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Orbit projections as fibrations

ORBIT PROJECTIONS AS FIBRATIONS

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Abstract. The orbit projection $\pi\colon M \to M/G$ of a proper G-manifold M is a fibration if and only if all points in M are regular. Under additional assumptions we show that π is a quasifibration if and only if all points are regular. We get a full answer in the equivariant category: π is a G-quasifibration if and only if all points are regular.

Keywords: orbit projection, proper G-manifold, fibration, quasifibration

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1. INTRODUCTION

A continuous map is called Hurewicz (Serre) fibration if it has the homotopy lifting property for all topological spaces (CW complexes). In Section 2 we give a simple proof of the fact that the orbit projection $\pi\colon M \to M/G$ of a proper smooth G-manifold M is a Hurewicz (Serre) fibration if and only if all points in M are regular. It seems that this unsurprising equivalence has not explicitly appeared in literature up to now. It is well-known that if M has only one orbit type, then π is a locally trivial fiber bundle. Our proof of the other direction basically uses the existence of slices at any point and the fact that for isotropy subgroups H the projection $G \to G/H$ is a fibration. Hence, the result generalizes to proper locally smooth G-spaces M. Moreover, it has its analog in the category of G-spaces and G-equivariant maps.

In Section 3 we investigate when the orbit projection π is a quasifibration, i.e., the canonical inclusion of each fiber in the corresponding homotopy fiber is a weak homotopy equivalence. We show that under certain conditions π is a quasifibration if

and only if all points of M are regular. In the equivariant category we get a full answer: The orbit projection π is a G-quasifibration if and only if all points of M are regular. Its proof uses some deep theorems from equivariant homotopy theory.

2. Orbit projections as fibrations

2.1. Fibrations ([2]). Let E and B be topological spaces. A continuous map $p\colon E \to B$ is called a *Hurewicz fibration* if it has the *homotopy lifting property*: For each topological space X, each continuous $f\colon X \times \{0\} \to E$ and each homotopy $\varphi\colon X \times I \to B$ of $p \circ f$, there exists a homotopy $\bar\varphi$ of f covering φ. If $p\colon E \to B$ has the homotopy lifting property for all CW complexes X, it is called a *Serre fibration*. The fibration is *regular* if $\bar\varphi$ can always be selected to be stationary with φ, i.e., for each $x \in X$ such that $\varphi(x,t)$ is constant as a function of t, the function $\bar\varphi(x,t)$ is constant as well.

$$
\begin{array}{ccc}
X \times \{0\} & \xrightarrow{\ f\ } & E \\
\downarrow & \overset{\bar\varphi}{\nearrow} & \downarrow{\scriptstyle p} \\
X \times I & \xrightarrow[\ \varphi\ ]{} & B
\end{array}
$$

A locally trivial fiber bundle is a regular Serre fibration; if the base is paracompact then it is a regular Hurewicz fibration.

For a Serre fibration $p\colon E \to B$ with fiber $F = p^{-1}(b_0)$ and $p(e_0) = b_0$ the following homotopy sequence is exact:

$$(2.1) \quad \ldots \to \pi_n(F, e_0) \to \pi_n(E, e_0) \to \pi_n(B, b_0) \to \pi_{n-1}(F, e_0) \to \ldots \to \pi_0(B, b_0)$$

If $p\colon E \to B$ is a Hurewicz (Serre) fibration and B is path connected (and all fibers are CW complexes), then any two fibers belong to the same homotopy type.

2.2. Compact transformation groups. Let G be a compact Lie group and let M be a G-manifold, i.e., M is a paracompact Hausdorff smooth manifold and the action $G \times M \to M$, $(g,x) \mapsto g.x$ is smooth. Endow the orbit space M/G with the quotient topology. Then M/G is paracompact and Hausdorff, and the orbit projection $\pi\colon M \to M/G$ is continuous, open, closed, and proper (e.g. [1], [3]). In particular, it follows that if M is (path) connected, then M/G is (path) connected. The orbits $G \cdot x$ which are exactly the fibers of π are compact smooth submanifolds of M. A point $x \in M$ is called stationary if $G \cdot x = \{x\}$.

Proposition. *Let G be a compact Lie group and let M be a path connected G-manifold containing a stationary point. The orbit projection $\pi\colon M \to M/G$ is a Hurewicz (Serre) fibration if and only if every point in M is stationary.*

P r o o f.   Suppose that $\pi\colon M \to M/G$ is a Serre fibration. Since M/G is path connected and all fibers are CW complexes, all fibers of π belong to the same homotopy type. There is one fiber consisting of one point only, namely the stationary point in M. It follows that all orbits consist of one point only, since all orbits are closed manifolds. Hence each point in M is stationary.   □

Corollary. *Let $\varrho\colon G \to \mathrm{GL}(V)$ be a representation of a compact Lie group G. The orbit projection $\pi\colon V \to V/G$ is a Hurewicz (Serre) fibration if and only if ϱ is trivial.*

2.3. Proper G-manifolds ([3]). Let G be a Lie group and let M be a proper G-manifold, i.e., the mapping $G \times M \to M \times M$, $(g, x) \mapsto (g.x, x)$ is proper. Examples are G-manifolds where G is compact, or properly discontinuous actions on manifolds. Again M/G is paracompact and Hausdorff, and the orbit projection π is continuous, open, and closed. In a proper G-manifold all isotropy subgroups $G_x = \{g \in G\colon g.x = x\}$ are compact. We denote by M_{reg} the set of regular points x in M, i.e., points x allowing an invariant open neighborhood U such that for all $y \in U$ there exists an equivariant map $f\colon G \cdot x \to G \cdot y$, or equivalently, $G_x \subseteq g G_y g^{-1}$ for some $g \in G$. Orbits through regular points are said to be of principal orbit type. The orbit types in M are the conjugacy classes (H) of the isotropy subgroups $H \subseteq G$. The inclusion relation on the family of isotropy subgroups induces a pre-order $(H) \lesssim (K)$ on the family of orbit types. For compact G, it is a partial ordering. If M/G is connected, then there is precisely one principal orbit type (H) and it satisfies $(H) \lesssim (K)$ for all isotropy subgroups $K \subseteq G$. If M is a Riemannian G-manifold, the regular points in M are exactly those whose slice representation $G_x \to \mathrm{O}(T_x(G \cdot x)^\perp)$ is trivial, where $T_x(G \cdot x)^\perp$ denotes the orthogonal complement of $T_x(G \cdot x)$ in $T_x M$. The set M_{reg} is open and dense in M, and $M_{\mathrm{reg}} \to M_{\mathrm{reg}}/G$ is a locally trivial fiber bundle.

Theorem. *Let M be a proper G-manifold. The orbit projection $\pi\colon M \to M/G$ is a Hurewicz (Serre) fibration if and only if $M = M_{\mathrm{reg}}$.*

P r o o f.   Suppose that $\pi\colon M \to M/G$ is a Serre fibration. There exists a G-invariant Riemannian metric making M a proper Riemannian G-manifold. By the differentiable slice theorem [9], for each $x \in M$ there exists a slice S_x such that the G-invariant neighborhood $G \cdot S_x$ of x is G-equivariantly diffeomorphic to the twisted

product $G \times_{G_x} S_x$. It follows that $G \cdot S_x/G \cong S_x/G_x$ is an open neighborhood of $\pi(x)$ in the orbit space M/G. The slice S_x can be chosen to be the diffeomorphic image of an open ball around the origin in the vector subspace $T_x(G \cdot x)^\perp$ of $T_x M$. Evidently, the restriction $\pi|_{G \cdot S_x} \colon G \cdot S_x \to G \cdot S_x/G$ is a Serre fibration as well.
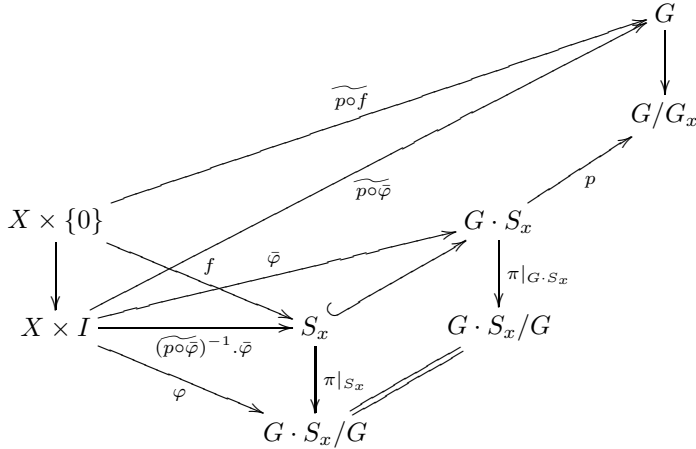
We claim that also $\pi|_{S_x} \colon S_x \to G \cdot S_x/G \cong S_x/G_x$ is a Serre fibration. Let $f \colon X \times \{0\} \to S_x$ be continuous and let $\varphi \colon X \times I \to G \cdot S_x/G$ be a homotopy of $\pi|_{S_x} \circ f$. Since $\pi|_{G \cdot S_x} \colon G \cdot S_x \to G \cdot S_x/G$ is a Serre fibration, there exists a homotopy $\bar{\varphi} \colon X \times I \to G \cdot S_x$ of f covering φ. Consider the projection $p \colon G \cdot S_x \cong G \times_{G_x} S_x \to G/G_x$ of the fiber bundle associated with the principal bundle $G \to G/G_x$ and the compositions $p \circ f$ and $p \circ \bar{\varphi}$. Now $p \circ f$ is constant and equals $eG_x \in G/G_x$ and thus allows a lift into G, e.g., $\widetilde{p \circ f} = e$. Since $G \to G/G_x$ is a fibration, there exists a homotopy $\widetilde{p \circ \bar{\varphi}} \colon X \times I \to G$ of $\widetilde{p \circ f}$ covering $p \circ \bar{\varphi}$. It follows that $(\widetilde{p \circ \bar{\varphi}})^{-1}.\bar{\varphi} \colon X \times I \to S_x$ is a homotopy of f covering φ. Hence the claim is proved.

We may view $\pi|_{S_x} \colon S_x \to G \cdot S_x/G \cong S_x/G_x$ as the orbit projection of the G_x-manifold S_x. Since we may consider the G_x-manifold S_x as a linear representation of a compact Lie group, the G_x-action on S_x must be trivial by Corollary 2.2. Since x was arbitrary, the statement follows. $\qquad\square$

Remark. If the orbit projection $\pi \colon M \to M/G$ is a Hurewicz (Serre) fibration, then it is regular.

2.4. *G*-**fibrations** ([10]). A G-equivariant continuous map $p \colon E \to B$ between G-spaces E and B is called a *G-fibration* if it is a fibration in the category of G-spaces and G-equivariant maps: For each G-space X, each G-equivariant continuous map $f \colon X \times \{0\} \to E$ and each G-equivariant homotopy $\varphi \colon X \times I \to B$ of $p \circ f$, there

exists a G-equivariant homotopy $\bar{\varphi}$ of f covering φ. The G-action on I is trivial. It is easy to verify that a G-fibration is a fibration in the usual sense. More precisely, one can show that $p\colon E \to B$ is a G-fibration if and only if $p^H\colon E^H \to B^H$ is a fibration for each closed subgroup $H \subseteq G$. Note that $E^H = \{e \in E\colon h \cdot e = e \text{ for all } h \in H\}$ and p^H denotes the restriction $p|_{E^H}$.

Theorem. *Let M be a proper G-manifold. The orbit projection $\pi\colon M \to M/G$ is a Hurewicz (Serre) G-fibration if and only if $M = M_{\mathrm{reg}}$.*

P r o o f.   Suppose that $M = M_{\mathrm{reg}}$. Let S_x be a slice at $x \in M$. Then $G \cdot S_x \cong G/G_x \times S_x$, $G \cdot S_x/G \cong S_x$, and $\pi|_{G \cdot S_x}\colon G/G_x \times S_x \to S_x$ is given by $([g], s) \mapsto s$. Hence $\pi|_{G \cdot S_x}$ is obviously a G-fibration; a G-equivariant homotopy of f covering φ is given by $\bar{\varphi} = (\mathrm{pr}_{G/G_x} \circ f, \varphi)$. Since M/G is paracompact and Hausdorff, one can then show that π is a G-fibration analogously with Hurewicz's uniformization theorem [2].

Since each G-fibration is a fibration, the other implication is an immediate consequence of Theorem 2.3. $\square$

2.5. Proper locally smooth actions ([1])**.** Let G be a Lie group and M a proper G-space, i.e., M is paracompact and Hausdorff and the G-action is continuous. Let $G \cdot x$ be an orbit in M and let V be a Euclidean vector space on which G_x operates orthogonally. Then a *linear tube* about $G \cdot x$ in M is a G-equivariant embedding onto an open neighborhood of $G \cdot x$ of the form $G \times_{G_x} V \to M$. A G-space M is called *locally smooth* if there exists a linear tube about each orbit. In that case M must be a topological manifold. It follows from the differentiable slice theorem [9] that proper G-manifolds in the sense of 2.3 are locally smooth.

The definition of local smoothness can be extended to manifolds M with boundary. For this we require, for orbits $G \cdot x$ lying on the boundary of M, tubes of the form $G \times_{G_x} V^+ \to M$, where $V^+ = \{y \in \mathbb{R}^n\colon y_1 \geqslant 0\}$ and G_x acts orthogonally on V^+ (in particular, the y_1-axis is stationary).

Properness guarantees that all isotropy groups G_x are compact. Hence the orbits in each G_x-space V (or V^+) are compact manifolds and therefore CW complexes. It follows that the arguments in 2.2, 2.3, and 2.4 are applicable and we obtain

Theorem. *Let M be a proper locally smooth G-space (with boundary). The orbit projection $\pi\colon M \to M/G$ is a Hurewicz (Serre) $(G\text{-})$fibration if and only if $M = M_{\mathrm{reg}}$.*

3.1. Quasifibrations ([4]). A continuous map $p\colon E \to B$ with B path connected is called a *quasifibration* if the induced map $p_*\colon \pi_n(E, p^{-1}(b), e) \to \pi_n(B, b)$ is an isomorphism for all $b \in B$, $e \in p^{-1}(b)$ and $n \geqslant 0$, or equivalently, if the inclusion of each fiber $p^{-1}(b)$ into the homotopy fiber F_b of p over b is a weak homotopy equivalence. The fiber $p^{-1}(b)$ is included in

$$F_b = \{(e, \gamma) \in E \times C^0(I, B)\colon \gamma(0) = p(e),\ \gamma(1) = b\}$$

as the pairs (e, γ) with $e \in p^{-1}(b)$ and γ the constant path at b. If B is not path connected, then $p\colon E \to B$ is a *quasifibration* if the restriction of p over each path component of B is a quasifibration. For quasifibrations the homotopy sequence (2.1) is exact. All fibers of a quasifibration $p\colon E \to B$ with B path connected belong to the same weak homotopy type. Hurewicz and Serre fibrations are quasifibrations.

Lemma 3.2. *Let M be a proper G-manifold with a connected orbit space M/G. Let k be the least number of connected components of isotropy groups of dimension $m := \min\{\dim G_x\colon x \in M\}$. Then the following conditions are equivalent:*

(1) $G \cdot x$ *is a principal orbit.*

(2) G_x *is of dimension m and has k connected components.*

P r o o f. Let $x \in M$ be such that G_x has minimal dimension and the least number of connected components for this dimension in all of M. Let S_x be a slice at x. For any $y \in G \cdot S_x$ we have $y \in g.S_x = S_{g.x}$ and thus $G_y \subseteq G_{g.x} = gG_xg^{-1}$ for some $g \in G$. By the choice of x we find $G_y = gG_xg^{-1}$, which shows that $G \cdot x$ is principal. The converse implication follows from the fact that there is precisely one principal orbit type, if M/G is connected. $\qquad\square$

Theorem 3.3. *Let M be a proper G-manifold. Let one of the following conditions be satisfied:*

(1) G *is finite.*

(2) G *is compact, connected, and simply connected.*

(3) G *is compact and there exists a connected and simply connected orbit in each path component of M/G.*

(4) *There exists a weakly contractible orbit in each path component of M/G.*

Then the orbit projection $\pi\colon M \to M/G$ is a quasifibration if and only if $M = M_{\mathrm{reg}}$.

P r o o f. Suppose that $\pi\colon M \to M/G$ is a quasifibration. We may suppose that M/G is path connected, by restricting π over each path component of M/G and treating them separately. Then all orbits belong to the same weak homotopy type.

We claim that each of the four conditions in the theorem implies that all occurring isotropy groups have the same dimension and the same number of connected components. If G is finite, all orbits and thus all isotropy groups have the same cardinality. Assume that G is compact. Let $G \cdot x$ and $G \cdot y$ be distinct orbits. Since they are compact manifolds, we find $\dim G \cdot x = \dim G \cdot y$ and, consequently, $\dim G_x = \dim G_y$. If (2) is satisfied we may conclude from the homotopy sequences of the fibrations $G \to G/G_x \cong G \cdot x$ and $G \to G/G_y \cong G \cdot y$ that $\pi_0(G_x) \cong \pi_1(G/G_x) \cong \pi_1(G/G_y) \cong \pi_0(G_y)$. Assume that (3) holds true. Then each orbit is connected and simply connected. Let $G \cdot x$ be principal and $G \cdot y$ arbitrary. Without loss of generality $G_x \subseteq G_y$ and we have a locally trivial fiber bundle $G/G_x \to G/G_y$ with fiber G_y/G_x. The associated homotopy sequence yields that G_y/G_x is trivial, whence the statement. Finally, if condition (4) is fulfilled, all orbits are weakly contractible, whence any two isotropy groups have the same weak homotopy type, again by the homotopy sequence of $G \to G/G_x \cong G \cdot x$. Since all isotropy groups are compact manifolds, the claim follows.

Since M/G is connected, there is precisely one principal orbit type, namely the type corresponding to the isotropy group with minimal dimension and minimal number of connected components. By the claim, all points are regular. $\qquad \square$

3.4. G-quasifibrations. A G-equivariant continuous map $p\colon E \to B$ between G-spaces E and B is called a G-*quasifibration* if $p^H\colon E^H \to B^H$ is a quasifibration for each closed subgroup $H \subseteq G$. In particular, a G-quasifibration is a quasifibration. Any G-fibration is a G-quasifibration.

Corollary. *Let M be a proper G-manifold. Suppose that one of the conditions (1)–(4) in Theorem 3.3 is satisfied. Then the orbit projection $\pi\colon M \to M/G$ is a G-quasifibration if and only if $M = M_{\mathrm{reg}}$.*

P r o o f. The statement follows from Theorem 2.4 and Theorem 3.3. $\qquad \square$

3.5. Equivariant homotopy theory. We collect a few results from equivariant homotopy theory needed in the proof of Theorem 3.6.

For a definition of G-CW complexes see the cited references.

Result 3.5.1 ([12, 4.14], [7, 3.3.5]; see also [8])**.** *Suppose that X and Y are proper G-spaces. Let $f\colon X \to Y$ be a G-map, and let $y \in Y$ have isotropy group H. Then, regarding f as an H-equivariant map based at y, the homotopy fiber F_y of f has the H-homotopy type of an H-CW complex whenever X and Y have the G-homotopy type of G-CW complexes.*

It is proved in [5] that a proper G-manifold M has a G-CW structure. The orbit space M/G is triangulable by [11].

Result 3.5.2 ([6, 1.1], [10, II.2.7]). *A G-map $f\colon X \to Y$ of G-CW complexes is a G-homotopy equivalence if and only if for any subgroup $H \subseteq G$ which occurs as isotropy subgroup of X or Y the induced map $f^H\colon X^H \to Y^H$ is a homotopy equivalence.*

3.6. Let M be a proper G-manifold. For the orbit projection $\pi\colon M \to M/G$ consider the path fibration $p\colon E_\pi \to M/G$, where

$$E_\pi = \{(x,\gamma) \in M \times C^0(I, M/G)\colon \gamma(0) = \pi(x)\}$$

and $p(x,\gamma) = \gamma(1)$. The space $C^0(I, M/G)$ carries the compact-open topology and E_π inherits the subspace topology from $M \times C^0(I, M/G)$. Then $p\colon E_\pi \to M/G$ is a Hurewicz fibration with fibers

$$F_z = \{(x,\gamma) \in M \times C^0(I, M/G)\colon \gamma(0) = \pi(x),\ \gamma(1) = z\},$$

$z \in M/G$. The G-action on M induces a natural G-action on each fiber $\pi^{-1}(z)$ and on each homotopy fiber F_z for which the canonical inclusion $\pi^{-1}(z) \hookrightarrow F_z$ is equivariant.

Assume that M/G is path connected. Let $\pi^{-1}(u) = G \cdot x$ and $\pi^{-1}(z) = G \cdot y$ be distinct orbits in M and choose a path $\alpha\colon I \to M/G$ with $\alpha(0) = u$ and $\alpha(1) = z$. We have the diagram

$$\pi^{-1}(u) \hookrightarrow F_u \longrightarrow F_z \hookleftarrow \pi^{-1}(z)\ ,$$

where $F_u \to F_z$ given by $(x,\gamma) \mapsto (x, \alpha\gamma)$ is a homotopy equivalence. Note that each arrow in the diagram is G-equivariant.

Theorem. *Let M be a proper G-manifold and assume that M/G is path connected. The following conditions are equivalent:*
(1) *The orbit projection $\pi\colon M \to M/G$ is a G-quasifibration.*
(2) *The inclusion $\pi^{-1}(z) \hookrightarrow F_z$ is a homotopy equivalence allowing a G-equivariant homotopy inverse for all $z \in M/G$.*
(3) *Let (H) be the principal orbit type. For any orbit type (K) there is a (weak) homotopy equivalence $f\colon G/H \to G/K$, and f is G-equivariant.*
(4) *$M = M_{\mathrm{reg}}$.*

P r o o f.   We prove that (1) implies (2). If $\pi\colon M \to M/G$ is a G-quasifibration, then the fixed point maps $\pi^H\colon M^H \to M/G$ are quasifibrations for all closed subgroups $H \subseteq G$. Let $z \in M/G$ be arbitrary. The fiber $\pi^{-1}(z)$ and the homotopy fiber F_z are G-spaces in a canonical way, and we have $(\pi^H)^{-1}(z) = \pi^{-1}(z)^H$ and

$$F_z^H = \{(x,\gamma) \in M^H \times C^0(I, M/G)\colon \gamma(0) = \pi^H(x), \ \gamma(1) = z\}.$$

Since π^H is a quasifibration, the canonical inclusion $\pi^{-1}(z)^H \hookrightarrow F_z^H$ is a weak homotopy equivalence. By 3.5.1, we may conclude that the F_z^H are homotopy equivalent to CW complexes and that F_z has the G-homotopy type of a G-CW complex. By Whitehead's theorem, the inclusions $\pi^{-1}(z)^H \hookrightarrow F_z^H$ are homotopy equivalences. By 3.5.2 we obtain that the inclusion $\pi^{-1}(z) \hookrightarrow F_z$ is even a G-homotopy equivalence. Hence (2).

It is evident that (2) implies (3).

Let us assume that condition (3) is satisfied. We prove (4). Without loss of generality we may suppose that $H \subseteq K$ and have the commuting diagram

$$
\begin{array}{ccc}
H & \hookrightarrow G \longrightarrow & G/H \\
\downarrow & \downarrow{\scriptstyle \mathrm{id}} & \downarrow{\scriptstyle f} \\
K & \hookrightarrow G \longrightarrow & G/K
\end{array}
$$

Consequently, using the fact that $G \to G/H$ and $G \to G/K$ are fibrations, we obtain the commuting diagram

$$
\begin{array}{ccccccccc}
\pi_{n+1}(G) & \longrightarrow & \pi_{n+1}(G/H) & \longrightarrow & \pi_n(H) & \longrightarrow & \pi_n(G) & \longrightarrow & \pi_n(G/H) \\
\downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\
\pi_{n+1}(G) & \longrightarrow & \pi_{n+1}(G/K) & \longrightarrow & \pi_n(K) & \longrightarrow & \pi_n(G) & \longrightarrow & \pi_n(G/K)
\end{array}
$$

for each $n \geqslant 0$ where the rows are exact and all vertical arrows apart from the middle one are isomorphisms. By the five lemma, the vertical middle arrow is an isomorphism as well. It follows that $\pi_n(H) \cong \pi_n(K)$ for all $n \geqslant 0$ and, by Whitehead's theorem, we find that H and K are homotopically equivalent. Since H and K are compact Lie groups, we conclude that they have the same dimension and the same number of connected components. By Lemma 3.2, all points in M have to be regular.

Theorem 2.4 yields that (4) implies (1). $\square$

Corollary. *Let M be a proper locally smooth connected G-space (with boundary) and suppose that the principal orbits are of codimension 1. Then the orbit projection $\pi\colon M \to M/G$ is a quasifibration if and only if $M = M_{\mathrm{reg}}$.*

P r o o f. It is proved in [1, IV.8] that under these conditions either all orbits are principal or M is equivalent as a G-space to the mapping cylinder of the equivariant map $G/H \to G/K$ for (H) principal and $(H) < (K)$ or to the union of the two mapping cylinders of $G/H \to G/K_i$ for (H) principal and $(H) < (K_i)$, $i = 0, 1$. In the latter cases the orbit space M/G is isomorphic either to $[0, 1)$ or to $[0, 1]$, and the natural projection of a mapping cylinder identifies with π. This projection is a quasifibration if and only if the mapping inducing the mapping cylinder is a weak homotopy equivalence. By the implication $(3) \Rightarrow (4)$ in the forgoing theorem, the statement of the corollary follows. $\qquad\square$

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