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Book Reviews

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BOOK REVIEWS

T. Andreescu, D. Andrica: COMPLEX NUMBERS FROM A TO... Z. Birkhäuser, Boston, 2006, 321 pages, ISBN 0-8176-4326-5, EUR 51.–.

This book is intended for people who are interested in solving advanced problems of school mathematics.

Complex numbers are introduced as the ordered pairs $\{(x, y) \mid x, y \in \mathbb{R}\}$ and it is said that any complex number can be uniquely represented in the algebraic form $z = x + \mathrm{i}y$, where x, y are real numbers and $\mathrm{i}^2 = -1$. Then the arithmetic operations with complex numbers in algebraic form are defined.

Trigonometric form of a complex number ($z = r(\cos\varphi + \mathrm{i}\sin\varphi)$) is illustrated through polar coordinates in the plane. De Moivre formula, n^{th} root of a complex number, geometric interpretation of the n^{th} root and geometric analogy between complex numbers, vectors and operations with them are described with help of the trigonometric form.

The third chapter (Complex numbers and geometry) shows basic geometric ideas in the plane where points are represented as complex numbers. Then the authors describe, in an analytical way using determinants, the distance between two points, the angle between two lines, conditions for collinearity and orthogonality, congruent and similar triangles, equation of a line and equation of a circle.

More difficult geometric problems are shown in the fourth chapter. The authors define real and complex product of two complex numbers as the analogue of the scalar and vector product of vectors. Some previous tasks or problems of searching for important points in triangles or convex polygons are solved elegantly with barycentric coordinates.

The top of this book is the fifth chapter, Olympiad-caliber problems; many concrete problems are solved here using complex numbers—algebraic equations, polynomials, trigonometric problems and also combinatorics.

Answers and solutions to proposed problems are mentioned at the end of the book.

Rostislav Lenker, Praha

José Ferreirós: LABYRINTH OF THOUGHT. A HISTORY OF SET THEORY AND ITS ROLE IN MODERN MATHEMATICS. Second revised edition, Birkhäuser, Basel, 2007, 465 pages, EUR 60.–.

Labyrinth of Thought represents a comprehensive survey on the history of set theory; more than four hundred pages are divided into three parts:

First part, *The Emergence of Sets within Mathematics*, starts with a description of the German mathematical community in the 19th century. Great attention is paid to the work of Riemann, especially his notion of a manifold. While we usually tend to interpret it in a topological or differential-geometric context, it seems that Riemann used the word for a rather general set of objects corresponding to a given concept. Another prominent person in the story is Dedekind, probably the second most important figure in the emergence of set theory (next to Cantor). He formulated the general notion of a mapping, stated the criterion that a set is infinite iff it is equivalent to its proper part etc. The book analyzes his construction of the real numbers via the Dedekind cuts, and compares it with the approaches of Weierstrass (infinite series) and Cantor (fundamental sequences). As is well known, Cantor's interest in the theory of point sets originated in the research concerning Fourier series. In one of his earliest studies, he introduced the notions of a limit point and a derived set. When considering a set P and its derived sets of n-th order $P^{(n)}$, he suggested to define $P^{(\infty)}$ as the intersection of all derived sets of finite order, and then moved on to $P^{(\infty+1)}$, $P^{(\infty+2)}, \ldots, P^{(2\infty)}, \ldots, P^{(3\infty)}, \ldots, P^{(\infty^2)}$ etc. Other mathematicians also introduced notions which are now classified as belonging to set theory, topology, or measure theory; no distinction was made between these disciplines at that time.

Second part, *Entering the Labyrinth—Toward Abstract Set Theory*, starts with Cantor's discovery of non-denumerability of the real line in the 1870's. The author offers an interesting picture of the relations between Cantor and Dedekind. It was the latter who suggested to Cantor the idea that the result might be used to prove the existence of transcendental numbers (since the set of algebraic numbers is countable). In his publications, Cantor often didn't give proper credit to Dedekind for his contributions; this might have been the reason why Dedekind left some of Cantor's later letters unanswered. The next chapters discuss the work of Dedekind (concentrating on his definition of natural numbers based on the theory chains), Cantor's mature theory of sets, transfinite numbers, and his work on the continuum hypothesis. One of the interesting topics treated here (and also in the epilogue) is the theorem that a power set $P(A)$ always has a greater cardinality than the original set A. Although Cantor could have used it to justify his infinite hierarchy of transfinite numbers, he formulated the result in the language of functions $f : A \to \{0, 1\}$, and probably failed to recognize its importance.

Third part, *In Search of an Axiom System*, focuses on the period from 1890 to 1940. This was a period of a growing acceptance of Cantor's ideas, but also of the discovery of set-theoretic paradoxes and the attempts to resolve them. Zermelo's axiomatization of set theory, which was published in 1908, included also the controversial axiom of choice. Zermelo needed it in order to prove the well-ordering theorem, from which he deduced the comparability of any two sets with respect to their size (this fact was left without proof by Cantor). Russell's theory of types represented another way to avoid the paradoxes, but was criticized especially for the rather unnatural axiom of reducibility. The subsequent development of both theories was influenced by people such as von Neumann, Bernays, Gödel etc.; the progress in the axiomatization of logic also played an important role. The author observes that, although originally very different, both systems gradually shifted closer to each other. The last chapter also contains a summary of Gödel's results, and a glance toward the post-war period.

The second edition of the book includes a new 25-page epilogue, which discusses additional topics (especially the iterative and dichotomy conceptions of sets).

J. Ferreirós has carefully studied the available sources and his book contains a wealth of information on the history of set theory. Moreover, the text is well-balanced in presenting the facts as well as general reflections. Most of the book is accessible to anyone with a basic knowledge of set theory. Czech readers will appreciate numerous remarks concerning the work of Bolzano.

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