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A SELF-ADAPTIVE TRUST REGION METHOD FOR THE EXTENDED LINEAR COMPLEMENTARITY PROBLEMS*

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Abstract. By using some NCP functions, we reformulate the extended linear complementarity problem as a nonsmooth equation. Then we propose a self-adaptive trust region algorithm for solving this nonsmooth equation. The novelty of this method is that the trust region radius is controlled by the objective function value which can be adjusted automatically according to the algorithm. The global convergence is obtained under mild conditions and the local superlinear convergence rate is also established under strict complementarity conditions.

Keywords: extended linear complementarity, self-adaptive trust region method, global convergence, local superlinear convergence

MSC 2010: 65K05, 90C30

1. Introduction

The extended linear complementarity problem (XLCP) introduced by Mangasarian and Pang [19] is to find a pair of vectors x and y in $\mathbb{R}^n$ such that

$$Mx - Ny \in \mathcal{P}, \quad x \geqslant 0, \quad y \geqslant 0, \quad \langle x, y\rangle = 0, \tag{1}$$

where M and N are two real matrices of order $m \times n$, $\mathcal{P}$ is a polyhedral set in $\mathbb{R}^m$ and $\langle\cdot,\cdot\rangle$ denotes the usual inner product. In the special case when $m = n$ and $\mathcal{P}$ is a singleton, XLCP reduces to the horizontal linear complementarity which has been a subject of extensive research in recent years [2], [11], [13], [25], [28]. If one further assumes that N is the identity matrix, then the classical linear complementarity problem [5] is obtained.

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Many researchers have studied the XLCP. For example, Mangasarian and Pang [19] established a number of properties of XLCP and the following quadratic bilinear program (BLP):

$$\min\langle x, y\rangle \quad \text{such that} \quad Mx - Ny \in \mathcal{P}, \quad x \geqslant 0, \quad y \geqslant 0. \tag{2}$$

It was shown that if the matrix MN^T is copositive on the dual of the recession cone of the set $\mathcal{P}$, then every Kuhn-Tucker point of (2) is a solution of XLCP. A further study of XLCP and the associated BLP was undertaken by Gowda [12]. Recently, Andreani, Martínez [1] and Solodov [26] considered reformulating the XLCP as the unconstrained or nonnegative constrained optimization problem, and then gave some sufficient conditions for every stationary point of the optimization problem to be a solution of XLCP. More recently, Zhang and Xiu [29] considered the error bound result. We note that all these researchers concentrate on the theoretical study while there are few algorithms available.

The aim of this paper is to propose a solution method for the XLCP. For convenience, we assume $m = n$, and the polyhedral set $\mathcal{P}$ in $\mathbb{R}^n$ appearing in the statement of XLCP (1) is presented as

$$\mathcal{P} = \{u \in \mathbb{R}^n \colon\ Au \geqslant h\},$$

where A is an $n \times n$ real matrix and $h \in \mathbb{R}^n$. For this representation, the recession cone of the set $\mathcal{P}$ is the set

$$0^+\mathcal{P} = \{u \in \mathbb{R}^n \colon\ Au \geqslant 0\},$$

and its dual is

$$(0^+\mathcal{P})^* = \{v \in \mathbb{R}^n \colon\ \langle v, u\rangle \geqslant 0 \text{ for all } u \in 0^+\mathcal{P}\} = \{v = A^T\mu \text{ for some } \mu \geqslant 0\},$$

where A^T denotes the transpose of the matrix A. Finally, we recall that a square matrix Q is said to be copositive on a cone $\mathcal{K}$ if $\langle Qv, v\rangle \geqslant 0$ for all $v \in \mathcal{K}$.

Throughout this paper, we assume that the feasible set of XLCP is nonempty:

$$\{(x, y) \colon\ Mx - Ny \in \mathcal{P},\ x \geqslant 0,\ y \geqslant 0\} \neq \emptyset.$$

Relying on the discussion in [26] and using some NCP function (a function $\varphi\colon \mathbb{R}^2 \to \mathbb{R}$ is called an NCP function if for any $(a, b)^T \in \mathbb{R}^2$, $\varphi(a, b) = 0 \Leftrightarrow a \geqslant 0$,

$b \geqslant 0$, $ab = 0$, see [9], [10], [17] for example), we know that solving XLCP (1) is equivalent to solving the system of nonlinear equation

$$\Phi(x,y,z) = \begin{pmatrix} \Phi(x,y) \\ AMx - ANy - b - z \\ \Phi_+(z) \end{pmatrix} = 0 \tag{3}$$

where $\Phi(x,y) = (\varphi(x_1,y_1), \varphi(x_2,y_2), \dots, \varphi(x_n,y_n))^T \in \mathbb{R}^n$ with $\varphi(a,b) = a + b - \sqrt{a^2+b^2}$ for any $a, b \in \mathbb{R}$ and $\Phi_+(z) = (\varphi_+(z_1), \varphi_+(z_2), \dots, \varphi_+(z_n))^T \in \mathbb{R}^n$ with $\varphi_+(a) = \max(-a, 0)$ for any $a \in \mathbb{R}$. For convenience, we rewrite $w = (x^T, y^T, z^T)^T$ and accordingly we denote $\Phi(w) = \Phi(x,y,z)$. Then solving the XLCP is equivalent to solving the minimization problem

$$\min \Psi(w) = \frac{1}{2}\|\Phi(w)\|^2, \tag{4}$$

with the objective function value zero.

The trust region method is one of the most important methods for problem (4) arising in some important mathematical problems such as nonlinear complementarity and variational inequalities problems, see [4], [15], [16], [21], [27] for example. In trust region methods, the initial trust region radius plays an important role since it determines the direction and stepsize of the current iteration. However, in the traditional trust region methods, the initial radius is given randomly which effects the efficiency of the algorithm dramatically. So many self-adaptive trust region methods (for unconstrained optimization) were proposed in recent years [8], [14], [24], [30]. The main idea of these self-adaptive trust region methods is that the initial trust region radius is controlled by the gradient of the current point. Numerical tests showed that the self-adaptive method is encouraging.

Motivated by the idea of [8], [14], [24], [30], in this paper we propose a self-adaptive trust region method for problem (4) arising in the XLCP. In our method, the trust radius is controlled by the objective function value which can be adjusted automatically according to the algorithm. The global convergence of the algorithm is obtained under certain conditions and the local superlinear convergence rate is also obtained under the strict complementarity assumptions.

The paper is organized as follows: In Section 2, we give some basic preliminaries. In Section 3, we describe the algorithm model and prove its global convergence. The local superlinear convergence rate is proved in Section 4. Numerical tests are reported in Section 5 and the conclusion is given in Section 6.

2. Some preliminaries

Let $G\colon \mathbb{R}^n \to \mathbb{R}^n$ be locally Lipschitz continuous, according to Rademacher's theorem, G is differentiable almost everywhere. Let D_G be the set where G is differentiable, the B-differential of G at $x \in \mathbb{R}^n$ is defined by

$$\partial_B G(x) = \Big\{ H \in \mathbb{R}^{n\times n}\colon\ H = \lim_{\substack{x_k \to x,\\ x_k \in D_G}} G'(x_k) \Big\}.$$

The generalized Jacobian of G at x in the sense of Clarke [3] is defined by

$$\partial G(x) = \operatorname{conv} \partial_B G(x).$$

Furthermore, we write

$$\partial_c G(x)^T = \partial G_1(x)^T \times \partial G_2(x)^T \times \ldots \times \partial G_n(x)^T$$

for the C-subdifferential of G at x, where the right-hand side denotes a set of matrices whose ith column can be any element from the generalized gradient of $\partial G_i(x)^T$.

Similar to the technique of proof of Facchinei and Soares [7], Qi [22] and Qi and Sun [23], see also Kanzow and Pieper [18], we can obtain the following semismooth properties.

Lemma 2.1. *Assume that $\{w_k\}$ is a convergent sequence with a limit point $w^\star$. Then the function Φ defined by* (3) *is semismooth, which means*

$$\|\Phi(w_k) - \Phi(w^\star) - H_k(w_k - w^\star)\| = o(\|w_k - w^\star)\|$$

for any $H_k \in \partial_c \Phi(w_k)$.

Lemma 2.2. *The generalized gradient of the function $\varphi(a,b)$ at a point $(a,b) \in \mathbb{R}^2$ is equal to the set of all $(g_a, g_b)^T \in \mathbb{R}^2$ with*

$$(g_a, g_b) = \begin{cases} \Big(1 - \dfrac{a}{\|(a,b)\|}, 1 - \dfrac{b}{\|(a,b)\|}\Big) & \text{if } (a,b) \neq 0,\\ (1-\xi, 1-\zeta) & \text{if } (a,b) = 0, \end{cases}$$

where (ξ,ζ) is any vector satisfying $\|(\xi,\zeta)\| \leqslant 1$. The generalized gradient of the function φ_+ at a point a is equal to

$$\partial\varphi_+(a) = \begin{cases} 1 & \text{if } a < 0,\\ [0,1] & \text{if } a = 0,\\ 0 & \text{if } a > 0. \end{cases}$$

As a consequence of Lemma 2.2 we obtain the following result.

Theorem 2.1. *Let $w \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n$ be given. Then any matrix $H \in \partial_c \Phi(w)$ has the form*

$$H = \begin{pmatrix} D_a(x) & D_b(y) & 0 \\ AM & -AN & -I \\ 0 & 0 & D_a(z) \end{pmatrix}$$

where

$$D_a(x) := \operatorname{diag}(a_i(x)), \quad D_b(y) := \operatorname{diag}(b_i(y)), \quad D_a(z) := \operatorname{diag}(a_i(z))$$

and $(a_i(x), b_i(y)) \in (g_a, g_b)$ and $a_i(z) \in \varphi_+(z)$ are defined in Lemma 2.2.

In what follows we give conditions that guarantee the stationary point of (4) to be a solution of the XLCP (1). The proof can be found in [26].

Lemma 2.3. *Suppose that MN^T is copositive on $0^+\mathcal{P}$. Then the stationary point of* (4) *is a solution of XLCP* (1).

Lemma 2.4. *The function $\Psi(w)$ is continuously differentiable with $\nabla\Psi(w) = H^T\Phi(w)$, where $H \in \partial_C \Phi(w)$.*

3. Algorithm model and its global convergence

In this section we state our algorithm for solving the XLCP and prove its global convergence.

For a given iteration point w_k we will solve the following trust region subproblem:

$$\min Q_k(d) = \frac{1}{2}\|\Phi(w_k) + H_k^T d\|^2, \quad \text{such that } \|d\| \leqslant \Delta_k, \tag{5}$$

where $H_k \in \partial_C \Phi(w_k)$ and $\Delta_k > 0$ is the trust region radius.

Let d_k be a solution of problem (5). We denote

$$\mathrm{Ared}_k = \frac{1}{2}\|\Phi(w_k)\|^2 - \frac{1}{2}\|\Phi(w_k + d_k)\|^2, \quad \mathrm{Pred}_k = Q_k(0) - Q_k(d_k)$$

and compute $r_k = \mathrm{Ared}_k/\mathrm{Pred}_k$.

Algorithm 3.1.

Step 0. Given $w_0 \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n$, $\eta \in (0,1)$, $c \in (0,1)$, set $p_0 = 0$, $k = 0$.

Step 1. If $\Phi(w_k) = 0$ or $\nabla\Psi(w_k) = 0$, stop.

Step 2. Choose $H_k \in \partial_C \Phi(w_k)$, if H_k is nonsingular, set $M_k = \|H_k^{-1}\|$, otherwise, choose $\mu_k > 0$ such that $H_k + \mu_k I$ is nonsingular, set $M_k = \|(H_k + \mu_k I)^{-1}\|$ and solve the following trust region subproblem:

$$\min \frac{1}{2}\|\Phi(w_k) + H_k^T d\|^2, \text{ such that } \|d\| \leqslant c^{p_k} M_k \|\Phi(w_k)\|. \tag{6}$$

Denote the solution of (6) by d_k, if $d_k = 0$ stop.

Step 3. Compute r_k, if $r_k \geqslant \eta$, set $x_{k+1} = x_k + d_k$, $p_{k+1} = 0$, $k := k+1$, go to Step 1, otherwise, set $p_{k+1} = p_k + 1$, go to Step 2.

The kth iteration is called successful when $r_k \geqslant \eta$, otherwise, the kth iteration is called unsuccessful. The following result can be found in Powell [20].

Lemma 3.1. *Let d_k be computed by (6). Then the inequality*

$$\mathrm{Pred}_k \geqslant \frac{1}{2}\|\nabla\Psi(w_k)\| \min\left\{ c^{p_k} M_k \|\Phi(w_k)\|, \frac{\|\nabla\Psi(w_k)\|}{\|H_k^T H_k\|} \right\} \tag{7}$$

holds for all k.

Lemma 3.2. *Let $\{w_k\}$ be a sequence generated by Algorithm 3.1 and let $\{w_k\}_K$ be a subsequence converging to $w^\star$. Assume there exists a constant $\mu > 0$ such that $\mu_k < \mu$ for all k. If $\|\Phi(w^\star)\| \neq 0$ and $\|\nabla\Psi(w^\star)\| \neq 0$, then*

$$\limsup_{k\to\infty,\, k\in K} p_k < +\infty. \tag{8}$$

Proof. Similar to the technique of proof in [16] or [30]. □

Lemma 3.2 shows that Algorithm 3.1 is well-defined. In what follows we prove our main convergence result.

Theorem 3.1. *Let $\{w_k\}$ be a sequence generated by Algorithm 3.1, $w^\star$ be an accumulation point of $\{w_k\}$. Then we have $\|\Phi(w^\star)\| = 0$ or $\|\nabla\Psi(w^\star)\| = 0$.*

Proof. We assume that $\{w_k\}_K \to w^\star$. Since $w_{k+1} = w_k$ for all unsuccessful iterations k and since there are infinitely many successful iterations by Lemma 3.2, we can assume without loss of generality that all iterations $k \in K$ are successful. If $\|\Phi(w^\star)\| = 0$, the conclusion is proved. Otherwise, there exists a positive constant δ_0 such that $\|\Phi(w_k)\| > \delta_0$ for all $k \in K$. Suppose that $\|\nabla\Psi(w^\star)\| \neq 0$, then by the upper semicontinuity of the generalized Jacobian there exist two positive constants δ_1, δ_2 such that

$$\|\nabla\Psi(w_k)\| > \delta_1 \quad \text{and} \quad \|H_k^T H_k\| \leqslant \delta_2$$

for all $k \in K$. Since the iterations $k \in K$ are successful, we have $r_k \geqslant \eta$ for all $k \in K$. On the other hand, by the definition of M_k, there exists an $M > 0$ such that $M_k \geqslant M$ for all $k \in K$. Therefore we have

$$
\begin{aligned}
(9) \qquad \Psi(w_k) - \Psi(w_{k+1}) &\geqslant \eta \operatorname{Pred}_k \\
&\geqslant \frac{1}{2}\eta \|\nabla \Psi(w_k)\| \min\Big\{c^{p_k} M_k \|\Phi(w_k)\|, \frac{\|\nabla \Psi(w_k)\|}{\|H_k^T H_k\|}\Big\} \\
&\geqslant \frac{1}{2}\eta \delta_1 \min\Big\{c^{p_k} M \delta_0, \frac{\delta_1}{\delta_2}\Big\}
\end{aligned}
$$

for all $k \in K$. Since the function value sequence $\{\Psi(w_k)\}$ decreases monotonically and is bounded below from zero, it is convergent. From (9) we have

$$
\begin{aligned}
\sum_{k \in K} \frac{1}{2}\eta \delta_1 \min\Big\{c^{p_k} M \delta_0, \frac{\delta_1}{\delta_2}\Big\} &\leqslant \sum_{k \in K} \Psi(w_k) - \Psi(w_{k+1}) \\
&\leqslant \sum_{k=0}^{\infty} \Psi(w_k) - \Psi(w_{k+1}) < +\infty.
\end{aligned}
$$

This implies $p_k \to +\infty$, a contradiction to Lemma 3.2. $\square$

By Lemma 2.3 and Theorem 3.1 we know that if MN^T is copositive on $0^+\mathcal{P}$, then every accumulation point of the sequence generated by Algorithm 3.1 is a solution of the XLCP.

4. Local convergence rate

In this section we will analyze the local convergence of Algorithm 3.1. We first prove the nonsingularity of the Jacobian of $\Phi(w)$ at a solution of the XLCP.

Lemma 4.1. *Assume the matrix AMN^TA^T is positive definite, $w^\star = (x^\star, y^\star, z^\star)$ is a solution of the XLCP and $(x^\star, y^\star, z^\star)$ satisfies $x_i^\star + y_i^\star > 0$, $z^\star > 0$. Then the Jacobian $\Phi'(w^\star)$ is nonsingular. Furthermore, $w^\star = (x^\star, y^\star, z^\star)$ is the unique solution of the equation $\Phi(w) = 0$.*

Proof. The condition in the lemma implies that Φ is continuously differentiable. Define two index sets

$$
\mathcal{B}_1 = \{i \in \{1, 2, \ldots, n\} \colon x_i^\star > 0\}, \quad \mathcal{B}_2 = \{i \in \{1, 2, \ldots, n\} \colon y_i^\star > 0\};
$$

then $\mathcal{B}_1 \cup \mathcal{B}_2 = \{1, 2, \ldots, n\}$. Note that

$$
\Phi'(w^\star)^T = \begin{pmatrix} D_a(x^\star) & M^TA^T & 0 \\ D_b(y^\star) & -N^TA^T & 0 \\ 0 & -I & I \end{pmatrix}
$$

with

$$D_a(x^\star) = \mathrm{diag}\Big(\frac{\partial\varphi}{\partial a}(x_i^\star, y_i^\star)\Big), \quad D_b(x^\star) = \mathrm{diag}\Big(\frac{\partial\varphi}{\partial b}(x_i^\star, y_i^\star)\Big).$$

By strict complementarity, we have

$$\frac{\partial\varphi}{\partial a}(x_i^\star, y_i^\star) = \begin{cases} 0 & \text{if } i \in \mathcal{B}_1, \\ 1 & \text{if } i \in \mathcal{B}_2; \end{cases}$$
$$\frac{\partial\varphi}{\partial b}(x_i^\star, y_i^\star) = \begin{cases} 1 & \text{if } i \in \mathcal{B}_1, \\ 0 & \text{if } i \in \mathcal{B}_2. \end{cases}$$

Now assume that there exists a vector $q = (q^{(1)}, q^{(2)}, q^{(3)})$ such that

$$\Phi'(w^\star)^T q = 0.$$

Then we have

$$D_a(x^\star)q^{(1)} + M^T A^T q^{(2)} = 0, \tag{10}$$
$$D_b(x^\star)q^{(1)} - N^T A^T q^{(2)} = 0, \tag{11}$$
$$-q^{(2)} + q^{(3)} = 0. \tag{12}$$

By (10), (11) we have

$$(q^{(1)})^T D_a(x^\star) D_b(y^\star) q^{(1)} = -(q^{(2)})^T AMN^T A^T q^{(2)}. \tag{13}$$

Since the matrix $D_a(x^\star)D_b(y^\star)$ is positive semidefinite and AMN^TA^T is positive definite, (13) implies that $q^{(2)} = 0$ and therefore $q^{(3)} = 0$ is obtained immediately from (13).

Now (10), (11) become

$$D_a(x^\star)q^{(1)} = 0, \quad D_b(x^\star)q^{(1)} = 0. \tag{14}$$

Let $q^{(1)}_{\mathcal{B}_1}$ denote the $|\mathcal{B}_1|$ dimension subvector of $q^{(1)}$ consisting of the components $q_i^{(1)}$ $(i \in \mathcal{B}_1)$; $(D_a)_{\mathcal{B}_1}$, $(D_b)_{\mathcal{B}_1}$ denote the $|\mathcal{B}_1| \times |\mathcal{B}_1|$ diagonal matrix containing the diagonal entries a_{ii} $(i \in \mathcal{B}_1)$ for the matrix D_a, D_b. Similarly, we can define the subvector and submatrix associated with the set $\mathcal{B}_2$. Then from (14) we obtain that

$$(D_a)_{\mathcal{B}_1} q^{(1)}_{\mathcal{B}_1} = 0, \quad (D_b)_{\mathcal{B}_2} q^{(1)}_{\mathcal{B}_2} = 0,$$

which implies that

$$q^{(1)} = (q^{(1)}_{\mathcal{B}_1} q^{(1)}_{\mathcal{B}_2}) = 0.$$

This shows that the Jacobian $\Phi'(w^\star)$ is nonsingular.

Now, by the standard result of [6], there exists a constant $c_1 > 0$ such that

$$\|\Phi(x,y,z)\| \geqslant c_1\|(x,y,z) - (x^\star, y^\star, z^\star)\|$$

for all (x,y,z) sufficiently close to $(x^\star, y^\star, z^\star)$; this inequality shows that $(x^\star, y^\star, z^\star)$ is the locally unique solution of the equation $\Phi(x,y,z) = 0$. On the other hand, since the solution set is convex, it follows that $(x^\star, y^\star, z^\star)$ is also the global unique solution. This completes the proof. □

A point $w^\star \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n$ is said to be a BD regular solution of the XLCP if all elements in the generalized Jacobian $\partial\Phi(w^\star)$ are nonsingular.

By Lemma 4.1 we know that the generalized Newton direction exists and that it is the unique solution of the trust region subproblem (6), hence similarly to [16, Theorem 3.10] we get the following local convergence result.

Theorem 4.1. *Let $\{w_k\}$ be any sequence generated by Algorithm* 3.1. *If $\{w_k\}$ has an accumulation point $w^\star$ which is a BD-regular solution of the XLCP, then the whole sequence $\{w_k\}$ converges to $w^\star$ superlinearly.*

5. Numerical tests

In this section we test our algorithm on some typical test problems. The program code was written in MATLAB and run in MATLAB 7.1 environment. The test problems are LCP problems, i.e., N is an identity matrix and $\mathcal{P}$ is a singleton, which means we find $x \in \mathbb{R}^n$, $y \in \mathbb{R}^n$ such that

$$Mx - y = -q, \quad x \geqslant 0, \quad y \geqslant 0, \quad \langle x, y\rangle = 0,$$

where $q \in \mathbb{R}^n$.

The parameters are chosen as $\eta = 0.1$, $c = 0.5$. The stop criterion is $\|\nabla\Psi(w_k)\| < \varepsilon$ or $\|\Phi(w_k)\| < \varepsilon$ with $\varepsilon = 10^{-5}$. We compare our algorithm with the traditional trust region algorithm, where for the latter, we used some different initial trust region radii $\Delta_0 = 0.01, 0.1, 10$ respectively. The test results are summarized in Tab. 1, where we use No. to denote the number of our problems, Size denotes the size of our test problems, n_t denotes the number of the iterations, time denotes the CPU time used when the iteration is stopped. We also stop the execution when 5000 iterations were completed without achieving convergence and denote this situation by $*$. In the last row, we use AV to denote the average iteration for all test problems.

From Tab. 1 we see that our algorithm can solve these problems efficiently. Compared with the traditional algorithm, our algorithm is more efficient for problems 2, 9 and 10. For the others, although there exists a Δ_0 such that the traditional algorithm is more efficient, the Δ_0 is different for different problems, which shows that the traditional algorithm depends on the initial trust region radius. At last, from the average iteration for all test problems we see that our algorithm is the most efficient choice.

No.	Size	$\Delta_0 = 0.01$: n_t/time	$\Delta_0 = 0.1$: n_t/time	$\Delta_0 = 10$: n_t/time	adaptive: n_t/time
1	2	5/0.5938	159/0.3594	5/0.2344	5/0.5938
2	16	232/2.05156	*	124/1.1094	80/0.2188
3	4	175/0.4063	22/0.5156	94/2.1094	36/08281
4	3	99/0.6719	12/0.1094	6/0.0625	6/0.0781
5	3	1901/46.3438	13/0.1406	7/0.0781	11/0.1250
6	4	163/1.1094	17/0.1094	7/0.0781	5/0.0781
7	3	87/0.1.1563	10/0.1719	5/0.1250	5/0.0625
8	3	149/1.9531	18/0.2656	11/0.2031	12/0.1094
9	10	229/1.6719	949/7.3125	653/5.0938	124/0.1719
10	10	140/0.9531	859/6.7344	498/3.8125	120/0.2031
AV		318	228.8	141	40.4

Table 1. Numerical results.

The test problems are introduced as follows (see also [31]):

Problem 1: $M = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, $q = (-1, -1)$, the initial point is $(0, 0, \ldots, 0)^T$.

Problem 2: $M = \begin{pmatrix} 1 & 2 & 2 & \ldots & 2 \\ 0 & 1 & 2 & \ldots & 2 \\ 0 & 0 & 1 & \ldots & 2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & 1 \end{pmatrix}$, $q = (-1, \ldots, -1)$, the initial point is $(0, 0, \ldots, 0)^T$.

Problem 3: $M = \begin{pmatrix} 0 & 0 & 10 & 20 \\ 0 & 0 & 30 & 15 \\ 10 & 20 & 0 & 0 \\ 30 & 15 & 0 & 0 \end{pmatrix}$, $q = (-1, -1, -1, -1)$, the initial point is $(0, 0, \ldots, 0)^T$.

Problem 4: $M = \begin{pmatrix} 4 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 4 \end{pmatrix}$, $q = (1, 0, -1)$, the initial point is $(0, 0, \ldots, 0)^T$.

Problem 5: $M = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & -1 & 4 \end{pmatrix}$, $q = (1, 0, -1)$, the initial point is $(0, 0, \ldots, 0)^T$.

Problem 6: $M = \begin{pmatrix} 4 & 2 & 2 & 1 \\ 2 & 4 & 0 & 1 \\ 2 & 0 & 2 & 2 \\ -1 & -1 & -2 & 0 \end{pmatrix}$, $q = (-8, -6, -4, 3)$, the initial point is $(0, 0, \dots, 0)^T$.

Problem 7: $M = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 1 \end{pmatrix}$, $q = (0, 0, 1)$, the initial point is $(0, 0, \dots, 0)^T$.

Problem 8: $M = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -2 \\ 0 & 2 & 1 \end{pmatrix}$, $q = (0, 0, 1)$, the initial point is $(0, 0, \dots, 0)^T$.

Problem 9: $M = \begin{pmatrix} 4 & -2 & 0 & 0 & \dots & 0 \\ 1 & 4 & -2 & 0 & \dots & 0 \\ 0 & 1 & 4 & -2 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & 1 & 4 & -2 \\ 0 & \dots & 0 & 0 & 1 & 4 \end{pmatrix}$, $q = (-1, \dots, -1)$, the initial point is $(0, 0, \dots, 0)^T$.

Problem 10: $M = \begin{pmatrix} 4 & -1 & 0 & 0 & \dots & 0 \\ -1 & 4 & -1 & 0 & \dots & 0 \\ 0 & -1 & 4 & -1 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & -1 & 4 & -1 \\ 0 & \dots & 0 & 0 & -1 & 4 \end{pmatrix}$, $q = (-1, \dots, -1)$, the initial point is $(0, 0, \dots, 0)^T$.

6. Conclusion

In this paper we consider a self-adaptive trust region method for the extended linear complementarity problem. Under certain conditions, the global and local convergence are obtained. The novelty of our algorithm is that the trust region radius can be adjusted automatically according to the objective function; thus we can avoid choosing the initial trust region radius blindly. From the numerical tests we can see the efficiency of the proposed algorithm. How to obtain the local convergence result without the strictly complementarity assumption deserves further study.

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