# Book Reviews

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# BOOK REVIEWS

Kubáčková, L.: FOUNDATIONS OF EXPERIMENTAL DATA ANALYSIS.
CRC Press, Inc., Boca Raton, Florida and Ister Science Press, Ltd., Bratislava 1992, 368 pp.
ISBN 0-8493-7736-6 (CRC Press), ISBN 80-900496-6-8 (Ister Science Press)

The book is an English translation of the successful Slovak monograph by Kubáčková, L.: Metódy spracovania experimentálnych údajov, Veda, Bratislava 1990, 324 pp. (for more details see a book review in Math. Slovaca **43** (1993), 108–109).

The material is concentrated into three main parts:

(1) Preliminary definitions from probability theory and mathematical statistics.
(2) Estimation theory.
(3) Analysis of a set of measures data: hypotheses testing.

The book is completed by five tables and a conclusion. The list of references is made with accent to Slovak and Czech sources.

In comparing with the Slovak edition, we can say that the present monograph is very well translated into English and, therefore, it will be accessible for a larger scientific, practical and student audience in the whole world.

*Anatolij Dvurečenskij, Bratislava*

Pázman, A.: NONLINEAR STATISTICAL MODELS.
Kluwer Academic Publishers, Dordrecht-Boston-London and
Ister Science Press, Bratislava 1993, 259 pp.
ISBN 0-7923-2247-9 (Kluwer Academic Publishers), ISBN 80-88683-00-9 (Ister Science Press)

The monograph is devoted mainly to nonlinear regression models with the exception of the first and the last chapter.

*Chapter* 1 (Linear regression models, pp. 7–33) is a preparatory chapter that recapitulates basic knowledge, including the variance-minimizing experimental designs, and emphasizes the geometrical approach important for further reading of the book.

*Chapter* 2 (Linear methods in nonlinear regression models, pp. 34–54). If the intrinsic curvature of a regression model equals zero, i.e., the mean value surface is a linear manifold in a sample space, then some procedures of linear estimation, mainly those based on projection matrices in a sample space, can be successfully used. In addition to that, linear approximations of nonlinear regression models, a test of linear or intrinsically linear models against a nonlinear alternative and confidence regions for an unknown vector parameter by linear methods are treated here.

*Chapter* 3 (Univariate regression models, pp. 55–79). The nonlinear regression model with an unknown scalar parameter is investigated. This chapter is important for understanding the philosophy of the book. The author introduces here procedures developed in further chapters for models with a vector parameter. Mainly procedures for determining the approximate density and approximate moments of the estimator, and comments on entropy or other measures of information are to be pointed out.

*Chapter* 4 (The structure of a multivariate regression model and properties of $L_2$ estimators,. pp. 80–112). Titles of sections such as Regular and singular models, Geometrical properties of a regular regression model – the curvature of the model, Properties of singular regression models: the regression model as a differentiable manifold, The existence and uniqueness of $L_2$ estimator characterize the contents of the chapter. (From the mathematical viewpoint, this chapter is of a basic importance and unavoidable even though not so attractive for applied statisticians as the other chapters.)

*Chapter* 5 (Nonlinear regression models: computation of estimators and curvatures, pp. 113–130). The methods of numerical computation of estimates are demonstrated; the Gauss-Newton method, the gradient method, the Newton method, quasigradient methods, the Levenberg-Marquardt method are mentioned here with comments which a statistician, unfamiliar with these methods, will appreciate. The last part of the chapter is devoted to curvature arrays and a numerical manipulation of them.

*Chapter* 6 (Local approximation of probability densities and moments of estimators, pp. 131–153). The asymptotic properties of estimators: the first- and the second-order approximations are demonstrated. Simultaneously, the attention of the reader is drawn to the fact that this approach is local and to no global property can be caught on in this way.

*Chapter* 7 (Global approximation of densities of $L_2$ estimators, pp. 154–191). It is one of the most attractive parts of the book. The probability density of estimators on the interior and on the boundary of the parameter space, of the posterior modus estimator, of the estimator with a non-correct covariance matrix, and of the estimator when the error distribution is not normal are determined. The use of the Riemannian curvature tensor in improved approximate densities ends the chapter.

*Chapter* 8 (Statistical consequences of global approximations especially in flat models, pp. 192–214). It is shown that a deeper insight into confidence regions is possible in the framework of flat models (there exists a regular reparametrization of such a model, which makes the Fisher information matrix constant). Different approaches to determining confidence regions in nonlinear (also non-flat) models, an estimator of the standard deviation, and optimum experimental design in nonlinear models are treated here as well.

*Chapter* 9 (Nonlinear exponential families, pp. 215–247). Another direction of a development of nonlinear theory is shown and commented here. The close connection between mathematical statistics and differential geometry is enlightened here and the role of the class of exponential distributions is exposed.

The reviewed monograph is an outstanding result of a several-years research and should not be missing in any library of statisticians.

*Lubomír Kubáček, Bratislava*

Dvurečenskij, A.: GLEASON'S THEOREM AND ITS APPLICATIONS.
Kluwer Academic Press, Dordrecht-Boston-London and Ister Science Press, Bratislava 1993, xv+325 pp.
ISBN 0-7923-1990-7 (Kluwer Academic Publishers), ISBN 80-900486-1-7 (Ister Science Press)

The modern probability theory is based on the Kolmogorovian approach, which can be traced back to 1933. Approximately at the same time, an alternative and more general approach was suggested by von Neumann. The latter approach includes the Kolmogorov approach describing "classical" experiments, as well as quantum probability, describing quantum mechanical experiments. This more general approach is based on a Hilbert space (complex, separable), and the set of all random events – the projection lattice of the Hilbert space – has a more general structure than a Boolean algebra, which is a basic structure in the classical probability approach.

The idea that quantum mechanical experiments have an event structure different from a Boolean algebra was emphasized in the paper "The logic of quantum mechanics" by Birkhoff and von Neumann in 1936, which gave rise to the quantum logic approach to quantum theory.

The well-known Mackey axioms, introduced in 1963, gave an axiomatic basis for the quantum logic approach. One of the basic problems posed by G. Mackey was the problem of description of all probability measures on the projection lattice $L(H)$ of a complex, separable Hilbert space $H$, which is the quantum logic in the von Neumann approach. This problem was solved by the Gleason theorem in 1957, and this deep theorem became a cornerstone of the quantum logic approach.

The present book, as the author himself characterized it, " ... gathers the facts on Gleason's theorem and its applications that has been proved by many experts as well as the author" and "the aim is to give a reader a beauty of both Gleason's theorem and Gleason measures, and to indicate the many directions of their applications". This is exactly what can be said, in general.

In more details, the book consists of five chapters.

*Chapter* 1 is devoted to basic properties of Hilbert spaces. The choice of facts is aimed to the presentation of Gleason's theorem. Some non-traditional solutions of familiar facts are presented, and some results known only from papers are collected there.

*Chapter* 2 is devoted to quantum logic theory. It contains the complete solution of the problem of existence of joint distributions of observables on general quantum logics, with the emphasis on the particular case of the projection lattice of a Hilbert space. Author's contribution to the solution of this problem is fundamental.

In *Chapter* 3, Gleason's theorem is explained and the so-called elementary proof suggested by R. Cooke, M. Keane and W. Moran in 1985 is presented. This proof concerns the case of three dimensional real space, and provides a crucial step for the full proof of Gleason's theorem. Then a generalization of Gleason's theorem for signed and infinite-valued measures is introduced. An interesting relation between non-real measurable cardinals and validity of Gleason's theorem in nonseparable Hilbert spaces is shown. As an application, various kinds of convergence of measures and the product of Gleason's measures are studied.

In *Chapter* 4, relations between Gleason's theorem and completeness criteria for inner product spaces are studied. The author found a series of new completeness criteria, which are connected with properties of several interesting subfamilies of subspaces of inner product spaces. An example is the well-known Amemyia and Araki result, that an inner product space is complete if and only if the set of all subspaces which are equal to their biorthogonal forms an orthomodular lattice. Another criterion is, for example, the existence of at least one completely additive state implies completeness.

In *Chapter* 5, orthogonal vector measures and their connections with quantum logics are studied. Gleason's theorem proved to be an important tool there. The rest of the chapter is devoted to so-called "non-classical" orthomodular spaces, which are generalized inner product spaces over a field different from the classical ones, that is, real, complex or quaternionic. As was shown by Keller, although the logic (projection lattice) of these spaces cannot be embedded into the logic of any classical Hilbert space, Gleason's theorem can also be applied there to the description of states.

The book also contains illustrating exercises and presents some open problems.

*Sylvia Pulmannová, Bratislava*

Wang, Z. - Klir, G. J.: FUZZY MEASURE THEORY.
Plenum Press, New York-London 1992, 354 pp.
ISBN 0-306-44260-4

The authors present recent results of fuzzy measure theory. Fuzzy measure theory is a generalization of classical measure theory, where the additivity axiom of classical measures is replaced by weaker axioms of monotonicity and continuity. The notion of a fuzzy measure is due to Sugeno (1974). However, note that the non-additive set functions (vanishing in the empty set) were investigated under various names by several authors, e.g. Denneberg, Dobrakov, Pap, Schmeidler, Šipoš.

The nine chapters of the book are supplemented with six Appendices and Authors and Subject Indexes.

*Chapter* 1 includes an introduction and a short historical overview of fuzzy measure theory.

*Chapter* 2 is devoted to the relevant prerequisites from the set theory.

The essence of fuzzy measure theory is covered in Chaps. 3–8. Namely, *Chapter* 3 presents the basic definitions and results on general fuzzy measures and special types, such as $\lambda$-fuzzy measures, quasi-measures, belief and plausibility measures, possibility and necessity measures, etc.

*Chapter* 4 is devoted to the extensions of fuzzy measures of several kinds.

*Chapter* 5 is devoted to several structural characteristics for set functions, such as null-additivity, autocontinuity, uniform autocontinuity, etc.

*Chapter* 6 is devoted to the measurable functions on fuzzy measurable spaces, including several types of convergences (e.g., of types "almost" and "pseudo-almost").

*Chapter* 7 deals with fuzzy integrals, generalizing the original Sugeno integral for non-negative measurable functions, including the convergence theorems, a transformation theorem, fuzzy measures defined by fuzzy integrals, etc.

*Chapter* 8 deals with Pan-integrals, including Pan-additions and Pan-multiplications, definition and basic properties of Pan-integrals, and a transformation theorem.

The applicability of the theory is illustrated by simple examples in *Chapter* 9.

Individual chapters are accompanied by notes (with relevant bibliographical and historical information), and several exercises.

The *Appendices* A and B are devoted to some relevant concepts and results regarding classical measure theory and fuzzy set theory.

The *Appendices* C and D are glossaries of key concepts and symbols.

The *Appendices* E and F contain three reprinted articles opening new directions in fuzzy measure theory (App. E) and three reprinted articles describing significant applications of fuzzy measure theory (App. F).

There are several minor slips and misprints, which can be taken as a hidden exercise for a reader. For example, see Theorem 3.14 on p. 58, Example 4.1 on p. 73 or Example 7.4 on p. 140. Further, it would be desirable to include more results from non-additive measure and integral theories. On the other hand, it should be mentioned that this is the first book devoted purely to fuzzy measure theory and it includes several new mathematical results and concepts.

The book is accessible to readers even not familiar with classical measure theory, including researches and students who do not major in mathematics. It can be recommended to everybody who wants to enrich his capability to properly model the intricacies of the real world. For the interested readers we recommend also a recent book of Dieter Denneberg: *Non-additive measures and integrals.*

*Radko Mesiar, Bratislava*