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Special issue dedicated to professor K. R. Rajagopal

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SPECIAL ISSUE DEDICATED TO PROFESSOR K. R. RAJAGOPAL

1. Introduction

This special issue of *Applications of Mathematics* is dedicated to Professor K. R. Rajagopal on the occasion of his sixtieth birthday. By publishing the special issue we wish to acknowledge his significant achievements in the field of mechanics and thermodynamics of continuous media, his contributions to the development of this journal, and, last but not least, his close collaboration with the Czech scientific community.

Professor Rajagopal, besides being a distinguished scholar, is our colleague, mentor and friend, and it is a great honour for us to write a foreword to the special issue dedicated to him. On the other hand, we feel that we will not be able to communicate the readers the whole depth and width of Professor Rajagopal's scientific activities that have resulted in more than four hundred research publications. The impressive list of Rajagopal's seminal contributions to continuum mechanics includes, among others, the following topics: a study of fundamental concepts in thermodynamics [7], thermodynamic criteria for stability of certain classes of non-Newtonian fluids [6], [5], a new approach to constitutive modelling by maximization of entropy production and by the notion of natural configuration [21], [26], [22], [23], [24], [25], [27], [28], boundary conditions for differential type fluids and boundary conditions in mixture theory [16], [15], electrorheological fluids [29], [20], a systematic study of implicit constitutive theory [17], [28], incompressible materials with material coefficients depending on the mean normal stress, constitutive theory of asphalt [10], and biomechanics [33], [1]. We prefer to leave the comprehensive discussion of these achievements to more qualified people as we rather pinpoint two issues that are in our opinion relevant to this journal and its audience. First, we would like to comment on Professor Rajagopal's opinion concerning applications of mathematics in natural sciences, and, second, on his relation to the Czech scientific community and this journal.

1.1. Applications of mathematics. In the famous essay on the nature of intellectual work of mathematicians, von Neumann [32] wrote:

> As a mathematical discipline travels far from its empirical source, or still more, if it is a second and third generation only indirectly inspired by ideas coming from "reality", it is beset with very grave dangers. It becomes more and more purely æstheticizing, more and more purely *l'art pour l'art*. This need not be bad, if the field is surrounded by correlated subjects, which still have closer empirical connections, or if the discipline is under the influence of men with an exceptionally well-developed taste. But there is a grave danger that the subject will develop along the line of least resistance, that the stream, so far from its source, will separate into a multitude of insignificant branches, and that the discipline will become a disorganized mass of details and complexities. In other words, at a great distance from its empirical source, or after much "abstract" inbreeding, a mathematical subject is in danger of degeneration.

Professor Rajagopal quoted this paragraph when writing an introduction to a special issue of *Computers and Mathematics with Applications* [18]. If we read von Neumann's essay [32], the above text continues, after few sentences, by the following paragraph:

> In any event, whenever this stage is reached, the only remedy seems to me to be the rejuvenating return to the source: the re-injection of more or less directly empirical ideas. I am convinced that this was a necessary condition to conserve the freshness and the vitality of the subject and that this will remain equally true in the future.

If we combine these statements, and if we recall Rajagopal's dislike of applying fancy mathematics in bad physical context or without proper understanding of the physical nature of the problem studied, we get a feeling about his sentiments concerning applied mathematics. The endeavour of an applied mathematician should be to use appropriate mathematical methods, and—if necessary—to develop new mathematical methods in connection with good physics, or more generally, in connection with deep understanding of the studied phenomenon.

The passion for doing good physics necessarily implies that the person having this passion sometimes inevitably criticizes preoccupations concerning methods and approaches to some well established problems, and hence challenges the ruling paradigm. Let us note that the word critical has, in the context of Professor Rajagopal's work, the right original meaning, critical simply means[1] "involving or exercising careful judgement or observation; nice, exact, accurate, precise, punctual", without any trace of nowadays distorted meaning "the passing of unfavourable judgement; fault-finding, censure".

[1] The definitions are taken from [31].

In some sense challenging a well established conceptual approach to a long studied phenomenon is more demanding than discussing possible approaches to a new one. Certainly, Professor Rajagopal has the ability to go against the mainstream, and criticize inappropriate approaches regardless of the authority backing them up. This sometimes leads to the situation that he is criticizing his own earlier works that he now considers to be imprecise. The criticism often inspires the development of new approaches, (mathematical) methods and techniques.

We think that the ability to criticize is essential in science (and not only in science), and should be mentioned side by side with the ability to get new and "positive" results. It is, however—due to its controversy—rarely mentioned in laudatory texts like this one, but we would like to make an exception, and point out this aspect of Rajagopal's scientific work. Some of the recent works that include sound criticism of mainstream methods and approaches are for example [14], [19], [30], and [4]—the titles of these works are self-explanatory.

1.2. Interaction with the Czech scientific community. Professor Rajagopal has tight relations to the Czech scientific community. Collaboration with Czech scientists started in December 1992 when Professor Rajagopal delivered, as one of the key speakers, a series of lectures on non-Newtonian fluids during the Second international school on mathematical theory in fluid mechanics at Paseky. This visit initiated his close and fruitful cooperation with Professor Jindřich Nečas and his students and colleagues that led to intensive joint research focused on the analysis of partial differential equations governing flows of non-Newtonian fluids and deformations of non-linear solids, as well as the development of a thermodynamically consistent framework to model the behaviour of mixtures and other complex materials. The cooperation resulted in more than forty research papers, see for example joint works [13], [20], [8], [11], [19], [9], [3], and [12] to name a few.

During almost twenty-year long cooperation, Professor Rajagopal invited several teachers and students to long-term research stays at University of Pittsburgh and later on at Texas A&M University in College Station. Conversely, during his visits in the Czech Republic, he delivered several compact courses on finite elasticity, mixture theory and behaviour of geomaterials.

In 2006, a few years after Professor Nečas had passed away, the mathematical modelling group initiated (with colleagues from the Charles University, the Czech Technical University and the Institute of Mathematics of the Academy of Sciences of the Czech Republic) a joint research centre that bears Nečas' name—Jindřich Nečas Centre for Mathematical Modelling. Professor Rajagopal has cordially supported this project (he is a member of the steering committee of the Centre), and continues to collaborate with researchers and especially young postgraduate and postdoctoral

researchers in the Centre, see for example [3] and [2]. He has also frequently acted as a referee of theses written by master's and doctoral students of mathematical modelling at Charles University.

Professor Rajagopal is a very active member of the Editorial board of *Applications of Mathematics* and he is concerned with the development of the journal. The journal should, to his opinion, follow the scope outlined by Professor Ivo Babuška (the founder of the journal)—the journal should be focused on applications of mathematical methods in various branches of science and engineering. Professor Rajagopal regularly contributes to the journal and also encourages young people to contribute to the journal. We would like to mention that we highly appreciate that he submitted to the journal his original research paper [17] in which he presented his ideas concerning the implicit constitutive theory—the theory that provides a concept that has recently attracted a lot of attention in the research oriented on the modelling of response of complex materials.

2. Content of the special issue

We have invited several mathematical analysts having a close relation to Professor Rajagopal to contribute to this special issue, and we think that all contributions submitted to the special issue closely observe the maxim expressed in the previous paragraphs—do applied mathematics with good understanding of the phenomenon studied. Many of the contributors are persons who can be considered Professor Rajagopal's students, and who have recently started their scientific career. We hope that they will in the future follow the maxim and do high quality research.

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