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ON A CANCELLATION RULE FOR SUBDIRECT PRODUCTS OF LATTICE ORDERED GROUPS AND OF *GMV*-ALGEBRAS

Ján Jakubík

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ABSTRACT. The notion of internal subdirect decomposition can be defined in each variety of algebras. In the present note we prove the validity of a cancellation rule concerning such decompositions for lattice ordered groups and for GMV-algebras. For the case of groups, this cancellation rule fails to be valid.

1. Introduction

Cancellation rules concerning direct product decompositions of some types of algebraic structures have been investigated in several papers; cf. e.g., [1], [9], [11]–[18].

In the present note we deal with a cancellation rule (denoted by (c_2)) concerning subdirect decompositions of lattice ordered groups and of GMV-algebras.

The basic definitions on subdirect products of algebraic structures are recalled in Section 2 below.

Suppose that $\mathcal{V}$ is a variety of algebras and $A, X, Y \in \mathcal{V}$. If A is a subdirect product of X and Y, then we write $A = (\mathrm{sub})X \times Y$.

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We say that the *cancellation rule* (c$_1$) *is valid in* $\mathcal{V}$ if, whenever $A, X, X_1, Y, Y_1 \in \mathcal{V}$ and $A \simeq (\mathrm{sub})X \times Y$, $A \simeq (\mathrm{sub})X_1 \times Y_1$, and $Y \simeq Y_1$, then $X \simeq X_1$.

In view of a well-known Birkhoff's theorem, each subdirect product decomposition of an algebra A is determined, up to isomorphisms, by a system $\{\rho_i\}_{i \in I}$ of congruence relations on A such that $\bigwedge_{i\in I} \rho_i = \rho_0$, where ρ_0 is the least element of the set $\operatorname{con} A$ of all congruence relations on A. (Cf. [2].)

We are interested in two-factor subdirect decompositions. Let $\rho_1, \rho_2 \in \operatorname{con} A$, $\rho_1 \wedge \rho_2 = \rho_0$. For $\rho \in \operatorname{con} A$ and $a \in A$ we put $a(\rho) = \{a' \in A : a'\rho a\}$. Consider the mapping $\varphi\colon A \to A/\rho_1 \times A/\rho_2$ defined by $\varphi(a) = (a(\rho_1), a(\rho_2))$ for each $a \in A$. Then φ determines an isomorphism of A into a subdirect product of A/ρ_1 and A/ρ_2. We express this fact by writing

$$A = (\mathrm{int\,sub})X_1 \times X_2, \tag{1}$$

where $X_1 = A/\rho_1$ and $X_2 = A/\rho_2$. We say that (1) is an internal subdirect decomposition of A (determined by the congruence relations ρ_1 and ρ_2).

The internal subdirect decomposition (1) is said to satisfy the condition

(m) (or the maximality condition) if, whenever $\rho_{11} \in \operatorname{con} A$, $\rho_{11} > \rho_1$ and

$$A = (\mathrm{int\,sub})(A/\rho_{11}) \times (A/\rho_2), \tag{2}$$

then $\rho_1 = \rho_{11}$. In such a case, (1) is called an *m-subdirect decomposition.*

We say that the cancellation rule (c$_2$) is valid for the variety $\mathcal{V}$ if, whenever (1) and

$$A = (\mathrm{int\,sub})X_1' \times X_2 \tag{1'}$$

are m-subdirect decompositions, then $X_1 \simeq X_1'$.

We remark that if $\rho_1, \rho_2, \rho_3 \in \operatorname{con} A$ such that $\rho_1 \wedge \rho_2 = \rho_0$ and $\rho_1 > \rho_3$, then we have

$$A = (\mathrm{int\,sub})(A/\rho_1) \times (A/\rho_2),$$
$$A = (\mathrm{int\,sub})(A/\rho_3) \times (A/\rho_2)$$

and $G/\rho_1 \neq G/\rho_3$; thus the maximality condition cannot be omitted in our consideration.

It is easy to verify (cf. Section 2 below) that a variety $\mathcal{V}$ satisfies the cancellation rule (c$_1$) if and only if each algebra of $\mathcal{V}$ has exactly one element.

We prove that the cancellation rule (c$_2$) is valid for each variety of lattice ordered groups and each variety of GMV-algebras. On the other hand, (c$_2$) fails to be valid for the variety of all groups.

We also show that if $\mathcal{V}$ is a variety of lattice ordered groups or a variety of GMV-algebras and if for some $A \in \mathcal{V}$ the relation (1) is valid, then there exists $\rho_{11} \in \operatorname{con} A$ with $\rho_{11} \geqq \rho_1$ such that A has an m-subdirect decomposition

$$A = (\operatorname{int}\operatorname{sub}) X_{11} \times X_2,$$

where $X_{11} = A/\rho_{11}$.

2. Preliminaries

For fixing the notation, we recall the basic definitions concerning subdirect products of algebras.

Assume that $(X_i)_{i \in I}$ is an indexed system of algebras belonging to a variety $\mathcal{V}$. The direct product

$$X = \prod_{i \in I} X_i$$

is defined in the usual way. If $I = \{1, 2, \ldots, n\}$, then we apply the notation $X = X_1 \times \cdots \times X_n$.

The elements of X are written in the form $x = (x_i)_{i \in I}$; we say that x_i is the component of x in X_i and we denote it also by $x(X_i)$. For $Z \subseteq X$ and $i \in I$ we put $Z(X_i) = \{z(X_i) : z \in Z\}$.

Let A be a subalgebra of X such that for each $i \in I$ the relation $A(X_i) = X_i$ is valid. Then A is said to be a *subdirect product of the indexed system* $(X_i)_{i \in I}$; we express this fact by writing

$$A = (\operatorname{sub}) \prod_{i \in I} X_i.$$

In the case $I = \{1, 2, \ldots, n\}$ we write $A = (\operatorname{sub}) X_1 \times \cdots \times X_n$.

For $B \in \mathcal{V}$ and $\rho \in \operatorname{con} B$, the quotient algebra B/ρ is defined in the standard way. For ρ and ρ_1 in $\operatorname{con} B$ we write $\rho \leqq \rho_1$ if $b(\rho) \subseteq b(\rho_1)$ for each $b \in B$.

Now let us consider the cancellation rule (c_1). If $\mathcal{V}$ is a variety such that each algebra belonging to $\mathcal{V}$ has exactly one element, then the cancellation rule (c_1) obviously holds.

Assume that $\mathcal{V}$ is a variety containing an algebra X_0 such that X_0 has more than one element. There exists a set I such that I is infinite and $\operatorname{card} I > \operatorname{card} X_0$. For each $i \in I$ we put $X_i = X_0$. Further, we set

$$X = \prod_{i \in I} X_i, \quad Y = X = Y_1, \quad X_1 = X_0.$$

Then for $A = X \times Y$ we have

$$A \simeq (\mathrm{sub})X \times Y, \quad A \simeq (\mathrm{sub})X_1 \times Y_1, \quad Y \simeq Y_1,$$

but X fails to be isomorphic to X_1. Therefore the cancellation rule (c_1) is not valid for the variety $\mathcal{V}$.

We denote by $\mathcal{V}_g$ the variety of all groups. The following example shows that the cancellation rule (c_2) does not hold for the variety $\mathcal{V}_g$.

Let $\mathbb{R}$ be the additive group of all reals. Put $X = Y = \mathbb{R}$, $G = X \times Y$. The elements of G will be denoted by (x, y) with $x \in X$, $y \in Y$. We put $Z = \{(x, y) \in G : x = y\}$. Then Z is a subgroup of G and $Z \simeq X$. Since A is abelian, Z is a normal subgroup of G.

For $g_i = (x_i, y_i)$ $(i = 1, 2)$ we put $g_1 \rho_1 g_2$ if $x_1 = x_2$, and $g_1 \rho_2 g_2$ if $y_1 = y_2$. Further, we set $g_1 \rho_3 g_2$ if $g_1 - g_2 \in Z$. We get $\rho_3 \in \operatorname{con} A$. Then we clearly have

$$A = (\mathrm{int\,sub})(A/\rho_1) \times (A/\rho_2). \qquad (\alpha)$$

If $g_1, g_2 \in A$ and $g_1 \rho_2 g_2$, $g_1 \rho_3 g_2$, then $g_1 = g_2$. Hence $\rho_2 \wedge \rho_3 = \rho_0$. This yields

$$A = (\mathrm{int\,sub})(A/\rho_3) \times (A/\rho_2). \qquad (\beta)$$

The following steps show that both (α) and (β) are m-subdirect decompositions of A.

a) Suppose that $\rho_4 \in \operatorname{con} A$, $\rho_4 \geqq \rho_1$, $\rho_4 \wedge \rho_2 = \rho_0$. By way of contradiction, assume that $\rho_4 > \rho_1$. Hence there exists $g = (x, y) \in A$ such that $0 \rho_4 g$ and $x \neq 0$. Put $g_1 = (0, y)$. We have $0 \rho_1 g_1$, whence $0 \rho_4 g_1$, and thus $0 \rho_4 (g - g_1)$. But $g - g_1 = (x, 0)$ and thus $0 \rho_2 (g - g_1)$. This yields $\rho_4 \wedge \rho_2 \neq \rho_0$, which is a contradiction. Hence (α) is an m-subdirect decomposition.

b) Suppose that $\rho_5 \in \operatorname{con} A$, $\rho_5 \geqq \rho_1$, $\rho_5 \wedge \rho_2 = \rho_0$. Further, assume that $\rho_5 > \rho_3$. Hence there exists $g \in A$ such that $0 \rho_5 g$, $g = (x, y)$ and $x \neq y$. Put $g_1 = (y, y)$. Then $0 \rho_3 \rho_1$, thus $0 \rho_5 g_1$ and so $0 \rho_5 (g - g_1)$. We obtain $g - g_1 = (x - y, 0)$, whence $0 \rho_2 (g - g_1)$ and $g - g_1 \neq 0$. Thus $\rho_5 \wedge \rho_2 \neq \rho_0$, and we arrived at a contradiction. Therefore (β) is an m-subdirect decomposition.

We obviously have $A/\rho_1 \neq A/\rho_3$. In view of (α) and (β) we conclude that the variety $\mathcal{V}_g$ does not satisfy the cancellation rule (c_2).

3. The condition (c_2) for lattice ordered groups

For lattice ordered groups we apply the terminology and the notation as in [2]. Thus the group operation in a lattice ordered group is denoted by the symbol $+$; the commutativity of this is not assumed to be valid. Let $\mathcal{G}$ be the class of all lattice ordered groups.

Assume that $G \in \mathcal{G}$; consider an internal subdirect decomposition

$$G = (\operatorname{int\,sub}) A \times B. \tag{1}$$

Hence there are $\rho_1, \rho_2 \in \operatorname{con} G$ such that $A = G/\rho_1$ and $B = G/\rho_2$. The mapping $\varphi\colon G \to A \times B$ corresponding to (1) is defined by $\varphi(g) = (g(\rho_1), g(\rho_2))$ for each $g \in G$.

There is a one-to-one correspondence between ℓ-ideals of G and congruence relations on G. If ρ is a congruence relation corresponding to an ℓ-ideal X, then for $g_1, g_2 \in G$ we have $g_1 \rho g_2$ iff $g_1 - g_2 \in X$.

Let X_1 and X_2 be ℓ-ideals of G and ρ_1, ρ_2 be the corresponding congruence relations. Then $\rho_1 \leqq \rho_2$ iff $X_1 \subseteq X_2$. This yields

$$X_1 \cap X_2 = \{0\} \iff \rho_1 \wedge \rho_2 = \rho_0.$$

Let $Z \subseteq G$. The *polar* $Z^\perp$ of Z is defined by

$$Z^\perp = \bigl\{g \in G : \ |g| \wedge |z| = 0 \text{ for each } z \in Z\bigr\}.$$

Each polar is a convex ℓ-subgroup of G.

Lemma 3.1. *Let Z be an ℓ-ideal of G. Then $Z^\perp$ is an ℓ-ideal of G as well.*

Proof. It suffices to verify that $Z^\perp$ is normal, i.e., that for each $x \in G$ and $z \in Z^\perp$ the relation $-x + z + x \in Z^\perp$ is valid. There exist $x_1, x_2 \in G^+$ with $x \quad x_1 - x_2$. Similarly, there exist $z_1, z_2 \in (Z^\perp)^+$ such that $z = z_1 - z_2$. From this we easily obtain that if suffices to prove that $-x + z + x \in Z^\perp$ is valid for each $x \in G^+$ and each $z \in (Z^\perp)^+$.

By way of contradiction, assume that there exist $x \in G^+$ and $z' \in (Z^\perp)^+$ such that $-x + z' + x \notin Z^\perp$. Then we must have $z' > 0$, whence $-x + z' + x > 0$. Further, there exists $z \in Z$ with $z \wedge (-x + z' + x) > 0$. From this we obtain

$$(x + z - x) \wedge z' > 0.$$

Put $z_1 = x + z - x$. Since Z is an ℓ-ideal, we get $z_1 \in Z$. Therefore $z_1 \wedge z' > 0$; we arrived at a contradiction. □

Consider the relation (1). There are ℓ-ideals A_1 and B_1 in G such that ρ_1 corresponds to A_1 and ρ_2 corresponds to B_1. Put $C = B_1^\perp$. In view of 3.1, C is an ℓ-ideal; let ρ_3 be the congruence relation which corresponds to C. Denote $\overline{A} \quad G/\rho_3$.

We have $C \cap B_1 = \{0\}$, whence $\rho_3 \wedge \rho_2 = \rho_0$. Thus the relation

$$G = (\operatorname{int\,sub}) \overline{A} \times B \tag{2}$$

is valid.

Lemma 3.2. *The relation* (2) *is an m-subdirect decomposition of G.*

P r o o f. Assume that we have a subdirect decomposition

$$G = (\operatorname{int}\operatorname{sub})A' \times B, \tag{3}$$

where B is as above and $A' = G/\rho_4$ with $\rho_4 \in \operatorname{con} G$ such that $\rho_4 > \rho_3$. Let c' be an ℓ-ideal of G having the property that ρ_4 corresponds to C'. In view of (3) we have $\rho_4 \wedge \rho_2 = \rho_0$, whence $C' \cap B_1 = \{0\}$. Thus $|c'| \wedge |b_1| = 0$ for each $c' \in C'$ and $b_1 \in B_1$. Hence $C' \subseteq B_1^{\perp} = C$. This yields $\rho_4 \leqq \rho_3$. Summarizing, we get $\rho_4 = \rho_3$ and therefore (2) is an m-subdirect decomposition. □

Under the notation as above, we also have $\rho_1 \wedge \rho_2 = \rho_0$, hence $A_1 \cap B_1 = \{0\}$ and thus $A_1 \subseteq B_1^{\perp} = C$; therefore $\rho_1 \subseteq \rho_3$.

From this and from 3.2 we conclude that the assertion concerning subdirect decompositions of ℓ-groups formulated at the end of Section 1 is valid.

Lemma 3.3. *Assume that* (1) *is valid and let us apply the notation as above. Then the following conditions are equivalent:*

(i) (1) *is an m-subdirect decomposition;*

(ii) $A_1 = B_1^{\perp}$.

P r o o f. Suppose that (i) is valid. Consider the relation (2). Since $\rho_3 \geqq \rho_1$, in view of the maximality condition we obtain $\rho_3 = \rho_1$, whence $A_1 = C$. Thus $A_1 = A_2^{\perp}$.

Conversely, suppose that (ii) holds. Then $A_1 = C$, thus $A - \overline{A}$. According to 3.2, (i) is valid. □

Corollary 3.4. *If* (1) *and*

$$G = (\operatorname{int}\operatorname{sub})A' \times B \tag{1'}$$

are m-subdirect decompositions, then $A = A'$.

Therefore we have:

Theorem 3.5. *The variety of $\mathcal{G}$ of all lattice ordered groups satisfies the cancellation rule* (c_2).

As a consequence we obtain that each subvariety of $\mathcal{G}$ satisfies (c_2) as well.

In the following Section we will apply Theorem 3.5 for proving an analogous result on GMV-algebras.

4. The cancellation rule (c_2) for GMV-algebras

The non-commutative generalization of the notion of MV-algebra was introduced in [6] and [7] (under the name of pseudo MV-algebra) and, independently, in [19] (under the name of generalized MV-algebra or, shortly, GMV-algebra).

A GMV-algebra can be defined as an algebraic structure $\mathcal{A} = (A; \oplus, ^-, ^\sim, 0, 1)$ of type $(2,1,1,0,0)$ such that the axioms (A1)–(A8) from [6] are satisfied.

If the operation $\oplus$ is commutative, then the unary operations $^-$ and $^\sim$ coincide; in this case $\mathcal{A}$ turns out to be an MV-algebra; for MV-algebras, cf. [3].

Let $x, y \in A$; we put $x \leqq y$ if $x^- \oplus y = 1$. Then $(A; \leqq)$ is a distributive lattice with the least element 0 and the greatest element 1.

An element u of a lattice ordered group G is a *strong unit* if for each $g \in G$ there exists $n \in \mathbb{N}$ such that $g \leqq nu$. In such a case, (G, u) is called a *unital lattice ordered group.*

For a unital lattice ordered group (G, u) consider the interval $A = [0, u]$ and for each $x, y \in A$ put

$$x \oplus y = (x + y) \wedge u, \tag{1}$$

$$x^- = u - x, \quad x^\sim = -x + u, \quad 1 = u. \tag{2}$$

Then $(A; \oplus, ^-, ^\sim, 0, 1)$ is a GMV-algebra which will be denoted by $\Gamma(G, u)$.

In [4] it was proved that for each GMV-algebra $\mathcal{A}$ there exists a unital lattice ordered group (G, u) such that $\mathcal{A} = \Gamma(G; u)$; the relation $\leqq$ in $\mathcal{A}$ coincides with the partial order defined in G.

In what follows, we assume that $\mathcal{A}$ is a GMV-algebra and that (G, u) is a unital lattice ordered group with $\mathcal{A} = \Gamma(G, u)$.

Let $\mathcal{J}(G)$ be the system of all ℓ-ideals of G; this system is partially ordered by the set-theoretical inclusion. It is well known that the mapping $\operatorname{con} G \to \mathcal{J}(G)$ defined by $\rho \mapsto 0(\rho)$ is an isomorphism of $\operatorname{con} G$ onto $\mathcal{J}(G)$.

A *normal ideal* of $\mathcal{A}$ is defined to be a nonempty subset X of A such that

(i) X is closed with respect to the operation $\oplus$,

(ii) if $x \in X$, $x_1 \in A$ and $x_1 \leqq x$, then $x_1 \in X$;

(iii) $a \oplus X = X \oplus a$ for each $a \in A$.

Let $\mathcal{NJ}(\mathcal{A})$ be the system of all normal ideals of $\mathcal{A}$; we suppose that it is partially ordered by the set-theoretical inclusion. The mapping $\operatorname{con} \mathcal{A} \to \mathcal{NJ}(\mathcal{A})$ defined by $\rho \mapsto 0(\rho)$ is an isomorphism of $\operatorname{con} \mathcal{A}$ onto $\mathcal{NJ}(\mathcal{A})$ (cf. [6], [19]).

Lemma 4.1. (Cf. [5].) *For each $Y \in \mathcal{J}(G)$ we put $\psi(Y) = Y \cap A$. Then ψ is an isomorphism of $\mathcal{J}(G)$ onto $\mathcal{N}\mathcal{J}(A)$.*

Let $\rho^1 \in \operatorname{con} G$. Put $0(\rho^1) = Y$. There exists a uniquely determined $\rho \in \operatorname{con} \mathcal{A}$ with $0(\rho) = \psi(Y)$.

Lemma 4.2. (Cf. [1].) *The mapping $\chi\colon \operatorname{con} G \to \operatorname{con} \mathcal{A}$ defined by $\chi(\rho^1) = \rho$ for each $\rho^1 \in \operatorname{con} G$ is an isomorphism of $\operatorname{con} G$ onto $\operatorname{con} \mathcal{A}$.*

Subdirect product decompositions of MV-algebras have been investigated in [8]. In [10] it was remarked that the main result of [8] can be generalized for GMV-algebras. The notation applied in [8] and [10] was different from that used in the present paper; in our present notation [10, Proposition 3.4, Lemma 3.5 can be formulated as follows:

Lemma 4.3. (Cf. [10].) *Assume that*

$$G = (\operatorname{int sub}) \prod_{i \in I} (G/\rho^i).$$

Then

$$\mathcal{A} = (\operatorname{int sub}) \prod_{i \in I} (\mathcal{A}/\chi(\rho^i)$$

and for each $i \in I$, $\mathcal{A}/\chi(\rho^i)$ is isomorphic to $\Gamma(G/\rho^0, u(\rho^i))$.

Lemma 4.4. (Cf. [10].) *Assume that*

$$\mathcal{A} = (\operatorname{int sub}) \prod_{i \in I} (\mathcal{A}/\rho_0^i).$$

Put $\rho^i = \chi^{-1}(\rho_0^i)$ for each $i \in I$. Then

$$G = (\operatorname{int sub}) \prod_{i \in I} (G/\rho^i).$$

In view of 4.2 and 4.3 we obtain:

Proposition 4.5. *Let $G = (\operatorname{int sub})(G/\rho_1) \times (G/\rho_2)$ be an m-subdirect decomposition. Put $\rho_i' = \chi(\rho_i)$ $(i = 1, 2)$. Then $\mathcal{A} = (\operatorname{int sub})(\mathcal{A}/\rho_1') \times (\mathcal{A}/\rho_2')$ is an m-subdirect decomposition.*

Similarly, in view of 4.2 and 4.4 we have:

Proposition 4.6. *Let $\mathcal{A} = (\operatorname{int sub})(G/\rho_1^1) \times (G/\rho_2^1)$ be an m-subdirect decomposition. Put $\rho_i = \chi^{-1}(\rho_i^1)$ $(i = 1, 2)$. Then $G = (\operatorname{int sub})(G/\rho_1) \times (G/\rho_2)$ is an m-subdirect decomposition.*

THEOREM 4.7. *The variety $\mathcal{G}_{mv}$ of all GMV-algebras satisfies the cancellation rule* (c_2).

P r o o f. This is a consequence of 3.5 and of 4.2–4.6. □

In view of 4.7, each variety of GMV-algebras satisfies (c_2).

Also, the assertion concerning subdirect decompositions of GMV-algebras formulated at the end of Section 1 is valid.

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