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OSCILLATORY PROPERTIES OF NONLINEAR DIFFERENTIAL SYSTEMS WITH RETARDED ARGUMENTS

RUDOLF OLACH* — HELENA ŠAMAJOVÁ**

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ABSTRACT. This paper deals with oscillatory properties of n-dimensional nonlinear differential systems with retarded arguments when $n \geq 3$ is odd. The problem of oscillation of all solutions is treated.

1. Introduction

We will consider the systems of nonlinear differential inequalities with retarded arguments of the form

$$\begin{aligned} y_i'(t) - p_i(t) y_{i+1}(t) &= 0\,, \qquad i = 1,2,\dots,n-2\,,\\ y_{n-1}'(t) - p_{n-1}(t) \big|y_n\big(h_n(t)\big)\big|^{\alpha} \operatorname{sgn}\big[y_n\big(h_n(t)\big)\big] &= 0\,,\\ y_n'(t) \operatorname{sgn}\big[y_1\big(h_1(t)\big)\big] + p_n(t)\big|y_1\big(h_1(t)\big)\big|^{\beta} &\leq 0\,, \end{aligned} \tag{1}$$

where the following conditions are always assumed:

$n \geq 3$ is odd, $\alpha > 0$, $\beta > 0$;

$p_i \colon [a,\infty) \to [0,\infty)$, $a \in \mathbb{R}$, $i = 1,2,\dots n$, are continuous functions and not identically zero on any subinterval of $[a,\infty)$;

$\int_a^\infty p_i(t)\,\mathrm{d}t = \infty$, $i = 1,2,\dots,n-1$;

$h_1 \colon [a,\infty) \to \mathbb{R}$, $h_n \colon [a,\infty) \to \mathbb{R}$ are continuous nondecreasing functions and $h_1(t) < t$, $h_n(t) < t$ on $[a,\infty)$;

$\lim\limits_{t\to\infty} h_1(t) = \infty$, $\lim\limits_{t\to\infty} h_n(t) = \infty$.

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By W we will denote the set of all solutions $y(t) = (y_1(t), \dots, y_n(t))$ of the system (1) which exist on some ray $[T_y, \infty) \subset [a, \infty)$ and satisfy

$$\sup\Big\{ \sum_{i=1}^{n} |y_i(t)| : \ t \ge T \Big\} > 0$$

for any $T \ge T_y$.

The oscillatory problem of two-dimensional differential systems with retarded arguments was studied by Ševelo and Varech [5] and in the other papers cited therein. The three-dimensional differential systems with deviating arguments were treated by Špániková in [6]. Our interest is focused on Marušiak's paper [2] where the author considered n-dimensional nonlinear differential systems with retarded arguments and investigated their oscillatory and asymptotic properties. It is to be pointed out that there is no oscillatory result for the system (1) in the case when $n \ge 3$ is odd. It is the reason why our attention in this paper is concentrated on that problem. In addition, Theorems 1 and 2 extend the result of [2; Theorem 3].

2. Main results

DEFINITION 1. A solution $y \in W$ is called *oscillatory* if each component has arbitrarily large zeros. A solution $y \in W$ is called *nonoscillatory* (resp. *weakly nonoscillatory*) if each component (resp. at least one component) is eventually of a constant sign.

We define $I_0 = 1$ and

$$I_k(t, s; p_k, \dots, p_1) = \int_s^t p_k(x) I_{k-1}(x, s; p_{k-1}, \dots, p_1)\,\mathrm{d}x\,, \qquad k = 1, \dots, n-2\,.$$

LEMMA 1. *Suppose that*

$$y = (y_1, \dots, y_n) \in W \tag{2}$$

is a nonoscillatory solution of (1) *in the interval* $[a, \infty)$ *and*

$$(-1)^{n+i} y_i(t) y_1(t) > 0 \qquad on \quad [t_0, \infty)\,, \ \ t_0 \ge a\,, \quad i = 2, \dots, n\,. \tag{3}$$

Then

$$|y_1(h_1(t))| \ge |y_n(h_n(t))|^{\alpha} \int_{h_1(t)}^{t} p_{n-1}(x) I_{n-2}(x, h_1(t); p_{n-2}, \dots, p_1)\,\mathrm{d}x \tag{4}$$

for all large t.

P r o o f. Let $t_0 \le s \le t$. It is evident that

$$y_1(s) = y_1(t) - \int_s^t y_1'(x)\,\mathrm{d}x = y_1(t) - \int_s^t p_1(x) y_2(x)\,\mathrm{d}x\,.$$

The second integral can be calculated by parts. Denote

$$v(x) = \int_s^x p_1(\tau)\,\mathrm{d}\tau = I_1(x,s;p_1)\,, \qquad u(x) = y_2(x)\,.$$

Then we have

$$\begin{aligned} y_1(s) &= y_1(t) - y_2(t) I_1(t,s;p_1) + \int_s^t y_2'(x) I_1(x,s;p_1)\,\mathrm{d}x \\ &= y_1(t) - y_2(t) I_1(t,s;p_1) + \int_s^t y_3(x) p_2(x) I_1(x,s;p_1)\,\mathrm{d}x\,. \end{aligned}$$

Applying further $(n-3)$ times the method by parts on the integral above we obtain the following identity

$$\begin{aligned} y_1(s) = {}& \sum_{j=0}^{n-2} (-1)^j y_{j+1}(t) I_j(t,s;p_j,\dots,p_1) \\ &+ \int_s^t p_{n-1}(x) \left|y_n\big(h_n(x)\big)\right|^{\alpha} \operatorname{sgn}\big[y_n\big(h_n(x)\big)\big] I_{n-2}(x,s;p_{n-2},\dots,p_1)\,\mathrm{d}x\,, \\ & \qquad t_0 \le s \le t\,. \end{aligned}$$

In view of (3) and the monotonicity of $y_n(t)$, we obtain for $T \ge t_0$ sufficiently large,

$$\begin{aligned} y_1(s)\operatorname{sgn}[y_1(s)] = {}& \sum_{j=0}^{n-2} (-1)^j y_{j+1}(t) \operatorname{sgn}[y_1(t)] I_j(t,s;p_j,\dots,p_1) \\ &+ \int_s^t p_{n-1}(x) \left|y_n\big(h_n(x)\big)\right|^{\alpha} I_{n-2}(x,s;p_{n-2},\dots,p_1)\,\mathrm{d}x\,, \\ & \qquad T \le s \le t\,, \end{aligned}$$

$$\left|y_1\big(h_1(t)\big)\right| \ge \left|y_n\big(h_n(t)\big)\right|^{\alpha} \int_{h_1(t)}^t p_{n-1}(x) I_{n-2}\big(x, h_1(t); p_{n-2},\dots,p_1\big)\,\mathrm{d}x\,, \qquad t > T\,.$$

The lemma is proved. □

The next notation will be used:

$$\bar{p}_i(t) = \min\{p_i(s):\ h_1(t) \le s \le t\}\,, \qquad t \ge a\,,\ \ i = 1,\dots,n-1\,,$$
$$P_{n-1}(t) = \bar{p}_{n-1}(t) \cdot \bar{p}_{n-2}(t) \cdots \bar{p}_1(t)\,.$$

The next lemma follows from [2; Theorem 3].

LEMMA 2. *Suppose that* $0 < \alpha\beta < 1$ *and*

$$\int^{\infty} \big(h_1(t)\big)^{(n-1)\beta} p_n(t) \big[P_{n-1}\big(h_1(t)\big)\big]^{\beta}\, \mathrm{d}t = \infty\,. \tag{5}$$

Then every nonoscillatory solution of system (1) *has the property*

$$\lim_{t\to\infty} y_k(t) = 0\,, \qquad k = 1, 2, \dots, n\,,$$

and (3) *holds.*

LEMMA 3. *Consider the differential inequality*

$$y'(t)\operatorname{sgn}\big[y\big(\tau(t)\big)\big] + p(t)\big|y\big(\tau(t)\big)\big|^{\lambda} \le 0\,, \qquad t \ge a\,, \tag{6}$$

where $0 < \lambda < 1$, $p \in C\big([a,\infty),[0,\infty)\big)$, $p \not\equiv 0$, $\tau \in C\big([a,\infty),(0,\infty)\big)$ *is nondecreasing function,* $\lim\limits_{t\to\infty} \tau(t) = \infty$, $\tau(t) < t$ *for* $t \ge a$ *and*

$$\int^{\infty} p(t)\, \mathrm{d}t = \infty\,. \tag{7}$$

Then every nonoscillatory solution of (6) *tends to zero as* $t \to \infty$.

Proof. Suppose that y is a positive solution of (6) and $y\big(\tau(t)\big) > 0$ for $t \ge t_1 \ge a$. Then $y'(t) < 0$ for $t \ge t_1$. So $\lim\limits_{t\to\infty} y(t) = L \ge 0$ exists. We show that $L = 0$. If $L > 0$ we get

$$y(\infty) - y(t_1) \le -\int_{t_1}^{\infty} p(s)\big[y\big(\tau(s)\big)\big]^{\lambda}\, \mathrm{d}s,$$

$$y(t_1) \ge L + \int_{t_1}^{\infty} p(s)\big[y\big(\tau(s)\big)\big]^{\lambda}\, \mathrm{d}s$$

$$\ge L + L^{\lambda} \int_{t_1}^{\infty} p(s)\, \mathrm{d}s,$$

and this is a contradiction to condition (7). Thus $y(t) \to 0$ as $t \to \infty$.

Now assume that y is a negative solution of (6) and $y(\tau(t)) < 0$ for $t \geq t_1 \geq t_0$. Then $y'(t) > 0$ for $t \geq t_1$, $y(t)$ is increasing and $\lim\limits_{t\to\infty} y(t) = L \leq 0$ exists. We claim that $L = 0$. If $L < 0$ we obtain

$$-y(t_1) \geq -y(\infty) + \int_{t_1}^{\infty} p(s)\left|y(\tau(s))\right|^{\lambda} \,\mathrm{d}s\,,$$

$$\begin{aligned} -y(t_1) &\geq -L + \int_{t_1}^{\infty} p(s)\left|y(\tau(s))\right|^{\lambda} \,\mathrm{d}s \\ &\geq -L + (-L)^{\lambda} \int_{t_1}^{\infty} p(s)\,\mathrm{d}s\,, \end{aligned}$$

which is a contradiction to (7). Thus $y(t) \to 0$ as $t \to \infty$. □

Lemma 4. *Assume that $0 < \lambda < 1$ and conditions of Lemma 3 are satisfied. Then the functional inequality*

$$y'(t) + p(t)\left|y(\tau(t))\right|^{\lambda} \operatorname{sgn} y(\tau(t)) \leq 0\,, \qquad t \geq a\,, \tag{8}$$

cannot have an eventually positive solution and

$$y'(t) + p(t)\left|y(\tau(t))\right|^{\lambda} \operatorname{sgn} y(\tau(t)) \geq 0\,, \qquad t \geq a\,, \tag{9}$$

cannot have an eventually negative solution.

Proof. Assume that y is a positive solution of (8) on $[t_1, \infty)$, $t_1 \geq a$. Lemma 3 implies that

$$y(t) \to 0 \qquad \text{as} \quad t \to \infty\,.$$

From inequality (8) it follows that there exists $t_2 \geq t_1$ such that y is decreasing on $[t_2, \infty)$. We have

$$\left[y(\tau(t))\right]^{\lambda} \geq \left[y(t)\right]^{\lambda}, \qquad t \geq t_3 \geq t_2\,.$$

From (8)

$$-y'(t) \geq p(t)\left[y(\tau(t))\right]^{\lambda} \geq p(t)\left[y(t)\right]^{\lambda}, \qquad t \geq t_3\,.$$

Then we get

$$\int_{y(t)}^{y(t_3)} \frac{\mathrm{d}u}{u^{\lambda}} = \int_{t_3}^{t} \frac{-y'(s)}{[y(s)]^{\lambda}} \,\mathrm{d}s \geq \int_{t_3}^{t} p(s)\,\mathrm{d}s\,.$$

Letting $t \to \infty$ we have

$$\infty > \int_{0}^{y(t_3)} \frac{\mathrm{d}u}{u^{\lambda}} \geq \int_{t_3}^{\infty} p(s)\,\mathrm{d}s\,,$$

which contradicts condition (7).

Assume that y is a negative solution of (9) on $[t_1, \infty)$, $t_1 \ge t_0$. By Lemma 3 we have

$$y(t) \to 0 \qquad \text{as} \quad t \to \infty .$$

Inequality (9) implies that there exists $t_2 \ge t_1$ such that y is increasing on $[t_2, \infty)$. Then we get

$$\left|y\big(\tau(t)\big)\right|^{\lambda} \ge |y(t)|^{\lambda} , \qquad t \ge t_3 \ge t_2 .$$

From (9) we have

$$y'(t) \ge p(t)\left|y\big(\tau(t)\big)\right|^{\lambda} \ge p(t)|y(t)|^{\lambda} , \qquad t \ge t_3 .$$

Then we obtain

$$\int\limits_{y(t_3)}^{y(t)} \frac{\mathrm{d}u}{|u|^{\lambda}} = \int\limits_{t_3}^{t} \frac{y'(s)}{|y(s)|^{\lambda}} \,\mathrm{d}s \ge \int\limits_{t_3}^{t} p(s) \,\mathrm{d}s .$$

Letting $t \to \infty$ we get

$$\infty > \int\limits_{y(t_3)}^{0} \frac{\mathrm{d}u}{|u|^{\lambda}} \ge \int\limits_{t_3}^{\infty} p(s) \,\mathrm{d}s ,$$

which contradicts condition (7).

The next lemma is in [2] as Lemma 1. □

LEMMA 5. *Let $y = (y_1, \dots, y_n) \in W$ be a weakly nonoscillatory solution of (1). Then y is nonoscillatory.*

THEOREM 1. *Suppose that $0 < \alpha\beta < 1$, (5) holds and*

$$\int^{\infty} p_n(s) \left[\int\limits_{h_1(s)}^{s} p_{n-1}(x) I_{n-2}\big(x, h_1(s); p_{n-2}, \dots, p_1\big) \,\mathrm{d}x \right]^{\beta} \mathrm{d}s = \infty .$$

Then all solutions of system (1) are oscillatory.

P r o o f . Assume that the system (1) has a solution $y = (y_1, \dots, y_n) \in W$ of which at least one component is eventually of a constant sign. Then by Lemma 5, y is nonoscillatory. We may suppose that $y_1(t) > 0$ for $t \ge t_0 \ge a$. By Lemma 2 the solution y has the property

$$\lim_{t\to\infty} y_k(t) = 0 , \qquad k = 1, 2, \dots, n ,$$

and (3) holds. Applying Lemma 1 in the nth inequality of the system (1) we obtain

$$y_n'(t)+p_n(t)\left[\int_{h_1(t)}^{t} p_{n-1}(x)I_{n-2}\big(x,h_1(t);p_{n-2},\dots,p_1\big)\,\mathrm{d}x\right]^{\beta} y_n^{\alpha\beta}\big(h_n(t)\big)\le 0\,, \tag{10}$$

$t\ge T\ge t_0$, where T is sufficiently large. With regard to the fact that $0<\alpha\beta<1$, by Lemma 4, the inequality (10) cannot have a positive solution. This contradicts the fact that $y_n(t)>0$ for $t\ge T$.

Now assume that $y_1(t)<0$, $t\ge t_0\ge a$. By Lemma 2 the solution y satisfies (3). Applying Lemma 1 in the nth inequality of the system (1) we get

$$-y_n'(t)+p_n(t)\left[\int_{h_1(t)}^{t} p_{n-1}(x)I_{n-2}\big(x,h_1(t);p_{n-2},\dots,p_1\big)\,\mathrm{d}x\right]^{\beta}\big|y_n\big(h_n(t)\big)\big|^{\alpha\beta}\le 0\,,$$

$$y_n'(t)-p_n(t)\left[\int_{h_1(t)}^{t} p_{n-1}(x)I_{n-2}\big(x,h_1(t);p_{n-2},\dots,p_1\big)\,\mathrm{d}x\right]^{\beta}\big|y_n\big(h_n(t)\big)\big|^{\alpha\beta}\ge 0\,,$$

$$y_n'(t)+p_n(t)\left[\int_{h_1(t)}^{t} p_{n-1}(x)I_{n-2}\big(x,h_1(t);p_{n-2},\dots,p_1\big)\,\mathrm{d}x\right]^{\beta}\cdot$$
$$\cdot\big|y_n\big(h_n(t)\big)\big|^{\alpha\beta}\operatorname{sgn}\big[y_n\big(h_n(t)\big)\big]\ge 0\,,$$

$t\ge T\ge t_0$, where T is sufficiently large. Lemma 4 implies that above inequality cannot have a negative solution, which contradicts $y_n(t)<0$ for $t\ge T$. The proof is complete. □

LEMMA 6. *Suppose that assumptions* (2) *and* (3) *hold. Then*

$$\big|y_1\big(h_1(t)\big)\big|\ge\frac{\big(t-h_1(t)\big)^{n-1}}{(n-1)!}P_{n-1}(t)\big|y_n\big(h_n(t)\big)\big|^{\alpha} \tag{11}$$

for all large t.

Proof. We may assume that $y_1(t)>0$ for $t\ge t_0\ge a$. In view of (4) we get

$$\begin{aligned}\big|y_1\big(h_1(t)\big)\big| &\ge \big|y_n\big(h_n(t)\big)\big|^{\alpha}\bar p_{n-1}(t)\int_{h_1(t)}^{t} I_{n-2}\big(x,h_1(t);p_{n-2},\dots,p_1\big)\,\mathrm{d}x\\ &=\big|y_n\big(h_n(t)\big)\big|^{\alpha}\bar p_{n-1}(t)\int_{h_1(t)}^{t}\int_{h_1(t)}^{x} p_{n-2}(s)I_{n-3}\big(s,h_1(t);p_{n-3},\dots,p_1\big)\,\mathrm{d}s\,\mathrm{d}x\,.\end{aligned}$$

Denote

$$u(x) = x\,, \qquad v(x) = \int\limits_{h_1(t)}^{x} p_{n-2}(s) I_{n-3}\big(s, h_1(t); p_{n-3}, \dots, p_1\big)\,\mathrm{d}s$$

and integrating by parts we obtain

$$\begin{aligned}
&\big|y_1\big(h_1(t)\big)\big| \\
&\quad \geq \big|y_n\big(h_n(t)\big)\big|^{\alpha} \bar{p}_{n-1}(t) \Bigg[t \int\limits_{h_1(t)}^{t} p_{n-2}(s) I_{n-3}\big(s, h_1(t); p_{n-3}, \dots, p_1\big)\,\mathrm{d}s \\
&\qquad - \int\limits_{h_1(t)}^{t} x p_{n-2}(x) I_{n-3}\big(x, h_1(t); p_{n-3}, \dots, p_1\big)\,\mathrm{d}x \Bigg] \\
&\quad = \big|y_n\big(h_n(t)\big)\big|^{\alpha} \bar{p}_{n-1}(t) \int\limits_{h_1(t)}^{t} (t-x) p_{n-2}(x) I_{n-3}\big(x, h_1(t); p_{n-3}, \dots, p_1\big)\,\mathrm{d}x \\
&\quad \geq \big|y_n\big(h_n(t)\big)\big|^{\alpha} \bar{p}_{n-1}(t) \bar{p}_{n-2}(t) \int\limits_{h_1(t)}^{t} (t-x) I_{n-3}\big(x, h_1(t); p_{n-3}, \dots, p_1\big)\,\mathrm{d}x \\
&\quad \geq \dots \geq \big|y_n\big(h_n(t)\big)\big|^{\alpha} \bar{p}_{n-1}(t) \dots \bar{p}_1(t) \int\limits_{h_1(t)}^{t} \frac{(t-x)^{n-2}}{(n-2)!}\,\mathrm{d}x\,.
\end{aligned}$$

Calculating the above integral we have

$$\big|y_1\big(h_1(t)\big)\big| \geq \frac{\big(t - h_1(t)\big)^{n-1}}{(n-1)!} P_{n-1}(t) \big|y_n\big(h_n(t)\big)\big|^{\alpha}\,, \qquad t \geq T\,,$$

where T is sufficiently large. □

THEOREM 2. *Suppose that $0 < \alpha\beta < 1$, (5) holds and*

$$\int^{\infty} \big(s - h_1(s)\big)^{(n-1)\beta} P_{n-1}^{\beta}(s) p_n(s)\,\mathrm{d}s = \infty\,.$$

Then all solutions of system (1) are oscillatory.

Proof. Assume that the system (1) has a solution $y = (y_1, \dots, y_n) \in W$ of which at least one component is nonoscillatory. Then by Lemma 5, y is nonoscillatory. We may suppose that $y_1(t) > 0$ for $t \geq t_0 \geq a$. Due to Lemma 2 the solution y has the property

$$\lim_{t\to\infty} y_k(t) = 0\,, \qquad k = 1, 2, \dots, n$$

and (3) holds.

Applying (11) in the nth inequality of (1) we get

$$y_n'(t) + \frac{\big(t - h_1(t)\big)^{(n-1)\beta}}{\big[(n-1)!\big]^{\beta}} P_{n-1}^{\beta}(t) p_n(t) \big|y_n\big(h_n(t)\big)\big|^{\alpha\beta} \le 0\,, \qquad t \ge T \ge t_0\,, \tag{12}$$

where T is sufficiently large. According to the condition $0 < \alpha\beta < 1$, by Lemma 4, the inequality (12) cannot have a positive solution. This is a contradiction with property (3).

Assume that $y_1(t) < 0$, $t \ge t_0 \ge a$. Then for solution y, (3) holds. Applying (11) in the nth inequality of (1) we have

$$-y_n'(t) + \frac{\big(t - h_1(t)\big)^{(n-1)\beta}}{\big[(n-1)!\big]^{\beta}} P_{n-1}^{\beta}(t) p_n(t) \big|y_n\big(h_n(t)\big)\big|^{\alpha\beta} \le 0\,,$$

$$y_n'(t) + \frac{\big(t - h_1(t)\big)^{(n-1)\beta}}{\big[(n-1)!\big]^{\beta}} P_{n-1}^{\beta}(t) p_n(t) \big|y_n\big(h_n(t)\big)\big|^{\alpha\beta} \operatorname{sgn}\big[y_n\big(h_n(t)\big)\big] \ge 0\,,$$

$$t \ge T \ge t_0\,,$$

where T is sufficiently large. By Lemma 4 the above inequality cannot have a negative solution. This contradicts (3). □

REFERENCES

[1] ERBE, L. H. KONG, Q.—ZHANG, B. G.: *Oscillation Theory for Functional Differential Equations*, Marcel Dekker, New York-Basel-Hong Kong, 1995.

[2] MARUŠIAK, P.: *On the oscillation of nonlinear differential systems with retarded arguments*, Math. Slovaca **34** (1984), 73–88.

[3] MARUŠIAK, P. OLACH, R.: *Functional Differential Equations*, University of Žilina/ /EDIS, 2000. (Slovak)

[4] OLACH, R.: *Observation of a Feedback mechanism in a population model*, Nonlinear Anal. **41** (2000), 539–544.

[5] ŠEVELO, V. N. VARECH, N. V.: *On the conditions of the oscillation of the solutions of differential system with retarded arguments.* In: Qualitative Methods of Theory of Differential Equations with Deviating Argument, Math. Inst. Ukrain. Acad. Sci., Kiev, 1977, pp. 26 44. (Russian)

[6] ŠPÁNIKOVÁ, E.: *Oscillatory properties of the solutions of three-dimensional nonlinear differential systems with deviating arguments*, Acta Math. Univ. Comenian. **54-55** (1998), 173–183.

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