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# A CONSTRUCTION OF A CW-DECOMPOSITION OF S-CUBES WHICH ARE MANIFOLDS

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## Introduction

Let $I^n = \{x \in R^n; |x_i| \leqq 1, \ i = 1, 2, \ldots, n\}$ be the $n$-dimensional cube, $J_i^n = \{x \in I^n; |x_i| = 1\}$ its $i$-th double-face and let $s_i$: $I^n \to I^n$, $x \mapsto (x_1, \ldots, x_{i-1}, -x_i, x_{i+1}, \ldots, x_n)$ be the symmetry of $I^n$ with respect to the hyperplane $x_i = 0$. Denote by $G_n$ the group generated by the set $\{s_1, \ldots, s_n\}$ of symmetries. Since for every $u \in G_n$ we have $u^2 = \mathrm{id}$, the group $G_n$ is commutative and $G_n \cong (Z_2)^2$. Every $u \in G_n$, $u \neq \mathrm{id}$, can be uniquely written in the form $u = s_{i_1} \circ s_{i_2} \circ \ldots \circ s_{i_k} = s_{i_1 i_2 \ldots i_k}$, where $i_1 < i_2 < \ldots < i_k$. Put $N_n = \{1, 2, \ldots, n\}$. Then there is a bijective map $\tau_n$: $G_n \to 2^{N_n}$, $\tau_n(s_{i_1 i_2 \ldots i_k}) = \{i_1, i_2, \ldots, i_k\}$, $\tau_n(\mathrm{id}) = \emptyset$.

Now according to [4] we recall the definition of an s-cube.

Let $u^1, \ldots, u^n \in G_n$. An *s-cube* $X = I^n/(u^1, \ldots, u^n)$ is a factor space $I^n/T$, where $T$ is an equivalence relation on $I^n$ defined by

$$x \ T \ y \ \text{ if and only if } x = y \text{ or there are } i_1, \ldots, i_k \in N_n$$

such that $x, y \in \bigcap_{j=1}^{k} J_{i_j}^n$ and $y = u^{i_1} \circ u^{i_2} \circ \ldots \circ u^{i}(x)$.

The integer $n$ is called the *dimension* of the s-cube $X$. The s-cube $X$ will be alternatively written in the form $X = I^n/(U_1, \ldots, U_n)$, where $U_i = \tau_n(u^i)$, $i \in N_n$.

In the paper [1] a CW-decomposition $\mathscr{F}^n$ of the $n$-dimensional cube $I^n$ is introduced in such a way that for any given s-cube $X = I^n/(u^1, \ldots, u^n)$ the equivalence relation $T$ is a cellular one[1]) on the CW-space $(I^n, \mathscr{F}^n)$ and a CW-decomposition $\mathscr{F}^n/T$ of $I^n/T$ is constructed. Since for every s-cube $X = I^n/T$ $T$ is the cellular equivalence relation on $(I^n, \mathscr{F}^n)$, by the growing $n$ the number of cells of $\mathscr{F}^n/T$ increases very rapidly. The practical computation shows that for $n \geqq 4$ the CW-decomposition $\mathscr{F}^n/T$ of $I^n/T$ is of very little use for the computation of the homology $H(X)$ of $X$.

[1]) See [3], page 32.

In the present paper a construction of a simpler CW-decomposition $\mathcal{H}$ of such $n$-dimensional s-cube $X$, which is a manifold, is given. The number of cells of $\mathcal{H}$ is much smaller than that of $\mathcal{F}^n$. E.g., for the s-cube $I^n/(s_{12\ldots n}, \ldots, s_{12\ldots n})$ which is homeomorphic to $RP^n$ we have card $\mathcal{F}^n = \frac{1}{2}(5^n - 3^n) + 1$ and card $\mathcal{H} = n+1$. Moreover, $\mathcal{H}$ is the standard CW-decomposition $\{e^0, e^1, \ldots, e^n\}$ of $RP^n$. Since the CW-decomposition $\mathcal{H}$ is just cut for the form of the s-cube $X$, it seems to be one of the best CW-decompositions of $X$ for the computation of $H(X)$.

## 1. Basic properties of s-cubes

We shall make use of the paper [4].

Let $X = I^n/(u^1, \ldots, u^n)$ be an s-cube. The s-cube $X$ is called an *r-cube* if for every $i, j \in N_n$ $u^i = s_j$ implies $u^j = s_j$. Every s-cube is homeomorphic to some r-cube ([4], Prop. 2.10), hence we can limit ourselves in our considerations only to r-cubes.

An r-cube $Y = I^n/(v^1, \ldots, v^n)$ has the *property "M"* if for each nonempty subset $P \subset N_n$ such that

i) $\forall i, j \in P$: $i \neq j \Rightarrow v^i \neq v^j$

ii) $\forall i \in P$: card $V_i \neq 1$

we have

$$P \cap \tau_n \left( \prod_{j \in P} v^j \right) \neq \emptyset$$

According to [4], Th. 3.18, an r-cube is a manifold if and only if it has the property "M".

## 2. o-cubes and their distribution characteristic

Let $X = I^n/(U_1, \ldots, U_n)$ be an r-cube and $M_j = \{x \in N_n;\ U_x = U_j\}$, $j \in N_n$. For the future construction of the CW-decomposition $\mathcal{H}$ it is suitable to arrange sets $U_1, \ldots, U_n$ in some appropriate order.

**Definition 2.1.** *Let $X = I^n/(U_1, \ldots, U_r)$ be an r-cube.*

*a) The r-cube $X$ is called an ordered cube (shortly an o-cube) if the following conditions are satisfied:*

*1) card $U_1 \leqq$ card $U_2 \leqq \ldots \leqq$ card $U_n$*

*2) there are integers $\alpha_1, \ldots, \alpha_s$, $1 \leqq \alpha_1 < \alpha_2 < \ldots < \alpha_s = n$, such that $M_{\alpha_1} = \{1, 2, \ldots, \alpha_1\}$, $M_{\alpha_2} = \{\alpha_1 + 1, \alpha_1 + 2, \ldots, \alpha_2\}, \ldots, M_{\alpha_s} = \{\alpha_{s-1} + 1, \alpha_{s-1} + 2, \ldots, \alpha_s\}$.*

*3) If card $U_{\alpha_i} = 1$, then $U_{\alpha_i} = \{\alpha_i\}$ for $i \in N_s$.*

*b) Let $X$ be an o-cube, $s, \alpha_1, \ldots, \alpha_s$, the integers defined in part a) and let*

*$p$, $q$, $0 \leqq p \leqq q \leqq s$, be such integers that card $U_{\alpha_i}=1$ for $p<i \leqq q$ and card $U_{\alpha_i} \neq 1$ otherwise. Put $\beta_1=\alpha_1$, $\beta_2=\alpha_2-\alpha_1$, ..., $\beta_s=\alpha_s-\alpha_{s-1}$. An $(2s+2)$-touple $(p, q; \alpha_1, \dots, \alpha_s; \beta_1, \dots, \beta_s)$ will be called the distribution characteristic of the o-cube X. The set $\{\alpha_{i-1}+1, \alpha_{i-1}+2, \dots, \alpha_i\}$, $i \in N_s$, will be henceforward denoted by $Q_i$, where $\alpha_0=0$ by definition.*

Example 2.2. r-cubes $X_1=I^5/(s_{124}, s_{124}, s_{12345}, s_{124}, s_{12345})$, $X_2=I^6/(\mathrm{id}, s_2, s_2, s_{456}, s_{456}, s_{456})$ are not o-cubes because the conditions 1), 2) for $X_1$, resp. the condition 3) for $X_2$ from Definition 2.1 are not satisfied. An r-cube $I^8/(\mathrm{id}, \mathrm{id}, s_5, s_5, s_5, s_{167}, s_{167}, s_{12345678})$ is an o-cube with the distribution characteristic $(1, 2; 2, 5, 7, 8; 2, 3, 2, 1)$.

Making use of [4], Prop. 1.3., it is not difficult to see that every r-cube is homeomorphic to some o-cube; it is sufficient to find only a suitable permutation of the coordinates. Since every s-cube is homeomorphic to some r-cube, we have the following

**Proposition 2.3.** *Every s-cube is homeomorphic to some o-cube.*

## 3. Representation of o-cubes by o-balls

Let $B^n=\{x \in R^n; \sqrt{x_1^2+\dots+x_n^2} \leqq 1\}$ be the standard $n$-dimensional ball. In this section we introduce a special type of factor spaces of the products of balls. Similarly to s-cubes we call them s-balls. We also introduce some special types of these spaces and prove that every o-cube is homeomorphic to some o-ball.

**Definition 3.1.** *Let $n$, $s$, $s \leqq n$, be integers and let $\beta_1, \dots, \beta_s \in N_n$, $\sum_{i=1}^{s} \beta_i=n$. Choose $u^1, \dots, u^s \in G_n$ in such a way that $u^i \neq u^j$ for all $i \neq j$. An s-ball $X=B^{\beta_1} \times \dots \times B^{\beta_s}/(u^1, \dots, u^s)$ is a factor space $B^{\beta_1} \times \dots \times B^{\beta_s}/T_B$, where $T_B$ is an equivalence relation on $B^{\beta_1} \times \dots \times B^{\beta_s}$ defined by*

*$x\, T_B\, y$ if and only if $x=y$ or there is a nonempty subset $M$ of $N_s$ such that $x, y \in \bigcap_{i \in M} J(\beta_1, \dots, \beta_s; i, n)$ and $y=\prod_{i \in M} u^i(x)$, where $J(\beta_1, \dots, \beta_s; i, n) = B^{\beta_1} \times \dots \times B^{\beta_{i-1}} \times \partial B^{\beta_i} \times B^{\beta_{i+1}} \times \dots \times B^{\beta_s}$, $i \in N_s$.*

The s-ball $X$ will be alternatively written in the form $B^{\beta_1} \times \dots \times B^{\beta_s}/(U_1, \dots, U_s)$, where $U_1=\tau_n(u^i)$, $i \in N_s$. The sums $\sum_{i=1}^{k} \beta_i$ will be denoted henceforward by $\alpha_k$, $k \in N_s$ and we put $\alpha_0=0$ by definition.

**Definition 3.2.** *an s-ball $X=B^{\beta_1} \times \dots \times B^{\beta_s}/(u^1, \dots, u^s)$ is called a regular ball (r-ball) if for every $i \in N_s$, $j \in N_n$ $\left(n=\sum_{i=1}^{s} \beta_i\right)$ $u^i=s_j$ implies $\alpha_{i-1}<j \leqq \alpha_i$.*

**Definition 3.3.** *a) An r-ball* $X = B^{\beta_1} \times \ldots \times B^{\beta_s}/(U_1, \ldots, U_s)$ *is called an ordered ball (o-ball) if the following conditions are satisfied:*

1) *card* $U_1 \geqq$ *card* $U_2 \leqq \ldots \leqq$ *card* $U_s$
2) *If card* $U_j = 1$, *then* $U_j = \{\alpha_j\}$, $j \in N_s$.

*b) Let* $X = B^{\beta_1} \times \ldots \times B^{\beta_s}/(U_1, \ldots, U_s)$ *be an o-ball and let* $p, q$, $0 \leqq p \leqq q \leqq s$, *be such integers that card* $U_i = 1$ *for* $p < i \leqq q$ *and card* $U_i \neq 1$ *otherwise. An* $(2s+2)$*-touple* $(p, q; \alpha_1, \ldots, \alpha_s; \beta_1, \ldots, \beta_s)$ *will be called the distribution characteristic of the o-ball X. The set* $\{\alpha_{i-1}+1, \alpha_{i-1}+2, \ldots, \alpha_i\}$ *we shall denote in future by* $R_i$, $i \in N_s$.

**Definition 3.4.** *Let* $X = B^{\beta_1} \times \ldots \times B^{\beta_s}/(u^1, \ldots, u^s)$ *be an o-ball and* $(p, q; \alpha_1, \ldots, \alpha_s; \beta_1, \ldots, \beta_s)$ *its distribution characteristic. The o-ball X has the property "M" if for every nonempty subset P of* $N_s$ *with card* $U_i \neq 1$ *for all* $i \in P$ *we have*

$$A \in \tilde{P} \Rightarrow A \cap \tau_n \left(\prod_{i \in P} u^{\alpha_i}\right) \neq \emptyset$$

*where*

$$\tilde{P} = \left\{A;\ A \subset \bigcup_{i \in P} R_i,\ \text{card}\,(A \cap R_i) = 1 \ \text{ for all } i \in P\right\} \tag{1}$$

Let $X = I^n/(U_1, \ldots, U_n)$ be a given o-cube, $(p, q; \alpha_1, \ldots, \alpha_s; \beta_1, \ldots, \beta_s)$ its distribution characteristic. Now we are going to find an o-ball $Y$ with the same distribution characteristic which is homeomorphic to $X$.

Let $F_i$: $I^i \to B^i$ be the standard homeomorphism defined by the radial extension (see [2], p. 55). We show that the map

$$\begin{gathered} F\colon\ I^n \to B^{\beta_1} \times \ldots \times B^{\beta_s}, \\ x \mapsto (F_{\beta_1}(x_1, \ldots, x_{\alpha_1}), \ldots, F_{\beta_s}(x_{\alpha_{s-1}+1}, \ldots, x_{\alpha_s})) \end{gathered} \tag{2}$$

induces a continuous map

$$\begin{gathered} \tilde{F}\colon\ I^n/(U_1, \ldots, U_n) \to B^{\beta_1} \times \ldots \times B^{\beta_s}/(U_{\alpha_1}, \ldots, U_{\alpha_s}), \\ [x] \mapsto [F(x)] \end{gathered} \tag{3}$$

It suffices to prove that $\tilde{F}$ is well-defined. Let $[x] = [y]$ for $x, y \in I^n$, $x \neq y$. Then there are $i_1, \ldots, i_k \in N_n$ such that $x, y \in \bigcap_{j=1}^{k} J^n_{i_j}$ and $y = u^{i_1} \circ \ldots \circ u^{i_k}(x)$. Without loss of generality we can suppose that $u^{i_p} \neq u^{i_q}$ for $p, q \in N_k$, $p \neq q$. Let $M = \{i \in N_s;\ \exists j \in N_k,\ i_j \in Q_i\}$.[1]) Then $F(x)$, $F(y) \in \bigcap_{i \in M} J(\beta_1, \ldots, \beta_s;\ i, n)$ and $F(y) = \left(\prod_{i \in M} u^{\alpha_i}\right)(F(x))$, because $u^{\alpha_i} = u^j$ for all $j \in Q_i$. Hence $\tilde{F}[x] = \tilde{F}[y]$.

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[1]) For $Q_i$ see Definition 2.1.

**Lemma 3.5.** *The map $\bar{F}$, defined by* (3), *is a homeomorphism.*

Proof. Since the map $\bar{F}$ is onto, the space $I^n/(U_1, \ldots, U_n)$ is compact and the space $B^{\beta_1}\times\ldots\times B^{\beta_s}/(U_{\alpha_1}, \ldots, U_{\alpha_s})$ is Hausdorff, it suffices to prove that $\bar{F}$ is injective. Let $[F(x)]=[F(y)]$ for some $x, y\in I^n$, $F(x)\neq F(y)$. Then there is a nonempty subset $M$ of $N$, such that $F(x)$, $F(y)\in\bigcap_{i\in M} J(\beta_1, \ldots, \beta_s; i, n)$ and $F(y)=\left(\prod_{i\in M} u^{\alpha_i}\right)(F(x))$. Let $i\in M$. Then

$$F(x),\ F(y)\in B^{\beta_1}\times\ldots\times B^{\beta_{i-1}}\times\partial B^{\beta_i}\times B^{\beta_{i+1}}\times\ldots\times B^{\beta_s}$$

Denote by $q_i$ an element from $Q_i$ such that $x, y\in J^n_{q_i}$. Then $x, y\in\bigcap_{i\in M} J^n_{q_i}$ and $y=\prod_{i\in M} u^{q_i}(x)$. Hence $[x]=[y]$.

**Lemma 3.6.** *The homeomorphism $\bar{F}$ given by* (3) *preserves the property "M".*

Proof. Let $I^n/(u^1, \ldots, u^n)$ be an o-cube with the property "M", with the distribution characteristic $(p, q; \alpha_1, \ldots, \alpha_s; \beta_1, \ldots, \beta_s)$ and let $P\neq\emptyset$ be a subset of $N$, such that card $U_{\alpha_i}\neq 1$ for all $i\in P$. Let $A\in\bar{P}$, where $\bar{P}$ is defined by formula (1), in which $R_i=Q_i$. Then

1) $\emptyset\neq A\subset N_n$
2) card $U_i\neq 1$ for every $i\in A$
3) $U_i\neq U_j$ for all $i, j\in A$, $i\neq j$

Since the o-cube $i^n/(u^1, \ldots, u^n)$ has the property "M", we have

$$A\cap\tau_n\left(\prod_{i\in A} u^i\right)\neq\emptyset$$

But $\prod_{i\in A} u^i=\prod_{i\in P} u^{\alpha_i}$ and the assertion follows.

We know that the homeomorphism $\bar{F}$ preserves also the distribution characteristic. Then with respect to Lemma 3.5 and Lemma 3.6 we have the following

**Proposition 3.7.** *Let $X=I^n/(u^1, \ldots, u^n)$ be an o-cube with the property "M" and with the distribution characteristic $(p, q; \alpha_1, \ldots, \alpha_s; \beta_1, \ldots, \beta_s)$. Then $X$ is homeomorphic to the o-ball $Y=B^{\beta_1}\times\ldots\times B^{\beta_s}/(v^{\beta_1}, \ldots, v^{\beta_s})$ which has the property "M" and the same distribution characteristic as $X$.*

## 4. A construction of the CW-decomposition $\mathscr{H}$ of an s-cube which is a manifold

Let $X=I^n/(u^1, \ldots, u^n)$ be an s-cube which is a manifold. Then $X$ is homeomorphic to some $r$-cube $X_1$ and according to Proposition 2.3 $X_1$ is homeomorphic to some o-cube $X_2$. Since $X_2$ is a manifold, it has the property "M". Proposition 3.7 says now that the o-cube $X_2$ is homeomorphic to an o-ball $Y$ with the property "M". Thus, there exists a homeomorphism $H$: $X \to Y$, so it suffices to construct the CW-decomposition $\mathscr{H}$ of the o-ball $Y$ only.

Let $(0, q; \alpha_1, \ldots, \alpha_s; \beta_1, \ldots, \beta_s)$ be the distribution characteristic of the o-ball $Y = B^{\beta_1} \times \ldots \times B^{\beta_s}/(v^1, \ldots, v^s) = B^{\beta_1} \times \ldots \times B^{\beta_s}/T_B$ and let $p_B$: $B^{\beta_1} \times \ldots \times B^{\beta_s} \to B^{\beta_1} \times \ldots \times B^{\beta_s}/T_B$ be the canonical projection. Now a CW-decomposition $\mathscr{C}$ of $B^{\beta_1} \times \ldots \times B^{\beta_s}$ will be constructed in such a way that $T_B$ will be a cellular equivalence relation on the CW-space $(B^{\beta_1} \times \ldots \times B^{\beta_s}, \mathscr{C})$

Denote by $\mathscr{C}_k$ the well-known CW-decomposition

$$\{e^0_{-1}, e^0_1, e^1_{-1}, e^1_1, \ldots, e^{k-1}_{-1}, e^{k-1}_1, e^k_0\}$$

of the $k$-ball $B^k$ with the characteristic maps

$$\begin{aligned} &f^j_{\pm 1}\colon\ B^j \to B^k,\ x \mapsto (x_1, \ldots, x_j, \pm\sqrt{1 - x_1^2 - \ldots - x_j^2}, 0, \ldots, 0) \\ &f^k_0\colon\ B^k \to B^k,\ x \mapsto x \end{aligned} \tag{4}$$

$j = 0, 1, \ldots, k-1$. This CW-decomposition of $B^k$ induces the product CW-decomposition $\mathscr{C}$ of $B^{\beta_1} \times \ldots \times B^{\beta_s}$. It consists of cells

$$e^{p_1}_{q_1} \times \ldots \times e^{p_s}_{q_s} \tag{5}$$

where $p_i \leqq \beta_i$ and $q_i \in \{-1, 0, 1\}$, $i \in N_s$. The cell (5) will be denoted by $e(p_1, \ldots, p_s; q_1, \ldots, q_s)$ and its characteristic map by $f(p_1, \ldots, p_s; q_1, \ldots, q_s)$. In particular, the cell $e(\beta_1, \ldots, \beta_s; 0, \ldots, 0)$ will be shortly denoted by $e^n$ and its characteristic map by $f^n$.

Let $e \in \mathscr{C}$ be an arbitrary cell, $e \neq e^n$, and let $G(e)$ be the group generated by the set

$$\{u^i;\ i \in N_s,\ e \subset J(\beta_1, \ldots, \beta_s; i, n)\}$$

The next Lemma follows immediately from the definition of an s-ball.

**Lemma 4.1.** *Let* $e \in \mathscr{C}$, $e \neq e^n$. *Then* $p_B^{-1}(p_B(e)) = \bigcup_{u \in G(e)} u(e)$.

To prove that $T_B$ is a cellular equivalence relation on the CW-space $(B^{\beta_1} \times \ldots \times B^{\beta_s}, \mathscr{C})$, we shall need the following

**Lemma 4.2.** *Let* $e \in \mathscr{C}$, $e \neq e^n$ *and let* $u \in G(e)$. *Then*

1) $u(e) \in \mathscr{C}$
2) $u(e) \cap e = \emptyset$ or $u|e = \mathrm{id}$.

Proof: Let $(0, p; \alpha_1, \ldots, \alpha_s; \beta_1, \ldots, \beta_s)$ be the distribution characteristic of $Y$, $S(e) = \{i \in N_s;\ e \subset J(\beta_1, \ldots, \beta_s; i, n\}$. Then $u$ can be written in the form $u = v \circ w$, where

$$v = \prod_{i \in P} u^i, \quad w = \prod_{i \in Q} u^i,$$

$P \subset N_q \cap S(e)$, $Q \subset (N_s - N_q) \cap S(e)$. Let us denote

$$P' = \{\alpha_{i-1} + p_i + 1\,;\, i \in P\}, \quad Q' = \{\alpha_{i-1} + p_i + 1\,;\, i \in Q\}.$$

Now put $S = P' \cup Q'$, $S^* = S \cap \tau_n(u)$. With respect to (4) we have sign $x_i$ = sign $y_i$ for all $x, y \in e$ and $i \in S$. Let for $x \in e$ $z = u(x)$. Then for every $i \in S^*$ we have sign $z_i = -$sign $x_i$, hence $u(e) = e(p_1, \ldots, p_s; q_1^*, \ldots, q_s^*)$, where $q_i^* = q_i$ for $i \notin S^*$, $q_i^* = -q_i$ for $i \in S^*$. So we have shown that $u(e) \in \mathscr{E}$ and that $e \cap u(e) = \emptyset$ if $S^* \neq \emptyset$. We shall discuss 3 cases:

1) $Q \neq \emptyset$, 2) $P \neq \emptyset$, $Q = \emptyset$, 3) $P = Q = \emptyset$.

1) Since the o-ball $B^{\beta_1} \times \ldots \times B^{\beta_s}/(u^1, \ldots, u^s)$ has the property "M" and $Q' \in \tilde{Q}^1$), we have $Q' \cap \tau_n \left(\prod_{i \in Q} u^i\right) = Q' \cap \tau_n(w) \neq \emptyset$.

Hence $S^* \neq \emptyset$ and $e \cap u(e) = \emptyset$.

2) If $P' \cap \tau_n(u) \neq \emptyset$, we have $s^* \neq \emptyset$ and $e \cap u(e) = \emptyset$. If $P' \cap \tau_n(u) = \emptyset$, we have $u|e = \mathrm{id}$.

3) In this case $u = \mathrm{id}$.

**Theorem 4.3.** *The equivalence relation $T_B$ is cellular*[2]) *on the CW-space* $(B^{\beta_1} \times \ldots \times B^{\beta_s}, \mathscr{E})$.

Proof: Let $e$ be an arbitrary cell in $\mathscr{E}$. If $e = e^n$, then $p_B^{-1}(p_B(e)) = e$. If $e \neq e^n$, according to Lemma 4.1 and Lemma 4.2, part 1), the set $p_B^{-1}(p_B(e))$ is a union of mutually homeomorphic cells of $\mathscr{E}$. Making use of assertion 2) of Lemma 4.2 and of the definition of an s-ball we get that $p_B$ maps every cell $e \in \mathscr{E}$ homeomorphically on $p_B(e)$.

According to [3], Prop. 5.8, p. 60, we have the following corollary of Theorem 4.3.

Corollary. The set $\mathscr{H} = \{p_B(e); e \in \mathscr{E}\}$ is a CW-decomposition of the o-ball $B^{\beta_1} \times \ldots \times B^{\beta_s}/T_B$. The map $p_B \circ f(p_1, \ldots, p_s; q_1, \ldots, q_s)$ is characteristic for the cell $p_B(e(p_1, \ldots, p_s; q_1, \ldots, q_s))$.

Example 4.4. Using the previous results we construct a CW-decomposition $\mathscr{H}$ of the o-ball $Y$ which is homeomorphic to the s-cube $X = I^3/(s_2, s_{123}, s_3)$. By [4], Lemma 1.4, $X$ is homeomorphic to an r-cube $X_1 = I^3/(s_{123}, s_{123}, s_3)$ and by [4], Prop. 1.3, $X_1$ is homeomorphic to an o-cube $X_2 = I^3/(s_1, s_{123}, s_{123})$. The o-cube $X_2$ has the property "M" and the distribution characteristic (0, 1; 1, 3; 1, 2). By Proposition 3.7 the o-cube $X_2$ is homeomorphic to an o-ball $Y = B^1 \times B^2/(s_1, s_{123}) = B^1 \times B^2/T_B$. The CW-decomposition $\mathscr{E}$ of $B^1 \times B^2$ consists of the following 15 cells: $e(0, 0; \pm 1, \pm 1)$, $e(0, 1; \pm 1, \pm 1)$, $e(0, 2; \pm 1, 0)$, $e(1, 0; 0, \pm 1)$, $e(1, 1; 0, \pm 1)$, $e(1, 2; 0, 0)$. The CW-decomposition $\mathscr{H}$ of $Y$ has 6 cells: $p_B(e(0, 0; 1, 1))$, $p_B(e(1, 0; 0, 1))$, $p_B(e(0, 1; 1, 1))$, $p_B(e(1, 1; 0, 1))$, $p_B(e(0, 2; 1, 0))$, $p_B(e(1, 2; 0, 0))$.

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1) See Definition 3.4

2) See [3], p. 32

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## КОНСТРУКЦИЯ CW-РАЗБИЕНИЯ s-КУБОВ, КОТОРЫЕ ЯВЛЯЮТСЯ МНОГООБРАЗИЯМИ

Jozef Tvarožek

Резюме

Пусть $X$ — $n$-мерный s-куб, который является многообразием. В статье построено CW-разбиение $\mathscr{K}$ s-куба $X$, которое позволяет вычислить $H(X)$ тоже для $n \geqq 4$.