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BOOK REVIEWS

Kurzweil, J.:
HENSTOCK-KURZWEIL INTEGRATION:
ITS RELATION TO TOPOLOGICAL VECTOR SPACES.
Series in Real Analysis 7.
World Scientific, Singapore 2000, vii + 136 pp.
ISBN 981-02-4207-7/hbk

In the book the space P of primitives of Kurzweil-Henstock integrable functions is considered. Recall that F is primitive to f if $F(K) = \int_K f \, dt$ for any non-degenerate closed subinterval K of a given compact interval $I = [a, b]$.

The aim of the book is a topological characterization of so-called E-convergence of a sequence (F_i) of primitives to a primitive $F \in P$. The convergence can be characterized by two conditions. The first one is the pointwise convergence $f_i \to f$, where F_i are primitives of f_i and F is a primitive of f. The second condition is a uniformity condition: for all $i, j \in \mathbb{N}$ and a given sequence (δ_j) of gauges $\delta_j : I \to (0, \infty)$ there holds

$$\left| \int_I f_i \, dt - \sum_k f_i(t_k)|J_k| \right| < 2^{-j}$$

whenever $\{(t_k, J_k) : k = 1, \dots, p\}$ is a δ_j-fine partition (i.e. $\bigcup_k J_k = I$, J_k are non-overlaping, $t_k \in J_k$ and $J_k \subset (t_k - \delta_j(t_k), t_k + \delta_j(t_k))$ for any k).

The main result of the book is the construction of a topology $\mathcal{T}$ on the space P of all primitives such that E-convergence of (F_i) to F implies its convergence in $(P, \mathcal{T})$ and such that $(P, \mathcal{T})$ is complete and $(P, \mathcal{T})$ is a topological vector space. Of course, the corresponding theory has a more general counterpart.

The book is self-contained, in the *first Chapter* basic information is exposed about the Kurzweil integral (with hints of proofs and references). The main result presented here is the characterization of integrable functions and their primitives. The same is considered in *Chapter 2*, of course, the corresponding gauges δ_i are supposed to be Borel measurable. This result is important for the proof (in *Chapter 8*) of the completeness of the space $(P, \mathcal{T})$.

In *Chapter 3* a new kind of convergence in P is introduced, so-called Q-convergence, and it is proved that E-convergence implies Q-convergence. The concept is generalized in *Chapter 4*. Instead of the Banach space $A \supset P$ of all additive and continuous functions which map subintervals of I to the reals, an arbitrary Banach space X is considered. Instead of the set of all sequences of gauges an arbitrary set D is taken and the Q-convergence is considered with respect to a mapping Q from D to the set of all convex balanced closed subsets of X such that for any $d_1, d_2 \in D$ there exists $d_3 \in D$ such that $Q(d_1) + Q(d_2) \subset Q(d_3)$. A sequence (x_i) Q-converges to x if there exists $d \in D$ such that $x_i \in Q(d)$ for any i and $x_i \to x$ in X.

Chapter 7 contains the construction (in the general setting) of a topology $\mathcal{T}$ on $\mathbf{P} = \bigcup\{Q(d) : d \in D\}$ such that Q-convergence implies $\mathcal{T}$-convergence. It follows that in the special case $\mathbf{P} = P$ of the set P of primitives, E-convergence implies $\mathcal{T}$-convergence. *Chapter 8* contains the complete discussion about this special but the most important case.

Chapters 5 and *6* are not directly related to the proof of the main theorem. *Chapter 5* in the abstract setting is concerned in the case $D = \mathbb{N}$ and *Chapter 6* contains some important facts about Q-convergence in the space P of primitives.

Some open problems are included in *Chapter 9*.

Although the results of the book are very interesting and profound, the book can be read successfully without preliminary knowledge. It is written with a great didactical mastery, clearly and precisely. It can be recommended not only for specialists on integration theory, but also for a large group of readers, mainly for postgradual students.

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