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ROBUST DECENTRALIZED H_2 CONTROL OF MULTI–CHANNEL DESCRIPTOR SYSTEMS WITH NORM–BOUNDED PARAMETRIC UNCERTAINTIES

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This paper considers a robust decentralized H_2 control problem for multi-channel descriptor systems. The uncertainties are assumed to be time-invariant, norm-bounded, and exist in both the system and control input matrices. Our interest is focused on dynamic output feedback. A necessary and sufficient condition for an uncertain multi-channel descriptor system to be robustly stabilizable with a specified H_2 norm is derived in terms of a strict nonlinear matrix inequality (NMI), that is, an NMI with no equality constraint. A two-stage homotopy method is proposed to solve the NMI iteratively. First, a decentralized controller for the nominal descriptor system is computed by imposing block-diagonal constraints on the coefficient matrices of the controller gradually. Then, the decentralized controller is modified, again gradually, to cope with the uncertainties. On each stage, groups of variables are fixed alternately at the iterations to reduce the NMI to linear matrix inequalities (LMIs). An example is given to show the efficiency of this method.

Keywords: decentralized control, descriptor systems, parametric uncertainty, homotopy method, nonlinear matrix inequality

AMS Subject Classification: 93A14, 93B51, 93B52, 93B40, 93C15

1. INTRODUCTION

Much attention has been paid to analysis and design of descriptor systems since such systems have extensive applications in various fields [2, 4, 7, 10, 11, 14]. Descriptor systems are also referred as singular systems, implicit systems, generalized state-space systems, differential-algebraic systems. A large number of results for descriptor systems have been obtained both in the areas of centralized control and decentralized control. For centralized control, Takaba et al. studied robust H_2 performance of uncertain descriptor systems using LMI method [9]. Ikeda et al. considered H_2 control problem for descriptor systems and derived a strict LMI condition which is necessary and sufficient for H_2 control [5]. Yang et al. studied the design problem of standard H_2 optimal controller for continuous-time descriptor systems. Parameterization of all stabilizing controller of the system is given using the method of linear fractional transformation [13]. Furthermore, by means of two H_2

generalized algebraic Riccati equations and two generalized Lyapunov equations respectively, a design of standard H_2 optimal controller and the calculation of optimal H_2 norm are presented.

As for decentralized control, Ikeda et al. considered centralized design of decentralized stabilizing controllers for interconnected descriptor systems [6]. The design problem was formulated as a feasibility problem for a bilinear matrix inequality (BMI), and to solve the BMI, a homotopy method was proposed, where the interconnections between subsystems are increased gradually from zeros to the given magnitudes. Wo and Zou considered decentralized robust stabilization problem for large scale descriptor systems with parametric uncertainty [12].

Up to now, the robust output feedback control problem for multi-channel descriptor systems by LMI method still remains open. The model of multi-channel descriptor systems includes the model of interconnected descriptor systems. Therefore, the study of multi-channel descriptor systems has practical value. Zhai et al. studied decentralized H_2 control problem for multi-channel descriptor systems and proposed strict LMI conditions for designing low order decentralized controller [16]. In that context, the uncertainties are not considered in the descriptor systems.

Based on the observation that there is very few existing result considering decentralized H_2 controller design for multi-channel descriptor systems with parametric uncertainties, this paper considers a robust decentralized H_2 control problem for multi-channel descriptor systems with parametric uncertainties. The uncertainties are assumed to be time-invariant, norm-bounded, and exist in both the system and control input matrices. Our interest is focused on dynamic output feedback. A necessary and sufficient condition for an uncertain multi-channel descriptor system to be robustly stabilizable with H_2 norm, is developed in terms of a strict nonlinear matrix inequality (NMI), that is, an NMI with no equality constraint. Then, a two-stage homotopy method is proposed to solve the NMI iteratively. The idea of the two-stage homotopy method [3] has been proposed by Chen et al. in solving a sufficient condition for a robust decentralized H_∞ controller to exist for interconnected systems, where the dimensions of local controllers are the same as those of corresponding subsystems. Here, a decentralized controller for the nominal descriptor system is first computed by imposing block-diagonal constraints on the coefficient matrices gradually. Then, the decentralized controller is modified, again gradually, to cope with the uncertainties. On each stage, groups of variables are fixed alternately at the iterations to reduce the NMI to LMIs. A given example shows the efficiency of this method.

This paper is organized as follows. Section 2 is devoted to the formulation of the robust decentralized H_2 control problem. Section 3 presents a necessary and sufficient condition for a robust decentralized H_2 controller to exist. In Section 4, a two-stage solution algorithm for the condition is proposed using homotopy methods. An example is presented in Section 5, which demonstrates the usefulness of the proposed algorithm. Finally, Section 6 concludes the paper.

2. PROBLEM DESCRIPTION

We consider an N-channel descriptor system with uncertainties, which is described by

$$\begin{cases} E\dot{x} = (A+\delta A)x + B_1 w + \sum_{i=1}^{N}(B_{2i}+\delta B_{2i})u_i \\ z = C_1 x \\ y_i = C_{2i}x, \quad i = 1,2,\cdots,N \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$ is the descriptor variable, $w \in \mathbb{R}^q$ is the disturbance input, $z \in \mathbb{R}^p$ is the controlled output, and $u_i \in \mathbb{R}^{m_i}$ and $y_i \in \mathbb{R}^{l_i}$ are the control input and the measured output of channel $i(i = 1,2,\cdots)$, respectively. The matrices E and A are square, and E may be singular with the rank $r \le n$. The matrices B_1, B_{2i}, C_1 and C_{2i} are constant and of suitable dimensions, and δA and δB_{2i} respectively denote uncertainties in the system and control input matrices. We suppose that the uncertainties are described as

$$\begin{bmatrix} \delta A & \delta B_{21} & \cdots & \delta B_{2N} \end{bmatrix} = L\Delta \begin{bmatrix} F_1 & F_{21} & \cdots & F_{2N} \end{bmatrix} \tag{2}$$

where $L, F_1, \ldots, F_{2N}$ are known constant matrices, and Δ is an unknown constant matrix satisfying

$$\Delta^T \Delta \le I. \tag{3}$$

Furthermore, to ensure fitness of the H_2 control problem, we assume that the system (1) satisfies the following condition [5]

$$\ker E \subset \ker C_1.$$

Remark 1. It is known that systems having direct transmission paths from w and u to z and y can be transformed to the form of (1) where no direct transmission appears explicitly, by augmenting the descriptor variable [7]. Therefore, the system (1) is a general representation of descriptor systems.

We adopt a decentralized output feedback controller described by

$$\begin{cases} \dot{\hat{x}}_i = \hat{A}_i\hat{x}_i + \hat{B}_i y_i \\ u_i = \hat{C}_i\hat{x}_i + \hat{D}_i y_i \end{cases}, \quad i = 1,2,\cdots,N \tag{4}$$

where $\hat{x}_i \in \mathbb{R}^{\hat{n}_i}$ is the state of the ith local controller and $\hat{n}_i$ is a specified dimension. The matrices $\hat{A}_i, \hat{B}_i, \hat{C}_i, \hat{D}_i, i = 1,2,\cdots,N$ are constant and to be determined.

To write the closed-loop system composed of the system (1) and the controller (4) in a compact form, we denote matrices in (1) and (2) as

$$\begin{cases} B_2 = \begin{bmatrix} B_{21} & B_{22} & \cdots & B_{2N} \end{bmatrix} \\ C_2 = \begin{bmatrix} C_{21}^T & C_{22}^T & \cdots & C_{2N}^T \end{bmatrix}^T \\ \delta B_2 = \begin{bmatrix} \delta B_{21} & \delta B_{22} & \cdots & \delta B_{2N} \end{bmatrix} \\ F_2 = \begin{bmatrix} F_{21} & F_{22} & \cdots & F_{2N} \end{bmatrix} \end{cases} \tag{5}$$

and define the state and matrices of (4) as

$$\begin{cases} \hat{x} = [\ \hat{x}_1^T \quad \hat{x}_2^T \quad \cdots \quad \hat{x}_N^T\ ]^T \\ \hat{A}_D = \mathrm{diag}\{\hat{A}_1, \hat{A}_2, \cdots, \hat{A}_N\},\ \hat{B}_D = \mathrm{diag}\{\hat{B}_1, \hat{B}_2, \cdots, \hat{B}_N\} \\ \hat{C}_D = \mathrm{diag}\{\hat{C}_1, \hat{C}_2, \cdots, \hat{C}_N\},\ \hat{D}_D = \mathrm{diag}\{\hat{D}_1, \hat{D}_2, \cdots, \hat{D}_N\} \end{cases} \tag{6}$$

$$G_D = \begin{bmatrix} \hat{A}_D & \hat{B}_D \\ \hat{C}_D & \hat{D}_D \end{bmatrix}. \tag{7}$$

Furthermore, we introduce the notations

$$\begin{aligned} &\tilde{x} = [\ x^T \quad \hat{x}^T\ ]^T, \quad \tilde{E} = \begin{bmatrix} E & 0_{n\times\hat{n}} \\ 0_{\hat{n}\times n} & I_{\hat{n}} \end{bmatrix}, \quad \tilde{A} = \begin{bmatrix} A & 0_{n\times\hat{n}} \\ 0_{\hat{n}\times n} & 0_{\hat{n}\times\hat{n}} \end{bmatrix}, \\ &\tilde{B}_1 = \begin{bmatrix} B_1 \\ 0_{\hat{n}\times q} \end{bmatrix}, \quad \tilde{B}_2 = \begin{bmatrix} 0_{n\times\hat{n}} & B_2 \\ I_{\hat{n}} & 0_{\hat{n}\times m} \end{bmatrix}, \quad \tilde{C}_1 = [\ C_1 \quad 0_{p\times\hat{n}}\ ], \\ &\tilde{C}_2 = \begin{bmatrix} 0_{\hat{n}\times n} & I_{\hat{n}} \\ C_2 & 0_{l\times\hat{n}} \end{bmatrix}, \ \delta\tilde{A} = \begin{bmatrix} \delta A & 0_{n\times\hat{n}} \\ 0_{\hat{n}\times n} & 0_{\hat{n}\times\hat{n}} \end{bmatrix}, \ \delta\tilde{B}_2 = \begin{bmatrix} 0_{n\times\hat{n}} & \delta B_2 \\ 0_{\hat{n}\times\hat{n}} & 0_{\hat{n}\times m} \end{bmatrix}, \\ &\tilde{L} = \begin{bmatrix} L \\ 0_{\hat{n}\times l} \end{bmatrix}, \quad \tilde{F}_1 = [\ F_1 \quad 0_{l\times\hat{n}}\ ], \quad \tilde{F}_2 = [\ F_2 \quad 0_{l\times\hat{n}}\ ] \end{aligned} \tag{8}$$

where $\hat{n} = \sum_{i=1}^N \hat{n}_i, m = \sum_{i=1}^N m_i, l = \sum_{i=1}^N l_i$, so that the closed-loop system is written as

$$\begin{aligned} \tilde{E}\dot{\tilde{x}} &= [\tilde{A} + \delta\tilde{A} + (\tilde{B}_2 + \delta\tilde{B}_2)G_D\tilde{C}_2]\tilde{x} + \tilde{B}_1 w \\ &= [\tilde{A} + \tilde{L}\Delta\tilde{F}_1 + (\tilde{B}_2 + \tilde{L}\Delta\tilde{F}_2)G_D\tilde{C}_2]\tilde{x} + \tilde{B}_1 w \\ z &= \tilde{C}_1\tilde{x}. \end{aligned} \tag{9}$$

We denote the transfer function from the disturbance input w to the controlled output z of the closed-loop system by $T_{zw}(s)$. We say that the uncertain descriptor system (1) is robustly stabilizable with H_2 norm γ if there exists a controller (4) so that the closed-loop system is stable and $\|T_{zw}(s)\|_2 < \gamma$ holds for any Δ satisfying (3), where γ is a specified positive number. The control problem of this paper is to design a decentralized controller (4) realizing such a closed-loop system.

3. EXISTENCE CONDITION FOR DECENTRALIZED H_2 CONTROLLER

To obtain the condition under which the closed-loop system (9) is stable with H_2 norm γ, we employ the following lemmas. Let $V, U \in \mathbb{R}^{n\times(n-r)}$ be matrices of full column ranks and composed of bases of Null E and Null E^T, respectively. We decompose E as $E = E_L E_R^T$, where E_L and E_R are of full column ranks.

Lemma 1. (Ikeda et al. [5]) Consider a linear descriptor system

$$\begin{cases} E\dot{x} = Ax + Bw \\ z = Cx \end{cases} \tag{10}$$

where $x \in \mathbb{R}^n$ is the descriptor variable, $w \in \mathbb{R}^q$ is the input, $z \in \mathbb{R}^p$ is the output, and E, A, B, C are constant matrices of suitable dimensions. Then, for a given constant $\gamma > 0$, the system (10) is stable and satisfies

$$\|C(sE - A)^{-1}B\|_2 < \gamma \tag{11}$$

if and only if there exist a symmetric matrix P and a matrix S which satisfy the LMIs

$$\begin{aligned} (PE + USV^T)^T A + A^T(PE + USV^T) + C^T C < 0 \\ \text{trace}[B^T PB] < \gamma^2. \end{aligned} \tag{12}$$

Lemma 2. (Petersen [8]) Suppose that Ξ, L, and F are matrices of suitable dimensions and Ξ is symmetric. Then,

$$\Xi + L\Delta F + F^T \Delta^T L^T < 0 \tag{13}$$

holds for all Δ satisfying $\Delta^T \Delta \leq I$, if and only if there exists a scalar $\varepsilon > 0$ such that

$$\Xi + \varepsilon LL^T + \varepsilon^{-1} F^T F < 0. \tag{14}$$

From these lemmas, a necessary and sufficient condition for the existence of a robust decentralized H_2 controller is derived as follows.

Theorem 1. For a given constant $\gamma > 0$, the uncertain descriptor system (1) is robustly stabilizable with H_2 norm γ via a decentralized controller (4) composed of $\hat{n}_i$-dimensional local controllers, if and only if there exist a matrix G_D of (7), a symmetric matrix $\tilde{P} \in \mathbb{R}^{(n+\hat{n})\times(n+\hat{n})}$, a matrix $\tilde{S} \in \mathbb{R}^{(n-r)\times(n-r)}$, and a scalar $\varepsilon > 0$ such that

$$\begin{aligned} &J(G_D, \tilde{P}, \tilde{S}, \varepsilon) = \phi_1 + \phi_u < 0 \\ &\text{trace}[\tilde{B}_1^T \tilde{P} \tilde{B}_1] < \gamma^2 \end{aligned} \tag{15}$$

hold, where

$$\begin{aligned} \phi_1 &= (\tilde{P}\tilde{E} + \tilde{U}\tilde{S}\tilde{V}^T)^T(\tilde{A} + \tilde{B}_2 G_D \tilde{C}_2) \\ &\quad + (\tilde{A} + \tilde{B}_2 G_D \tilde{C}_2)^T(\tilde{P}\tilde{E} + \tilde{U}\tilde{S}\tilde{V}^T) + \tilde{C}_1^T \tilde{C}_1 \\ \phi_u &= \varepsilon(\tilde{P}\tilde{E} + \tilde{U}\tilde{S}\tilde{V}^T)\tilde{L}\tilde{L}^T(\tilde{P}\tilde{E} + \tilde{U}\tilde{S}\tilde{V}^T)^T \\ &\quad + \varepsilon^{-1}(\tilde{F}_1 + \tilde{F}_2 G_D \tilde{C}_2)^T(\tilde{F}_1 + \tilde{F}_2 G_D \tilde{C}_2) \\ \tilde{V} &= \begin{bmatrix} V \\ 0_{\hat{n}\times(n-r)} \end{bmatrix}, \quad \tilde{U} = \begin{bmatrix} U \\ 0_{\hat{n}\times(n-r)} \end{bmatrix}. \end{aligned} \tag{16}$$

P r o o f . From Lemma 1 for the system (9), we see that the uncertain multi-channel descriptor system is robustly stabilizable with the disturbance attenuation level γ, if and only if there exist a matrix G_D of (7), a symmetric matrix $\tilde{P}$, and a matrix $\tilde{S}$ such that

$$\begin{aligned} &\phi_1 + [(\tilde{P}\tilde{E} + \tilde{U}\tilde{S}\tilde{V}^T)\tilde{L}]\Delta[\tilde{F}_1 + \tilde{F}_2 G_D \tilde{C}_2] \\ &+ \{[(\tilde{P}\tilde{E} + \tilde{U}\tilde{S}\tilde{V}^T)\tilde{L}]\Delta[\tilde{F}_1 + \tilde{F}_2 G_D \tilde{C}_2]\}^T < 0 \\ &\text{trace}[\tilde{B}_1^T \tilde{P} \tilde{B}_1] < \gamma^2 \end{aligned} \tag{17}$$

hold. Based on Lemma 2, inequalities (17) hold for any Δ satisfying (3) if and only if there exist $G_D, \tilde{P}, \tilde{S}$ and $\varepsilon > 0$ such that (15) hold. This completes the proof. □

4. COMPUTATION ALGORITHM

The existence condition (15) for a robust decentralized H_2 controller is an NMI with the variables $G_D, \tilde{P}, \tilde{S}$ and ε, which is generally very hard to solve. In order to overcome this difficulty, we adopt the idea of the homotopy method. For this purpose, we first decompose $J(G_D, \tilde{P}, \tilde{S}, \varepsilon)$ of (15) into the nominal part $J_0(G_D, \tilde{P}, \tilde{S})$ and the perturbation part $J_u(G_D, \tilde{P}, \tilde{S}, \varepsilon)$ generated by the uncertainties, as

$$J(G_D, \tilde{P}, \tilde{S}, \varepsilon) = J_0(G_D, \tilde{P}, \tilde{S}) + J_u(G_D, \tilde{P}, \tilde{S}, \varepsilon) \tag{18}$$

where

$$J_0(G_D, \tilde{P}, \tilde{S}) = \phi_1 \tag{19}$$

and

$$J_u(G_D, \tilde{P}, \tilde{S}, \varepsilon) = \phi_u. \tag{20}$$

4.1. A solution algorithm for nominal descriptor system

We propose a two-stage homotopy method. On the first stage, we consider only the nominal part. In this case, the NMI (15) is a BMI $J_0(G_D, \tilde{P}, \tilde{S}) < 0$, $\text{trace}[\tilde{B}_1^T \tilde{P} \tilde{B}_1] < \gamma^2$. To solve this, we introduce a real number λ varying from 0 to 1 and define a matrix function as

$$H_0(G_D, \tilde{P}, \tilde{S}, \lambda) = J_0((1-\lambda)G_F + \lambda G_D, \tilde{P}, \tilde{S}) \tag{21}$$

where

$$G_F = \begin{bmatrix} A_F & B_F \\ C_F & D_F \end{bmatrix} \tag{22}$$

and A_F, B_F, C_F, D_F are coefficient matrices of a centralized H_2 controller for the performance γ with the same dimension as the decentralized one (4). The idea of the matrix function $H_0(G_D, \tilde{P}, \tilde{S}, \lambda)$ of (21) is that we gradually impose block-diagonal constraints on the coefficient matrices of the controller. Then,

$$H_0(G_D, \tilde{P}, \tilde{S}, \lambda) = \begin{cases} J_0(G_F, \tilde{P}, \tilde{S}), \lambda = 0 \\ J_0(G_D, \tilde{P}, \tilde{S}), \lambda = 1 \end{cases} \tag{23}$$

and the problem of finding a solution to (15) is embedded in the parametrized family of problems

$$H_0(G_D, \tilde{P}, \tilde{S}, \lambda) < 0, \quad \text{trace}[\tilde{B}_1^T \tilde{P} \tilde{B}_1] < \gamma^2, \quad \lambda \in [0, 1]. \tag{24}$$

We start computation of the solution to (24) with $\lambda = 0$ where we need a centralized H_2 controller as the initial value in the homotopy method for the decentralized controller. Here, we suggest using the design method [5] of an H_2 controller for a descriptor system with an appropriate modification. That is, we first compute a centralized H_2 controller in a descriptor form, and then convert it to a state equation. The dimension of the centralized controller obtained in the state space is r.

Following the discrete method [1], we let M be a positive integer and consider $(M+1)$ points $\lambda_k = k/M(k = 0, 1, \cdots, M)$ in the interval [0,1] to generate a family of problems

$$H_0(G_D, \tilde{P}, \tilde{S}, \lambda_k) < 0, \quad \text{trace}[\tilde{B}_1^T \tilde{P} \tilde{B}_1] < \gamma^2. \tag{25}$$

If the problem at the kth point is feasible, we denote the obtained solution by $(G_{Dk}, \tilde{P}_k, \tilde{S}_k)$. Then, we compute a solution $(G_{D,k+1}, \tilde{P}_{k+1}, \tilde{S}_{k+1})$ of $H_0(G_D, \tilde{P}, \tilde{S}, \lambda_{k+1}) < 0$, $\text{trace}[\tilde{B}_1^T \tilde{P} \tilde{B}_1] < \gamma^2$ by solving it as an LMI with variables being fixed as $\tilde{P} = \tilde{P}_k, \tilde{S} = \tilde{S}_k$ or $G_D = G_{Dk}$. If the family of problems $H_0(G_D, \tilde{P}, \tilde{S}, \lambda_k) < 0, \text{trace}[\tilde{B}_1^T \tilde{P} \tilde{B}_1] < \gamma^2, k = 1, 2, \cdots, M$ are all feasible, a solution of the NMI (15) is obtained at $k = M(\lambda = 1)$. If it is not the case, that is, both $H_0(G_D, \tilde{P}, \tilde{S}, \lambda_{k+1}) < 0$, and $\text{trace}[\tilde{B}_1^T \tilde{P} \tilde{B}_1] < \gamma^2$ are infeasible for some k, we consider more points in the interval $[\lambda_k, 1]$ by increasing M, and repeat the procedure from the solution $(G_{Dk}, \tilde{P}_k, \tilde{S}_k)$ at $\lambda = \lambda_k$.

The above idea is summarized as a computation algorithm for a decentralized H_2 output feedback controller.

Step 1. Initialize M to a certain positive integer, and set a certain upper bound $M_{\max}$ for M. Set $k = 0$. Let $G_{Dk} = \hat{G}_{D0}$ and $\tilde{P}_k = \hat{\tilde{P}}_0, \tilde{S}_k = \hat{\tilde{S}}_0$ in accordance with the initial centralized controller.

Step 2. Set $k := k+1$ and $\lambda_k = k/M$. Compute a solution G_D of $H_0(G_D, \tilde{P}_{k-1}, \tilde{S}_{k-1}, \lambda_k) < 0$. If it is not feasible, go to Step 3. If it is feasible, set $G_{Dk} = G_D$, and compute a solution $(\tilde{P}, \tilde{S})$ of $H_0(G_{Dk}, \tilde{P}, \tilde{S}, \lambda_k) < 0$. Then, set $\tilde{P}_k = \tilde{P}, \tilde{S}_k = \tilde{S}$ and go to Step 5.

Step 3. Compute a solution $(\tilde{P}, \tilde{S})$ of $H_0(G_{D,k-1}, \tilde{P}, \tilde{S}, \lambda_k) < 0, \text{trace}[\tilde{B}_1^T \tilde{P} \tilde{B}_1] < \gamma^2$. If it is not feasible, go to Step 4. If it is feasible, set $\tilde{P}_k = \tilde{P}, \tilde{S}_k = \tilde{S}$, and compute a solution G_D of $H_0(G_D, \tilde{P}_k, \tilde{S}_k, \lambda_k) < 0$. Then, set $G_{Dk} = G_D$, and go to Step 5.

Step 4. Set $M := 2M$ under the constraint $M \le M_{\max}$, set $\tilde{P}_{2(k-1)} = \tilde{P}_{k-1}, \tilde{S}_{2(k-1)} = \tilde{S}_{k-1}, \tilde{G}_{D,2(k-1)} = \tilde{G}_{D,k-1}, k := 2(k-1)$ and go to Step 2. If we cannot increase M any more, we conclude that this algorithm does not converge.

Step 5. If $k < M$ go to Step 2. If $k = M$ the obtained $(G_{DM}, \tilde{P}_M, \tilde{S}_M)$ is a solution of (24).

If the dimension $\hat{n}$ of the decentralized controller (4) is equal to r, we can apply the computation method of Zhai et al. [15], which has been proposed for state-space models. If we obtain a solution at $\lambda = 1$, it gives us a decentralized H_2 controller for the nominal system. If the dimension $\hat{n}$ of the controller (4) is less than r, we can also use the technique of Zhai et al. [15]. More precisely, we define $\hat{G}_D$ by augmenting the matrix G_D as

$$\hat{G}_D = \left[\begin{array}{cc|c} \hat{A}_D & 0_{\hat{n}\times(r-\hat{n})} & \hat{B}_D \\ * & -I_{r-\hat{n}} & ** \\ \hline \hat{C}_D & 0_{m\times(r-\hat{n})} & \hat{D}_D \end{array}\right] \tag{26}$$

where the notations $*, **$ are any submatrices, and $\hat{A}_D, \hat{B}_D, \hat{C}_D, \hat{D}_D$ were defined in (6). We note that the r-dimensional controller defined by $\hat{G}_D$ of (26) is equivalent to the $\hat{n}$-dimensional decentralized controller defined by G_D of (7) if we extract their controllable and observable parts. Then, replacing with $\hat{G}_D$ in the matrix function (21), we solve the feasibility problem of (24) by the homotopy method.

If the dimension $\hat{n}$ of the controller (4) is greater than r, we define $\hat{G}_F$ by augmenting the matrix G_F as

$$\hat{G}_D = \left[\begin{array}{cc|c} A_F & 0_{\hat{n}\times(\hat{n}-r)} & B_F \\ * & -I_{\hat{n}-r} & ** \\ \hline C_F & 0_{m\times(\hat{n}-r)} & D_F \end{array}\right] \tag{27}$$

where A_F, B_F, C_F and D_F were defined in (22). We note that the $\hat{n}$-dimensional centralized controller defined by $\hat{G}_F$ of (27) is equivalent to the r-dimensional centralized controller defined by G_F of (22) if we extract their controllable and observable parts. In this case, replacing G_F with $\hat{G}_F$ in (21), we solve the feasibility problem of (24) by the homotopy method.

Remark 2. In this paper, we consider local controllers of specified dimensions. Then, we may not stabilize the system even if the sum of dimensions of the local controllers is equal to that of the system. For this reason, we deal with the case $\hat{n} > r$ as well as $\hat{n} \le r$. Furthermore, by considering higher dimensional local controllers, we may achieve a better disturbance attenuation level.

Remark 3. Though $-I$ was set at the (2,2)-block of $\hat{G}_D$ in (26) and $\hat{G}_F$ in (27) for simplicity, it can be any stable matrix and variable.

4.2. A solution algorithm for uncertain descriptor system

Now, we suppose that a solution $(G_D, \tilde{P}, \tilde{S})$ of (24) at $\lambda = 1$ on the first stage has been obtained, which we denote by $(\hat{G}_{D0}, \hat{\tilde{P}}_0, \hat{\tilde{S}}_0)$. That is, we have a solution of the decentralized H_2 control problem for the case of no uncertainty. Then, we proceed to the second stage, where we take into account uncertainties in the multi-channel descriptor system (1). In order to compute a solution of NMI (15), we again employ a homotopy method in a different way. We introduce a real number $\tilde{\lambda} \in [0,1]$ and define the matrix function

$$H_1(G_D, \tilde{P}, \tilde{S}, \varepsilon, \tilde{\lambda}) = J_0(G_D, \tilde{P}, \tilde{S}) + \tilde{\lambda} J_u(G_D, \tilde{P}, \tilde{S}, \varepsilon). \quad (28)$$

Then,

$$H_1(G_D, \tilde{P}, \tilde{S}, \varepsilon, \tilde{\lambda}) = \begin{cases} J_0(G_D, \tilde{P}, \tilde{S}), \tilde{\lambda} = 0 \\ J(G_D, \tilde{P}, \tilde{S}, \varepsilon), \tilde{\lambda} = 1 \end{cases} \quad (29)$$

and the problem of finding a solution to (15) is embedded in the parametrized family of problems

$$H_1(G_D, \tilde{P}, \tilde{S}, \varepsilon, \tilde{\lambda}) < 0, \quad \mathrm{trace}[\tilde{B}_1^T \tilde{P} \tilde{B}_1] < \gamma^2, \quad \tilde{\lambda} \in [0,1]. \quad (30)$$

We note that the solution to (30) with $\tilde{\lambda} = 0$ has been already obtained as $(\hat{G}_{D0}, \hat{\tilde{P}}_0, \hat{\tilde{S}}_0)$ on the first stage.

To solve the matrix inequalities (30), we apply the Schur complement and consider two equivalent matrix inequalities shown as

$$\begin{gathered} H_{11}(G_D, \tilde{P}, \tilde{S}, \varepsilon, \tilde{\lambda}) = \begin{bmatrix} \phi_1 + \phi_{u1} & (\tilde{F}_1 + \tilde{F}_2 G_D \tilde{C}_2)^T \\ \tilde{F}_1 + \tilde{F}_2 G_D \tilde{C}_2 & -\varepsilon \tilde{\lambda}^{-1} I \end{bmatrix} < 0 \\ \mathrm{trace}[\tilde{B}_1^T \tilde{P} \tilde{B}_1] < \gamma^2 \\ \phi_{u1} = \varepsilon \tilde{\lambda} (\tilde{P}\tilde{E} + \tilde{U}\tilde{S}\tilde{V}^T) \tilde{L} \tilde{L}^T (\tilde{P}\tilde{E} + \tilde{U}\tilde{S}\tilde{V}^T)^T \end{gathered} \quad (31)$$

and

$$\begin{gathered} H_{12}(G_D, \tilde{P}, \tilde{S}, \varepsilon^{-1}, \tilde{\lambda}) = \begin{bmatrix} \phi_1 + \phi_{u2} & (\tilde{P}\tilde{E} + \tilde{U}\tilde{S}\tilde{V}^T)\tilde{L} \\ \tilde{L}^T (\tilde{P}\tilde{E} + \tilde{U}\tilde{S}\tilde{V}^T)^T & -\varepsilon^{-1} \tilde{\lambda}^{-1} I \end{bmatrix} < 0 \\ \mathrm{trace}[\tilde{B}_1^T \tilde{P} \tilde{B}_1] < \gamma^2 \\ \phi_{u2} = \varepsilon^{-1} \tilde{\lambda} (\tilde{F}_1 + \tilde{F}_2 G_D \tilde{C}_2)^T (\tilde{F}_1 + \tilde{F}_2 G_D \tilde{C}_2). \end{gathered} \quad (32)$$

We note that if we fix $\tilde{P}$ and $\tilde{S}$, then $H_{11}(G_D, \tilde{P}, \tilde{S}, \varepsilon, \tilde{\lambda}) < 0$ becomes an LMI with respect to G_D and ε. On the other hand, if we fix G_D, then $H_{12}(G_D, \tilde{P}, \tilde{S}, \varepsilon^{-1}, \tilde{\lambda}) < 0$ is an LMI in $\tilde{P}, \tilde{S}$ and ε^{-1}. We use this property in the homotopy method of the second stage.

Following the discrete method [1], we let M be a positive integer and consider $(M+1)$ points $\tilde{\lambda}_k = k/M (k = 0, 1, \cdots, M)$ in the interval $[0,1]$ to generate a family of problems

$$H_1(G_D, \tilde{P}, \tilde{S}, \varepsilon, \tilde{\lambda}_k) < 0, \quad \mathrm{trace}[\tilde{B}_1^T \tilde{P} \tilde{B}_1] < \gamma^2. \quad (33)$$

If the problem at the kth point is feasible, we denote the obtained solution by $(G_{Dk}, \tilde{P}_k, \tilde{S}_k)$. Then, we compute a solution $(G_{D,k+1}, \tilde{P}_{k+1}, \tilde{S}_{k+1})$ of $H_{11}(G_D, \tilde{P}, \tilde{S}, \varepsilon, \tilde{\lambda}_{k+1}) < 0$ or $H_{12}(G_D, \tilde{P}, \tilde{S}, \varepsilon^{-1}, \tilde{\lambda}_{k+1}) < 0$, $\mathrm{trace}[\tilde{B}_1^T \tilde{P} \tilde{B}_1] < \gamma^2$ by solving it as an

LMI with variables being fixed as $\tilde{P} = \tilde{P}_k, \tilde{S} = \tilde{S}_k$ or $G_D = G_{Dk}$. If the family of problems $H_1(G_D, \tilde{P}, \tilde{S}, \varepsilon, \tilde{\lambda}_k) < 0$, $\text{trace}[\tilde{B}_1^T \tilde{P} \tilde{B}_1] < \gamma^2$, $k = 1, 2, \cdots, M$ are all feasible, a solution of the NMI (15) is obtained at $k = M(\tilde{\lambda} = 1)$. If it is not the case, that is, both $H_{11}(G_D, \tilde{P}_k, \tilde{S}_k, \varepsilon, \tilde{\lambda}_{k+1}) < 0$ and $H_{12}(G_D, \tilde{P}, \tilde{S}, \varepsilon^{-1}, \tilde{\lambda}_{k+1}) < 0$, $\text{trace}[\tilde{B}_1^T \tilde{P} \tilde{B}_1] < \gamma^2$ are infeasible for some k, we consider more points in the interval $[\tilde{\lambda}_k, 1]$ by increasing M, and repeat the procedure from the solution $(G_{Dk}, \tilde{P}_k, \tilde{S}_k)$ at $\tilde{\lambda} = \tilde{\lambda}_k$.

We formulate this idea of the second stage as an algorithm for computing a robust decentralized H_2 controller.

Step 1. Initialize M to a certain positive integer, and set a certain upper bound $M_{\max}$ for M. Set $k = 0$. Let $G_{Dk} = \hat{G}_{D0}$ and $\tilde{P}_k = \hat{\tilde{P}}_0, \tilde{S}_k = \hat{\tilde{S}}_0$ using the solution of the first stage.

Step 2. Set $k := k+1$ and $\tilde{\lambda}_k = k/M$. Compute a solution (G_D, ε) of $H_{11}(G_D, \tilde{P}_{k-1}, \tilde{S}_{k-1}, \varepsilon, \tilde{\lambda}_k) < 0$. If it is not feasible, go to Step 3. If it is feasible, set $G_{Dk} = G_D$ and compute a solution $(\tilde{P}, \tilde{S}, \varepsilon^{-1})$ of $H_{12}(G_{Dk}, \tilde{P}, \tilde{S}, \varepsilon^{-1}, \tilde{\lambda}_k) < 0, \text{trace}[\tilde{B}_1^T \tilde{P} \tilde{B}_1] < \gamma^2$. Then, set $\tilde{P}_k = \tilde{P}, \tilde{S}_k = \tilde{S}$ and go to Step 5.

Step 3. Compute a solution $(\tilde{P}, \tilde{S}, \varepsilon^{-1})$ of $H_{12}(G_{D,k-1}, \tilde{P}, \tilde{S}, \varepsilon^{-1}, \tilde{\lambda}_k) < 0, \text{trace}[\tilde{B}_1^T \tilde{P} \tilde{B}_1] < \gamma^2$. If it is not feasible, go to Step 4. If it is feasible, set $\tilde{P}_k = \tilde{P}, \tilde{S}_k = \tilde{S}$, and compute a solution (G_D, ε) of $H_{11}(G_D, \tilde{P}_k, \tilde{S}_k, \varepsilon, \tilde{\lambda}_k) < 0$. Then, set $G_{Dk} = G_D$, and go to Step 5.

Step 4. Set $M := 2M$ under the constraint $M \leq M_{\max}$, set $\tilde{P}_{2(k-1)} = \tilde{P}_{k-1}$, $\tilde{S}_{2(k-1)} = \tilde{S}_{k-1}, \tilde{G}_{D,2(k-1)} = \tilde{G}_{D,k-1}, k := 2(k-1)$ and go to Step 2. If we cannot increase M any more, we conclude that this algorithm does not converge.

Step 5. If $k < M$ go to Step 2. If $k = M$ the obtained $(G_{DM}, \tilde{P}_M, \tilde{S}_M, \varepsilon)$ is a solution of the NMI (15).

Remark 4. The convergence of the algorithm depends on the choice of the initial controllers, and a possible improvement is to consider random generation of the initial controllers since the subcontroller's size is small and can be parametrized.

Remark 5. When it is necessary, we can try to obtain a tight H_2 norm γ by considering the generalized eigenvalue problem (EVP): "minimize γ^2, s. t. (24) or (30), respectively".

5. AN EXAMPLE

We present an example to demonstrate the efficiency of the proposed design method. We deal with a two-channel descriptor system, where the coefficient matrices of the nominal descriptor system are

$$E = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ 2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 0.6 & -1 & 0 \\ -4 & 0 & 0.8 & 0 \\ 0 & 0 & 1.5 & -1 \\ -1 & 1 & 0 & 0 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1 \\ 0 \\ 0.5 \\ 0 \end{bmatrix}, \quad B_{21} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad B_{22} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 1 & 0 & 0 & 1 \end{bmatrix}, \quad C_{21} = \begin{bmatrix} -2 & 3 & -2 & 1 \end{bmatrix},$$
$$C_{22} = \begin{bmatrix} 2 & -1 & 1 & 0 \end{bmatrix}.$$

and the uncertainties are defined by

$$L = [\ -0.2 \quad 0.3 \quad 0.1 \quad 0.3\ ]^T, \quad F_1 = [\ 0.5 \quad 0.1 \quad 0.6 \quad -0.2\ ],$$
$$F_2 = [\ 0.6 \quad 0.5\ ].$$

Here, we consider the case $r < \hat{n}$ and set the disturbance attenuation level to be achieved as 3. We design a decentralized H_2 controller composed of two local controllers (4) whose dimensions are $\hat{n}_1 = 2$ and $\hat{n}_2 = 2$. We note that the total dimension of the local controller is $\hat{n} = \hat{n}_1 + \hat{n}_2 = 4$, which is higher than the dimension $r = 3$ of the controlled system.

To compute an initial value for the homotopy method of the first stage, we use the design method [5] of an H_2 controller for a descriptor system with an appropriate modification as stated above. Then, we obtain a centralized controller in the state equation with the coefficient matrices

$$G_F = \left[\begin{array}{ccc|cc} -5.2 & 2.45 & -0.87 & 0.35 & 0.97 \\ 4.45 & -7.19 & -2.00 & 1.6 & 0.08 \\ 3.74 & -1.82 & -1.31 & -0.19 & 0.69 \\ \hline -2.53 & 0.08 & -0.79 & 0.24 & -0.61 \\ 3.12 & -1.51 & -1.05 & -0.49 & 1.25 \end{array}\right].$$

We compute the solution of r-dimensional centralized H_2 controller and augment it as $\hat{G}_F$ of (27), that is

$$\hat{G}_F = \left[\begin{array}{cccc|cc} -5.2 & 2.45 & -0.87 & 0 & 0.35 & 0.97 \\ 4.45 & -7.19 & -2.00 & 0 & 1.6 & 0.08 \\ 3.74 & -1.82 & -1.31 & 0 & -0.19 & 0.69 \\ 1 & 1 & 1 & -1 & 1 & 1 \\ \hline -2.53 & 0.08 & -0.79 & 0 & 0.24 & -0.61 \\ 3.12 & -1.51 & -1.05 & 0 & -0.49 & 1.25 \end{array}\right].$$

An initial solution $(\tilde{P}_0, \tilde{S}_0)$ for $H_0(G_D, \tilde{P}, \tilde{S}, \lambda) < 0, \mathrm{trace}[\tilde{B}_1^T \tilde{P} \tilde{B}_1] < \gamma^2$ is computed by replacing $\hat{G}_D$ with this G_F in $J_0(\hat{G}_D \tilde{P}, \tilde{S}) < 0, \mathrm{trace}[\tilde{B}_1^T \tilde{P} \tilde{B}_1] < \gamma^2$ as

$$\hat{P}_0 = \begin{bmatrix} 11.45 & 0.45 & 0.12 & -10.29 & -3.52 & -1.34 & -2.36 & 3.25 \\ 0.45 & 20.2 & 15.5 & -8.26 & 5.02 & 6.09 & -4.00 & 5.23 \\ 0.12 & 15.5 & 1.01 & 18.17 & 1.41 & 1.19 & -0.80 & -5.85 \\ -10.29 & -8.26 & 18.17 & 21.84 & -0.18 & -0.94 & -5.86 & -0.73 \\ -3.52 & 5.02 & 1.41 & -0.18 & 33.39 & -2.46 & 4.2 & -0.02 \\ -1.34 & 6.09 & 1.19 & -0.94 & -2.46 & 17.81 & 2.57 & -2.43 \\ -2.36 & -4.00 & -0.8 & -5.86 & 4.20 & 2.57 & 33.62 & -0.81 \\ 3.25 & 5.23 & -5.85 & -0.73 & -0.02 & -2.43 & -0.81 & 7.23 \end{bmatrix}$$

$$\tilde{S}_0 = -8.29$$

Then, for the nominal descriptor system, we design a decentralized H_2 controller composed of two local controllers (4) of the dimensions $\hat{n}_1 = 2$ and $\hat{n}_2 = 2$. Augmenting G_D as $\hat{G}_D$ of (26), we obtain the coefficient matrices of a decentralized H_2 controller by using the homotopy method with $M = 16$ as

$$\hat{G}_{D0} = \left[\begin{array}{cccc|cc} -7.68 & 14.49 & 0 & 0 & -3.8 & 0 \\ 3.50 & -11.91 & 0 & 0 & 3.57 & 0 \\ 0 & 0 & -23.95 & -7.61 & 0 & 2.76 \\ 0 & 0 & -10.6 & -6.70 & 0 & -3.12 \\ \hline -8.33 & 14.60 & 0 & 0 & -3.74 & 0 \\ 0 & 0 & -0.74 & 0.40 & 0 & 2.43 \end{array}\right]$$

and $(\hat{\tilde{P}}_0, \hat{\tilde{S}}_0)$ by computing $(\tilde{P}, \tilde{S}) < 0$ of $J_0(G_D, \tilde{P}, \tilde{S}) < 0$, $\mathrm{trace}[\tilde{B}_1^T \tilde{P} \tilde{B}_1] < \gamma^2$ with $G_D = \hat{G}_{D0}$ as

$$\hat{\tilde{P}}_0 = \begin{bmatrix} 3.46 & -5.28 & -0.10 & -13.19 & -3.00 & -0.19 & -0.46 & 3.17 \\ -5.28 & 8.60 & 16.04 & -5.67 & -1.72 & 9.51 & -1.06 & 3.22 \\ -0.10 & 16.04 & 7.47 & 14.62 & -6.01 & 0.92 & 0.56 & -3.71 \\ -13.19 & -5.67 & 14.62 & 21.08 & -0.09 & 0.35 & -3.98 & 7.66 \\ -3.00 & -1.72 & -6.01 & -0.09 & 20.37 & 9.57 & 0.64 & -0.97 \\ -0.19 & 9.51 & 0.92 & 0.35 & 9.57 & 17.16 & 1.37 & -3.34 \\ -0.46 & -1.06 & 0.56 & -3.98 & 0.64 & 1.37 & 6.40 & -2.83 \\ 3.17 & 3.22 & -3.71 & 7.66 & -0.97 & -3.34 & -2.83 & 10.68 \end{bmatrix}$$

$$\hat{\tilde{S}}_0 = -6.19.$$

On the second stage, we compute a robust decentralized H_2 controller for the uncertain system using the algorithm proposed in Section 4. With $M = 8$, we obtain the coefficient matrices

$$\hat{G}_D = \left[\begin{array}{cccc|cc} -4.61 & 1.89 & 0 & 0 & 0 & 0 \\ 0.22 & -2.18 & 0 & 0 & 0.70 & 0 \\ 0 & 0 & -25.16 & -9.03 & 0 & 1.73 \\ 0 & 0 & -9.16 & -5.96 & 0 & -3.79 \\ \hline -5.25 & 1.23 & 0 & 0 & 0.16 & 0 \\ 0 & 0 & -0.48 & 0.63 & 0 & 2.69 \end{array}\right].$$

6. CONCLUSION

This paper considered a robust decentralized H_2 control problem for uncertain multi-channel systems. Based on a version of the strict bounded real lemma for descriptor systems, an NMI condition with no equality constraint was derived for a robust decentralized H_2 controller to exist, which achieved a specified H_2 norm. A two-stage homotopy method was proposed to solve the NMI iteratively. The method started with the solution to the centralized H_2 control problem. The first homotopy stage considered decentralized controller with no uncertainty, and the second homotopy stage considered the presence of uncertainty. The proposed method allowed for controllers which have dimension either higher or lower than that of the centralized H_2 controller. An example was given to illustrate the proposed method.

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