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VARIATION OF THE PRODUCTION OF SOLAR RADIATION DURING ELEVEN-YEAR CYCLE OF SOLAR ACTIVITY IN THE WAVE-LENGTH RANGE 620 Å TO 11 000 Å

KOLÍSÁNÍ PRODUKCE SLUNEČNÍHO ZÁŘENÍ BĚHEM JEDENÁCTILETÉHO CYKLU SLUNEČNÍ ČINNOSTI V OBORU VLNOVÝCH DÉLEK 620 Å — 11 000 Å.

ВАРИАЦИЯ ПРОДУКЦИИ СОЛНЕЧНОГО ИЗЛУЧЕНИЯ В ТЕЧЕНИЕ ОДИННАДЦАТИЛЕТНЕГО ЦИКЛА СОЛНЕЧНОЙ ДЕЯТЕЛЬНОСТИ В ОБЛАСТИ ДЛИН ВОЛН 620 Å—11 000 Å

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1. INTRODUCTION

Short-term variation and long-term changes of the production of the radiative solar energy have been qualitatively shown by a number of authors, both for total radiation and for different monochromatic ranges. However, these results have not been elaborated by the universal method up to the present time and no physical interpretation has been presented.

This paper deals with changes of solar radiation during an eleven-year solar cycle and with their dependence on the wave-length. The Sun being the source of the radiation of all bodies of the solar system, the methods of the determination of variation of the solar constant are very different and they concern a wide spectral range. The most important ones are as follows:

a) direct measurements of the solar constant;
b) investigation of influence of solar radiation on solar-terrestrial phenomena;
c) investigation of influence of solar radiation on the brightness of planets and their satellites;
d) investigation of influence of solar radiation on the brightness of comets.

Concrete methods giving results suitable for qualitative elaboration are described in paragraphs 3—7.

2. METHOD OF ELABORATION

As individual solar cycles are not equally active, the changes of the solar constant must be expressed as a function of some parameter of solar activity. Studying the influence of ultraviolet solar radiation on the critical frequency of individual ionospheric regions Allen [5, 6] succesfully used sunspot numbers.

The suitability of a sunspot number as a parameter of solar activity is also demonstrated by the photographs of the Sun obtained on March 13, 1959, in the Lyman-alpha line by a camera placed in a Aerobee-Hi rocket [1]: the maximum intensity of the solar disc corresponds to a group of sunspots conspicuous on the integral light photograph.

Therefore, we shall investigate the energy of the Sun $I_\odot$ in the arbitrary wavelength, λ, as the following function of the suspot number, R:

$$I_\odot(\lambda) = a(\lambda)[1 + k_\lambda R], \tag{1}$$

where k_λ is the variation coefficient; $a(\lambda)$ denotes the zero-point and its value depends on the choice of units.

3. DIRECT MEASUREMENTS OF THE SOLAR CONSTANT

These measurements have been obtained at the Smithsonian Institute in the United States and its branch institutes. Results have been published in a number of papers by Abbot and his collaborators. Paper [2] directly concerns our problem; it studies the time-course of the total solar constant during the years 1920—1939 by means of harmonic analysis and elaborates the variations of the radiation of different wave-lengths from 3500 Å to the infrared region by comparison of two groups of extreme values of very reliably determined monochromatic solar constants. Abbot found besides thirteen shorter periods a period of 23 years in the time-course of the total solar constant from the years 1920—1939. This period can be used as a basis for determining the variation of monochromatic solar constants, which are given only in arbitrary units in paper [2].

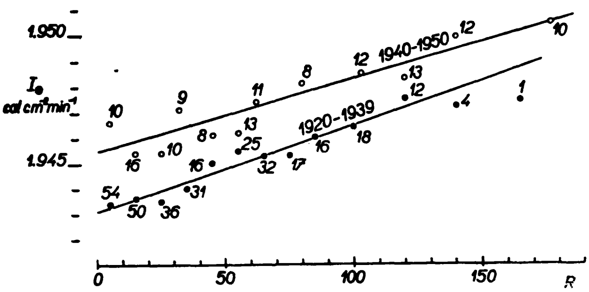

Fig. 1. Regression straight line of the dependence of the total solar constant on sunspot numbers during years 1920—1939 and 1940—1950.

The regression straight line of $I_\odot$ regarding R of the correlation between the total solar constant and sunspot numbers for the shown time-interval is illustrated in Fig. 1. Plotting the ratio $\frac{I_\odot}{a}$ from the formula (1) against R, its slope is equal to

$$k_r = 0.0000174 \pm 0.0000012 \text{ m. e.} \tag{2}$$

The correlation coefficient, ψ, of the dependence and the resulting variation coefficient, k_Σ, which is computed from the formula

$$k_\Sigma = \operatorname{tg} \frac{1}{2} \operatorname{arctg} k_r \frac{1 + \frac{1}{\psi^2}}{1 - \left(\frac{k_r}{\psi}\right)^2}, \tag{3}$$

are given in summary Tab. 3.

The elaboration of measurements of monthly means of total solar constants from the years 1940—1950 has been carried out in the same way by Aldrich [3]. The slope of the regression straight line of $\frac{I_\odot}{a}$ regarding R in this period results in

$$k_r = 0.0000145 \pm 0.0000019 \text{ m. e.,} \tag{4}$$

which agrees very well with (2) (see Fig. 1). The figures at the individual points denote the number of months included in them.

To be able to link a number of relative variations to the variation of the total solar constant, it is necessary to determine such an "effective" wave-length λ_o of total solar radiation, where the variation coefficient of corresponding monochromatic radiation is identical with that of the total solar constant k_Σ; then it follows that

$$k_\Sigma = \frac{\int\limits_0^\infty k_\lambda I_\odot(\lambda)\,\mathrm{d}\lambda}{\int\limits_0^\infty I_\odot(\lambda)\,\mathrm{d}\lambda}. \tag{5}$$

In this equation energies $I_\odot(\lambda)$ are virtually weights of coefficients k_λ. From the dependence of monochromatic radiation on wave-length published in Abbot's paper [2] it follows that the relation between the variation coefficient of the total solar constant and that of monochromatic fluxes may be expressed

$$k_\lambda = k_\Sigma \left(\frac{\lambda}{\lambda_o}\right)^\eta, \tag{6}$$

where

$$\eta = -3.2 \pm 0.2. \tag{7}$$

By inserting for k_λ from (6) into (5) we obtain the following expression for the "effective" wave-length of the total solar constant $I_\odot$:

$$\lambda_o = I_\odot^{-1/\eta} \left[\int\limits_0^\infty I_\odot(\lambda).\lambda^\eta\,\mathrm{d}\lambda\right]^{1/\eta}. \tag{8}$$

If we know the energy distribution in the spectrum of the Sun we can easily determine λ_o. Values published in [4] were taken for intensities $I_\odot(\lambda)$; they are plotted at the bottom of Fig. 2. Function $k_\lambda I_\odot(\lambda)$ computed by using the value of k_Σ from Tab. 3 is given at the top of Fig. 2. We then obtain

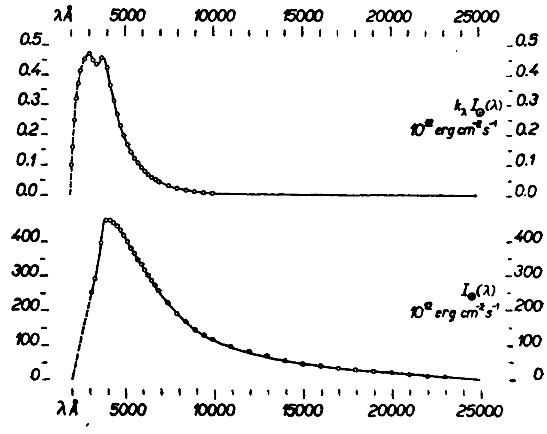

Fig. 2. Curve of absolute amplitudes of solar radiation variation in the range of wave-lengths 2000 Å to 26 000 Å (at the top) and energy distribution in continuous solar spectrum (at the bottom).

$$\lambda_o = 4850 \text{ Å} \tag{9}$$

as the most probable value of effective wave-length of total solar constant.

Variation coefficients k_λ are again included in Tab. 3. Finally, let us underline that Abbot and his collaborators took direct measurements of monochromatic solar radiation beginning with a wave-length of 3500 Å.

An independent investigation of changes of long-wave ultraviolet solar radiation was carried out by Pettit [14]. With the help of the method of Bouguer's straight lines he studied the ratio of monochromatic intensities of 3200 Å and 5000 Å. But the results of his measurements from the years 1933—1938 are not too reliable as the unfavourable influence of the atmosphere on the determination of the exact slope of these straight lines has not been quite eliminated. The results of the elaboration of these measurements according to equation (1) are shown in summary Tab. 3.

4. VARIATION OF THE SOLAR CONSTANT DETERMINED FROM THE CHANGES OF ELECTRON DENSITY OF IONOSPHERIC REGIONS

Considering terrestrial phenomena the correspondence between solar activity and the degree of ionisation of ionospheric layers has been found to be the best so far. According to Chapman's theory [7] of the structure of a ionospheric layer the expression for its maximum electron concentration is

$$N = \left(\frac{S_o \cos \chi}{\alpha . H . e} \right)^{1/2}, \tag{10}$$

where S_o is the number of photons falling on 1 cm²/sec on the boundary of the atmosphere, χ is the zenith distance of the Sun, α recombination coefficient, H parameter of the height of the layer and e the basis of Napier's logarithmi. As the maximum electron concentration depends on the critical frequency of the ionospheric layer f_e according to the formula

$$N \sim f_e^2$$

and as we can obviously put

$$S_o \sim I_\odot(\lambda),$$

we have the opportunity of determining the solar constant in the corresponding wave-length range from the formula

$$I_\odot(\lambda) \sim H . \alpha . \frac{f_c^4}{\cos \chi} . \tag{11}$$

The differences of the value of the characteristic number $f_c^4/\cos \chi$ of the E-region in the years 1937/38 (maximum of solar activity) and 1933/34 (minimum) were already noticed by Appleton and Naismith [8] and a number of other authors. Measurements of the course of the characteristic number of E- and F_1-regions have been elaborated in paper [9]. A detailed study of the critical frequencies of ionospheric regions E, F_1 and F_2 during the years 1937—1947 was carried out by Allen [5, 6]. He elaborated the measurements in two different ways: the intensity of radiation, S_o, was expressed in form (1) and derived from the somewhat generalized equation (11):

$$S_o \sim f_c^n, \tag{12}$$

where he inserted for n both $n = 4$ according to theory [7] and such a value which best corresponded to observations. For the E-region he got

$$n_E = 3.72$$

and for the F_1-region

$$n_{F_1} = 4.30.$$

The mechanism of the F_2-region ionization is different from those previously mentioned, so that the exponent in equation (12) is about $n \sim 2$.

The problem of determination of wave-lengths of ionization radiation, more exactly that of ionization potentials of photo-chemical reactions in individual regions is solved in papers [10, 11, 12]. The situation is more complicated for the F_2-region, as we do not know the contribution of the ionization effect of corpuscular radiation towards the ionization of the region. Therefore we cannot use the F_2-region for the determination of variation of wave character radiation.

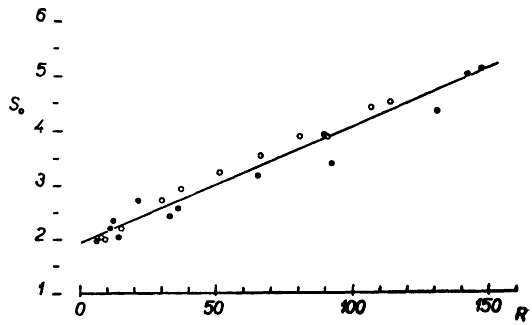

Fig. 3. Yearly means of the changes of short-wave solar radiation affecting the E-region ionization in the dependence on sunspot numbers (1931—1954).

The investigation of changes of the E-region electron density during the years 1931—1954 was carried out by Hulburt [32]. He assumed — in agreement with our basic equation (1) — that the radiation consists of a quiet sun background on which a flux of radiation directly proportional to the sunspot number, R, is superposed. Dependence $I_\odot = I_\odot(R)$ is represented in Fig. 3; the yearly means are plotted, open circles denote odd cycles, full ones even cycles. The variation coefficient derived from odd cycles is equal to

$$k_\lambda = 0.0123 \pm 0.0006$$

and the correlation coefficient $\psi(R, I_\odot) = 0.993 \pm 0.004$. For even cycles we get analogously

$$k_\lambda = 0.0108 \pm 0.0009$$

with the correlation coefficient $\psi(R, I_\odot) = 0.979 \pm 0.012$. Both values of the variation coefficient are in good agreement.

The most probable wave-lengths of radiation ionizing the E- and F_1-regions as well as the results of measurements from Allen's papers [5, 6] and from that of Hulburt [32] are tabulated in summary Tab. 3.

Theoretically it is possible to derive the solar radiation variation in a wave-length range between 1500 Å and 2500 Å from the change in the ozone content of the upper atmosphere during the eleven-year cycle on the basis of the method advanced by Wulf and Deming for the determination of the vertical distribution of ozone in the atmosphere [13]. In practice, this way has not yet been used. The physical processes at smaller heights above the surface of the Earth cannot be used for the determination of changes of the solar radiation as initial agent, considering their considerable inertia.

5. DETERMINATION OF THE VARIATION OF THE SOLAR CONSTANT FROM CHANGES IN THE BRIGHTNESS OF URANUS AND NEPTUNE

During the years 1953 to 1958 systematic photoelectric measurements of the brightness of the planets Uranus and Neptune were taken at the Lowell Observatory; the spectral range of measurements corresponds to B-magnitude

Tab. 1

B-magnitudes of Uranus and Neptune in mean opposition during years 1953—1958

year	B-magnitude	
	Uranus	Neptune
	m	m
1953.0 — 1953.5	6.070	8.269
1953.5 — 1954.5	6.072	8.266
1954.5 — 1955.5	6.075	8.251
1955.5 — 1956.5	6.056	8.245
1956.5 — 1957.5	6.060	8.241
1957.5 — 1958.5	6.056	8.235

of Johnson's and Morgan's photometrical system (U, B, V) with $\lambda_{eff} = 4350$ Å [15]. The details of the method are discussed in papers [16, 17, 18]. The results of measurements are included in Tab. 1. The dependence of B-magnitude of Uranus and Neptune on the sunspot number is shown in Fig. 4 and 5, and resulting coefficients k are tabulated in Tab. 3.

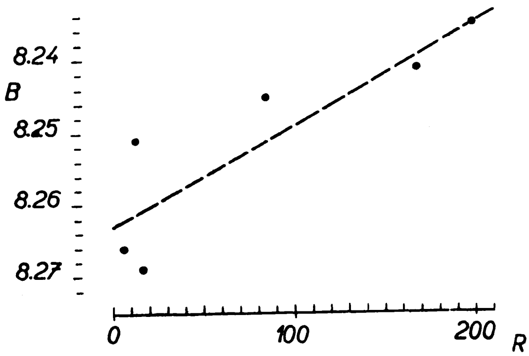

Fig. 4. Yearly means of the changes of Uranus' brightness in the dependence on sunspot numbers (1953—1958).

6. DETERMINATION OF THE VARIATION OF THE SOLAR CONSTANT FROM CHANGES IN THE BRIGHTNESS OF COMETS

The investigation of the dependence of absolute brightness of 563 comets published in Vsekhsviatsky's "Catalogue of Absolute Magnitudes of Comets" [19, 20] on the phase of the solar cycle was carried out by the author [21, 22]. The physical interpretation of the ascertained dependence, which is published together with the results in paper [23], is based on the conception of the comet dust-gas model. In paper [22] it is proved by consideration of qualitative character that the changes of the total brightness of a comet are affected firstly by those of its gaseous component. The balance of this molecular radiation can be written in the form

$$\frac{\mathrm{d}H_{10}}{\mathrm{d}\Phi} = A_1 + \frac{\mathrm{d}R}{\mathrm{d}\Phi} \exp\left[\frac{0.4}{\mathrm{mod}} H_{10}\right]\left(A_2 + A_3 I_{\odot}^{-3/2}\right), \tag{13}$$

where H_{10} is the average absolute magnitude of comets in the given phase of solar cycle Φ, R is the corresponding sunspot number and A_1, A_2, A_3 are constants in which coefficients of the photodissociation process, life-time of radiating molecules and their effective cross-section are included; besides, the last two coefficients contain the variation coefficient of solar radiation exciting gaseous molecules in cometary atmosphere.

From ionization potentials of the most frequent molecules in cometary atmospheres [24] the following most probable effective wave-length of exciting solar UV-radiation holds

$$\lambda_{eff} = 900\ \text{Å}.$$

The results for the average solar cycle are included in the summary table.

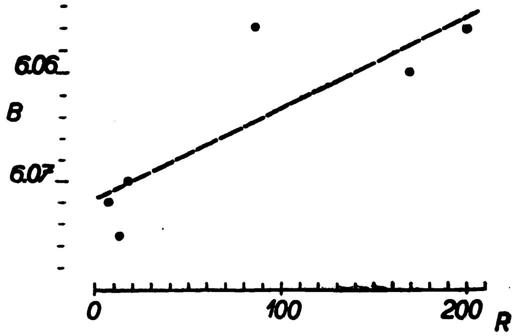

Fig. 5. Yearly means of the changes of Neptune's brightness in the dependence on sunspot numbers (1953—1958).

7. ROCKET RESEARCH OF THE LYMAN-ALPHA EMISSION AND OTHER REGIONS OF UV-RANGE OF THE SOLAR SPECTRUM

The rocket research of UV-radiation of the Sun falls methodically under the paragraph about direct measurements of solar radiation, but in contrast to methods of solar constant measurements this research has a special characteristic features arising partly from the technical aspect of the problem partly from the physical structure of the Sun in the studied range of radiation.

The great disadvantage of these results, which were ascertained mainly before the opening of the IGY, is the fact that they record the solar radiation for a very short time; the determination of the changes is thus very inaccurate.

The greatest progress was reached in the measurement of the Lyman-alpha emission; twelve measurements were obtained by means of photon counters, thermoluminescent phosphors, ion chambers and photometry of spectrograms during the years 1949—1956 [33, 34, 35, 36, 37, 38, 39]. As responses of individual types of indicators are different the results obtained by various method are not comparable. The most continuous series of measurements was taken by ion chambers; their ionization thershold is at 1340 Å. On the short wave-length side the response of ion chambers is limited at 1100 Å. More than 95 per cent of the solar radiation between these two wave-lengths is concentrated in the Lyman-alpha line [40]. The summary of the results obtained by ion chambers is listed in Tab. 2. The individual columns show: launching time M. S. T., Lyman-alpha emission intensity, sunspot number for the launching date, sunspot number smoothed-out from thirteen days (six before and six after launching date) and references. Even though the correlation between the sunspot number and emission intensity is low enough, the agreement using $R^{(13)}$ is much better.

Tab. 2

The Lyman-alpha emission energy measured by ion chambers during years 1955—1956

Date, M. S. T.	$L\alpha$ erg.cm^{-2}.s^{-1}	R	$R^{(13)}$	ref.
1955 X. 18.66	4.7—8.7	0	29	[38]
1955 X. 21.72	4.0 ±0.8	23	42	[38]
1955 XI. 4.35	9.2 ±3	52	90	[38]
1956 VII. 17.80	6.1 ±0.3	98	125	[39]
1956 VII. 25.88	6.7 ±0.3	90	99	[39]

It seems, the suitability of sunspot numbers as statistical parameter of solar activity is verified.

At all events the value of Lyman-alpha emission variation, which is given in Tab. 3, must be considered to carry very low weight regarding the following unfavourable circumstances:

a) correlation coefficient is very low, so that the correlation ratio is only about 1,4;

b) intensity measurements of the Lyman-alpha emission included great observational errors as may be seen from Tab. 2;

c) correlation coefficient is derived from a very short time interval (about one year);

d) the mean of Lyman-alpha intensity values ought to be plotted against $R^{(13)}$ for elimination of observational errors and real short-term fluctuation of emission;

e) the Lyman-alpha emission originates in the chromosphere, so that the found variation of monochromatic solar constant differs by its character from continuous photosphere radiation ending practically at 1500 Å [44].

The situation concerning other emissions of Lyman series is more difficult. On a spectrogram which showed the Lyman-alpha emission as well as the Lyman-beta emission Tousey [40] estimated their intensities as 0,4 erg.cm^{-2}.s^{-1} and 0,03 erg.cm^{-2}.s^{-1} respectively, so that their ratio was about 1000 : 75, while on the later spectrogram Violett and Resnse [41] estimated it as 1000 : 60; at the same time the Lyman-alpha intensity was about 3,4 erg.cm^{-2}.s^{-1} [42].

Several experiments were carried out to determine the intensity distribution across the solar disc in the Lyman-alpha line. The results obtained by Miller, Mercury and Rense [43] did not give uniform conclusion in this question. On the other hand Johnson, Malitson, Purcell and Tousey [37] on the basis of the Lyman-alpha line images obtained on February 21, 1955, came to the conclusion that neither limb brightening nor darkening exist to within about 20 %, which was the attainable accuracy. Similarly, any limb brightening effect, theoretically assumed by de Jager [45], was not found on the later images by Mercury, Miller, Rense and Stuart [46, 47]; the very first-quality image obtained on March 19, 1959 [1] gives negative result too.

8. LIST OF NUMERICAL RESULTS AND EMPIRICAL FORM OF THE DEPENDENCE OF VARIATION COEFFICIENT ON WAVE-LENGTH

Tab. 3 contains the list of variation coefficients k_λ of solar radiation, derived by the methods described in paragraphs 3—7. The individual columns give:

λ_{eff} — effective wave-length of solar radiation;
k_λ — variation coefficient of solar radiation; the numerical value describes its relative change corresponding to the change of sunspot number for a unit; assuming for an average solar cycle the maximum (monthly) sunspot number $R_M = 100$ and the minimum one $R_m = 0$, the relative amplitude of the intensity expressed in per cents is equal to $10^4\, k_\lambda$; its mean error is given contingently;
log k_λ — its Brigg's logarithm with mean error contingently;
aut. — name of the author publishing the measurements results, and references;
meth. — the main feature of the method determining the variation coefficient k_λ;
int t — time-interval during which measurements were taken;
note — note giving e. g. some special character of the method or elaboration;
sign. — denotation on Fig. 6;
$\psi(R,I_\odot)$ — correlation coefficient of the relation and its mean error as far as it was possible to derive it;
$\frac{\psi}{\Delta\psi}$ — correlation ratio;
$O—C_1$ — residuals between values observed and computed from (14) considering (I);
$O—C_2$ — residuals between values observed and computed from (14) considering (II).

The dependence of k_λ on the wave-length is graphically represented in Fig. 6. Denotation is done according to column 8 of Tab. 3.

Fig. 6 shows that the variation coefficient can be with a sufficient accuracy expressed in the form

$$\log k_\lambda = A + B \log \frac{\lambda}{\lambda_o} + C \left(\log \frac{\lambda}{\lambda_o}\right)^2 . \tag{14}$$

Assuming, that the form of this dependence in the range between 1300 Å and 3000 Å, where no measurements are at our disposal, corresponds to the character of that in neighbouring ranges, the coefficients A, B and C can be derived for the whole range between 620 Å and 11000 Å; in such a case we get

$$\left.\begin{aligned} A &= -\,4.242 \pm 0.022, \\ B &= -\,3.36 \pm 0.11, \\ C &= -\,0.75 \pm 0.15, \end{aligned}\right\} \quad \text{(I)}$$

if λ and k_λ are expressed in the same units as in Fig. 6. The effective wave-length of the total solar constant is again denoted as λ_o, so that $A = \log k_\Sigma$, using symbolics of paragraph 3. Using measurements from visual and partly from infrared ranges only, the values of coefficients are as follows:

Tab. 3

Synopsis of measured variation coefficients of solar constant during

λ_{eff}(Å)	k_λ	log k_λ	aut.	meth.
620	0.0124	—1.91	Allen [5, 6]	from influence of ionization UV-radiation changes on changes of critical frequency of F_1-region
	0.0168	—1.77		
740	0.0097	—2.01	Allen [5, 6]	from influence of ionization UV-radiation changes on changes of critical frequency of E-region
	0.0088	—2.05		
	0.0110 ± 0.0007	−1.96 ± 0.03	Hulburt [32]	
900	0.0080 ± 0.0001	−2.10 ± 0.01	Sekanina [23]	from course of cometary brightness during cycle
1216	0.0049 ± 0.0038	−2.31 ± 0.34	BCFK [38], CFKK [39]	Lyman-alpha emission measured by ion chambers
3200	0.00025 ± 0.00014	−3.60 ± 0.24	Pettit [14]	from ratio of intensities λ 3200 : λ 5000
4350	0.000079 ± 0.000023	−4.10 ± 0.13	Johnson, Iriate [15]	from B-magnitude variation of Uranus
	0.000132 ± 0.000039	−3.88 ± 0.13		from B-magnitude variation of Neptune
4850	0.0000515 ± 0.0000053	−4.29 ± 0.05	Abbot, Aldrich, Hoover [2]	total solar constant variation
3500	0.000172	—3.76	Abbot, Aldrich, Hoover [2]	from the ratio of variation of monochromatic solar constant to that of total solar constant
3600	0.000162	—3.79		
3710	0.000119	—3.92		
3850	0.000089	—4.05		
3970	0.000100	—4.00		
4130	0.000070	—4.15		
4310	0.000089	—4.05		
4520	0.000055	—4.26		
4750	0.000046	—4.34		
5030	0.000037	—4.43		
5350	0.000043	—4.37		
5740	0.000034	—4.47		
6240	0.000034	—4.47		
6860	0.0000137	—4.86		
7220	0.0000122	—4.91		
7640	0.0000183	—4.74		
8120	0.0000107	—4.97		
8630	0.0000077	—5.11		
9220	0.0000062	—5.21		
9860	0.0000030	—5.52		
10620	0.0000030	—5.52		

eleven-year cycle in the wave-length range 620 Å to 11 000 Å

int t	note	sign.	ψ (R, $I_\odot$)	$\frac{\Delta\psi}{\psi}$	O—C_1	O—C_2
1937—1947	for n = 4 in formula $S_0 \sim f_c^n$	▼			—0.07	—
	for n = 4,30 in formula $S_0 \sim f_c^n$	▽			+0.07	—
1937—1947	for n = 4 in formula $S_0 \sim f_c^n$	▲			—0.01	—
	for n = 3,72 in formula $S_0 \sim f_c^n$	△			—0.06	—
1931—1954	for n = 4 in formula $S_0 \sim f_c^n$	■	0.978 ± 0.009	108.7	+0.04	—
1610—1954	mean from odd and even cycles	O=			+0.09	—
1955—1956	instrument response 1100 Å — 1345 Å	◍	0.481 ± 0.343	1.4	+0.18	—
1933—1938		■	0.262 ± 0.116	2.3	+0.06	+0.09
1953—1958		● U	0.850 ± 0.113	7.5	—0.02	—0.02
		● N	0.872 ± 0.098	8.9	+0.20	+0.20
1920—1939	changes in 23-year cycle	⊙	0.479 ± 0.044	10.9	—0.05	—0.05
1929—1939	total solar constant variation determined from Abbot's, Aldrich's and Hoover's measurements [2] was used as a zero-point	o—o—o			+0.02	+0.04
					+0.03	+0.04
					—0.06	—0.05
					—0.14	—0.13
					—0.04	—0.04
					—0.14	—0.14
					+0.02	+0.02
					—0.12	—0.12
					—0.13	—0.14
					—0.14	—0.14
					+0.01	+0.01
					+0.02	+0.01
					+0.14	+0.13
					—0.10	—0.11
					—0.07	—0.08
					+0.19	+0.19
					+0.06	+0.06
					+0.02	+0.02
					+0.03	+0.03
					—0.17	—0.17
					—0.05	—0.03

$$
\left.\begin{aligned}
A &= -4.236 \pm 0.029, \\
B &= -3.27 \pm 0.22, \\
C &= -1.18 \pm 1.05,
\end{aligned}\right\} \quad \text{(II)}
$$

that is almost the same result.

In the following Tab. 4 values of the exponent ε of the approximate relation

$$k \sim \lambda^{\varepsilon(\lambda)} \tag{15}$$

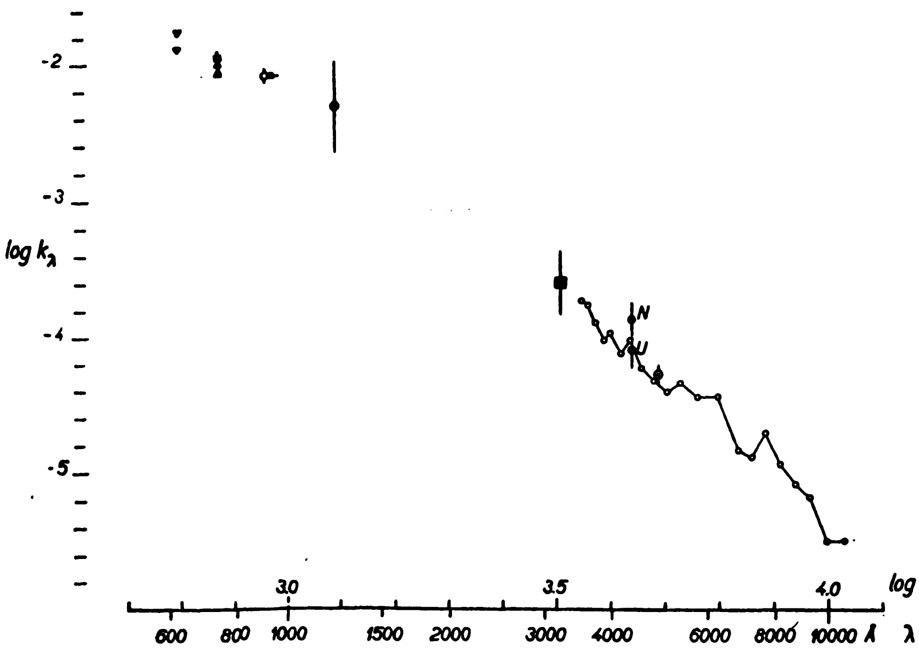

Fig. 6. Measured values of variation coefficient in the dependence on wave-length in the range of 620 Å to 11 000 Å.

for different wave-lengths are given; they are computed from (14) using results (I) and (II). We can see the exponent reaches the values within the boundaries —2,7 and —3,6 in the investigated wave-length range. For $\lambda\lambda < 3000$ Å tabulated values of $\varepsilon(\lambda)$ hold good only under the same assumptions taken into account when deriving the coefficients (I). Regarding the approximate character of the empirical formula (14), the exponent $\varepsilon(\lambda)$ for $\lambda \sim 10\,000$ Å must be considered with reserve too.

9. INTENSITY DISTRIBUTION OF CONTINUOUS RADIATION OVER THE SOLAR DISC

To explain the form of the dependence of the variation coefficient k_λ on the wave-length it is necessary to admit a certain expression for emergent monochromatic flux of radiation on the solar surface [25a]. Let us denote ϑ angle between the direction of radiation and normal to the solar surface, τ_ν optical

Tab. 4

Exponent ε of dependence of variation coefficient on wave-length

λ (Å)	$\varepsilon(\lambda)$	
	I	II
620	—2.69	—
740	—2.75	—
900	—2.81	—
1216	—2.91	—
2000	—3.08	—
2500	—3.15	—
3200	—3.23	—3.06
3500	—3.26	—3.11
4350	—3.32	—3.21
4850	—3.36	—3.27
5500	—3.40	—3.33
6500	—3.46	—3.42
8000	—3.53	—3.52
10000	—3.60	—3.64

depth for radiation of a given frequency ν, $I_\nu(\tau_\nu, \vartheta)$ intensity of radiation in a given depth and of a given direction, and $J_\nu(\tau_\nu)$ source function, then the transfer equation for stellar atmospheres has the form of

$$\cos\vartheta \frac{\mathrm{d}\, I_\nu(\tau_\nu, \vartheta)}{\mathrm{d}\tau_\nu} = I_\nu(\tau_\nu, \vartheta) - J_\nu(\tau_\nu) \tag{16}$$

and for the intensity on the surface of photosphere results:

$$I_\nu(0, \vartheta) = \int\limits_0^\infty J_\nu \exp\left[-\tau_\nu \sec\vartheta\right] \sec\vartheta \, \mathrm{d}\tau_\nu. \tag{17}$$

Assuming that in each point of the atmosphere characterized by temperature T_{τ_ν} the conditions of the local thermal equilibrium are fulfilled the source function is equal to Planckian one of a given temperature:

$$J_\nu(\tau_\nu) = B_\nu(\tau_\nu) = \frac{2\,h\nu^3}{c^2}\left\{\exp\left[\frac{h\nu}{kT_{\tau_\nu}}\right] - 1\right\}^{-1}, \tag{18}$$

h is Planck's constant, k Boltzmann's constant and c the velocity of light. Let us assume further the continuous absorption coefficient $\varkappa_\nu$ in the solar atmosphere to be independent of optical depth, so that

$$\tau_\nu = \frac{\varkappa_\nu}{\bar{\varkappa}} \cdot \tau, \tag{19}$$

where $\bar{\varkappa}$ is the opacity of solar matter. Both these assumptions are, on the whole, in good agreement with reality. By inserting expressions from equations (18) and (19) into (17) we get

$$I_\nu(0,\vartheta) = \frac{2ckc_2}{\lambda^5}\int\limits_0^\infty \frac{\varkappa_\nu}{\bar{\varkappa}}\sec\vartheta \exp\left[-\frac{\varkappa_\nu}{\bar{\varkappa}}\sec\vartheta\right]\cdot$$
$$\cdot\left\{\exp\left[\alpha\left\{\frac{3}{4}(\tau+q(\tau))\right\}^{-1/4}\right]-1\right\}^{-1} d\tau, \qquad (20)$$

where

$$\alpha = \frac{c_2}{\lambda T_e} = 24770\ \lambda^{-1}[\text{Å}],$$

when using according to [25b]:

$$T_e = 5780 \pm 40\ °\text{K},$$

$$c_2 = \frac{hc}{k};$$

the dependence of the temperature on the optical depth [25c] is

$$T_\tau^4 = \frac{3}{4} T_e^4 [\tau + q(\tau)], \qquad (21)$$

where various approximations are used for $q(\tau)$. The first approximation gives $q(\tau) = \frac{2}{3}$ [25d], which is quite sufficient for practical purposes [26a].

10. THE CHANGES OF THE VARIATION COEFFICIENT OF CONTINUOUS RADIATION IN THE CASE OF NON-VARIABILITY OF CONTINUOUS ABSORPTION COEFFICIENT

If we assume the linear dependence of the source function $J_\nu(\tau)$ on the optical depth, the following expression for the emergent flux of radiation F_ν can be written

$$F_\nu = I_\nu\left(0, \frac{\pi}{2}\right)\cdot\left(1+\frac{2}{3}\beta_\nu\right), \qquad (22)$$

where

$$I_\nu\left(0,\frac{\pi}{2}\right) = \lim_{\vartheta\to\pi/2} I_\nu(0,\vartheta) = \frac{2ckc_2 . \lambda^{-5}}{e^u - 1},$$
$$\beta_\nu = \frac{3}{8}\frac{\bar{\varkappa}}{\varkappa}\cdot\frac{u}{1-e^{-u}} \qquad (23)$$

and

$$u = \alpha\sqrt[4]{2} = 29460\lambda^{-1}\ [\text{Å}].$$

By putting (23) and $\frac{\bar{\varkappa}}{\varkappa_\nu} \equiv 1$ into (22) the flux we are looking for is equal to

$$F_\nu = \frac{2ckc_2\lambda^{-5}}{e^u - 1}\left(1 + \frac{1}{4}\frac{u}{1-e^{-u}}\right) \qquad (24)$$

and the variation coefficient

Tab. 5

Theoretical values of functions giving the course of variation coefficient under two different assumptions

λ (Å)	$\frac{\varkappa_\nu}{\bar{\varkappa}}$ from I_λ (0)	$\frac{\varkappa_\nu}{\bar{\varkappa}}$ from F_λ	$1 - \frac{1}{\Phi}\frac{\partial \Phi}{\partial \alpha}$	$G(\alpha)$	$H(u)$
3000	1.34	1.32	0.96	7.93	9.11
3500	1.36	1.29	0.97	6.87	7.74
4000	0.67	0.66	0.91	5.63	6.72
4500	0.66	0.64	0.89	4.90	5.93
5000	0.72	0.67	0.91	4.50	5.30
5500	0.73	0.70	0.93	4.19	4.79
6000	0.78	0.75	0.94	3.88	4.36
6500	0.82	0.79	0.96	3.66	4.00
7000	0.84	0.80	0.96	3.40	3.70
7500	0.86	0.87	0.97	3.20	3.43
8000	0.98	0.96	0.98	3.04	3.20
8500	1.00	1.00	0.99	2.88	3.01
9000	1.12	1.07	1.00	2.75	2.82
9500	1.09	1.04	1.00	2.61	2.66
10000	1.03	0.93	1.00	2.48	2.53

$$k_\lambda = \frac{1}{F_\nu} \cdot \frac{\Delta F_\nu}{\Delta R},$$

is in the case of fairly high values of u, when $e^{-u} \to 0$ (practically for $\lambda <$ < 11000 Å), represented by the formula

$$k_\lambda = u \frac{3+u}{4+u} \cdot \frac{1}{T_e} \cdot \frac{\Delta T_e}{\Delta R}. \tag{25}$$

Values of the function

$$H(u) = u \cdot \frac{3+u}{4+u} \tag{26}$$

within 3000 Å to 11 000 Å are included in Tab. 5. Exponent $\varepsilon(\lambda)$ defined by relation (15) results about

$$\varepsilon(\lambda) \sim -1,$$

which is in variance with the data of Tab. 4. In Fig. 7 values of function $H(u) \sim k_\lambda$ are plotted by open circles.

11. THE CHANGES OF THE VARIATION COEFFICIENT OF CONTINUOUS RADIATION IN THE CASE OF VARIABILITY OF THE CONTINUOUS ABSORPTION COEFFICIENT

In the last paragraph we assumed the linear dependence of the source function $J_\nu(\tau)$ on the optical depth, as well as the independence of the continuous absorption coefficient on the wave-length.

It turns out that the former assumption can sometimes lead to quite incorrect results [27] and the latter assumption is not fulfilled either, which follows from the papers studying the dependence of the continuous absorption coefficient on the wave-length both theoretically [28, 29] and experimentally [30].

By eliminating these two assumptions equation (20) gives for emergent monochromatic flux the expression [30]

$$F_\nu = 2\int_0^\infty B_\nu(T_\tau).E_2\left(\frac{\varkappa_\nu}{\bar{\varkappa}}.\tau\right)\cdot\frac{\varkappa_\nu}{\bar{\varkappa}}\,\mathrm{d}\tau, \tag{27}$$

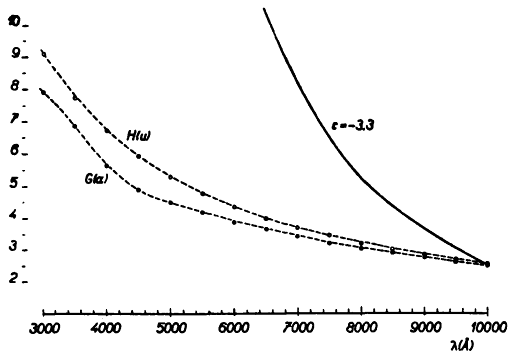

Fig. 7. Comparison of the form of empirical dependence of variation coefficient on wave-length with that derived theoretically in the case of "quiet" photosphere.

where T_τ is given by (21) and E_2 is integrallogarithm:

$$E_2(x) = \int_1^\infty \frac{\mathrm{e}^{-xw}}{w^2}\,\mathrm{d}w.$$

Further, the Planckian distribution for T_τ can be expressed by that corresponding to effective temperature of the Sun, so that equation (27) can be written in the form

$$F_\nu = B_\nu(T_e).\,\Phi\left(\alpha, \frac{\varkappa_\nu}{\bar{\varkappa}}\right). \tag{28}$$

Function Φ in this equation is equal to

$$\Phi = 2(\mathrm{e}^\alpha - 1)\int_0^\infty \frac{E_2\left(\frac{\varkappa_\nu}{\bar{\varkappa}}\cdot\tau\right)}{\exp\left[\alpha\left\{\frac{3}{4}\left[\tau + q(\tau)\right]\right\}^{-1/4}\right] - 1}\cdot\frac{\varkappa_\nu}{\bar{\varkappa}}\cdot\mathrm{d}\tau \tag{29}$$

and its values are tabulated by Burkhardt [31]. The variation coefficient in the case of $\mathrm{e}^{-\alpha} \to 0$ according to (28) is given by the expression

$$k_\lambda = \alpha\left(1 - \frac{1}{\Phi}\cdot\frac{\partial\Phi}{\partial\alpha}\right)\cdot\frac{1}{T_e}\cdot\frac{\Delta T_e}{\Delta R}. \tag{30}$$

Functions Φ and $\frac{\partial\Phi}{\partial\alpha}$ can be determined with sufficient accuracy from nomogram $\Phi = \Phi\left(\alpha, \frac{\varkappa_\nu}{\bar{\varkappa}}\right)$[30]. The dependence of the continuous absorption coefficient on the wave-length was determined by Münch [30] both from the spectral distribution of intensity in the centre of the solar disc, $I_\lambda(0)$, and from the emergent monochromatic fluxes, F_λ. His results are included in the second and third columns of Tab. 5 for wave-lengths between 3000 Å and 10 000 Å. The auxiliary expression, $1 - \frac{1}{\Phi}\frac{\partial\Phi}{\partial\alpha}$, is given in column 4, column 5 and 6 list functions

$$G(\alpha) = \alpha\left(1 - \frac{1}{\Phi}\frac{\partial\Phi}{\partial\alpha}\right), \tag{31}$$

and $H(u)$ from equation (26). In Fig. 7 the dependence of function $G(\alpha) \sim k_\lambda$ on the wave-length is represented by full circles. The average slope of the empirical dependence in the range of 3000 Å — 11 000 Å ($\bar{\varepsilon} = -3.3$; see paragraph 8) is shown by a conspicuous curve in the same figure.

It is apparent from Fig. 7 that the introduction of the variable continuous absorption coefficient does not practically affect the form of the dependence of the variation coefficient on the wave-length. Therefore the interpretation of the obtained k_λ-curve only by radiation of a "quiet" photosphere does not lead to sufficient agreement between theory and observation. The introduction of the continuous absorption coefficient as a function of optical depth would only negligibly change the form of the theoretical dependence (30) as the function $\varkappa_\nu = \varkappa_\nu(\tau)$ changes very inappreciably; this fact was found by Münch[30] on the basis of the theory elaborated by Chandrasekhar [48] in the case of the Negative Hydrogen Ion, which is the main agent of continuous absorption in wave-lengths between 3000 Å and 11 000 Å.

12. CONTRIBUTION OF ACTIVE REGIONS TO THE TOTAL RADIATION OF THE SUN IN VARIOUS WAVE-LENGTHS

Let us note that the basic formula (1) determined from experience leads us to the consideration of the influence of active regions on total solar radiation. Faculous areas are the most effective constituent of these regions in the wave-length range 3000 Å to 11 000 Å.

Therefore we can presuppose that the relative variation of effective temperature of the Sun during the eleven-year cycle found from observations which, according to (30), is expressed by

$$\left(\frac{\Delta T_e}{T_e}\right)_{obs} = \frac{k_\lambda}{\alpha}\Delta R \cdot \left(1 - \frac{1}{\Phi}\frac{\partial\Phi}{\partial\alpha}\right)^{-1}, \quad \Delta R \sim 100, \tag{32}$$

is the consequence of two relative variations: that of the effective temperature of a "quiet" solar photosphere, $\left(\frac{\Delta T_e}{T_e}\right)_\odot$, and that of the effective temperature of faculous areas, $\left(\frac{\Delta T_e}{T_e}\right)_F$; the contribution of both constituents to an observed result (32) changes with the wave-length. The contribution of the variation of the effective temperature of faculae in wave-length λ can be characterized by the γ_λ-coefficient:

$$\left(\frac{\Delta T_e}{T_e}\right)_{obs} = \gamma_\lambda\left(\frac{\Delta T_e}{T_e}\right)_F + (1 - \gamma_\lambda) \cdot \left(\frac{\Delta T_e}{T_e}\right)_\odot; \tag{33}$$

it has the meaning of weight and we can write it in the form:

$$\gamma_\lambda = \bar{\varphi}_\lambda - 1, \tag{34}$$

where $\bar{\varphi}_\lambda$ is the ratio of average emergent intensities of faculae and photosphere

$$\bar{\varphi}_\lambda = \frac{[\bar{I}_\lambda]_F}{[\bar{I}_\lambda]_\odot}. \tag{35}$$

To derive $\bar{\varphi}_\lambda$ it is firstly necessary to insert $[I_\lambda(\vartheta)]_\odot$ and $[I_\lambda(\vartheta)]_F$ into the general expression for average intensity

$$\bar{I}_\lambda = 2 \int_0^{\frac{1}{2}\pi} I_\lambda(\vartheta) \cos\vartheta \sin\vartheta \, d\vartheta. \tag{36}$$

As very great accuracy is not demanded we can use the model of linear dependence of source function on optical depth; in this case the ratio of the intensity of the photosphere, $[I_\lambda(\vartheta)]_\odot$, to that in the centre of the disc, $[I_\lambda(0)]_\odot$, is given by the well-known relation

$$\frac{[I_\lambda(\vartheta)]_\odot}{[I_\lambda(0)]_\odot} = \frac{1 + \beta_\lambda \cos\vartheta}{1 + \beta_\lambda}, \tag{37}$$

where for $\lambda < 11\,000$ Å the β_λ-coefficient is derived according to (23) by

$$\beta_\lambda = \frac{3}{8} u \frac{\bar{\varkappa}}{\varkappa_\lambda}.$$

By inserting (37) into (36) we obtain after integration:

$$[\bar{I}_\lambda]_\odot = [I_\lambda(0)]_\odot \cdot \frac{1 + \frac{2}{3}\beta_\lambda}{1 + \beta_\lambda}. \tag{38}$$

To apply equation (36) to faculae too we must know at least the approximative form of function $\varphi_\lambda = \varphi_\lambda(\vartheta)$. Its values can be derived on the basis of the few photoelectric measurements of intensities of faculae at our disposal so far at various distances from the centre of the solar disc. Measurements suitable for elaboration were taken by Richardson [49] in wave-lengths of 4330 Å and 5780 Å, by Krat [50] in those of 3900 Å and 5000 Å, and by Wormell [51] in total solar light. Johnson's new results of the energy distribution in the continuous spectrum of the Sun [52, 53] give the value of 7120 Å as an effective wave-length of the total solar radiation; that means function $\varphi_\lambda = \varphi_\lambda(\vartheta)$ is at our disposal for five wave-lengths. These relations are graphically represented in Fig. 8. Even when measurements are not too accurate, the expression of dependence $\varphi_\lambda = \varphi_\lambda(\vartheta)$ in the form

$$\varphi_\lambda = 1 + a_\lambda(1 - \cos\vartheta) \tag{39}$$

seems to be quite suitable; the a_λ-coefficient in this formula is independent of angle ϑ. Using (39), (35), (36) and (34) we get for the γ_λ-coefficient

$$\gamma_\lambda = q \cdot \frac{a_\lambda}{3} \frac{1 + \frac{1}{2}\beta_\lambda}{1 + \frac{2}{3}\beta_\lambda}, \tag{40}$$

where q is a parameter depending on the vibration of the image of photometried faculae. For a perfectly calm image is $q = 1$; under the assumption that the vibration complies with the Gaussian law distribution, Krat [50] using the most probable values of its parameters found that the real contrast between the intensities of faculae and the neighbouring photosphere is about 2,5times more than that observed; thus with regard to equations (39) and (35), we can immediatelly write $q = 2{,}5$ in equation (40). The influence of the diffused light on the change of contrast appeared to be negligible.

By finding quantities $\left(\frac{\Delta T_e}{T_e}\right)_{obs}$ and the γ_λ-coefficients from the material and by applying the method of least squares to five conditional equations of the form (33) we obtain the following values of relative variation amplitude of effective temperature of the photosphere and faculae during the eleven-year cycle:

$$\left(\frac{\Delta T_e}{T_e}\right)_\odot = 0.00007 \pm 0.00021,$$
$$\left(\frac{\Delta T_e}{T_e}\right)_\odot = 0.0062 \pm 0.0010; \qquad (41)$$

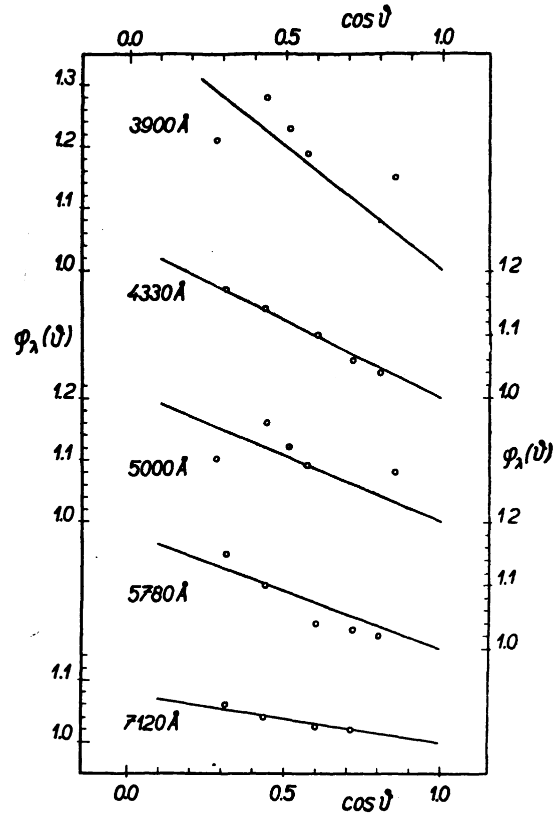

Fig. 8. Ratio between intensity of faculae and neighbouring photosphere in the dependence on the distance from the centre of the solar disc, in various wave-lengths.

using the following values of effective temperature of the photosphere [25b] and faculous areas [26b]

$$(T_e)_\odot = 5780 \pm 40\ °K,$$
$$(T_e)_F = 6000 \pm 100\ °K,$$

the corresponding variations result in:

$$\left.\begin{aligned}(\Delta T_e)_\odot &= 0.4 \pm 1{,}2\ °K,\\ (\Delta T_e)_F &= 37 \pm 6\ °K.\end{aligned}\right\} \quad (42)$$

We therefore may draw the conclusion that the effective temperature of the photosphere probably does not systematically change during the cycle, and that the variation coefficient obtained from the material is in close mutual causality with the contribution of faculous areas to the total radiation of the solar disc within the wave-lengths 3000 Å to 11 000 Å. From equations (33) and (32) with respect to (41) it follows approximately that

Tab. 6

Determination of the ratio of intensity of faculae and photosphere in various wave-lengths

λ_{eff} Å	k_λ	$\left(\frac{\Delta T_e}{T_e}\right)_{vos}$	a_λ	γ_λ		
				O	C	$O—C$
			m. e.	m. e.		
3900	0.000114	0.00165	$+0.398 \pm 0.051$	0.273 ± 0.024	0.259	+0.014
4330	0.000082	0.00135	$+0.243 \pm 0.007$	0.166 ± 0.003	0.210	—0.044
5000	0.000052	0.00097	$+0.210 \pm 0.035$	0.145 ± 0.016	0.148	—0.003
5780	0.000032	0.00067	$+0.182 \pm 0.027$	0.127 ± 0.013	0.098	+0.029
7120	0.000015	0.00058	$+0.077 \pm 0.006$	0.055 ± 0.003	0.051	+0.004

$$\gamma_\lambda \sim \lambda k_\lambda$$

within the studied range of wave-lengths.

Tab. 6 contains results of elaboration of Krat's, Richardson's and Wormell's measurements of contrast between faculae and the neighbouring surface of the Sun; ascertained variations of monochromatic solar constants according to (14) using values (II), $\left(\frac{\Delta T_e}{T_e}\right)_{obs}$ following from (32), contrast coefficients a_λ and resulting coefficients γ_λ (using $q = 2{,}5$) determined from (40) on the basis of measurements as well as computed from (33) using numerical values of (41) are tabulated for each effective wave-length; the last column $O—C$ shows sufficient agreement.

The general form of equation (33) gives the following expression for the γ_λ-coefficient within the wave-lengths 3000 Å to 11 000 Å:

$$\gamma_\lambda = 0.0113\left\{3.57 \times 10^{-7}.\lambda^{6.43} \exp\left[—2.72 (\log \lambda)^2\right] \cdot \left(1 - \frac{1}{\Phi} \cdot \frac{\partial \Phi}{\partial \alpha}\right)^{-1} - 1\right\}. \tag{43}$$

The course of this dependence is represented in Fig. 9 where the measured values of the γ_λ-coefficient are also plotted.

Even when equation (43) is only an interpolation formula for a given range of wave-lengths (Fig. 9) the results following from its solution for limited values of the γ_λ-coefficient are still interesting:

for $\gamma_\lambda \to 0 \ldots\ldots \lambda_1 \sim 13300$ Å,

for $\gamma_\lambda \to 1 \ldots\ldots \lambda_2 \sim 1900$ Å.

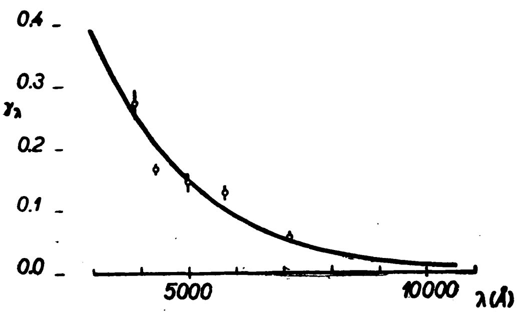

Fig. 9. Coefficient of the contribution of faculae to the total change of effective temperature in the dependence on the wave-length.

That means: the contrast between the photosphere and faculae quite disappears in the infrared range, while the contribution of faculae to the total radiation of the solar disc is considerable in the ultraviolet range about 2000 Å. This fact is in qualitative agreement with statement [44] about the decrease to zero of the continuous radiation of the photosphere at 1500 Å. As the greatest contribution to the radiation of faculae originates in the hottest (i. e. upper) layers of matter we must presuppose that the "effective" geometrical depth decreases with the decreasing wave-length and taking the visible limb of the solar disc as zero-point it must reach negative values. A great number of emission lines discovered in the ultraviolet range below 1800 Å [41, 42], which originate in the solar chromosphere, support this conclusion. In this wave-length range the interpretation which holds good within 3000 Å to 11 000 Å is meaningless.

13. VARIATIONS OF THE SHORT ULTRAVIOLET SOLAR RADIATION

Although the character of the solar radiation in the wave-length range below 2000 Å considerably differs from that of long ultraviolet, visual and infrared radiation, elaborating the material we applied in both cases the same method of determination of variation coefficient k_λ. As this method, according to paragraph 4, gave good results even for investigations of the changes of the ionization degree of ionospherical regions affected by solar radiation, we can conclude that the sunspot numbers are a general enough parameter of solar activity.

Except for the Lyman-alpha emission the list of variation coefficients of wave-lengths < 2000 Å (Tab. 3) includes the data obtained entirely by indirect methods. The corresponding effective wave-lengths are always only weighted means of a number of wave-lengths affecting investigated phenomena.

To consider all these emissions in the interpretation would considerably complicate the calculations and therefore we shall assume that each of the investigated phenomena was affected by a certain fictional emission, the position in the spectrum and the variation coefficient of which are given by values λ_{ef} and k_λ of Tab. 3 respectively.

Assuming the thermal character of chromospherical emission, the intensity distribution in the line can be written [25e] as follows:

$$I_\lambda(x, v) = B_\lambda(T)\left[1 - e^{-C(x) \cdot H(\alpha', v)}\right], \tag{44}$$

where x is the distance from the solar limb in which the line originates, α' is the expression of

$$\alpha' = \frac{\gamma}{4\pi c \Delta\lambda_D},$$

γ is the damping constant, c the light-speed and $\Delta\lambda_D$ the Doppler's width of the line; v in equation (44) gives the distance from the centre of the line

$$v = \frac{\Delta\lambda}{\Delta\lambda_D}.$$

The Planckian distribution can be written in an investigated range with great accuracy in the form of

$$B_\lambda(T) = 2hc^2\lambda^{-5}.e^{-\frac{c_2}{\lambda T}}, \tag{45}$$

T now denotes the temperature of the chromosphere. Finally, functions $C(x)$ and $H(\alpha', v)$ are given by the expressions:

$$C(x) = \frac{\pi e^2}{mc^2} \cdot \frac{f}{\Delta\lambda_D} \cdot n_0 \left(\frac{2a}{\alpha}\right)^{1/2} \cdot e^{-\alpha x}, \tag{46}$$

$$H(\alpha', v) = e^{-v^2} - \frac{2\alpha'}{\pi^{1/2}} [1 - 2v\mathrm{F}(v)], \tag{47}$$

where m and e are the mass and charge of electron respectively, f oscillator strength, a the radius of the Sun, n_0 and α are parameters of the concentration changes of emitted atoms n with the height h above the photosphere surface:

$$n = n_0 . e^{-\alpha h}.$$

The function $F(v)$ in (47) is equal to

$$F(v) = e^{-v^2} \int_0^v e^{u^2} \mathrm{d}u$$

and its values are tabulated by Miller and Gordon [54].

The intensity of the emitted energy in the line is then expressed as

$$I_\lambda(x) = 2B_\lambda(T) \int_0^\infty \left(1 - e^{-C(x) . H(\alpha', v)}\right) \mathrm{d}v. \tag{48}$$

Let us assume that

$$\frac{\partial}{\partial T} \int_0^\infty \left(1 - e^{-C(x) . H(\alpha', v)}\right) \mathrm{d}v \approx 0 \tag{49}$$

and derive for a given line the amplitude of temperature corresponding to the variation coefficient, k_λ, defined by equation (1). As quantity $k_\lambda(R_{max} - R_{min})$ is of the same order as a unit for wave-lengths < 2000 Å the corresponding amplitude of intensity, $I_{\lambda max} - I_{\lambda min}$, cannot be determined by a simple derivation. If we denote

$$\int_0^\infty \left(1 - e^{-C(x) . H(\alpha', v)}\right) \mathrm{d}v = \Omega(x),$$

the following relation holds for the resulting amplitude:

$$\Delta I_\lambda = 2hc^2\lambda^{-5} . \Omega(x) . \exp\left[-\frac{c_2}{\lambda T_{min}}\right] \cdot \left\{\exp\left[\frac{c_2}{\lambda}\left(\frac{1}{T_{min}} - \frac{1}{T_{max}}\right)\right] - 1\right\}, \tag{50}$$

so that the expression for the variation coefficient has the form of:

$$k_\lambda = \frac{1}{\Delta R} \left\{\exp\left[\frac{c_2}{\lambda}\left(\frac{1}{T_{min}} - \frac{1}{T_{max}}\right)\right] - 1\right\}. \tag{51}$$

If we further denote

$$K(\lambda) = \lambda \log (1 + \Delta R . k_\lambda), \ \Delta R \sim 100,$$

the correlation coefficient $\psi(\lambda, K)$ gives the degree of reality of the dependence of $\Omega(x)$ on the temperature, i. e. it shows us to what extent our assumption was correct. Values of function $K(\lambda)$ empirically enumerated from the variation

Tab. 7

Function $K(\lambda)$ in wave-length range 620 Å to 1220 Å

λ (Å)	k_λ	$K(\lambda)$
620	0.0124	217
620	0.0168	265
740	0.0097	218
740	0.0088	203
740	0.0110	238
900	0.0080	230
1216	0.0049	210

coefficients in the range of 620 Å to 1220 Å are listed in Tab. 7. The resulting correlation coefficient is equal to

$$\psi(\lambda, K) = -0.40 \pm 0.38,$$

so that the correlation is very low and assumption (49) is fulfilled to a first approximation. The most probable value of K is

$$K = 226 \pm 21 \text{ (m. e.)}$$

and the amplitude of temperature during the cycle,

$$\Delta T = T_{\max} - T_{\min},$$

is expressed by the relation:

$$\Delta T = \frac{T_{\min}}{\dfrac{\operatorname{mod} c_2}{K \, . \, T_{\min}} - 1} \, . \tag{52}$$

Goldberg [55], Athay and Thomas [56] dealt with the determination of the temperature of those regions radiating in Lyman-alpha; on the basis of the total energy of this emission line and its width they obtained the value $T \approx 10000$ °K which demontrates a large height above the boundary of the solar photosphere. Inserting it into (52) we obtain for the amplitude of temperature

$$\Delta T = 380 \pm 50 \text{ °K}, \tag{52'}$$

if the error of the temperature is ± 500 °K.

Comparing (51) with (15) the following expression has to hold for exponent ε between wave-lengths 620 Å and 1220 Å for $\lambda \neq 1000$ Å:

$$\varepsilon(\lambda) = \frac{0.168 + \log \left\{ \exp \left[\dfrac{520}{\lambda} \right] - 1 \right\}}{\log \lambda - 3} \tag{53}$$

and $\varepsilon = -1.28$ for $\lambda = 1000$ Å. Computed values for a few wave-lengths are listed in Tab. 8. The average value in the investigated range of wave-lengths following from (53) is

$$\bar{\varepsilon} = -1.31.$$

Tab. 8

Exponent ε of dependence of variation coefficient on wave-length in range 620 Å to 1220 Å

λ (Å)	ε (λ)
620	—1.38
740	—1.34$_5$
900	—1.31
1000	—1.28
1216	—1.20
620—1220	—1.31

Its direct determination from the data of Tab. 3 gives

$$\bar{\varepsilon} = -1.56 \pm 0.24 \text{ (m. e.)}.$$

The agreement between both results is sufficient. At the same time we draw the conclusion that the character of the dependence of the variation coefficient on the wave-length (which is represented in numerical values of constants too) is mutually so different in wave-length ranges < 2000 Å and > 3000 Å that it is immaterial to look for the general formula, which would express this relation in the whole range of wave-lengths of orders 500 Å to 15 000 Å using the same coefficients. For the same reason, constants A, B, C of system (I) from paragraph 8 have only a formal character. A comparison of the data from Tab. 4 and 8 proves it quite objectively.

14. CONCLUSIONS REACHED SO FAR AND FURTHER PROSPECTS OF RESEARCH

In the first part of the paper the results of a number of direct and indirect methods determining the changes of the solar radiation at time-intervals comparable with the length of the eleven-year cycle are elaborated. The main product of this part of the paper is the empirically ascertained dependence of the variation coefficient on the wave-length. Numerical values of the variation coefficient determine the relative change of solar energy production of a certain wave-length (or a range of them) corresponding to the change of sunspot number for a unit. A large degree of correlation found in the majority of investigated causes of determining the variation coefficient shows the suitability of sunspot numbers as the general statistical parameter of solar radiation and attributes to the variation coefficient a clear and important physical meaning. Those of the cases when the degree of correlation was low concerned material containing either considerable intrinsic uncontrollable errors (Pettit's measurements) or material of little extent (Lyman-alpha emission).

The physical interpretation was established separately for wave-lengths > 3000 Å and < 2000 Å owing to the different character of the radiation in both ranges; in the former mainly continuous radiation takes place, in the latter emission line radiation.

The following conclusion can be drawn in the problem of the dependence of the variation coefficient on the wave-length in the range of $\lambda\lambda$ > 3000 Å:

a) radiation changes affected by changes of effective temperature of "quiet" photosphere following from the solution of the transfer equation show considerably different dependence on the wave-length to that found from material; no better agreement is reached by introducing a variable continuous absorption coefficient;

b) agreement between theory and measurements can be reached by the assumption that the observed variation of the effective temperature of the Sun is affected by two constituents: the "quiet" photosphere and the active regions, viz. faculous areas. Calculation shows that the error of variation of the photosphere temperature is greater than its amplitude, so that probably no systematical changes of the photosphere temperature exist during a solar cycle. On the other hand, the variation of temperature of faculous areas is inconsiderable but real;

c) the given results were found under the assumption that the conditions of local thermal equilibrium in the photosphere are fulfilled and on the basis of measurements of the contrast between faculae and the neighbouring photosphere in various distances from the solar limb.

The variation of intensity of emissions in the UV-range of $\lambda\lambda < 2000$ Å was interpreted on the basis of the theory of energy distribution in emission as a consequence of the electron temperature changes under two assumptions: that they have a thermal character and their profiles are independent of temperature changes. Theory agrees very well with the empirical results even in this simple form.

However, a considerable want of systematic measurements in a short-wave UV-range of radiation is still felt and the majority of results calls for verification. The determination of the variation coefficient from long observational series (over several solar cycles) with more precision as well as the finding of more reliable effective wave-lengths are essential. This problem is very important for radiation affecting the ionization of ionospheric regions. It is well-known that the E-region ionization is affected by X-rays [40] too, on the other hand Nicolet believes that the intensity of Lyman-beta is quite sufficient to ionize O_2 to the extent necessary to control the changes of maximum electron density of E-region [40]. Similarly, the effective wave-length of radiation exciting gaseous molecules in cometary atmospheres is uncertain enough (within about ± 100 Å).

From the point of view of theory, the deviations from the condition of thermal equilibrium in various regions of the chromosphere have to be investigated in detail and the gradient of electron temperature must be determined. Therefore, expression (52′) gives the amplitude of the temperature only in certain regions of the chromosphere, i. e. the regions most effectively contributing to the Lyman-alpha emission production.

The last important problem for further investigations of this character is an enlargement of the wave-length range on both sides, i. e. into very short UV-radiation and X-rays as well as into radio-waves, and the determination of the character of the variation coefficient in these ranges.

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Souhrn

V prvních odstavcích práce jsou zpracovávány výsledky měření slunečního záření v různých vlnových délkách v oboru 620 Å — 11 000 Å, a to jak přímých tak i nepřímo získaných. Jako parametru sluneční činnosti je užito relativního čísla slunečních skvrn a za charakteristiku kolísání energie záření v různých vlnových délkách je zvolen tzv. koeficient variace, udávající relativní změnu intensity záření při změně relativního čísla o jednotku. V práci je konstruován graf závislosti koeficientu variace na vlnové délce a jsou určeny konstanty vztahu z materiálu metodou nejmenších čtverců.

Fysikální interpretace nalezeného vztahu je rozdílná pro oblast vlnových délek větších než 3000 Å a menších než 2000 Å vzhledem k tomu, že charakter záření je v obou případech odlišný. V oboru delších vlnových délek lze nalezené změny v produkci záření interpretovat jako změny efektivní teploty, přijmeme-li, že se na nich podílí jednak „klidná" fotosféra a jednak aktivní oblasti, konkrétně fakulová pole. Řešením příslušných normálních rovnic obdržíme pro amplitudu teplotních změn fakulí hodnotu asi 40 °K, zatím co teplota fotosféry během cyklu patrně žádným systematickým změnám nepodléhá.

V oblasti vlnových délek menších než 2000 Å je spojité záření fotosféry zanedbatelné vůči řadě intensivních emisí chromosféry (popř. i korony). Proto je nyní interpretace založena na teorii rozložení intensity v emisních čarách. Za předpokladu, že emise mají termický charakter, je odvozen výraz pro amplitudu teploty chromosféry. V práci udaná hodnota amplitudy, jež platí pro teplotu v těch výškách nad povrchem fotosféry, v nichž se vytváří čára L_α, představuje — vzhledem k tomu, že gradient teploty chromosféry a jeho průběh není dosud uspokojivě znám — víceméně jen příklad.

V závěru práce jsou vedle výsledků shrnuty i hlavní potíže, jež dosud nepříznivě ovlivňují řešení problému a stručně vytyčeny další perspektivy výzkumu v tomto směru.

Резюме

В первых параграфах обрабатываются результаты измерений солнечного излучения в разных длинах волн с 620 Å до 11 000 Å, произведенных прямыми и непрямыми методами. Относительного числа солнечных пятен использовано как параметра солнечной деятельности, затем что изменения энергии излучения в разных длинах волн характеризуются так называемым коэффициентом вариации; его числовое значение дает относительное изменение интенсивности излучения соответствующее изменению относительного числа на единицу. В работе построен график зависимости коэффициента вариации от длины волны и постоянные отношения определяются по материалу методом наименьших квадратов.

Физическая интерпретация обнаруженного отношения различна для областей длин волн больших чем 3000 Å и меньших чем 2000 Å ввиду того, что характер излучения в обеих случаях иной. В области больших длин волн обнаруженные изменения в продукции излучения можно интерпретировать изменениями эффективной температуры если принять, что наряду с „невозмущенной" фотосферой участвуют в них и активные области, а именно факелы. Решением соответствующих нормальных уравнений получается для амплитуды температурных изменений факелов значение около 40 °К, тогда как температура фотосферы в течение цикла по-видимому никаким систематическим изменениям не подвергается.

В области длин волн меньших чем 2000 Å непрерывным излучением фотосферы можно пренебречь по сравнению с рядом интенсивных эмиссий хромосферы (или же и короны). Потому интерпретация основывается на теории распределения интенсивности в эмиссионных линиях. В предположении, что эмиссии обладают термическим характером, дается выражение для амплитуды температуры хромосферы. Ее значение, приведенное в работе и соответствующее температуре в тех высотах над поверхностью фотосферы, в которых образуется линия L_α, представляет собой — ввиду того что градиент температуры хромосферы и его ход нет пока удовлетворительно известен — более или менее пример.

В конце работы наряду с результатами резюмированы и основные трудности, которые пока неблагоприятно влияют на решение проблемы, и кратно поставлены дальнейшие перспективы исследований такого рода.