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PROFESSOR MICHAL GREGUŠ SEPTUAGENARIAN

Professor Michal Greguš, a distinguished Slovak mathematician, celebrated his 70th birthday on 22 December 1996. His work has significantly influenced the scientific, educational and cultural life in Slovakia.

He was born on 22 December 1926 in Zbehy (district Nitra) in a family of a railwayman. In 1946, he entered Comenius University in Bratislava and studied Mathematics and Physics, in 1946–1950 at the Faculty of Natural Sciences. Except a short period of work at the Slovak Technical University in Bratislava and the Military Technical Academy in Brno, his life and his activities have been closely connected with Comenius University.

In 1957, he received his CSc. (Candidate of Science) degree from the Faculty of Science of Purkyně University, Brno, and, at the same Faculty, he defended his DrSc. thesis at the age of 39 years. In 1959, he was appointed Associated Professor and, in 1965, Full Professor of Mathematics.

Professor Greguš played an important role in the organization of education. In 1959, he was appointed Head of the Department of Mathematics. He was Vice-Dean in the years 1959–1962, then the Dean of the Faculty of Natural Sciences, and from 1965 till 1968 he held the office of Vice-Chancellor of Comenius University. In these offices, he considerably contributed to the development of the Faculty of Natural Sciences, especially, to the construction of its new pavilions in Mlynská dolina. He also was one of the founders of the mathematical journal Acta Mathematica Universitatis Comenianae and was a member of its Editorial Board. In 1968, he was appointed Deputy Minister of Education, and, at this position, he remained till 1973. During the period 1973–1978 he was head of the Permanent Mission at the UNESCO.

After coming back to the University, he took part in preparing the establishment of the Faculty of Mathematics and Physics, and, after its foundation in 1980, he became its first Dean. The merits for the extraordinary increase of the activity of the Faculty goes to a great extent to Professor Greguš. This concerns not only education and research, but also cooperation of the Faculty with selected enterprises and research institutes, especially with Slovak Academy of Sciences, whose Member he has been. This cooperation was intended to help, on the one hand, to the cooperating institutions, and, on the other, to the Faculty in improving the training of the students for their future tasks. He had also great merits in appointing associated professors and full professors at the Faculty, many of them being without any political membership. Nowadays they form a core of teaching staff at the Faculty.

As a teacher, Professor Greguš has very close and friendly relation to his students. In his Seminar on the Theory of Linear Differential Equations, he had educated a number of excellent students. The creative atmosphere as well as the fruitful discussion were characteristic for this seminar. His experiences of many years of teaching Mathematics he has used in the textbook Ordinary Differential Equations [B3], of which he is a co-author. He also prepared lecture notes Partial Differential Equations [B2]. Both texts are characterized by the author's effort not only to present basic facts, but also to give the reader some applications of the mathematical results.

Professor Greguš was a Vice-President and, in the years 1990-93, President of the Union of Slovak Mathematicians and Physicists. For his merits, he was awarded a number of distinctions, among them the Golden Medal of the Faculty of Mathematics and Physics and of the Comenius

University. For his contributions to the international cooperation, he was granted medals of the Universities in Leningrad (St. Petersburg), Gent and Sofia.

Professor Greguš has had great merits in organizing the Czecho-Slovak conferences on differential equations and their applications EQUADIFF. The conferences have been organized by turns in Prague, Bratislava and Brno each fourth or fifth year since 1962. They were attended by hundreds of world specialists on differential equations, numerical methods and applications. Professor Greguš was the main organizer of the conferences held in Bratislava in 1966 and 1981.

However, the foundation stone of Professor Greguš's activity has always been his research in Mathematics. Under influence of the famous Italian mathematician G. Sansone in the fifties he started to study linear differential equations of the third order. Professor O. Boruvka had stimulated his research. He himself developed the theory of linear differential equations of the second order. Under his supervision M. Greguš joined these efforts with all his vigour. Within twenty years he developed the theory of the third order linear differential equation and educated a number of students, most of them now being already well-known mathematicians and his close collaborators. The result of his assiduous systematical and creative work is the monograph [B1] whose English version [B4] Third Order Linear Differential Equations contains an additional chapter on applications of the third order linear differential equations.

It is of advantage to write the homogeneous linear differential equation of the third order in the normal form

$$y''' + 2A(x)y' + \big[A'(x) + b(x)\big]y = 0\,, \tag{a}$$

where the functions A', b are continuous in a certain interval I. This form plays an important role in the transformation theory, and, moreover, the adjoint equation to (a) can be written in the form

$$z''' + 2A(x)z' + \big[A'(x) - b(x)\big]z = 0\,. \tag{b}$$

The function b is called the Laguerre invariant. The principal notion for the theory of equation (a) is the notion of the band of solutions. A two-dimensional subspace of solutions y of equation (a) satisfying $y(x_1) = 0$ is called *a band of solutions of the first kind of equation* (a) *at the point* x_1. Similarly, *bands of solutions of the second* ($y'(x_1) = 0$) and *of the third* ($y''(x_1) = 0$) *kinds* are introduced. Each band satisfies a second order equation

$$w(x)y'' - w'(x)y' + \big[w''(x) + 2A(x)w(x)\big]y = 0\,, \tag{c}$$

where w is a certain solution of equation (b). Under certain conditions $w(x) \neq 0$ either to the right, or to the left from x_1, and thus, the theorem on separation of zeros and, eventually, the whole theory of the 2nd order equations holds for the band of solutions. Another method of dealing with equation (a) consists in using the integral identities. The most known for solutions of equation (a) are the following ones:

$$y(x)y''(x) - \frac{1}{2}y'^2(x) + A(x)y^2(x) + \int_{x_0}^{x} b(t)y^2(t)\,\mathrm{d}\,\mathrm{t} = \mathrm{const}\,, \tag{d}$$

$$y''(x) + 2A(x)y(x) + \int_{x_0}^{x} [b(t) - A'(t)]y(t)\,\mathrm{d}\,\mathrm{t} = \mathrm{const}\,, \tag{e}$$

where $x_0 \in I$ is a fixed number, $x \in I$ is variable. These considerations helped to establish the properties of the zero-points of solutions of equation (a).

Among the problems, they have been solved will be mentioned the following ones:

1. A necessary and sufficient condition for equation (a) was found that every solution of (a) with at least one zero be oscillatory.
2. A sufficient condition for equation (a) was established that there exist at least one solution without zero points.
3. Sufficient conditions for the disconjugacy of equation (a) were found.

Another fundamental concept of the theory of oscillation of solutions of equation (a) is that of *a conjugate point*. Between conjugate points of two equations comparison theorems hold, on the basis of which, further criteria for oscillatoricity of equation (a) were derived. Another interesting problem was:

4. To construct an equation (a) whose all solutions oscillate.

The discussion about oscillatory properties of solutions of equation (a) has been completed by the investigation of the asymptotic behaviour of solutions of this equation, in particular, by a study of such properties of solutions in an interval $[a, \infty)$ as boundedness, existence of a limit at infinity, asymptotic formulae and the existence of solutions from the space L^1 or L^2.

An important part of Professor Greguš's research is a study of boundary value problems. He constructed the Green function for the multipoint boundary value problem, which makes it possible to transform nonlinear boundary value problems to nonlinear integral equations. Further he extended the Sturm oscillation theorem to equation (a) and solved the three-point boundary value problem for equation (a) whose coefficients depend on two parameters.

Many results for equation (a) have been transferred to the 3rd order equation in the general form

$$y''' + p_1(x)y'' + p_2(x)y' + p_3(x)y = 0\,. \tag{A}$$

Professor Greguš has also looked for impulses for his research not only within the theory, but also in considering mathematical models in physics, especially in the physics of plasma.

In the last decade, Professor Greguš has devoted mainly to the scientific work. During this period, he has written, either himself or with his collaborators (J. R. Graef, M. Gera and M. Greguš, Jr.), 15 papers. Instead of equation (a), he investigated the third order nonlinear differential equations which are more or less special cases of the equation

$$y''' + p(t, y, y', y'')y'' + q(t, y, y', y'')y' + r(t, y, y', y'')y = f(t, y, y', y'')\,, \tag{f}$$

where p, q, r and f are continuous functions. The basis of this research is the theory of the third order linear differential equations. If x is a solution of (f) on some interval, then it is a solution of the linear third order differential equation

$$y''' + p(t, x, x', x'')y'' + q(t, x, x', x'')y' + r(t, x, x', x'')y = f(t, x, x', x'')$$

on the same interval. This idea gives the possibility for the study of oscillatory and asymptotic properties of solutions of (f).

The most frequently solved problem was to find sufficient conditions under which solutions of equation (f) or of its special case with one zero are oscillatory. Closely related to that problem was the investigation of nonoscillatory solutions y when they have the property $y(t)y'(t) < 0$ in a neighbourhood of ∞. The relation between boundedness and oscillatoricity of solutions is investigated, too. Sufficient conditions for solutions to be from the class L^2 or $L^{\alpha+1}$ are also established.

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For his excellent results in research Professor Greguš became a well-known expert in the theory of third order nonlinear differential equations, who is often asked to solve problems in applied sciences, to review the papers written by the other authors and to consult the young colleagues. His results are frequently referred to in works of Slovak as well as foreign mathematicians.

It is difficult to briefly describe the many-sided and useful activities of Professor Greguš. However, it can be pointed out that he is always doing what is most important and is devoting himself to his work with all his well-known vigour and enthusiasm. Therefore we wish him for the future all the energy necessary to bring his plans to reality, to continue his work in good health and to have all opportunities to enjoy the results of his work.

Valter Šeda

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