through the plane of the dust atmosphere perpendicular to the radiusvector during the same period. Let us further denote N_t the number of particles running through a unit cross-section of this plane per unit of time, then the sought for ratio is

$$P = \frac{N_{coll}}{N_\Sigma} = \frac{\int_0^R 2\pi s . N_t \, ds}{\int_0^{D/2} 2\pi s . N_t \, ds}, \quad (5.22)$$

where R is the radius of cometary nucleus and D is the coma diameter measured perpendicularly to the radius-vector. If ϱ_x is the number of particles in a unit volume in this plane and v_x their velocity, we can write simply:

$$N_t = \varrho_x . v_x . \quad (5.23)$$

MOKHNACH's analysis of the space density distribution in the comet fountain model (MOKHNACH, 1956) gives for ϱ_x the expression as follows:

$$\varrho_x = \frac{n.v}{s^2\sqrt{v^4 - \Gamma^2 s^2}}, \tag{5.24}$$

where s is the distance from the nucleus in the plane of $x = 0$; v is the initial particle velocity and n the number of particles expelled in a unit of solid angle in a given direction per unit of time. According to the fountain model velocity v_x in the plane of $x = 0$ is independent of coordinates y and z being identical with the initial particle velocity v. Therefore

$$N_t = \frac{n}{s^2\sqrt{1 - \frac{\Gamma^2 s^2}{v^4}}}. \tag{5.25}$$

For $s = 0$ each of the two integrals of (5.22) are infinite and their ratio converges to the unit. To overcome this difficulty we will assume N_t = const $= N_t(R)$ in the region $\langle 0, R\rangle$ and the validity of (5.23) and (5.24) in the region $\left\langle R, \frac{D}{2}\right\rangle$. Since $R \ll D$ the resulting formula for P is

$$P = \frac{1}{1 + 2\ln\frac{4\xi_0}{R}}, \tag{5.26}$$

where the distance of the head-top from the comet nucleus, ξ_0, is introduced instead of D; $\xi_0 = \frac{1}{4} D$. Studying the dust atmosphere of the comet DOBROVOLSKY (1953c) found the formula for ξ_0 expressed through the parameters of gas:

$$\xi_0 = \frac{4}{3}\cdot\frac{c.m.R}{L}.v_0, \tag{5.27}$$

c is the velocity of light, m the mass of a molecule, R the effective radius of the nucleus, v_0 the velocity of gas and L the heat of evaporation per molecule. Expressing L in cal/mol and r in A. U. we get

$$P = \left[42.2 + 4.6\log\frac{\sqrt{\mu_0}}{L} - 0.46\log r\right]^{-1}, \tag{5.28}$$

μ_0 is the molecular mass of a molecule. The result is independent of both the effective radius of the comet nucleus and the dust-particle size. There is only a slight dependence of P on the heliocentrical distance and the kind of gas, as seen in Table 29. The average $\overline{P}$ lies near 0.036.

Within a time interval between t_0 and $t_0 + dt$ a certain amount of meteorites of mass dm_M fall on the surface of the comet nucleus and produces the ejection of tiny dust particles from it of the mass

$$\frac{M_v(t)}{M_0}\cdot\frac{dm_M(t)}{dt}.dt, \quad t \in \langle t_0, t_0 + dt\rangle. \tag{5.29}$$

Table 29
Secondary effect

Gas	P		
	r = 0.32 AU	r = 1 AU	r = 222 AU
CH_4	0.0335	0.0338	0.0351
NH_3	0.0366	0.0369	0.0382
H_2O	0.0378	0.0382	0.0396
CO_2	0.0346	0.0350	0.0364
C_2N_2	0.0342	0.0345	0.0358
Average	0.0353	0.0357	0.0370

After finishing the trajectories in the cometary atmosphere the P-th portion of the particles falls back on the nucleus surface. The corresponding period is called further the "life-time" of dust particles, τ_0. If the particles are pulled up with the gas into the cometary atmosphere immediately after their impact on the comet nucleus, the contribution of them to the total mass of the material in the coma within the interval $\langle t_0 + \tau_0, t_0 + \tau_0 + \mathrm{d}t \rangle$ is

$$\frac{M_p(t)}{M_0} \cdot \frac{\mathrm{d}m_M(t)}{\mathrm{d}t} . P(t)\, \mathrm{d}t,\ t \in \langle t_0, t_0 + \mathrm{d}t \rangle . \qquad (5.30)$$

Analogously, after the period of $j\tau$ the respective contribution will be

$$\frac{M_p(t)}{M_0} \cdot \frac{\mathrm{d}m_M(t)}{\mathrm{d}t} . [P(t)]^j\, \mathrm{d}t,\ t \in \langle t_0, t_0 + \mathrm{d}t \rangle . \qquad (5.31)$$

Adding all j and integrating over the whole period of occurrence of microscopic explosions we get, with respect to (5.4), (5.17) and (5.28):

$$\Delta^{\Sigma} M_{com} = \int_{t_0}^{t_1} \sum_{j=0}^{\infty} \left(\frac{M_p}{M_0} \cdot \frac{\mathrm{d}m_M}{\mathrm{d}t} . P^j \right) \mathrm{d}t \doteq \sum_{j=0}^{\infty} (\overline{P})^j \int_{t_0}^{t_1} \frac{M_p}{M_0} \cdot \frac{\mathrm{d}m_M}{\mathrm{d}t}\, \mathrm{d}t \doteq$$

$$\doteq 2\,(1 + \overline{P}) . \Delta M_{com} . \qquad (5.32)$$

Here t_1 is the moment $T + (T - t_0) = 2T - t_0$, T is the time of perihelion passage, t_0 the moment of beginning the pulverization process: $t_0 = t(r_0)$.

5.5. THE METEORITE-IMPACT LOSS OF THE COMET AS A FUNCTION OF THE HELIOCENTRIC DISTANCE

For analyzing the dependence of the amount of expelled matter on the heliocentrical distance (assuming the parabolical orbit) we shall divide the whole period of the pulverization process into three parts:

1) The period taking place immediately after beginning the dust emission due to meteorite impacts; its duration is equal to the life-time of dust particles expelled from the nucleus surface by meteorites (primary effect).

2) The period just following the preceding one, where both the primary and secondary effects are present.

3) The period beginning when the primary process of microscopic explosions ends; only the secondary effect continues.

Here we must realize that it is only from the statistical point of view that distance r_0 is the most probable one for starting the pulverization process. In fact, from the character of velocity $\bar{V}_{com}$ it follows that the spontaneous origin of microscopic explosions is a process taking place successively over the interval of several tens of A. U. The problem is in some degree idealized by introducing distance r_0, of course.

According to our definition the duration of the period, during which the volume of the dust in the atmosphere is entirely exchanged, is

$$\tau_t = \frac{v_t}{\Gamma} \sqrt{2}, \tag{5.33}$$

where v_t is the mean "thermal" velocity of dust particles within the interval of heliocentrical distances $\langle r_{t-\tau_t}, r_t \rangle$ and Γ is the acceleration due to the radiation pressure. Let us further denote

$$\tau_{0,t} = \frac{\bar{g}_t}{\Gamma} \sqrt{2} \tag{5.34}$$

the life-time of dust particles emitted from the nucleus surface by meteorite impacts; $\bar{g}_t$ is the mean velocity of particles expelled from the surface within $\langle r_{t-\tau_t}, r_t \rangle$ given by (5.7) and t is in both the expressions the moment of impact of a portion of particles (i. e. a P-th portion) back on the nucleus. Velocity v_t in (5.33) may be approximated by the formula of DOBROVOLSKY (1953c):

$$v_t = \left(\frac{m Q_0 R}{2 \varrho \delta L} \right)^{1/2} . r^{-1} . v_0^{1/2}, \tag{5.35}$$

m is the mass of a molecule, Q_0 solar constant, R effective radius of the cometary nucleus, ϱ dust-particle size, δ its mass density and L the heat of evaporation of molecules. This formula is close to that derived for the dust-particle velocity of the comet icy model by WHIPPLE (1951).

Let us introduce into (5.33) and (5.34) the average heliocentrical distance $\bar{r}_t$ of the interval $\langle r_{t-\tau_t}, r_t \rangle$. The relation between the average distance, $\bar{r}_t$, and the distance of the "last moment" of the same interval, r_t, may be with a relatively high accuracy written in the form

$$r_t = \bar{r}_t \mp \frac{1}{2} \Delta_t, \tag{5.36}$$

where the upper sign is valid before the passage of a comet through the perihelion, the lower after it, and

$$\Delta_t = (2 G M_\odot)^{1/2} . \left(1 - \frac{q}{r_t} \right)^{1/2} . r_t^{-1/2} . R_♁^{-3/2} . \tau_t . \tag{5.37}$$

In addition to this we can write

$$\Gamma = G M_\odot (1 + \mu) . \bar{r}_t^{-2} . R_♁^{-2}, \tag{5.38}$$

$1 + \mu$ is the ratio between the radiation pressure and solar gravitation force. Then (5.33) has a form of

$$\tau_t = \frac{(2GM_\odot)^{-1/2}.\left(1 - \frac{q}{r_t}\right)^{-1/2}.r_t^{3/2}.R_\delta^{3/2}}{\frac{1+\mu}{v_t}(GM_\odot)^{1/2}.\left(1 - \frac{q}{r_t}\right)^{-1/2}.r_t^{-1/2}.R_\delta^{-1/2} \mp 1} \tag{5.39}$$

The computation of $\tau_{0,t}$ is quite analogous.

All the time intervals will further be expressed by life-times τ. For this purpose we introduce the simplified denotation as follows (t is a zero-point):

$$t - \tau_t = t - \tau$$

$$t - \tau_t - \tau_{t-\tau_t} = t - 2\tau$$

$$t - \tau_t - \tau_{t-\tau_t} - \tau_{t-\tau_t-\tau_{t-\tau_t}} = t - 3\tau$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

Within the first of the three investigated periods, given by

$$t_0 \leq t \leq t_0 + \tau_{0,t'_0}$$

(t'_0 is the moment of impact back on the nucleus of the first particles expelled by meteorites), the total mass of photometrically effective dust particles is at a given moment, t:

$$\mathfrak{M}(t) = \int_{t_0}^{t} \frac{M_p}{M_0} \cdot \frac{\mathrm{d}m_M}{\mathrm{d}t}\,\mathrm{d}t. \tag{5.40}$$

The second investigated period is bounded by

$$t_0 + \tau_{0,t'_0} \leq t \leq t_0 + k\tau + \tau_{0,t'_0} = t_1,$$

where k is the number of subintervals τ_t placed between $t_0 + \tau_{0,t_0}$ and $t_1 = 2T - t_0$. Let us select the subinterval of

$$t_0 + (n-1)\,\tau + \tau_{0,t-(n-1)\tau} \leq t \leq t_0 + n\tau + \tau_{0,t-n\tau},$$

$n \leq k$ is the natural number. Then the total mass of dust particles present in the cometary atmosphere at a given moment, t, is

$$\mathfrak{M}(t) = \sum_{i=0}^{n-1} \int_{t-i\tau-\tau_{0,t-i\tau}}^{t-i\tau} \frac{M_p}{M_0} \cdot \frac{\mathrm{d}m_M}{\mathrm{d}t} P^i\,\mathrm{d}t + \int_{t_0}^{t-n\tau} \frac{M_p}{M_0} \cdot \frac{\mathrm{d}m_M}{\mathrm{d}t} P^n \mathrm{d}t. \tag{5.41}$$

As the variation of the integrand within the intervals given by the limits of the integral is quite small, the mean-value theorem may be applied, and (5.41) is then

$$\mathfrak{M}(t) = \sum_{i=0}^{n-1} \tau_{0,t-i\tau} . \overline{P}^i . \left(\frac{M_p}{M_0} \cdot \frac{\mathrm{d}m_M}{\mathrm{d}t}\right)_{\overline{T}_{t-i\tau}} + \int_{t_0}^{t-n\tau} \frac{M_p}{M_0} \cdot \frac{\mathrm{d}m_M}{\mathrm{d}t} P^n \mathrm{d}t, \tag{5.42}$$

where $\tau_{0,\,t-i\tau}$ is determined according to (5.39), using the reccurent formula

$$r_{t-(i+1)\tau} = r_{t-i\tau}\left[1 - \frac{1}{1 \pm \frac{1+\mu}{2v_{t-i\tau}} .(GM_\odot)^{1/2}.\left(1 - \frac{q}{r_{t-i\tau}}\right)^{-1/2} .r_{t-i\tau}^{-1/2} .R_\delta^{-1/2}}\right]. \tag{5.43}$$

Distance $\bar{r}_{t-i\tau}$ is given by another formula:

$$\bar{r}_{t-i\tau} = r_{t-i\tau}\left[1 - \frac{1}{2 \pm \frac{1+\mu}{\bar{g}_t} (GM_\odot)^{1/2}.\left(1 - \frac{q}{r_{t-i\tau}}\right)^{-1/2} .r_{t-i\tau}^{-1/2} .R_{\delta}^{-1/2}}\right]. \tag{5.44}$$

Only a few first terms in the series of (5.42) must be taken into account in practical computation.

Finally, in the third investigated period, where

$$t \geq t_1 = t_0 + k\tau + \tau_{0,t'_0},$$

we shall select the subinterval of

$$t_0 + (k+n)\,\tau + \tau_{0,t-(k+n)\tau} \leq t \leq t_0 + (k+n+1)\,\tau + \tau_{0,t-(k+n+1)\tau},$$

$n = 0, 1, 2, \ldots$ and then the respective mass of the dust is

$$\begin{aligned} \mathfrak{M}(t) = \int\limits_{t-n\tau-\tau_{0,t-n\tau}}^{t_1} \frac{M_p}{M_0} . \frac{\mathrm{d}m_M}{\mathrm{d}t} P^n \mathrm{d}t + \sum_{i=1}^{k} \int\limits_{t-(n+i)\tau-\tau_{0,t-(n+i)\tau}}^{t-(n+i)\tau} \frac{M_p}{M_0} . \frac{\mathrm{d}m_M}{\mathrm{d}t} P^{n+i}\,\mathrm{d}t\, + \\ + \int\limits_{t_0}^{t-(n+k+1)\tau} \frac{M_p}{M_0} . \frac{\mathrm{d}m_M}{\mathrm{d}t} P^{n+k+1}\,\mathrm{d}t. \end{aligned} \tag{5.45}$$

The next modification and solution are quite analogous to those in the preceding case.

The beginning of the first period is given by (5.12). Since

$$\tau_{0,t'_0} = \frac{2^{-3/2} . \frac{\bar{g}}{1+\mu} .GM_\odot.(\varepsilon_k^*)^{-2}}{1 + \frac{\bar{g}}{1+\mu} . \left(\frac{2}{\varepsilon_k^*}\right)^{1/2} . \left(1 - \frac{q.R_\delta.\varepsilon_k^*}{GM_\odot}\right)}, \tag{5.46}$$

the beginning of the second period is

$$r_{t'_0} = \frac{1}{2} . \frac{GM_\odot}{R_\delta \varepsilon_k^*}\left[1 - \frac{1}{1 + \frac{1+\mu}{\bar{g}} . \left(\frac{\varepsilon_k^*}{2}\right)^{1/2} . \left(1 + \frac{qR_\delta\varepsilon_k^*}{GM_\odot}\right)}\right], \tag{5.47}$$

while that of the third period is determined by (5.12) again.

5.6. NUMERICAL RESULTS

Further we shall apply the theory to a comet with parabolical orbit and perihelion distance of 0.32 A. U. From great parabolical or near-parabolical comets the Arend-Roland comet of 1957 had such an orbit. The very close orbits (within $\pm$ 0.05 A. U.) also belonged to some other bright "new" comets as 1957d, 1941 I, 1911 IV, 1903 IV, 1899 I, 1860 III, 1853 III and to 10 fainter comets of the 19th and 20th centuries. Moreover, there are three periodical comets with a similar perihelion distance, viz. 1886 V (period 745 years), 1883 II (period 64.6 years) and the well-known Encke comet (period 3.3 years).

The constants we shall use are as follows: $G = 6.67 . 10^{-8}$ CGS, $M_{\odot} = 1.993 . 10^{33}$ gm, $\varrho_{com} = \delta = 3.54$ gm/cm^3, $M_{com} = 1 . 10^{16}$ gm, $\varrho_M^{(0)} = 5 . 10^{-22}$ gm/cm^3 (see Section 5.3.), and we shall accept according to Stanyukovich (1960): $A = 16$, $A_0^{1/3} \doteq 1$, $c_x = 2$, $\varepsilon_k^* = 2 . 10^{10}$ erg/gm, $\eta \approx 1$. The dependence of $\bar{\eta}$ and $\bar{g}$ on the heliocentric distance for $\frac{1}{a} \to 0$ is represented in Figure 21. The following values of basic magnitudes result from the adopted constants: $X_0 = 1.27 . 10^5$ gm, $Y_0 = 0.74 . 10^4$ gm, $Z_0 = 6.10$, $r_0^{\infty} = 222$ A. U. and $\varepsilon = 0.0014$. After their inserting into (5.18) or (5.19) we get the total meteorite-impact loss of a comet mass given, as related to exponent γ, in Table 30. It is obvious that the influence of γ on $\Delta^{\Sigma} M_{com}$ is quite slight and no order changes take place.

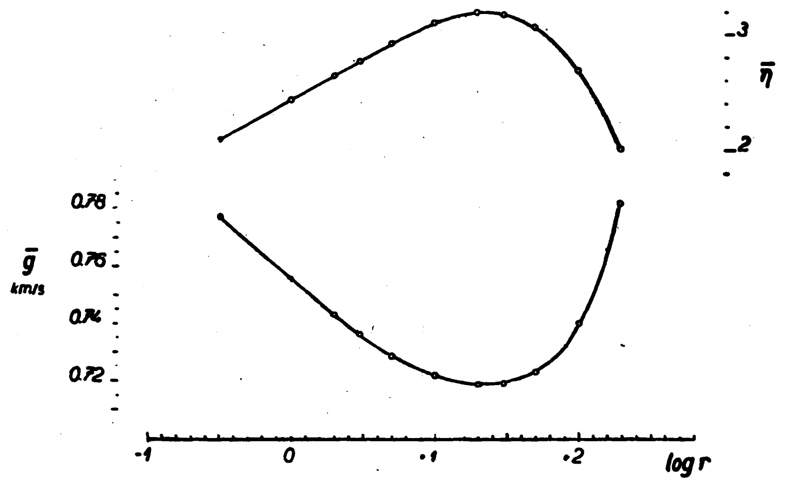


Fig. 21. Dependence of function $\bar{\eta}$ and velocity $\bar{g}$ on the heliocentric distance.

For the stationary state of interplanetary matter the dependence of the momentary amount of dust particles expelled from the comet nucleus into the atmosphere owing to the meteorite-impact process on the heliocentric distance is represented in Figure 22. There is only a slight perihelion asymmetry of the curve towards the post-perihelion period. Generally, such a curve is with quite sufficient accuracy described by the relation

$$\mathfrak{M}(r) = \text{const} . r^{1/3 - \gamma}, \tag{5.48}$$

which may be derived from (5.41) after introducing a few approximations. Relation (5.48) makes it possible to determine the photometrical exponent of

Table 30

Total meteorite-impact loss of a comet mass as related to exponent γ

γ	ΔM_{com}, gm
0	$1.06 \cdot 10^5$
$\frac{1}{2}$	$0.85 \cdot 10^5$
1	$1.05 \cdot 10^5$
$\frac{4}{3}$	$1.28 \cdot 10^5$
2	$2.16 \cdot 10^5$
$\frac{7}{3}$	$2.89 \cdot 10^5$

the dust coma characterizing the pulverization process. If, generally, γ is a function of r, we get

$$n_d(r) = \frac{3}{2} + r \frac{\mathrm{d}}{\mathrm{d}r} [\ln r^{\gamma(r)}], \tag{5.49}$$

or, if γ is a constant:

$$n_d = \frac{3}{2} + \gamma. \tag{5.50}$$

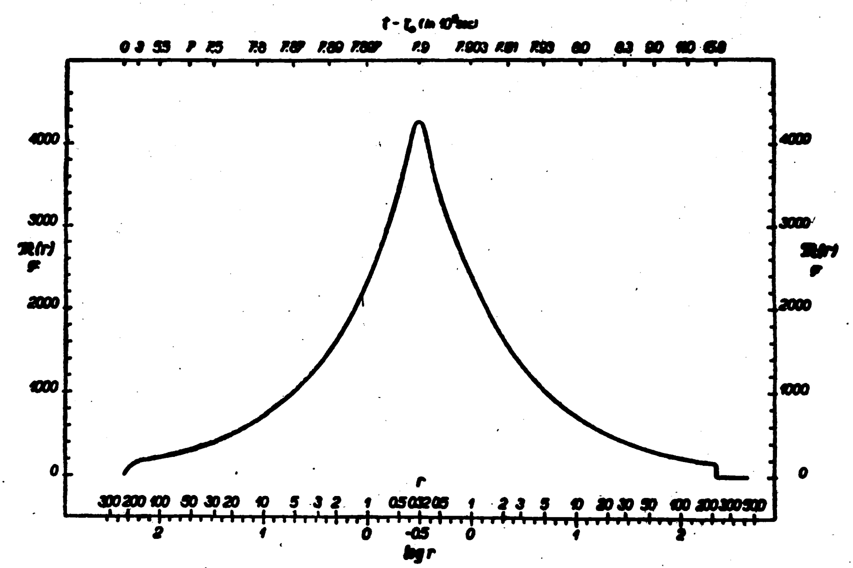


Fig. 22. The dependence of the comet-mass loss due to micrometeorite impacts on the heliocentric distance.

Relation (5.48) further indicates that for $\gamma < \frac{1}{2}$ the momentary amount of dust particles in the atmosphere increases with the heliocentrical distance, and corresponding exponent $n_d < 2$, while for $\gamma > \frac{1}{2}$ it decreases, and $n_d > 2$.

From Table 30 and Figure 22 it is obvious that the meteorite-impact process can in no way explain the amount of the dust ascertained spectroscopically and photometrically in the atmosphere of "new" comets — according to how they were defined by OORT and SCHMIDT (1951) — especially of comets with strong continuous spectra. The mass of photometrically effective dust particles in the cometary atmosphere at 1 A. U. is about 10^9 — 10^{10} gm for comets with strong continuous spectra (see Chapter Four), for other "new" comets it will be less by about one or two orders, while the pulverization process can fill the cometary atmosphere at the same moment only by about 10^3 gm of the dust. Just short-term bursts of the meteorite-impact frequency (see Section 5.3) may in case of some "new" comets perceptibly contribute to the amount of dust particles expelled into the atmosphere by the comet's internal forces. Furthermore, we must take into consideration that long-period comets are exposed to the pulverization process effects only during a very short period of their life. Another situation occurs in comets with the shortest orbital periods. To analyze this problem in detail it is necessary to investigate the range of validity of condition (5.13).

5.7. THE SPHERE OF THE PULVERIZATION-PROCESS EFFECTS AS RELATED TO THE FORM OF A COMETARY ORBIT

In order that the entire orbit of a comet should lie inside the sphere of the pulverization-process effects the following relation must hold good between the orbital period, P, and the perihelion distance, q:

$$q = \frac{4}{r_0^\infty} . P^{4/3}\left(1 + \frac{2P^{2/3}}{r_0^\infty}\right)^{-1}, \qquad (5.51)$$

P is expressed in years, $r_0^\infty = 222$ A. U. and q results in A. U. All the orbits, for which the left side of this relation is greater than the right side, lie inside the sphere. In the opposite case, let us introduce parameter $p < 1$, indicating the portion of the orbital period during which a comet is exposed to effects of the pulverization process:

$$p = \frac{1}{\pi}\left[\arccos\frac{a - r_0}{a - q} - \frac{1}{a}\sqrt{(r_0 - q)(2a - r_0 - q)}\right], \qquad (5.52)$$

where

$$r_0 = \frac{222\,a}{a + 111}$$

and a is the semi-major axis of the orbit. Parameter p may be easily derived from the nomogram in Figure 23.

The sphere of the pulverization-process effects is well seen in Figure 24, in which the p-isolines are sketched in system ($\log P$, $\log q$) and all the known comets with orbital periods shorter than 500 years are plotted. It is useful to

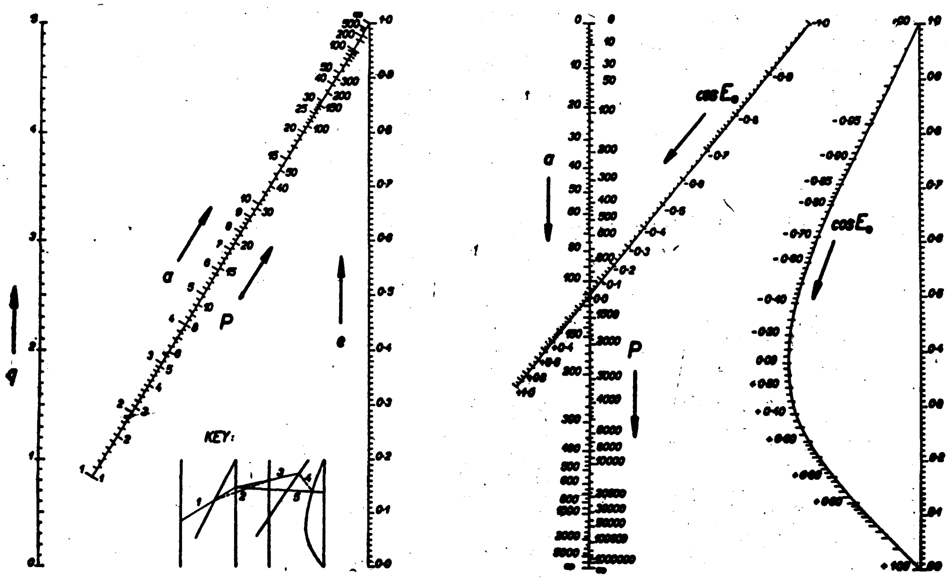


Fig. 23. Nomogram for determining parameter p.

extrapolate the system of p-isolines into the region of $p > 1$. For this purpose, p was defined as the ratio of the hypothetical period, P_h, to the real period, P; the physical meaning of P_h is clear from the relation

$$2P_h^{2/3} = q + r_0.$$

In such a way we get

$$p = 20.3\, q^{3/2}(1 + 0.05\sqrt{q}).P^{-1}. \tag{5.53}$$

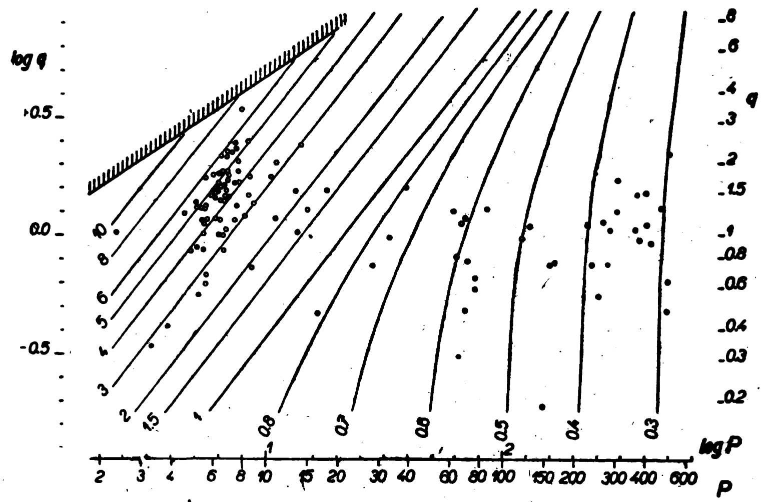


Fig. 24. The sphere of the pulverization process.

Hence, the p-parameter depends both on the orbital period and on the perihelion distance of a comet.

On the other side, the perihelion distance does not influence on the p-parameter of the long-period comets, for which (5.52) may be converted to the form of:

$$p = 496.3\left(1 - \frac{130}{a}\right).P^{-1}. \tag{5.54}$$

For $a = 1000$ A. U. this formula gives p with the accuracy of 1.2 per cent, and the deviation from (5.52) falls as the inverse 2.2 power of the semi-major axis; if the correction term in the brackets is omitted the accuracy for the above semi-major axis drops to 13.5 per cent, and the deviation falls now in inverse proportion to the semi-major axis. The additional correction term of $\frac{15400}{a^2}$ reduces the deviation to a half of the value valid for (5.54).

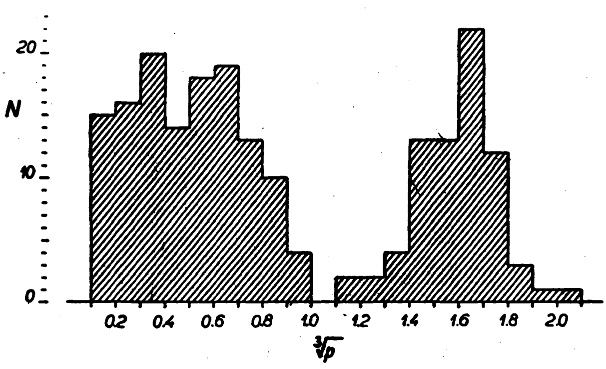


Fig. 25. Distribution of comets according to parameter p.

Figure 24 conspicuously separates the short-period comets of the Jupiter group and partly those of the Saturn group from the other periodic and all non-period comets, and indicates that there is no comet within the range of p between 0.85 and 1.6. As no selection effect may explain this blank we must assume that it is something like a "forbidden" region for comtes. The distribution of comets according to $\sqrt[3]{p}$ is represented in Figure 25. Comets with $\sqrt[3]{p} < 0.1$, not included in Figure 25, are the members of the Oort cloud. Table 31 gives the average absolute brightness of the comets inside the intervals of the distribution from Figure 25. Figure 25 as well as Table 31 indicate that the short-period comets form a special group with respect to the extent of the sphere of the pulverization-process effects.

Table 31

Distribution of absolute magnitudes of comets according to $\sqrt[3]{p}$

int $\sqrt[3]{p}$	H_0
	m
0.1—0.2	6.2
0.2—0.3	6.4
0.3—0.4	6.4
0.4—0.5	6.3
0.5—0.6	6.3
0.6—0.7	6.1
0.7—0.8	8.2
0.8—0.9	7.9
0.9—1.0	8.3
1.0—1.2	(11.6)
1.2—1.4	10.2
1.4—1.6	10.9
1.6—1.8	11.0
1.8—2.1	11.3

5.8. CONSIDERATIONS CONCERNING THE ENCKE COMET, AND CONCLUSIONS

The influence of the pulverization process on the short-period comets will be investigated on the Encke comet, for which $a = 2.22$ A. U., $q = 0.34$ A. U. and $P = 3.3$ years. The total meteorite-impact loss of the comet mass per orbital period is $\Delta^{\Sigma}M_{com} = 7.10^5$ gm (for $\gamma = 0$), i. e. it is comparable with that of parabolic comets.

The effect of "micrometeorite showers" (Section 5.3.) depends on both the probability and lasting of the meteorite shower — comet collision during the comet's orbital period, p_0, and the "frequency" of micrometeorites, or a mass density, ϱ_μ, inside such a shower. The contribution of the showers to the total pulverization-process loss of a comet mass during its orbital period is thus

$$\Delta^{\Sigma}M_{com} \cdot \frac{\varrho_\mu}{\varrho_M} \cdot p_0 \,.$$

Rocket and satellite probes indicate that the encounters with "micrometeorite showers" are relatively frequent (Berg, Meredith, 1956, Dubin, 1960, Komissarov, Nazarova, Neugodov, Poloskov, Rusakov, 1958, Manring, 1959, Nazarova, 1960). The ϱ_μ/ϱ_M ratio varies from 10^1 (Dubin, 1960) up to 10^6 (Berg, Meredith, 1956, Nazarova, 1960), p_0 is a very uncertain magnitude, probably ranged within a wide interval round $10^{-2\pm}$ (Dubin, 1960).

The momentary amount of the dust in the atmosphere of the Encke comet accumulated by the pulverization process at the distance of 1 A. U. is to be about $1.6.10^3$ gm which gives the absolute magnitude $H_{0d} = 23^m$. If the nucleus diameter of the Encke comet is estimated about 400 metres the corresponding absolute magnitude yields in $H_{0n} = 20^m$, while the integrated absolute magnitude of the comet is 12^m to 13^m in the present time.

The pulverization process itself cannot be generally sufficient for explaining the existence of the dust even in the atmosphere of the short-period comets (for comet Encke photometrical investigations give $H_{0d} \approx 18^m$), however, three phenomena may be due to collisions of the comet with interplanetary dust and especially with micrometeorite showers:

1) The pulverization process may be the initial stage in forming the dispersion dust layer analogous but much smaller than that of "new" comets. It is likely that the process has this function only if it may act continuously and in a sufficient intensity, i. e. only in comets with relatively high p (see Section 5.7.). For other comets it is, perhaps, ineffective in this line and the process of release of next gaseous and dust particles from the comet nucleus is stopped early in secular scale. Such comets ($p \approx 1$) have not a sufficient amount of the material in the atmosphere to radiate in a high degree, their brightness falls quickly and we may hardly discover them. For comets with $p \ll 1$ the pulverization process is quite immaterial and the disperison dust layer originates in some other way.

2) This process is probably also responsible for a number of well-known comet outbursts, especially for those with a sharp variation in the colour index and polarization degree. For $\varrho_\mu/\varrho_M = 10^4$ the brightness of the dust in the atmosphere rises by 10^m and, for example, in case of comet Encke it is comparable with the integrated magnitude of the comet.

3) "Micrometeorite showers" and interplanetary dust in general shape the face of the comet nucleus surface. Impact points certainly do not have a perfect random distribution on the comet surface, which — over a very long period — in this way obtains an irregular form. The same effect must take place in the case of asteroids in which it leads to the short-term variations in brightness.

CHAPTER SIX

DEVELOPMENT OF THE COMETARY ATMOSPHERE DURING THE APPROACH OF A COMET TO THE SUN

6.1. SKETCH OF THE PROBLEM

In the preceding chapters we have explained some photometrical and dimensional properties of cometary head, as well as physical conditions under which the processes are running. When investigating the development of the cometary atmosphere the properties of the comet must be compared with one another and described as a function of time. In large heliocentric distances the surface temperature of the comet nucleus is very low (—150° C or less), the temperature of deeper layers is even somewhat lower and the incident heat is not enough to vaporize frozen gases. The cometary atmosphere is filled by particles released owing to impacts of tiny bodies from outer space on the comet-nucleus surface. The amount of released particles is relatively small and the comet is therefore very faint. The comet is, however, approaching the Sun, the collisions become more frequent and, at the same time, the surface-layer temperature is increasing. Couplings between frozen gases and meteoritical material become weaker and, finally, in heliocentrical distances between 2 and 3 A. U., as a rule, the "thermal" release of first masses of gas takes place. The solar heat penetrates deeper layers of the nucleus and the amounts of evaporated gas rise quickly; the comet starts to increase its dimensions sharply. This process leads to the "saturation" of the whole region of the cometary atmosphere in heliocentric distances of 1 → 2 A. U. Near the Sun, an increase of the brightness of the coma takes place, but its dimensions are dropping. After the perihelion passage the development repeats in the reverse order.

6.2. QUANTITATIVE RELATIONS IN THE PROBLEM

The problem of the development of coma dimensions is connected with two other problems:

a) with the mechanism of the release of photometrically effective particles from surface layers of the comet nucleus;

b) with the orientation and its variation of the resultant of forces forming trajectories of evaporated particles.

According to what has been said in Section 6.1. it is advantageous to study separately three stages of the development of a cometary atmosphere:

I) the development proceeding before the beginning of spontaneous evaporation of gaseous molecules from the surface (corresponding heliocentric distances $r > r_1$);

II) the period of intense evaporation of molecules and forming gaseous coma (corresponding $r \in [r_2, r_1]$);

III) the dimension variation of fully developed coma (corresponding $r < r_2$).

Let us now derive the formulae for the variation of a linear coma diameter D_0

$$\Delta_r^{D_0} = \frac{\mathrm{d} \log D_0}{\mathrm{d} \log r} \tag{6.1}$$

within the individual stages of a coma development and compare expressions found theoretically with observations.

Within the two first stages when the coma is not "saturated" its dimensions may be derived by applying a comet fountain model. Let us denote ϱ the number of particles in a unit of volume, n the number of particles leaving the comet nucleus in a unit of solid angle in the given direction at a unit of time; then the apparent density of the coma in point (ξ, η) of the rectangular system with the comet nucleus as an origin of coordinates, the X-axis of which is oriented towards the Sun and the Z-axis towards the observer, is given by the expression:

$$N(\xi, \eta) = \int\limits_{\zeta_1}^{\zeta_2} \varrho(\xi, \eta, \zeta)\, \mathrm{d}\zeta . \tag{6.2}$$

On the assumption that all particles are of the same dimensions and have the same initial velocity g, and that the emission is isotropic and its importance invariant in time, the isophotes are circles (MOKHNACH, 1956)

$$N(\xi, \eta) = \frac{n\pi}{g} (\xi^2 + \eta^2)^{-1/2} . \tag{6.3}$$

The receiver is able to perceive only those isophotes brighter than a certain N_0; the coma diameter is then

$$D_0 = \frac{2\pi n}{g N_0} \tag{6.4}$$

and parameter (6.1)

$$\Delta_r^{D_0} = r\left(\frac{1}{n} \frac{\mathrm{d}n}{\mathrm{d}r} - \frac{1}{g} \frac{\mathrm{d}g}{\mathrm{d}r}\right). \tag{6.5}$$

6.2.1. **First stage**

Given that the surface temperature of the comet nucleus, T, is lower than that for which a strong spontaneous evaporation of gas starts, T_1. The corresponding heliocentrical distances are then, according to (2.3), expressed by

$$r > \left(\frac{T_0}{T_1}\right)^{1/\alpha} . \tag{6.6}$$

Consequently, no internal force of the comet is in action in this period, and all the physical processes proceeding in the atmosphere and on the nucleus surface of the comet are due to the external forces:

a) impacts of micrometeorites, belonging to the dust constituent of interplanetary matter;

b) impacts of particles of solar corpuscular radiation.

Generally, both the forces are able to release the gas as well as the dust from the surface layer of the comet nucleus.

The former of the two processes was described in Chapter Five, while the latter was dealt with by DOBROVOLSKY (1953c). Let us now compare the efficiency of the two processes.

The release of dust due to the micrometeorite-impact process is described by relation (5.48) with sufficient accuracy. Assuming the stationary state of interplanetary dust we get the magnitude of dust constituent of cometary radiation with respect to (5.50) and Fig. 22 in the form:

$$H_d = 22.8 + 6.2 \log r + 5 \log \Delta. \tag{6.7}$$

The release of gas proceeds in another way. As is known a cèrtain temperature must be attained for the process of thermo-dissociation to start. Consequently, a type of the chemical reaction determines the quantitative results. Let us, in accordance with RYIVES (1952) and DOBROVOLSKY (1953c), consider the reaction

$$2\ CH_4 = C_2 + 4\ H_2, \tag{6.8}$$

which takes place when the temperature exceeds 856° K (SHKLOVSKY, 1952).

If the part of an impact-energy of a particle spent for elastic collisions increasing the temperature of medium is Q, within the region of a radius of d,

$$d = 0.41 \left(\frac{Q}{\varrho_{com} . c T_m} \right)^{1/3}, \tag{6.9}$$

the maximum temperature due to the impact will be greater than T_m. Here ϱ_{com} is the mass density of the medium and c its specific heat. We will assume $\varrho_{com}\, c =$ $= 1$. As the afore-mentioned reaction needs at least an energy of

$$E_0 = 0.7 \text{ eV/molecule},$$

the number of CH_4-molecules dissociated by an impact of a particle is

$$n_g = x_0 . \frac{Q}{E_0}, \tag{6.10}$$

$x_0 \approx 0.1$ is the coefficient reducing the maximum number of dissociated molecules to the most probable real number. If the velocity of a comet relative to micrometeorites is V and the space concentration of the micrometeorites ϱ_M/M, the total number of dissociated CH_4-molecules on a unit surface per unit of time is

$$N_g = n_g . V . \frac{\varrho_M}{M} . \tag{6.11}$$

Further, the total number of C_2-molecules present in the cometary atmosphere at a given moment is

$$N_g^{\Sigma} = \frac{1}{2} N_g . 2\pi R^2 . \tau, \tag{6.12}$$

where τ is the life-time of C_2-molecules and R the radius of the comet nucleus. Here we assume that the process takes place on the illuminated hemisphere only.

According to POLOSKOV (1951) the apparent magnitude of the comet (i. e. of its gaseous coma) connects with the number of radiating molecules N_g^{Σ} in the way as follows:

$$H_g = -2.5 \log \left(2.76 . 10^{-34} \frac{N_g^{\Sigma}}{\Delta^2 r^2} \right). \tag{6.13}$$

Putting $M = 1.5 . 10^{-8}$ gm, $\varrho_M = 5 . 10^{-22}$ gm/cm^3, $R \approx 10^5$ cm, $\tau = 10^5 . r^2$ sec, $V = 4.2 . 10^6/\sqrt{r}$ cm/sec, and assuming that the whole impact-energy of a particle is spent for the increase of the temperature round the point of impact, the brightness of photometrically effective gas is

$$H_g = 22.2 + 6.2 \log r + 5 \log \Delta, \tag{6.14}$$

however, the real brightness will be certainly much smaller as the just mentioned assumption is probably far from being fulfilled.

The process just described proceeds continuously. In addition, the collisions of a comet with micrometeorite showers (Section 5.8.) may increase the intensities H_d and H_g by about 10^m in some periods and then the process can be efficient enough to yield an observable effect.

The solar corpuscular radiation produces a similar process. However, it does not proceed continuously because of a considerably non-homogeneity of the corpuscular-radiation field. For determining the amount of both the dust and the gas in the cometary atmosphere due to the corpuscular process a relation analogous to (6.11) may be applied:

$$N_x = n_x . V . q, \tag{6.15}$$

where $x = d$ in the case of dust and $x = g$ in that of gas, N_x is the total number of expelled dust particles or dissociated parent-molecules on a unit surface per unit of time, n_x the number of dust particles expelled (or that of parent-molecules dissociated) by an impact of a solar corpuscle, V the velocity of the corpuscular radiation relative to the comet, q the space concentration of the corpuscles inside the stream.

For n_d in (6.15) we can write

$$n_d = \frac{3Q}{4\pi\varepsilon . \varrho^3 . \varrho_{com}}, \tag{6.16}$$

Q is the part of an impact-energy of a corpuscle spent for the increase of the temperature of medium, ε the energy density necessary for breaking up the crystal lattice of the material, ϱ the dimension of a dust particle, ϱ_{com} the mass density of a comet monolithic nucleus. Rosenberg (1951) on the basis of the study of aurora-borealis spectrum determined the velocity of solar protons to be $3 . 10^8$ cm/sec. Dobrovolsky (1953c) asserts that a great deal of the impact-energy of protons is spent on the ionization of nucleus particles, and according to Slaighter (1952) only the last 10^4 eV of the total energy of proton is after its impact on the material spent on the increase of the temperature. The proton space concentration we shall write in the form of

$$q(r) = q_0 . r^{-\beta}, \tag{6.17}$$

where $q_0 = 10^2$ cm^{-3} (Shklovsky, 1952) and β depends on the forces acting within the cloud of corpuscles (the rate of dispersion).

On the other hand, from (3.11) and (4.19) we get, since $n_d = \nu$:

$$n_d = \frac{\Delta^2}{\Phi(\varrho) . \varrho^2 A_0} . 10^{0.4(\sigma - H_d)} . \tag{6.18}$$

Comparing (6.16) with (6.18) and putting respective numerical values we obtain finally

$$H_d = 11.6 + 2.5 \beta \log r + 5 \log \Delta. \tag{6.19}$$

The number of molecules, n_g, may be expressed formally in the form of (6.10). The result is

$$H_g = 10.7 + 2.5\beta \log r + 5 \log \Delta. \qquad (6.20)$$

The process produced by corpuscular radiation gives the values of the same order as the pulverization process due to the collisions with micrometeorite showers of the space density 10^4 times higher than that of interplanetary matter. However, there is an important difference between the two processes: while the meteorite-impact process is the more efficient the smaller heliocentric distances are considered, the corpuscle-impact process is stopped when the total number of molecules in the cometary atmosphere exceeds the order of 10^{37}, i. e. in heliocentrical distances about 3 A. U., because then the atmosphere stops being transparent for the fast protons.

All the numerical results just given are of an approximate character. It is likely that both the processes take part in the comet radiation in large heliocentrical distances.

For establishing derivatives (6.1) a variability of the initial particle velocity, g, must further be discussed.

The initial velocity of dust particles expelled by the meteorite-impact process, which is represented in Fig. 21, indicates practically no change with the heliocentric distance. Within heliocentric distances of 1 — 10 A. U., for instance, it changes as the inverse 1/40 power of the heliocentric distance. An analogous result may be expected in releasing the dust by the corpuscle-impact process. As for the release of the gas, in both the processes a certain (and constant) temperature must be reached for thermo-dissociation to start, so that also a certain (and constant) velocity, corresponding to the dissociation temperature, must be obtained by molecules. Thus, in (6.5) g may be always put equal to a constant.

From (6.13) and (6.14) on the one hand and from (6.19) and 6.20) on the other we can see that the amount of both the dust and gas in the cometary atmosphere changes for a given process in the same way. Therefore, we may further consider the dust only, when deriving the expressions for (6.5).

Whatever process produces the release of the dust the number of photometrically effective dust particles in the coma at the given moment is equal to (Section 3.3.):

$$\nu = \left[\nu_0 + \left(\frac{R}{\varrho}\right)^2\right].r^2.\exp\left[-\int_1^r \frac{n_d(r)}{r}\,\mathrm{d}r\right] - \left(\frac{R}{\varrho}\right)^2, \qquad (6.21)$$

n_d is the photometrical exponent of the dust part of the cometary atmosphere defined by the well-known formula. So we can write

$$\Delta_r^{D_\bullet} = \frac{\mathrm{d}}{\mathrm{d}\log r} \log\left\{\left[\nu_0 + \left(\frac{R}{\varrho}\right)^2\right].r^2.\exp\left[-\int_1^r \frac{n_d(r)}{r}\,\mathrm{d}r\right] - \left(\frac{R}{\varrho}\right)^2\right\}. \qquad (6.22)$$

As the case of $\frac{\nu_0}{(R/\varrho)^2} \rightarrow 0$ is physically impossible it must be $\frac{(R/\varrho)^2}{\nu_0} \lesssim 1$ and equation (6.22) can be written in an approximate form

$$\Delta_r^{D_\bullet} = (2 - n_d)\left\{1 + \frac{(R/\varrho)^2}{\nu_0}.r^{-2}.\exp\left[\int_1^r \frac{n_d(r)}{r}\,\mathrm{d}r\right]\right\}, \qquad (6.23)$$

where the latter term in curly brackets is in practice negligible regarding a unit.

In this way, comparing (5.50) with (6.23) we get for the meteorite-impact process approximately

$$\gamma(r) = \frac{1}{2} - \Delta_r^{D_0}, \tag{6.24}$$

while, comparing (6.19) with (6.23) we find for the corpuscle-impact process:

$$\beta(r) = 2 - \Delta_r^{D_0}. \tag{6.25}$$

Hence, the first stage of the development of a cometary atmosphere makes it possible to determine the average power of the change of the micrometeorite space concentration in a given range of heliocentric distances simply from (6.24). It must yield $\gamma \to 0_+$ to be in keeping with values of γ derived in another way (Section 5.3.). This is the first criterion of the correctness of our theoretical considerations in the present problem.

6.2.2. Second stage

At heliocentric distance r_1 an intense evaporation of molecules starts from the cometary nucleus into the atmosphere. This process can be mathematically described again by equation (6.3) or (6.4), so that the coma has the form of a circle. However, for n it must be now inserted from LEVIN's formula (Section 2.1.):

$$n = n_0 \left(\frac{kT}{2\pi m}\right)^{1/2} . \mathrm{e}^{-\frac{L}{R_0 T}}, \tag{6.26}$$

where n_0 is the concentration of molecules in the surface layer of the nucleus, L the heat of evaporation necessary to set free gas, R_0 the gas constant, T again the temperature of the nucleus surface, m the mass of a molecule, and k Boltzmann's constant. Further,

$$g^2 \sim r^{-\bar{\alpha}}, \tag{6.27}$$

where $\bar{\alpha}$ is identical with α of (2.3), if the initial velocity is that of a thermal character. If g is given by the kinetic energy obtained during the photo-dissociation process it is independent of the heliocentrical distance (LEVIN, 1947) and $\bar{\alpha} = 0$. According to (6.4), (6.26) and (6.27) the following relation holds good:

$$D_0 \sim r^{1/2(\bar{\alpha}-\alpha)} \exp\left[-Br^{\alpha}\right], \tag{6.28}$$

where $B = \dfrac{L}{R_0 T_0}$, T_0 is the temperature of the nucleus surface at heliocentric distance of 1 A. U. The expression for the derivative we are looking for is equal to:

$$\Delta_r^{D_0} = \tfrac{1}{2}(\bar{\alpha} - \alpha) - \alpha B r^{\alpha}. \tag{6.29}$$

From (6.29) the photometrical exponent of the gaseous part of cometary atmosphere, n_g, can be determined at heliocentrical distance $r_0 \in (r_2, r_1)$ within the second stage of the coma development:

$$n_g(r_0) = \frac{1}{2}\bar{\alpha} - (\Delta_r^{D_0})_{r_0}. \tag{6.30}$$

The second criterion of the correctness of our theoretical considerations is given by a comparison of the total photometrical exponent, n, determined from the form of the photometrical curve of a comet, with exponent n_g derived in the above-mentioned way at the same heliocentrical distance. The basic equation of the dust-gaseous model of a comet (Section 2.2.) gives the following relation valid for every heliocentrical distance (excepting minute heliocentrical distances in which the problem is more complicated):

$$n_g(r) \geq n(r), \tag{6.31}$$

where the equality corresponds to the pure gaseous model.

If the total photometrical exponent is determined at heliocentrical distance r'_0 close to r_0, exponent $n_g(r'_0)$ can be expressed through $(\Delta_r^{D_0})_{r_0}$ in the form:

$$n_g(r'_0) = \frac{1}{2}\bar{\alpha} - \left(\frac{r'_0}{r_0}\right)^{\alpha} \cdot (\Delta_r^{D_0})_{r_0}. \tag{6.32}$$

Consequently, the validity of relation (6.31) at heliocentrical distance r'_0 is a necessary condition for the correctness of our considerations.

6.2.3. Third stage

The dimensions of a cometary head enlarge with the increased number of photometrically effective molecules in a coma up to its "saturation" which is given by physical quantities as follows:

a) by an initial particle velocity, more exactly, by a particle velocity on the boundary of the nucleus sphere of activity;

b) by ratio $1 + \mu$ between accelerations of the solar gravitation field and the solar radiation pressure;

c) eventually, by ratio μ'/ξ^2 between the solar gravitation field and the repulsive force of the cometary nucleus.

Considering only the agents given under a) and b), the distance of the top of the cometary head from the comet nucleus is expressed by BESSEL's well-known formula (BESSEL, 1836):

$$\xi_0 = \frac{g^2 r^2}{2k^2 (1 + \mu)}, \tag{6.33}$$

k^2 is the universal gravitation constant; according to (6.27) we get the result:

$$\Delta_r^{D_0} = 2 - \alpha, \tag{6.34}$$

where again $\bar{\alpha} = \alpha$, or $\alpha = 0$ according to the character of velocity g.

ORLOV's new theory of the cometary head (ORLOV, 1935, 1937, 1945), which is based on assumption c), gives for the same parameter of a cometary head an expression considerably different from that given by (6.33)

$$\xi_0 = (2r)^{1/2} \cdot \left(\frac{\mu'}{1 + \mu}\right)^{1/4}, \tag{6.35}$$

so that the resulting derivate

$$\Delta_r^{D_0} = \frac{1}{2} \tag{6.36}$$

is independent of velocity g of individual comets.

Relation (6.35) was derived by ORLOV from the differential equation for the motion of a particle along the radius vector in the cometary atmosphere (Section 4.2.). By integrating it he got

$$\frac{\xi_0}{r^2} = -\frac{1}{r - \sqrt{M}} - \frac{\sqrt{M}}{r^2} + \frac{1}{r - \xi_0} + \frac{M}{r^2 \xi_0}, \qquad (6.37)$$

where M is the ratio

$$M = \frac{\mu'}{1 + \mu} = \text{const.} \qquad (6.38)$$

Moreover, it applies with an accuracy of the order of M that the radius of action of the comet, given by the point where the two accelerations equal, is

$$\xi_m \doteq \sqrt{M}. \qquad (6.39)$$

The numerical values of the respective magnitudes in (6.37) are of the following orders: $r \approx 1$ A. U., $\xi_0 \approx 10^{-3} - 10^{-4}$ A. U., $M \approx 10^{-12} - 10^{-16}$ (A. U.)^2. ORLOV felt justified by these value in neglecting some minor terms in a modified form of (6.37), and thus got formula (6.35).

This formula was criticized by DOBROVOLSKY (1953a). He objected that the error due to ORLOV's neglecting the minor terms was of the same order as ξ_0, which made of (6.35) a mere empirical formula. The correctness of (6.35) from the empirical viewpoint will be discussed in the next two sections. Now we prove its correctness from the mathematical point of view, provided we comply with the postulates of the differential equation. Modifying relation (6.37) we get the following cubic equation accurate to the order of $M\xi_0$:

$$\xi_0^3 + 2\sqrt{M}\,\xi_0^2 - 2r\sqrt{M}\,\xi_0 + Mr = 0. \qquad (6.40)$$

First we establish the exponents of relation

$$\xi_0 = \gamma r^{\alpha} M^{\beta}; \qquad (6.41)$$

to the order of M/ξ_0^2 we have

$$\alpha = \frac{1}{2}\left[1 + \frac{3}{4}\frac{\sqrt{M}}{\xi_0}\right] \approx \frac{1}{2} (\pm 0.1\ \%\ \text{max.}) \qquad (6.42)$$

and to that of ξ_0^2

$$\beta = \frac{1}{4}\left[1 + \frac{1}{2} \cdot \frac{\xi_0}{r}\right] \approx \frac{1}{4} (\pm 0.1\ \%\ \text{max.}). \qquad (6.43)$$

Hence we may write $\xi_0 \sim r^{1/2} M^{1/4}$. The γ-coefficient can be determined after inserting (6.41) into (6.40). We put $\gamma^2 = 2 - \varepsilon$, determine ε from

$$3 - \varepsilon = \sqrt{2 - \varepsilon}\,.\,r^{1/2}.\,M^{-1/4}.\,\varepsilon$$

and may write

$$\gamma = \sqrt{2} - \frac{3}{4} r^{-1/2} M^{1/4} \approx \sqrt{2}\ (\pm 0.1\ \%\ \text{max.}). \qquad (6.44)$$

Consequently, the Orlov formula is correct when only the infinitesimals higher than the third order are neglected. Expressions (6.42), (6.43) and (6.44) show that the first and third terms in equation (6.40) are of the order of $M^{1/2}$, while the second and fourth of that of M, and therefore negligible. This directly leads to equation (6.35), which is accurate to the order of $M^{1/2}$, that is within 1 to 3 per cent.

Hence, by the determination of $\Delta_r^{D_0}$ in the third stage of the coma development from the material we can arbitrate either formula, (6.33) or (6.35), describes correctly the variations of coma dimensions in this stage.

6.2.4. Scheme of the coma development

The scheme of the dependence of coma dimensions on the heliocentric distance according to the given theoretical conception is shown in Fig 26. As can be seen the passage between two neighbouring stages is characterized by breaks. In Fig. 26 two different forms of the third stage are considered; they are denoted as IIIa and IIIb, and represent relations (6.36) and (6.34) respectively. Numerical values of the parameters of the curve from Fig. 26 are included in Tab. 32; $\bar{\alpha}$ is taken from the paper of LEVIN (1948).

Heliocentrical distances r_1, r_2, corresponding to the breaks between individual stages of the development of a cometary atmosphere, are important parameters of the development curve. We shall call them the "limiting" heliocentrical distances. If we consider the complicated physical conditions inside the cometary nucleus as well as on its surface, we find that the numerical values of the "limiting" distances are determined by the factors as follows:

a) structure of the comet nucleus, which gives the course of the nucleus-surface temperature;

b) the amount and the way of deposition of gas in the nucleus;

c) chemical composition of the released gas and the dissociation-process starting early after evaporation from the nucleus;

d) the comet-nucleus dimensions.

A macroscopic mass distribution in the comet nucleus as well as its microscopic composition, i. e. porosity of the meteorical material, thermal conductivity, fusibility, etc., are understood by the structure of the nucleus. It is obvious that the more similar the nucleus to the monolith, the more difficult conduction of heat, the smaller distances r_1, r_2. The effect is the more conspicuous, the less porous material.

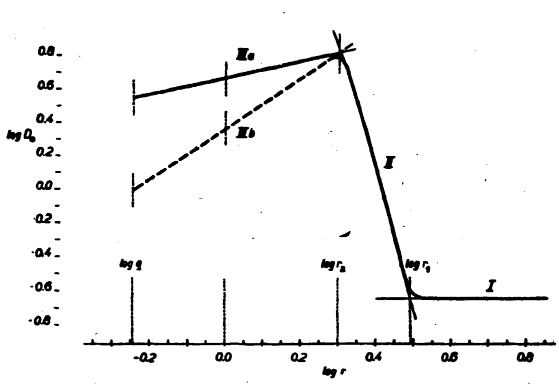


Fig. 26. Scheme of the coma development.

As the gas is, as a rule, the predominant agent of the cometary radiation, its total amount included in the comet is an important factor determining the absolute brightness of the comet. Even we could put both the quantities equal, if the way of deposition of gas in the comet nucleus had a significant influence. This problem consists of several parts, from which the most important question is if the gas in the comet nucleus is absorbed or adsorbed (Chapter One). In the latter case the release of gas is easier and the "limiting" distances become larger.

Table 32

Parameters of the scheme of the coma-development curve

Stage	I	II	IIIa	IIIb
int r, A. U.	$r > 3.1$	$2.0 < r < 3.1$	$q < r < 2.0$	$q < r < 2.0$
$D_0(0)$, km	10,000	20,000,000	200,000	100,000
γ	0.5	—	—	—
β	2.0	—	—	—
α	—	0.5	—	0.5
B	—	10.0	—	—

The "limiting" distances, further, depend on the repulsive pressure of the solar radiation, $1 + \mu$, acting on the radiating molecules, and on velocity g on the boundary of the sphere of the comet-nucleus activity. Since the radiation pressure is the function of the effective cross-section of the molecule and since the "initial" velocity, if that of the thermal character, depends on the mass of a mole of gas, if that obtained in the dissociation-process, depends on the individual links of this process, we arrive finally at the parent-molecules released from the nucleus as the further factor affecting the observed "limiting" heliocentrical distances.

After all, we must admit that the comet-nucleus dimensions, or, more exactly, its total mass, can also influence on the "limiting" distances. On the basis of the study of meteorites RICHTER (1953) found that 1 gramme of the meteoritic material contained about 10^{19} molecules. Consequently, a comet of greater mass should be, statistically, saturated by a greater amount of sorbed gas.

To a first approximation, the conditional equation for r_1 can be established from (6.4) by comparing (6.21) with (6.26), and that for r_2 by comparing relation (6.33) or (6.35) with (6.4) after inserting expression (6.26) into it.

6.3. MATERIAL AND TREATMENT OF OBSERVATIONAL DATA

On the basis of an appearance of a series of cometary heads described by various authors ORLOV (1913) drew the conclusion that the contour of a cometary head was better expressed by a catenary than by a parabola. If we denote the angular distance between the cometary nucleus and the top of a cometary head as E and the angular coma diameter measured across the nucleus perpendicularly to the radius-vector of a comet as D, the paraboloid is always characterized by the ratio

$$\frac{D}{E} = \frac{2\eta_0}{\xi_0} = 4, \tag{6.45}$$

where ξ_0 is the linear distance of the top of a head from the nucleus and η_0 is the parameter of a paraboloid. In the case of a catenary, ratio D/E depends on the choice of its parameter. ORLOV therefore introduces the reduction coefficient q to be able to apply his theory to ratios η_0/ξ_0 rather different from (6.45). If we denote the phase-angle Sun-comet-Earth as k then it holds good (ORLOV, 1945):

$$\xi_0 = \frac{\Delta}{2} . q . \sin \frac{D}{2} \sin k , \tag{6.46}$$

where Δ is the geocentrical distance of a comet and q depends on ratio D/E and on phase-angle k; its values are tabulated in the book by ORLOV (ibid.). The coma diameter is then equal to

$$D_0 = Q . D . \Delta , \tag{6.47}$$

where

$$Q = q \sin k .$$

The majority of comets usually appear as diffused nearly circular objects for which ratio D/E ranges from 2.0 to 2.5. Values of Q for both limiting values of ratio D/E are included in Table 33, from which it follows that $Q = 1$ can be put with a sufficient accuracy in equation (6.47).

Table 33

Dependence of $Q = q \sin k$ on the phase-angle

k	Q		k
	$D/E = 2.0$	$D/E = 2.5$	
30°	1.12	0.92	150°
35	1.11	0.95	145
40	1.10	0.97	140
45	1.09	0.98	135
50	1.08	1.00	130
55	1.06	1.00	125
60	1.05	1.00	120
65	1.03	1.00	115
70	1.02	1.00	110
75	1.01	1.00	105
80	1.01	1.00	100
85	1.00	1.00	95
90	1.00	1.00	90

For each of the three stages of a coma development the parameters of curve $D_0 = D_0(r)$ are further determined; its approximate form used is

$$\log D_0(r) = \log D_0(0) + \Delta_r^{D_0} . \log r , \tag{6.48}$$

which is quite sufficient to compare our theory with observational data.

The following material was used to verify our theoretical conclusions:

a) 507 visual measurements of coma dimensions of 22 comets obtained by BEYER (1950a, 1955, 1958, 1959) from the years 1947 — 1956;

b) 16 photographs of the comet 1942g taken at the Sonneberg Observatory (HOFFMEISTER, 1956);

c) 65 photographs of the Halley comet 1909c obtained by MtHamilton Crossley Reflector and MtWilson 60-inch Reflector (BOBROVNIKOFF, 1931);

d) 120 estimations of coma dimensions of the Encke comet from the years 1795 — 1914 collected by BOUŠKA and ŠVESTKA (1949).

6.4. NUMERICAL RESULTS AND DISCUSSION

Table 34 contains the resulting values of parameters of the development curve of coma dimensions of 23 comets studied; individual columns give the preliminary and definite designation of a comet, the stage of a coma development (denotation according to Fig. 26), the parameters of a coma development curve, the geometrical mean and the range of heliocentrical distances used, the time-orientation of measurements regarding the perihelion passage of a comet (A — prior to the perihelion passage, B — after the perihelion passage), the number of measurements, and a note if the comet is a short-period one. The coma diameter at $r = 1$ A. U. is expressed in minutes of arc; the values corresponding to the first and second stages are extrapolated next to all cases.

Curves $D_0 = D_0(r)$ of the following comets are represented in Figures 27 to 32: Halley 1909c, Whipple-Fedtke 1942g, Bester 1947k, Honda-Bernasconi 1948g, Minkowski 1950b, and Pons-Brooks 1953c.

The first stage of the coma development may be studied only in three comets; the results are included in Table 35, in which the exponents β and γ, and corresponding effective heliocentrical distance, r_{eff}, are given. In fact, for γ we obtain the values close to those derived in another way; nevertheless, the few measurements of D_0 at large

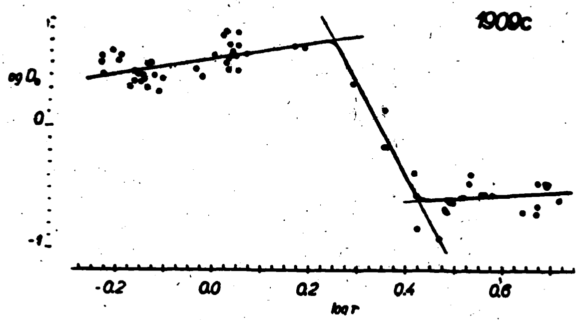


Fig. 27. **Development of the atmosphere of comet Halley 1909c.**

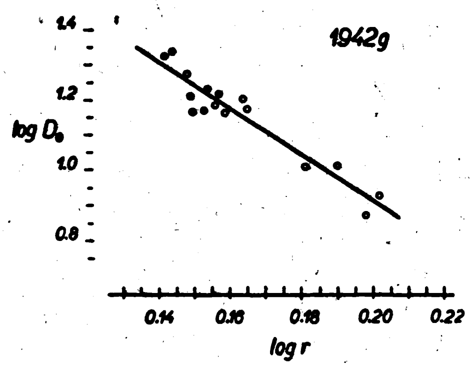


Fig. 28. **Development of the atmosphere of comet Whipple-Fedtke 1942g.**

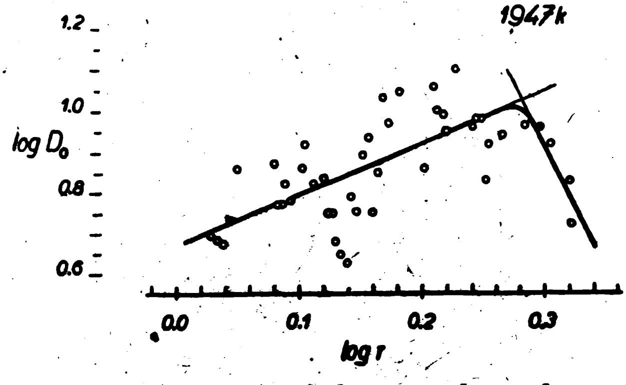


Fig. 29. **Development of the atmosphere of comet Bester 1947k.**

Table 34
List of the parameters of the coma development of 23 comets

Comet prel.	Comet def.	Stage	$\log D_0(0)$	$\Delta_r^{D_0}$	$\overline{\log r}$	int r	per.	N	Note
Encke	1795—1914	III	0.36 ± 0.02	+ 0.40 ± 0.08	9.905	0.40 — 1.6	*AB*	120	*P*/Encke
1909c	1910 II	I	9.30 ± 0.10	+ 0.23 ± 0.17	0.584	3.04 — 5.20	*AB*	20	*P*/Halley
		II	2.55 ± 0.28	— 7.39 ± 0.72	0.388	1.95 — 2.94	*A*	8	
		III	0.55 ± 0.01	+ 0.59 ± 0.10	9.940	0.59 — 1.55	*AB*	37	
1942g	1943 I	II	2.24 ± 0.06	— 6.66 ± 0.39	0.163	1.39 — 1.59	*B*	16	
1947i	1947 XI	I	0.35 ± 0.05	— 0.34 ± 0.35	0.134	1.11 — 1.49	*A*	5	*P*/Encke
		II	0.60 ± 0.03	— 5.76 ± 0.88	9.999	0.93 — 1.11	*A*	3	
		III	0.79 ± 0.02	+ 0.52 ± 0.32	9.936	0.75 — 0.93	*A*	7	
1947j	1947 VII	III	9.69 ± 0.26	+ 1.46 ± 0.90	0.286	1.88 — 2.03	*B*	12	*P*/Reinmuth 2
1947k	1948 I	II	2.60 ± 0.35	— 5.63 ± 1.13	0.305	1.92 — 2.10	*B*	5	
		III	0.67 ± 0.03	+ 1.22 ± 0.15	0.149	1.07 — 1.84	*B*	40	
1948a	1948 II	III	9.88 ± 0.15	+ 1.98 ± 0.67	0.218	1.60 — 1.70	*B*	7	
1948d	1948 V	II	3.26 ± 0.31	— 4.85 ± 0.56	0.541	3.26 — 3.83	*B*	20	
		III	0.60 ± 0.09	+ 0.54 ± 0.26	0.353	2.10 — 2.52	*B*	43	
1948g	1948 IV	III	0.49 ± 0.01	+ 0.47 ± 0.06	0.157	0.81 — 2.20	*B*	27	
1948l	1948 XI	II	2.18 ± 0.08	— 3.17 ± 0.19	0.400	2.17 — 3.06	*B*	18	
1949a	1950 I	II	3.75 ± 0.45	— 7.59 ± 1.06	0.420	2.55 — 2.74	*B*	22	
1949c	1949 IV	II	2.60 ± 0.16	— 5.57 ± 0.45	0.345	2.09 — 2.35	*A*	22	
1950b	1951 I	II	1.98 ± 0.12	— 2.71 ± 0.24	0.504	3.03 — 3.43	*A*	26	
1950e	1951 III	III	0.94 ± 0.04	+ 0.64 ± 0.16	9.728	0.41 — 0.64	*A*	10	*P*/Encke
1951f	1951 IV	II	0.83 ± 0.07	— 7.82 ± 1.15	0.060	1.11 — 1.19	*B*	9	*P*/Tuttle-Giacobini-Kresák
1951l	1952 III	II	2.60 ± 0.27	— 8.78 ± 1.18	0.232	1.67 — 1.73	*B*	4	*P*/Schaumasse
		III	0.73 ± 0.03	+ 0.54 ± 0.23	0.134	1.20 — 1.57	*B*	25	
1952d	1952 VI	III	0.44 ± 0.08	+ 1.40 ± 0.67	0.115	1.20 — 1.40	*B*	20	
1952e	1953 I	I	0.42 ± 0.44	+ 0.54 ± 1.51	0.291	1.91 — 2.00	*A*	6	
		II	3.42 ± 0.07	—10.40 ± 0.42	0.241	1.67 — 1.83	*A*	10	
1953c	1954 VI	III	0.78 ± 0.04	+ 0.55 ± 0.13	0.304	1.63 — 2.44	*A*	21	*P*/Pons-Brooks
		III	1.06 ± 0.02	+ 0.68 ± 0.15	0.054	0.90 — 1.57	*A*	21	
1953g	1954 X	III	0.78 ± 0.01	+ 0.80 ± 0.09	0.130	1.05 — 1.78	*A*	24	
1954d	1954 XII	III	0.42 ± 0.06	+ 1.17 ± 0.54	0.112	1.23 — 1.36	*A*	7	
1954h	1955 VI	II	4.01 ± 0.54	— 5.22 ± 0.89	0.610	3.94 — 4.15	*A*	16	
1955e	1955 III	III	0.79 ± 0.03	+ 0.39 ± 0.21	9.892	0.62 — 0.95	*B*	11	
1955f	1955 IV	II	6.03 ± 0.69	—14.92 ± 1.93	0.357	2.23 — 2.33	*B*	6	
		III	0.66 ± 0.10	+ 0.65 ± 0.43	0.234	1.52 — 1.96	*B*	10	
1956a	1956 IV	II	9.10 ± 0.49	—24.58 ± 1.40	0.351	2.16 — 2.31	*A*	7	*P*/Olbers
		III	0.72 ± 0.02	+ 0.52 ± 0.09	0.163	1.18 — 1.95	*A*	33	
		III	0.73 ± 0.05	+ 0.39 ± 0.26	0.175	1.19 — 1.85	*B*	10	

heliocentrical distances make it impossible to carry out the analysis of this problem in detail.

A comparison of the total photometrical exponent with that of the gas coma is given in Table 36; n_g is computed according to (6.32) at heliocentrical distance r'_0. Total exponents are taken over from BEYER's papers (BEYER, 1950a, 1955, 1958, 1959) for the majority of comets used, from the author's paper (SEKANINA, 1959; see Chapter Two, too) for comet 1942g, and from BOBROVNIKOFF's study (BOBROVNIKOFF, 1942) for comet 1909c; in addition the exponent computed by the author on the basis of HOLETSCHEK's material (HOLETSCHEK, 1916) for the last-mentioned comet was also taken into account. Of course — the time-orientations both of the n-material and of the n_g-material are the same for each comet of the set. From sixteen cases included in Table 36 eight give conspicuously $n_g > n$, eight give $n_g \approx n$ within errors but no case shows $n_g < n$, so that the second stage of a coma development fully supports the validity of relation (6.31). Exponent n_g was again computed using $\alpha = 0.5$. MARKOVICH (1958, 1959), however, asserts that this value is physically incorrect and that the most probable one must lie, in fact, between 0.1 and 0.2 (see Section 2.1., too). To ascertain the change of $n_g(r'_0)$ with the change of α, let us derive difference

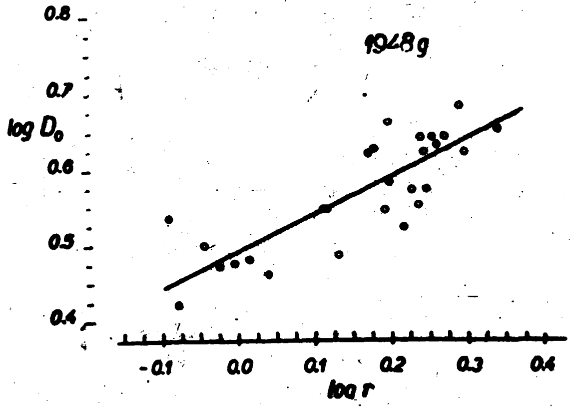


Fig. 30. Development of the atmosphere of comet Honda-Bernasconi 1948g.

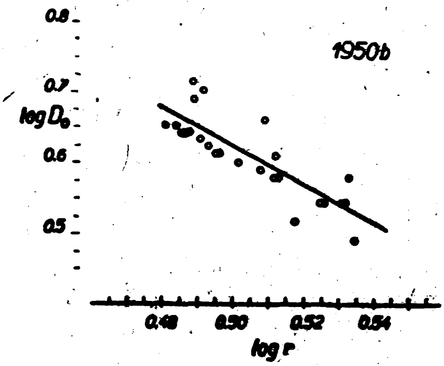


Fig. 31. Development of the atmosphere of comet Minkowski 1950b.

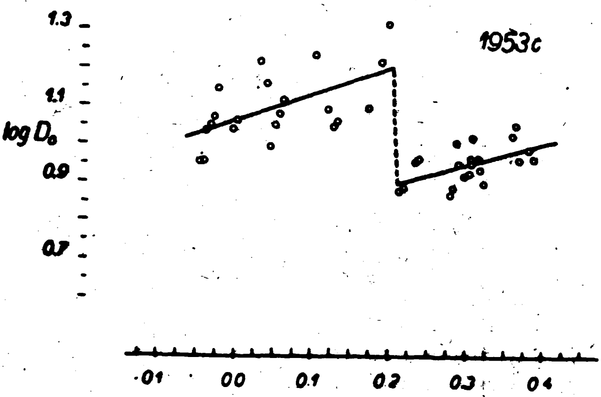


Fig. 32. Development of the atmosphere of comet Pons-Brooks 1953c.

$\Delta n_g = n_g(\alpha_1) - n_g(\alpha_2)$ from (6.32):

$$\Delta n_g = \Delta\alpha\left[\frac{1}{2} + \left(1 - \frac{r'_0}{r_0}\right) . (\Delta_r^{D_*})_{r_*}\right], \qquad (6.49)$$

where

$$\Delta\alpha = \alpha_1 - \alpha_2.$$

For $\Delta\alpha = 0.4$ the maximum $\Delta n_g = 0.6$ results from Tables 34 and 36, which is not more than an average error of n_g; therefore, the change of $\Delta\alpha$ does not affect our above-mentioned assertion.

Table 35
Numerical values of exponents β and γ

Comet	r_{eff}	$\beta(r_{eff})$	$\gamma(r_{ff})$
1909c	3.84	1.77 ± 0.17	0.27 ± 0.17
1947i	1.36	2.34 ± 0.35	0.84 ± 0.35
1952e	1.95	1.5 ± 1.5	0.0 ± 1.5

The coma development in the third stage makes it possible to collate expressions (6.33) and (6.35). From the data of Table 34 the most probable value of $\Delta_r^{D_*}$ follows as

$$\Delta_r^{D_*} = 0.66 \pm 0.23, \qquad (6.50)$$

which agrees quite well with (6.36); to reach an agreement between (6.50) and (6.34) it would be necessary to assume $\alpha = 1.3$, which is absurd. Consequently, the collected material conspicuously supports ORLOV's thoughts (ORLOV, 1945).

The problem of comet Pons-Brooks 1953c is very interesting in this connection. The third stage of its coma development occurred when the comet was

Table 36
Comparison of photometrical exponents n and n_g

Comet	r'_0	$n\ (r'_0)$	$n_g(r'_0)$
1909c	1.71	5.52 ± 0.21	6.43 ± 0.60
	2.16	6.81 ± 0.11	7.20 ± 0.68
1942g	1.52	6.00 ± 0.03	7.05 ± 0.40
1947i	1.04	6.32 ± 0.38	6.14 ± 0.90
1947k	1.43	2.97 ± 0.20	4.98 ± 0.95
1948d	2.84	4.44 ± 0.13	4.64 ± 0.51
1948l	2.17	3.66 ± 0.52	3.20 ± 0.18
1949a	2.69	5.36 ± 1.19	7.92 ± 1.07
1949c	2.41	5.53 ± 0.23	6.04 ± 0.47
1950b	2.97	1.54 ± 0.49	2.87 ± 0.23
1951f	1.15	11.05 ± 2.00	8.09 ± 1.15
1951l	1.47	6.57 ± 0.35	8.40 ± 1.09
1952e	1.83	13.73 ± 2.64	10.90 ± 0.43
1954h	4.12	4.33 ± 0.77	5.50 ± 0.90
1955f	1.83	7.24 ± 0.60	13.62 ± 1.73
1956a	1.52	6.38 ± 0.19	20.50 ± 1.15

at the heliocentrical distance of 2.4 A. U.; the changes of the coma diameter corresponded to ORLOV's law (6.36). At the heliocentrical distance of 1.6 A.U. a sudden enlargement of the coma dimensions occurred, so that they were about twice as large as before; in this situation the coma diameter changed again according to (6.36). We may assume that at $r = 1.6$ A. U. the change in the nature of excited gases occured, or a sudden, very intense and standing emission of gas from new bulky sources took place, or, which is the most probable case, both effects were present. The tumultuous activity of this comet is sufficiently illustrated in Fig. 32, in which a series of local fluctuations are superimposed on both linear paths.

Physical differences between periodical comets on the one hand and long-period and non-period comets on the other conspicuously affect data of the "limiting" heliocentrical distances, r_1, r_2 (Paragraph 6.2.4.). The coma-development curve (Fig. 26) for "new" comets is considerably shifted towards larger heliocentrical distances in comparison with that of periodical comets. In Table 37 heliocentrical distances r_1, r_2, difference $r_1 - r_2$, and the orbital period round the Sun, P, are given for eight comets, even though incompletely. An analysis in detail is again impossible owing to an insufficient abundancy of homogeneous material.

Table 37

"Limiting" heliocentrical distances

Comet	r_1	r_2	$r_1 - r_2$	P
				y
1947i	1.11	0.93	0.18	3.305
1951l	>1.73	1.59	>0.14	8.172
1956a	>2.31	2.16	>0.15	69.47
1909c	2.67	1.78	0.89	76.03
1952e	1.88	<1.67	>0.21	1,445
1947k	>2.10	1.91	>0.19	—
1955f	>2.33	2.21	>0.12	—
1948d	>3.83	3.11	>0.72	—

The ascertained effect could be expected and differences among periodical, "old" and "new" comets found on the basis of photometrical data (OORT, SCHMIDT, 1951, VANÝSEK, 1952) are quite supported in this way. This effect is in connection with the facts that the supplies of gas of periodical comets are present only in depths of a nucleus, and that the gases of the great heat of evaporation are the question. Systematic differences in values $r_1 - r_2$ can be explained analogously.

The same effect can be shown when using the average heliocentric distance of the second stage of development, $\bar{r}$, because $\bar{r} \approx \frac{1}{2}(r_1 + r_2)$. Distance $\bar{r}$ may be derived for 15 comets used; the result included in Table 38 convincingly shows the increase of $\bar{r}$ with increasing semi-major axis a of the cometary orbit.

If the coma diameter "permitted" by equation (6.35) is rather considerable and if the mean life-time of molecules is relatively short, then the third stage of the development need not occur. On the other hand, the second stage of development is absent only in those comets which radiate merely by reflex light of the Sun.

Table 38

Average heliocentrical distance $\bar{r}$ as related to the semi-major axis of the cometary orbit

int 1/a	$\bar{r}$	Comets
<0	2.91	1949a, 1950b
0	2.81	1947k, 1948d, 1949c, 1954h, 1955f
≪0.0010	2.51	1948l
0.0011—0.010	1.60	1942g, 1952e
0.011—0.10	2.23	1909c, 1956a
0.11—1.0	1.29	1947i, 1951f, 1951l

6.5. CONCLUSIONS

1) During the motion of a comet round the Sun the cometary atmosphere passes three stages of its development: the first stage is characterized by the action of external forces (collisions of a comet with micrometeorite showers, or solar proton streams), the second one begins by the intense evaporation of gas from the cometary nucleus due to internal comet forces, and ends when the coma attains the dimensions given by the mechanical theory; the third stage is that of a "saturated" atmosphere.

2) The first stage passes through within relatively large heliocentrical distances and the coma diameter changes only very slightly; this change determines the variation of the concentration of the dust in interplanetary space, or that of the space concentration within the solar-corpuscle streams with the heliocentric distance in a given range of r; but few homogeneous measurements of coma diameters make great difficulties to obtain reliable numerical results.

3) Considerable enlargement of the gas coma dimensions characterizes the second stage; now the change of the coma diameter is in close connection with the photometrical exponent of the gaseous coma; if the trend of the temperature of a nucleus surface is known the heat of evaporation of excited molecules can be derived.

4) The third stage occurs after the "saturation" of a cometary atmosphere and its dimensions always decrease with the approach the perihelion passage. The rate of this development in the majority of comets used is much better described by ORLOV's theory than by that of BESSEL and BREDIKHIN; in some cases the third stage need not take place.

5) The "limiting" heliocentrical distances separating the neighbouring stages of the coma development are in a conspicuous correlation with the orbital period of a comet and they give a qualitative picture about the supplies of gas within individual comets; the OORT-SCHMIDT classification of comets into "new" and "old" ones is independently supported; the analysis of the average heliocentrical distance of the second stage of the development gives the same effect as the analysis of the "limiting" distances.

6) The asymmetry of the curve of a coma development regarding the perihelion passage changes from case to case; it is obvious that this problem is in some connection with the molecular concentration drop in the surface layer of the nucleus as well as with the thermal inertia of the process of their evaporation analogously as the asymmetry of the photometrical curve (LEVIN, 1947).

CHAPTER SEVEN

GENERAL RESULTS AND CONCLUSIONS

The most important results of the investigated problems and general conclusions following from the analysis are here summarized. All the performed calculations indicate the ability of the comet dust-gas model, or somewhere even of its simplification, the gas model, to explain observed effects. The main items are as follows:

(1) The basic formula describing the properties of the photometrical curve of the comet dust-gas model is derived in a few forms, with various number of the so-called physical parameters. These are: (a) absolute magnitude of the comet model; (b) ratio between the intensity of both the constituents of the non-homogeneous model; (c) heat of evaporation of the gas forming the gaseous constituent of the model; (d) rate of the surface-temperature drop of the comet nucleus; (e, f, g) parameters of the solid constituent of the comet model, depending on such magnitudes as the initial amount of the dust layer, heat conductivity, specific heat, density, micro- and macroscopic structure of the meteoric material, etc.

(2) The photometrical formula with three unknown parameters, given under (a), (b) and (c) can satisfactorily describe the form of the photometrical curve of a comet with a weak continuous spectrum, or without a continuous spectrum at all, i. e. of a comet in which the photometrical efficiency of the gas is much greater than that of the dust. The general form of the photometrical formula must be applied to a comet with a strong or at least well-pronounced continuous spectrum, i. e. to a comet in which the photometrical efficiency of both the constituents is of the same order.

(3) Two comets have been selected to verify the methods derived. The Whipple-Fedtke comet of 1943 was one with no substantial influence of the dust on its brightness, while the Arend-Roland comet of 1957 was characterized by a strong continuous spectrum. The performed analysis has given a few interesting results described in Sections 2.5. and 3.8. respectively.

(4) The gas constituent is described by the generalized, well-known Levin formula, while a semiemiprical formula has been found to characterize the variation of the solid-constituent brightness on the heliocentric distance. In accordance with the formula the photometrical exponent of the dust coma reaches its maximum at about 1.3 A. U. on an average, and drops more quickly towards the small heliocentric distances. An exact form of the n_d-curve changes from case to case and it is hard to give any limits at present.

(5) A statistical analysis of the total photometrical exponents of the comets with the orbital period longer than 1,000 years has also been carried out. At least two different groups of comets have been found in accordance with the frequency distribution of n. A statistical dependence has been established of the photometrical exponent of the group of "typical" long-period comets on the heliocentric distance. Considerations concerning the differences between the pre- and post-perihelion periods have been discussed on the basis of the observational data available. In order that the empirical dependence $n_d = n_d(r)$ might be corrected for the phase-effect, the dependences of the phase-angle on both the heliocentric and geocentric distances have been investigated.

(6) To be able to study more quantitative relations between the comet's radiation and its physical structure, the functions of dust, $E(r)$, and that of gas, $G(r)$, have been introduced. The former represents a combination of the basic physical characteristics of the "solid" radiation constituent: (a) effective radius of the nucleus, i. e. the radius of a monolithic spherical nucleus; (b) mean dust particle dimensions; (c) equivalent thickness of the layer of photometrically effective dust, i. e. the thickness of the layer of a theoretically maximum concentration of particles at the surface of a monolithic nucleus; (d) entire number of photometrically effective dust particles in a cometary atmosphere; (e) albedo of the nucleus and dust particles; (f) phase-effect of the nucleus and dust particles; (g) mass density of the nucleus and dust particles, etc. The latter of the two functions consists of the basic characteristics of the "gas" radiation constituent: (a) heat of evaporation necessary to release a certain amount of gas molecules from the comet-nucleus surface; (b) mass of an average molecule; (c) concentration of molecules in the surface layer of the comet nucleus; (d) photometrical efficiency of a radiating molecule; (e) ratio between the number of radiating and evaporated molecules, etc.; and of some other magnitudes, as the absolute temperature of the comet nucleus at a unit of heliocentric distance.

(7) On the basis of the worked-out conception of the non-homogeneous cometary atmosphere and the Orlov considerations concerning the comet nucleus, the method of determining the total mass of the nucleus as well as of the dust layer has been derived and applied to a number of comets with strong continuous spectra. The mean diameter of investigated comets lies between 2.2 and 4.0 kilometres and the dust-cloud mass of comets with the strongest continuous spectra is 10^{10} gm on an average.

(8) The problem of physical consequences of a collision between the comet nucleus and a dust particle in interplanetary space is solved. On the basis of the physical theory worked-out by STANYUKOVICH the expression is derived for the total loss of the comet mass due to the pulverization process originated owing to collisions with micrometeorites. It is indicated that the pulverization process itself is not sufficient for explaining the amount of the dust in atmospheres of comets ascertained photometrically and spectroscopically. In case of short-period comets, which are exposed to effects of the process for their whole "life", it may be the initial stage in forming a thin dispersion dust layer. The process may also be responsible for a number of comet outbursts, and shapes the face of the surface of the comet nucleus.

(9) The dimensions of the cometary atmosphere as related to the heliocentric distance have been studied, too. They change during an approach of a comet to the Sun in three stages. The first stage is characterized by the action of external forces (collisions of a comet with micrometeorite showers, or solar proton streams). In this stage the cometary atmosphere passes through within relatively large heliocentrical distances and the coma diameter changes quite slightly. This change determines the variation of the concentration of the dust in interplanetary space, or that of the space concentration within the solar-corpuscle streams with the heliocentric distance in a given range of r. In the second stage of the development the intense evaporation of gas from the nucleus of a comet begins so that the dimensions of the head rapidly enlarge with the decrease of the heliocentrical distance. As soon as they reach the value given by the mechanical theory of cometary forms, the third stage occurs dur-

ing which the coma diameter falls relatively slowly with the further approach to the Sun. While the variation of the coma dimensions is a function of the photometrical exponent of the gas coma in the second stage, and therefore the coma diameter varies from case to case, a comparison of the theory with observations shows that the validity of ORLOV's formula is, in practice, general in the third stage, so that the coma diameter of all comets changes in the same way now. The "limiting" heliocentrical distances corresponding to breaks between individual stages of the development curve of "new" comets are considerably shifted towards larger heliocentrical distances in comparison with "old" comets.

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NĚKTERÉ PROBLÉMY KOMETÁRNÍ FYZIKY ŘEŠENÉ NA PODKLADĚ FOTOMETRICKÝCH ÚDAJŮ

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Souhrn

V práci je řešena řada otázek kometární fysiky. Fotometrická měření, která slouží za podklad pro studium problémů, jsou analysována z hlediska jednotné fysikální teorie, pracho-plynového modelu komety. Nejdůkladněji je v práci řešen jeden ze základních problémů fotometrie komet; charakter závislosti jasnosti komet na heliocentrické distanci. Velká pozornost je věnována vlivu prachu na fotometrické vlastnosti kometární atmosféry. Kromě toho je podrobně rozebrán i problém vzájemného působení mezi jádrem komety a meziplanetární hmotou a změny rozměrů kometární hlavy se vzdáleností od Slunce, jež úzce souvisí s fotometrickými parametry komety. Schopnost pracho-plynového modelu vysvětlit všechny uvedené efekty se plně potvrzuje.

НЕКОТОРЫЕ ПРОБЛЕМЫ ФИЗИКИ КОМЕТ РАССМАТРИВАЕМЫЕ НА ОСНОВАНИИ ФОТОМЕТРИЧЕСКИХ ДАННЫХ

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Резюме

В работе решен ряд вопросов физики комет. Фотометрические измерения, которые служат основой для изучения проблем, анализируются с точки зрения единой физической теории, пыле-газовой модели кометы. Наиболее обстоятельно в работе решена одна из основных проблем фотометрии комет, характер зависимости блеска комет от их расстояния до Солнца. Большое внимание обращается на влияние пыли на фотометрические свойства кометной атмосферы. Кроме того подробно изучена также проблема взаимодействия между ядром кометы и междупланетной материей, и изменения размеров головы кометы в зависимости от расстояния до Солнца, которые находятся в тесной связи с фотометрическими параметрами кометы. Способность пыле-газовой модели объяснить все приводимые эффекты полностью потверждается.