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PROFESSOR MARKO ŠVEC, OCTOGENARIAN

Professor Marko Švec, a distinguished Slovak mathematician, excellent university teacher who educated many generations of engineers and mathematicians, celebrated his 80th birthday on 10th October 1999. His work has significantly influenced scientific and educational life in Slovakia.

He was born on 10th October 1919 in Kmeťovo, in the district of Nové Zámky. He studied Mathematics and Physics at the Faculty of Natural Sciences, Slovak University in Bratislava. In the years 1944–1949 he taught at gymnasiums in Topoľčany and Bratislava. From the year 1949 till the year 1968 he was engaged at the Faculty of Electrical Engineering of the Slovak Technical University in Bratislava where till 1955 he was a Reader, and then an Associate Professor till in 1966 he was appointed Full Professor of Mathematics. In the period 1968–1994 he was a Professor of Mathematics at the Department of Mathematical Analysis at the Faculty of Natural Sciences and at the Faculty of Mathematics and Physics of the Comenius University in Bratislava, respectively (from the year 1980). In the years 1969–1972 and in 1974 in the frame of the UNESCO mission he taught at the University in Bahia in Brazil. From 1994 he has been at the Pedagogical Faculty of the Comenius University, where till 1999 he held the position of head of the Department of Mathematics. In 1957 he received his CSc. (Candidate of Science) degree from the Faculty of Natural Sciences of Purkyně University in Brno and, at the same faculty, he defended his DrSc. thesis in 1965.

First of all I would like to recall his endeavours as an excellent university teacher. He wrote (together with two colleagues) the first modern textbook Mathematics I and Mathematics II aimed for technical university students. In this tradition he published a textbook on integral equations and then, as a coauthor, the textbook "Ordinary Differential Equations". The latter contains his own results and for many years it will be a rich source of knowledge for all who are interested in this theory. As a coauthor he also wrote "Mathematical Analysis of Functions of a Real Variable".

At the beginning of his scientific career Prof. Marko Švec focused his interest on ordinary differential equations, especially on linear differential equations of higher order. This field of investigation is far from being simple, because the relations expressed by a differential equation of higher order are complicated; nevertheless it is important, as linear differential equations are the first step for studying nonlinear differential equations and functional differential equations.

In the nineties Prof. Marko Švec has continued his scientific work in several directions.

First of all he devoted his attention to the *study of oscillatory and asymptotic properties of nonlinear differential equations with quasiderivatives.* If we have a function $y(t)$ and positive continuous functions $a_i(t)$, $i = 0, 1, \dots, n$, in the interval $J = [t_0, \infty)$, then the functions defined by the relations

$$L_0 y(t) = a_0(t) y(t) , \qquad L_i y(t) = a_i(t) \big(L_{i-1} y(t)\big)' , \quad i = 1, 2, \dots, n , \tag{1}$$

are called respectively the *quasiderivative of order zero* and the *ith quasiderivative of the function* y *at the point* t. It is clear that in the case that the function $a_i(t)$ is only continuous,

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the existence of quasiderivatives does not depend on the existence of classical derivatives. However, if $a_i(t) \in C^{(n-i)}(J)$, $i = 0,1,\dots,n$, then the existence of all quasiderivatives up to the order n follows from the existence of $y^{(n)}(t)$. One of the fundamental results for linear differential equations (TRENCH) says that for each disconjugate linear differential equation in J there exists an n-tuple of positive functions $a_i(t) \in C^{(n-i)}(J)$, $i = 0,1,\dots,n$, such that

$$\int^{\infty} \frac{\mathrm{d}t}{a_i(t)} = \infty\,, \qquad i = 1,2,\dots,n-1\,, \tag{2}$$

and this equation can be written in the form

$$L_n y(t) = 0\,. \tag{3}$$

Hence equations with quasiderivatives are perturbed equations of a disconjugate differential equation and in addition to this, all functions $a_i(t)$ are only continuous.

In the paper [1], Prof. M. Švec investigated the differential equation with deviating argument

$$L_n y(t) + h\big(t, y(\varphi(t)), y'(\varphi(t)), \dots, y^{(n-1)}(\varphi(t))\big) = 0\,, \qquad n > 1\,, \tag{4}$$

where $h\colon J \times \mathbb{R}^n \to \mathbb{R}$, $\varphi\colon J \to \mathbb{R}$, $a_i\colon J \to (0,\infty)$, $i = 0,1,\dots,n$, are continuous functions. In addition to the assumption (2) a substantial assumption is the sign property of the function h, namely

$$y_0 h(t, y_0, y_1, \dots, y_{n-1}) > 0 \qquad \text{for all } t \in J \text{ and any } y_i \in \mathbb{R},\ i = 0,1,\dots,n-1,\ y_0 \neq 0\,, \tag{5}$$

or

$$y_0 h(t, y_0, y_1, \dots, y_{n-1}) < 0 \qquad \text{for the same values of } t,\ y_i,\ i = 0,1,\dots,n-1, \tag{6}$$

and

$$\lim_{t\to\infty} \varphi(t) = \infty\,. \tag{7}$$

Then for each nonoscillatory solution $y(t)$ of (4) there exists a number T_y such that on the interval $[T_y,\infty)$ each quasiderivative $L_i y(t)$, $i = 0,1,\dots,n$, has constant sign and therefore, $L_i y(t)$, $i = 0,1,\dots,n-1$, are monotone functions on $[T_y,\infty)$ so that $\lim_{t\to\infty} L_i y(t)$, $i = 0,1,\dots,n-1$, exist in the extended sense.

For the nonoscillatory solutions the following two cases are possible:

(a) $\lim_{t\to\infty} |L_i y(t)| = \infty$ for all $i = 0,1,\dots,n-1$, which implies that $\operatorname{sgn} L_{n-1} y(t) = \operatorname{sgn} L_i y(t)$, $i = 0,1,\dots,n-2$. In this case we say that $y(t)$ belongs to the *class* V_n.

(b) There exists $k \in \{0,1,\dots,n-1\}$ such that $\lim_{t\to\infty} L_k y(t)$ is finite, $\lim_{t\to\infty} L_i y(t) = \infty \cdot \operatorname{sgn} y(t)$, $i = 0,1,\dots,k-1$, and $\lim_{t\to\infty} L_i y(t) = 0$, $i = k+1,\dots,n-1$. Then we say that $y(t)$ belongs to the *class* V_k.

The classes V_k, $k = 0,1,\dots,n$, are disjoint and each nonoscillatory solution of (4) belongs to one and only one class V_k.

In the paper [1], the conditions are determined under which $\lim_{t\to\infty} L_k y(t) = 0$ for each solution $y(t) \in V_k$, $k \in \{0,1,\dots,n-1\}$, and conditions are given which guarantee that the class V_k, $k \in \{0,1,\dots,n-1\}$, is empty. The results obtained are applied to find a sufficient condition for the equation (4) to have property A or property B.

Let us recall the properties A and B. Equation (4) has *property* A if in the case n is even all solutions of (4) are oscillatory and in the case n is odd each solution of (4) is either oscillatory or $\lim_{t\to\infty} L_i y(t) = 0$, $i = 0,1,\dots,n-1$.

Equation (4) has *property B* if for n even each solution of (4) is either oscillatory or $\lim_{t\to\infty} L_i y(t) = 0$, $i = 0, 1, \dots, n-1$, or belongs to the class V_n and for n odd each solution of (4) is either oscillatory or belongs to the class V_n.

Similar problems were solved in [2] for the differential inclusion

$$L_n x(t) \in F\big(t, x(\varphi(t))\big), \qquad n > 1, \tag{8}$$

where $F(t,x)$ is a multifunction on $J \times \mathbb{R}$ such that

$$\begin{aligned}
&F(t,x) \text{ is upper semicontinuous on } J \times \mathbb{R};\\
&F(t,0) = \{0\};\\
&F(t,x)x < 0 \text{ for each } (t,x) \in J \times \mathbb{R},\ x \neq 0; \iff yx < 0 \text{ for each } y \in F(t,x);\\
&\text{or}\\
&F(t,x)x > 0 \text{ for each } (t,x) \in J \times \mathbb{R},\ x \neq 0; \iff yx > 0 \text{ for each } y \in F(t,x)
\end{aligned} \tag{9}$$

and φ and L_n have the same meaning as above.

Similarly $F(t,x) \geqq h(t,x) \iff y \geqq h(t,x)$ for each $y \in F(t,x)$. If $B \subset \mathbb{R}$, then $\|B\| = \inf\{|x| :\ x \in B\}$.

The idea of how to come from multifunctions to functions is the following: If $x(t)$ is a solution of (8) and, say,

$$0 \leqq G(t,|x|) \leqq \|F(t,x)\|, \qquad (t,x) \in J \times \mathbb{R},$$

G is a minorant function, then

$$0 \leqq G\big(t, |x(\varphi(t))|\big) \leqq \big\|F\big(t, x(\varphi(t))\big)\big\|$$

and, with respect to (8),

$$0 \leqq G\big(t, |x(\varphi(t))|\big) \leqq |L_n x(t)|, \qquad t > T_1.$$

Differential inclusions are very attractive for Prof. M. Švec.

In the paper [3], he studied equation (8) and, by means of the Ky Fan fixed point theorem for multivalued mappings, he found conditions under which there exist infinitely many positive and infinitely many negative solutions $x(t)$ of (8) which are asymptotic to the solutions of

$$L_n x(t) = 0, \tag{10}$$

more precisely, they satisfy

$$\lim_{t\to\infty} \frac{|L_0 x(t)|}{P_k(t,b)} = c_k, \qquad k \in \{0, 1, \dots, n-1\}, \tag{11}$$

where

$$\begin{aligned}
P_0(t,b) &= 1,\\
P_1(t,b) &= \int_b^t \frac{1}{a_1(s_1)}\, \mathrm{d}s_1,\\
&\ \vdots\\
P_{n-1}(t,b) &= \int_b^t \frac{1}{a_1(s_1)}\, \mathrm{d}s_1 \int_b^{s_1} \frac{1}{a_2(s_2)}\, \mathrm{d}s_2 \dots \int_b^{s_{n-2}} \frac{1}{a_{n-1}(s_{n-1})}\, \mathrm{d}s_{n-1},
\end{aligned} \tag{12}$$

is a fundamental system of solutions of (10).

He also found conditions under which there exist infinitely many positive and infinitely many negative solutions $x(t)$ of (8) such that

$$\lim_{t\to\infty} \frac{L_0 x(t)}{P_k(t,b)} = 0\,, \quad \lim_{t\to\infty} \frac{|L_0 x(t)|}{P_{k-1}(t,b)} = \infty\,, \qquad k \in \{1,2,\dots,n-1\}\,. \tag{13}$$

Such problems were solved in 1989 by T. KUSANO and M. ŠVEC for ordinary differential equations.

Prof. M. Švec is well-known also for his work on the equivalence of two differential systems and functional differential equations.

Two systems

$$x' = F(t,x) \tag{14}$$

and

$$y' = G(t,y) \tag{15}$$

are *equivalent* if to each solution x of the former system there exists a solution y of system (15) such that in some metric they are "near" and conversely if to any solution y of system (15) there exists a solution x of system (14) such that again x and y are "near" in this metric Prof. M. Švec considered different metrics and he studied for example asymptotic equivalence and integral equivalence.

In *ψ-asymptotic equivalence* the relation that the solution $x(t)$ of (14) be near the solution $y(t)$ of system (15) is given by

$$\lim_{t\to\infty} \frac{|x(t)-y(t)|}{\psi(t)} = 0\,, \qquad |\cdot| \text{ is the norm in } \mathbb{R}^n\,,$$

while (ψ,p) *integral equivalence* means that

$$\frac{|x(t)-y(t)|}{\psi(t)} \in L^p\big((t_0,\infty)\big)\,.$$

From the results attained by M. ŠVEC and A. HAŠČÁK it follows that the asymptotic behaviour of the solutions of perturbed, even very complicated systems, is similar to the behaviour of the solutions of simple systems if the perturbation term has a small growth in a certain sense. For their results concerning equivalence they both got an extraordinary prize of the Czechoslovak Academy of Sciences in 1986.

In the paper [4], Prof. M. Švec considered two differential inclusions

$$y^{(n)}(t) \in F\big(t, y(t)\big)\,, \tag{16}$$

$$x^{(n)}(t) \in G\big(t, x(t)\big)\,, \qquad n > 1\,, \tag{17}$$

where F and G are multifunctions which fulfil assumptions (9).

Differential inclusions (16) and (17) are *asymptotically equivalent* if for each solution $y(t)$ of (16) there exists a solution $x(t)$ of (17) such that

$$\lim_{t\to\infty} \big(y^{(i)}(t) - x^{(i)}(t)\big) = 0\,, \qquad i = 0,1,\dots,n-1\,, \tag{18}$$

and conversely. If they are asymptotically equivalent, then either both (16) and (17) possess or both do not possess the property A (or property B). Further sufficient condition is given which guarantees the asymptotic equivalence of (16) and (17).

The paper [6] deals with the periodic boundary value problem

$$L_4 x(t) + a(t)x(t) \in F\big(t, x(t)\big)\,, \qquad t \in [a,b]\,, \tag{19}$$

$$L_i(x(a)) = L_i\big(x(b)\big)\,, \qquad i = 0,1,2,3\,, \tag{20}$$

where in the quasiderivatives (1) the functions $a_0(t) = a_4(t) \equiv 1$, $a_1(t) = a_3(t)$, $t \in [a,b]$, $a(t) \geqq 0$ and $F(t,x)\colon [a,b] \times \mathbb{R} \to \{\text{nonempty convex compact subsets of } \mathbb{R}\}$.

The basic assumptions concerning $F(t,x)$ are:

(1) $F(t,x)$ is upper semicontinuous on $[a,b] \times \mathbb{R}$;
(2) To each measurable function $z(t)\colon [a,b] \to \mathbb{R}$ there exists a measurable selector $v(t)\colon [a,b] \to \mathbb{R}$ such that $v(t) \in F(t, z(t))$ a.e. on $[a,b]$.

The idea of the proof is the following. We consider a function $u \in Y = \{u(t) \in C([a,b]) : L_i u(a) = L_i u(b)\,,\ i = 0,1,2,3\}$. Then to $u \in Y$ belongs the set of all measurable selectors $Mu(t)$. Let $v(t) \in Mu(t)$, hence $v(t) \in F(t, u(t))$. Then we seek a solution $x(t)$ of the problem (20),

$$L_4 x(t) + a(t)x(t) = v(t)\,, \tag{21}$$

which is equivalent to the problem

$$x(t) = \int_a^b G(t,s)v(s)\,\mathrm{d}s \tag{22}$$

(because the Green function exists for the corresponding homogeneous problem) and there exists the multivalued operator

$$A\colon Y \to cf(Y) \quad (= \text{the set of all convex closed subsets of } Y)$$

defined by

$$Au(t) = \Big\{x(t) = \int_a^b G(t,s)v(s)\,\mathrm{d}s :\ v(t) \in Mu(t)\Big\}\,. \tag{23}$$

Evidently $Au(t) \subset Y$ and $Au(t) \neq \emptyset$ and convex. If $Au(t)$ is also a closed subset of Y, A is upper semicontinuous and $A(Y)$ is compact, then by the Ky Fan fixed point theorem the proof is complete.

The joint papers with D. Hricišáková deal with the generalized Liénard differential equation. In the former paper, the equation

$$x'' + p(t,x,x')x' + g(t,x) = 0 \tag{24}$$

is compared with the linear equation

$$z'' + P(t)z' + h(t)z = 0\,. \tag{25}$$

A sufficient condition is established that guarantees the implication: If (24) is oscillatory, then (25) is oscillatory, too. In [7], the boundary value problem

$$x'' + f(x)x' + g(x) = 0\,, \tag{26}$$

$$x(t_1) = 0\,, \quad x'(t_2) + F(x(t_2)) = 0\,, \qquad t_1 < t_2\,, \tag{27}$$

is studied, where $F(x) = \int_0^x f(s)\,\mathrm{d}s$. It is shown that the problem (26), (27) has only the trivial solution if $t_2 - t_1$ is sufficiently small.

The paper [9] is dealt with the following problem. If a selfadjoint equation of the fourth order

$$(L_4 y \equiv)\ \big(a_0(t)y''\big)'' + \big(a_1(t)y'\big)' + a_2(t)y = 0 \tag{28}$$

is nonoscillatory and a function $f(t) \geqq 0$ or $f(t) \leqq 0$ in the interval $[a,\infty)$, under which conditions is the non-homogeneous equation

$$L_4 y = f(t)$$

nonoscillatory?

Prof. M. Švec has gained by his results, by his endeavours for the benefit of his colleagues, students and the scientific community, admiration and respect of all of us. On occasion of his anniversary we congratulate him and we wish him good health, much strength and many years of happy life. May God bless him!

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