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# OSCILLATORY PROPERTIES OF THE FOURTH ORDER LINEAR DIFFERENTIAL EQUATION

JOZEF ROVDER

## 1. Introduction

In the present paper we consider the equation

$$y^{(iv)}+q(t)y'+r(t)y=0, \tag{1}$$

where the functions $q(t)$, $r(t)$ are continuous on an interval $[a, \infty)$. A solution $y(t)$ of (1) is said to be oscillatory if it has an infinite numbers of zeros on $[a, \infty)$ and nonoscillatory if there exists a number $c>a$ such that $y(t)\neq 0$ on $[c, \infty)$. The present paper is a continuation of paper [5] in which an asymptotic behaviour of (1) was studied. The aim of this work is to show an oscillatory behaviour of solutions of (1) provided that we know its asymptotic behaviour. Unlike other works, for example Mamrilla [2, 3], we do not require $g(t)$ and $r(t)$ to be of one sign. Further the symbol $L[a, \infty)$ will refer to the set of all complexvalued functions which are Lebesque integrable on $[a, \infty)$.

## 2. Preliminary results

**Lema 1.** (Hinton [1]) *Let $r(t)>0$ on $[a, \infty)$ and $r''(t)/r^{1+1/n}(t)$ be in* L$[a, \infty)$ *for $n=1, 2, \ldots, n$. Then*

(i) *$r^{1/n}(t)$ is not in* L$(a, \infty)$

(ii) *$[r'(t)/r^{1+1/n}(t)]'$ is in* L$[a, \infty)$.

(iii) *$[r'(t)/r^{1+1/2n}(t)]^2$ is in* L$[a, \infty)$.

**Lemma 2.** *Let $r''(t)/r^{1+1/n}(t)$ be in* L$[a, \infty)$, *$r(t)\neq 0$ on $[a, \infty)$. Let $\alpha\neq 0$, $\beta$ real numbers. Then*

$$\lim_{t\to\infty} |r(t)|^{\beta} \exp\left(-|\alpha| \int_b^t |r(s)|^{1/n}\, ds\right)=0$$

*for every number $b\geq a$.*

Proof. Let $r(t)>0$ on $[a, \infty)$. Then from Lemma 1 there follows

$$\int_b^\infty \left|\left[\frac{r'(t)}{r^{1+1/n}(t)}\right]'\right| \mathrm{d}t<\infty \quad \text{and hence} \quad \int_b^\infty \left[\frac{r'(t)}{r^{1+1\ n}(t)}\right]' \mathrm{d}t<\infty.$$

Since

$$\int_b^\infty \left[\frac{r'(t)}{r^{1+1/n}(t)}\right]' \mathrm{d}t=\lim_{t\to\infty}\frac{r'(t)}{r^{1+1/n}(t)}-K, \quad \text{where} \quad K=\frac{r'(b)}{r^{1+1\ n}(b)},$$

then $\lim\limits_{t\to\infty}\frac{r'(t)}{r^{1+1/n}(t)}$ exists. Let us denote it by $c$. We show that $c=0$. Suppose on the contrary that $c\neq 0$. Then

$$\lim_{t\to\infty}\left[\frac{r'(t)}{r^{1+1/n}(t)}\right]^2=c^2$$

and for the number $c^2/2>0$ there exists a number $t_0\geqslant b$ such that

$$\left|\left[\frac{r'(t)}{r^{1+1/n}(t)}\right]^2-c^2\right|<\frac{c^2}{2}$$

for all $t\geqslant t_0$, i.e.

$$\left[\frac{r'(t)}{r^{1+1/n}}\right]^2>\frac{1}{2}c^2.$$

Multiplying the last inequality by $r^{1/n}(t)$ we get

$$\left[\frac{r'(t)}{r^{1+1\ n}(t)}\right]^2>\frac{1}{2}c^2r^{1/n}(t)$$

which contradicts Lemma 1, because the left-hand side is in $\mathrm{L}[a, \infty)$, while the right-hand side is not in $\mathrm{L}[a, \infty)$. Hence

$$\lim_{t\to\infty}\frac{r'(t)}{r^{1+1/n}(t)}=0.$$

From this it follows that

$$\lim_{t\to\infty}\frac{\beta}{\alpha}\frac{r'(t)}{r^{1+1/n}(t)}=\lim\beta\frac{r'(t)}{r(t)}\cdot\frac{1}{\alpha\cdot r^{1/n}(t)}=0$$

for every real number $\alpha$, $\beta$, $\alpha\neq 0$. Then for the number $1/2$ there exists a number $t_1\geqslant 0$ such that

$$\left|\beta\cdot\frac{r'(t)}{r(t)}\cdot\frac{1}{\alpha r^{1/n}(t)}\right|<\frac{1}{2}$$

for every $t>t_1$, i.e.

$$\left|\beta \frac{r'(t)}{r(t)}\right| < \frac{1}{2}|\alpha| r^{1/n}(t)$$

and hence

$$\beta \frac{r'(t)}{r(t)} < \frac{1}{2}|\alpha| r^{1/n}(t).$$

Integrating the last inequality over $[t_1, t]$ we get

$$[\ln r(t)^{\beta}]_{t_1}^{t} < \frac{1}{2}|\alpha| \int_{t_1}^{t} r(s)^{1/n} \, ds,$$

i.e.

$$0 < \left[\frac{r(t)}{r(t_1)}\right]^{\beta} < \exp\left[\frac{1}{2}|\alpha| \int_{t_1}^{t} r(s)^{1/n} \, ds\right].$$

Multiplying the last inequality by $\exp\left[-|\alpha| \int_{t_1}^{t} r^{1/n}(s) \, ds\right]$ we obtain

$$0 < \left[\frac{r(t)}{r(t_1)}\right]^{\beta} \exp\left[-|\alpha| \int_{t_1}^{t} r^{1/n}(s) \, ds\right] < \exp\left[-\frac{1}{2}|\alpha| \int_{t_1}^{t} r^{1/n}(s) \, ds\right].$$

Since $\int_{t_1}^{\infty} r^{1/n}(s) \, ds = \infty$, then $\lim_{t \to \infty} \exp\left[-\frac{1}{2}|\alpha| \int_{t_1}^{t} r^{1/n}(s) \, ds\right] = 0$ and consequently

$$\lim_{t \to \infty} [r(t)]^{\beta} \exp\left[-|\alpha| \int_{t_1}^{t} r^{1/n}(s) \, ds\right] = 0. \tag{2}$$

Let now $r(t) < 0$. Then $-r(t) > 0$ and $-r(t)$ satisfies the assumptions of Lemma 1. Therefore

$$\lim_{t \to \infty} [-r(t)]^{\beta} \exp\left[-|\alpha| \int_{t_1}^{t} [-r(s)]^{1/n} \, ds\right] = 0. \tag{3}$$

From (2) and (3) we obtain the conclusion of Lemma 2.

## 3. Main results

From Lemma 2 and the results of paper [5] concerning the asymptotic properties of (1) we obtain the following theorems:

**Theorem 1.** *Let $q(t) \neq 0$ on $[a, \infty)$, $q''(t)/q^{4/3}(t)$ and $r(t)/q(t)$ be in* $\mathrm{L}[a, \infty)$. *Then the differential equation* (1) *has a fundamental system of solutions which consists of two nonoscillatory solutions $y_1$, $y_2$ and two oscillatory solutions $y_3$, $y_4$ with the properties:*

a) *If $q(t) > 0$, then*
$y_1 \to 1$, $y_1^{(i)} q^{-i/3} \to 0$ *as* $t \to \infty$, $i = 1, 2, 3$,
$y_2^{(i)} \to 0$ *as* $t \to \infty$, $(-1)^i y_2^{(i)} > 0$ *on* $[T, \infty)$, $i = 0, 1, 2, 3$,

$y_{3,4}^{(i)}$, $i=0, 1, 2, 3$ *are oscillatory and unbounded.*

b) *If* $q(t)<0$, *then*
$y_1 \to 1$, $y_1^{(i)} q^{-i/3} \to 0$ *as* $t\to\infty$, $i=1, 2, 3$,
$y_2^{(i)} \to -\infty$ *for* $i=0, 1, 2, 3$ *and* $t\to\infty$,
$y_{3,4}^{(i)}$ *are oscillatory and* $y_{3,4}^{(i)} \to 0$ *as* $t\to\infty$, $i=0, 1, 2, 3$.

**Theorem 2.** *Let* $r(t)\neq 0$ *on* $[a, \infty)$, $r''(t)/r^{5/4}(t)$ *and* $q(t)/r^{1/2}(t)$ *be in* $\mathrm{L}[a, \infty)$. *Then the following statements are valid*

a) *If* $r(t)>0$, *then the differential equation* (1) *has a fundamental system of oscillatory solutions* $y_k(t)$, $k=1, 2, 3, 4$ *such that* $y_{1,2}^{(i)}(t)$, $i=0, 1, 2, 3$ *approaches zero as* $t\to\infty$ *and* $y_{3,4}^{(i)}(t)$, $i=0, 1, 2, 3$ *are unbounded.*

b) *If* $r(t)<0$, *then*
$y_1^{(i)} \to \infty$ *as* $t\to\infty$, $i=0, 1, 2, 3$,
$y_2^{(i)} \to 0$ *as* $t\to\infty$, $(-1)^i y_2^{(i)}(t)>0$, $i=0, 1, 2, 3$,
$y_{3,4}^{(i)}$, $i=0, 1, 2, 3$ *are oscillatory and unbounded if*
$|r(t)|^{3/8} \to 0$, *and bounded if* $|r(t)|^{3/8} \to \infty$ *as* $t\to\infty$.

## 4. Proofs of theorems

Proof of Theorem 1. Let $q(t)>0$. From Theorem 4 in [5] it follows that equation (1) has a fundamental system of solutions $y_k$, $k=1, 2, 3, 4$ such that

$$\mathrm{dia}[q, q^{2/3}, q^{1/3}, 1](y_1, y_1', y_1'', y_1''')^{\mathrm{T}} q^{-1} \to (1, 0, 0, \acute{y})^{\mathrm{T}}, \tag{4}$$

$$\mathrm{dia}[q, q^{2/3}, q^{1/3}, 1](y_k, y_k', y_k'', y_k''')^{\mathrm{T}} \exp\left[-\tau_k \int_{t_0}^{t} q^{1/3}\, \mathrm{d}\delta\right] q^{-1/3} \to p_k, \tag{5}$$

where $k=2, 3, 4$, $\tau_2=-1$, $\tau_{3,4}=\frac{1}{2}\pm\frac{\sqrt{3}}{2}\mathrm{i}$, $p_k=(\bar{\tau}_k, 1, \tau_k, -\bar{\tau}_k)^{\mathrm{T}}$.

It is easily seen that (4) implies $y_1\to 1$ and for $i=1, 2, 3$, $y_1^{(i)} q^{-i/3} \to 0$. For the solution $y_2$ we get from (5)

$$y_2^{(i)} q^{(2-i)/3} \exp\left[\int_{t_0}^{t} q^{1/3}(s)\, \mathrm{d}s\right] \to (-1)^{i+1}, \quad i=0, 1, 2, 3. \tag{6}$$

It follows from Lemma 2 that $y_2^{(i)}\to 0$ for $i=0, 1, 2, 3$. From (6) it also follows that there exists a number $T>a$ such that $y_2<0$, $y_2'>0$, $y_2''<0$, $y_2'''>0$ on the interval $[T, \infty)$ and hence $y_2$ is a nonoscillatory solution of (1). For the other two solutions $y_{3,4}$ of equation (1) we get from (5)

$$y_3 q^{2/3} \exp\left[\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}\mathrm{i}\right) \int_{t_0}^{t} q^{1/3}(\delta)\, \mathrm{d}\delta\right] \to \frac{1}{2}-\frac{\sqrt{3}}{2}\mathrm{i},$$

i.e.

$$y_3=\left(\frac{1}{2}-\frac{\sqrt{3}}{2}\,\mathrm{i}\right) q^{-2/3} \exp\left[\left(\frac{1}{2}+\frac{\sqrt{3}}{2}\,\mathrm{i}\right) \int_{t_0}^{t} q^{1/3}(\delta)\, d\delta\right] [1+o(1)].$$

Similarly we get

$$y_4=\left(\frac{1}{2}+\frac{\sqrt{3}}{2}\,\mathrm{i}\right) q^{-2/3} \exp\left[\left(\frac{1}{2}-\frac{\sqrt{3}}{2}\,\mathrm{i}\right) \int_{t_0}^{t} q^{1/3}(\delta)\, d\delta\right] [1+o(1)].$$

Since $\frac{1}{2}-\frac{\sqrt{3}}{2}\,\mathrm{i}=\cos\frac{2}{3}\pi+\mathrm{i}\sin\frac{2}{3}\pi$ and $\frac{1}{2}+\frac{\sqrt{3}}{2}\,\mathrm{i}=\cos\frac{\pi}{3}+\mathrm{i}\sin\frac{\pi}{3}$, then for $y_{3,4}$ there holds

$$y_{3,4}=q^{-2/3} \exp\left[\frac{1}{2}\int_{t_0}^{t} q^{1/3}(\delta)\, d\delta\right] \left[\pm\cos\left(\frac{2}{3}\pi+\frac{\sqrt{3}}{2}\int_{t_0}^{t} q^{1/3}(\delta)\, d\delta\right)+\right.$$

$$\left.+\mathrm{i}\sin\left(\frac{2}{3}\pi+\frac{\sqrt{3}}{2}\int_{t_0}^{t} q^{1/3}(\delta)\, d\delta\right)\right] [1+o(1)].$$

If we denote by

$$\alpha(t)=q^{-2/3} \exp\left[\frac{1}{2}\int_{t_0}^{t} q^{1/3}(\delta)\, d\delta\right], \quad \beta(t)=\frac{2\pi}{3}+\frac{\sqrt{3}}{2}\int_{t_0}^{t} q^{1/3}(\delta)\, d\delta,$$

then for $u_3=\operatorname{Re} y_3$ and $u_4=\operatorname{Im} y_4$ we have

$$u_3=\alpha(t)\,[\cos\beta(t)+o(1)], \quad u_4=\alpha(t)\,[\sin\beta(t)+o(1)].$$

From Lemma 1 and 2 it follows that $\lim_{t\to\infty}\alpha(t)=\lim_{t\to\infty}\beta(t)=\infty$ and therefore $u_3(t)$, $u_4(t)$ are oscillatory and unbounded. It holds as well for $u_3^{(i)}(t)$ and $u_4^{(i)}(t)$, $i=1, 2, 3$. It is evident that the functions $y_1$, $y_2$, $u_3$, $u_4$ form a fundamental system of (1).

Let now $q(t)<0$. Then from Theorem 6 in [5] we have

$$\operatorname{dia}[q, q^{2/3}, q^{1/3}, 1](y_1, y_1', y_1'', y_1''')^{\mathrm{T}}\cdot q^{-1}\to(1, 0, 0, 0)^{\mathrm{T}} \tag{6}$$

$$\operatorname{dia}[q, q^{2/3}, q^{1/3}, 1]\cdot(y_k, y_k', y_k'', y_k''')^{\mathrm{T}}\cdot q^{-1/3}\cdot$$
$$\cdot\exp\left[\tau_k\int_{t_0}^{t} q^{1/3}(\delta)\, d\delta\right]\to p_k, \tag{7}$$

Where $p_k=(1, \tau_k, \tau_k^2, 1)^{\mathrm{T}}$, $k=2, 3, 4$ and $\tau_2=1$, $\tau_{3,4}=-\frac{1}{2}\pm\frac{\sqrt{3}}{2}\,\mathrm{i}$. From (6) we have $y_1\to 1$, $y_1^{(i)}q^{-i/3}\to 0$ as $t\to\infty$, $i=1, 2, 3$ and (7) gives

$$y_2^{(i)} q^{(2-i)/3} \exp\left[\int_{t_0}^{t} q^{1/3}(\delta)\, \mathrm{d}\delta\right] \to 1.$$

Applying Lemma 2 to the last relation we get $y_2^{(i)} \to -\infty$ as $t \to \infty$, $i = 0, 1, 2, 3$.
For the other two solutions we get from (7)

$$y_3 q^{2/3} \exp\left[\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}\mathrm{i}\right) \int_{t_0}^{t} q^{1/3}(\delta)\, \mathrm{d}\delta\right] \to 1,$$

i.e.

$$y_3 = q^{-2/3} \exp\left[\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\mathrm{i}\right) \int_{t_0}^{t} q^{1/3}(\delta)\, \mathrm{d}\delta\right] \cdot [1 + o(1)]$$

and similarly

$$y_4 = q^{-2/3} \exp\left[\left(\frac{1}{2} - \frac{\sqrt{3}}{2}\mathrm{i}\right) \int_{t_0}^{t} q^{1/3}(\delta)\, \mathrm{d}\delta\right] \cdot [1 + o(1)].$$

Similarly as in the case $q(t) > 0$ we get that $u_3 = \operatorname{Re} y_3$, $u_4 = \operatorname{Im} y_4$ and their derivatives are oscillatory, but they approach zero ($\alpha(t) \to 0$ as $t \to \infty$).

Proof of Theorem 2. Let $r(t) > 0$. It follows from Theorem 8 in [5] that the differential equation (1) has a fundamental system of solutions $y_k(t)$, $k = 1, 2, 3, 4$ with the properties

$$\begin{aligned} &\operatorname{dia}[r^{3/4}, r^{1/2}, r^{1/4}, 1] \cdot (y_k, y_k', y_k'', y_k''')^{\mathrm{T}} r^{-3/8} \cdot \\ &\cdot \exp\left[-\tau_k \int_{t_0}^{t} r^{1/4}(\delta)\, \mathrm{d}\delta\right] \to p_k, \end{aligned} \tag{8}$$

where $\tau_k$ are the roots of the equation $\tau^4 + 1 = 0$ and $p_k = (1, \tau_k, \tau_k^2, \tau_k^3)^{\mathrm{T}}$. Since the roots of this equation are $\tau_k = \pm\frac{\sqrt{2}}{2} \pm \frac{\sqrt{2}}{2}\mathrm{i}$, we have

$$y_1 r^{3/8} \exp\left[\left(\frac{\sqrt{2}}{2} + \mathrm{i}\frac{\sqrt{2}}{2}\right) \int_{t_0}^{t} r^{1/4}(\delta)\, \mathrm{d}\delta\right] \to 1,$$

$$y_2 r^{3/8} \exp\left[\left(\frac{\sqrt{2}}{2} - \mathrm{i}\frac{\sqrt{2}}{2}\right) \int_{t_0}^{t} r^{1/4}(\delta)\, \mathrm{d}\delta\right] \to 1$$

$$y_3 r^{3/8} \exp\left[\left(-\frac{\sqrt{2}}{2} - \mathrm{i}\frac{\sqrt{2}}{2}\right) \int_{t_0}^{t} r^{1/4}(\delta)\, \mathrm{d}\delta\right] \to 1$$

$$y_4 r^{3/8} \exp\left[\left(-\frac{\sqrt{2}}{2} + \mathrm{i}\frac{\sqrt{2}}{2}\right) \int_{t_0}^{t} r^{1/4}(\delta)\, \mathrm{d}\delta\right] \to 1.$$

If we divide these expressions into real and imaginar parts, we get that $u_1 = \operatorname{Re} y_1$, $u_2 = \operatorname{Im} y_2$ are oscillatory approach zero, while $u_3 = \operatorname{Re} y_3$, $u_4 = \operatorname{Im} y_4$ are oscillatory and unbounded. Similarly we obtain that the derivatives of the solutions $u_1$, $u_2$ are oscillatory and approach zero, and the derivatives of the solutions $u_3$, $u_4$ are oscillatory and unbounded.

Let now $r(t) < 0$. Then from Theorem 10 in [5] if follows that equation (1) has a fundamental system of solutions $y_k$, $k = 1, 2, 3, 4$ such that

$$\operatorname{dia}[(-r)^{3/4}, (-r)^{1/2}, (-r)^{1/4}, 1] \cdot (y_k, y_k', y_k'', y_k''')^{T} \cdot \\ \cdot (-r)^{-3/8} \exp\left[-\tau_k \int_{t_0}^{t} (-r)^{1/4}\, d\delta\right] \to p_k,$$

where $\tau_k$ are the roots the equation $\tau^4 - 1 = 0$ and $p_k = (1, \tau_k, \tau_k^2, \tau_k^3)^T$. Since the roots of the equation $\tau^4 - 1 = 0$ are $\tau_1 = 1$, $\tau_2 = -1$, $\tau_{3,4} = \pm i$, then for $k = 0, 1, 2, 3$ there holds

$$y_1^{(k)} |r|^{3/8} \exp\left[-\int_{t_0}^{t} |r^{1/4}(\delta)|\, d\delta\right] \to 1,$$

$$y_2^{(k)} |r|^{3/8} \exp\left[\int_{t_0}^{t} |r^{1/4}(\delta)|\, d\delta\right] \to (-1)^k,$$

$$y_3^{(k)} |r|^{3/8} \exp\left[-i \int_{t_0}^{t} |r^{1/4}(\delta)|\, d\delta\right] \to (i)^k,$$

$$y_4^{(k)} |r|^{3/8} \exp\left[i \int_{t_0}^{t} |r^{1/4}(\delta)|\, d\delta\right] \to (-i)^k.$$

From these relations it follows that $y_1$, $y_2$ are nonoscillatory and $y_1^{(k)}(t) \to \infty$, $y_2^{(k)}(t) \to 0$, $y_1^{(k)}(t) > 0$, $(-1)^k y_2^{(k)} > 0$. We cannot apply Lemma 2 for investigating the solutions $y_3$, $y_4$, because $\operatorname{Re} \tau_{3,4} = 0$. However, there is valid for $y_{3,4}^{(k)}$:

$$y_3^{(k)} = (i)^k |r|^{-3/8}\, exp \left[-i \int_{t_0}^{t} |r^{1/4}(\delta)|\, d\delta\right] \cdot [1 + o(1)],$$

$$y_4^{(k)} = (-i)^k |r|^{-3/8} \exp\left[i \int_{t_0}^{t} |r^{1/4}(\delta)|\, d\delta\right] \cdot [1 + o(1)],$$

and therefore

$$y_3(t) = |r|^{-3/8} \left[\cos\left(\int_{t_0}^{t} |r|^{1/4}\, d\delta\right) - i \sin\left(\int_{t_0}^{t} |r|^{1/4}\, d\delta\right)\right] [1 + o(1)],$$

$$y_4(t) = |r|^{-3/8} \left[\cos\left(\int_{t_0}^{t} |r|^{1/4}\, d\delta + i \sin\left(\int_{t_0}^{t} |r|^{1/4}\, d\delta\right)\right] [1 + o(1)].$$

From these expressions we have that $u_3 = \operatorname{Re} y_3$, $u_4 = \operatorname{Im} y_4$ are oscillatory and unbounded if $|r|^{3\ 8} \to 0$ as $t \to \infty$ and $u_3$, $u_4$ are oscillatory and bounded if $|r|^{3\ 8} \to \infty$ as $t \to \infty$. We can easily prove that the same properties are valid for the derivatives of $u_3$, $u_4$.

## 5. Discussion

In paper [2] Mamrilla proved the following statements for the differential equation

$$(\mathrm{a}_1) \qquad y^{(iv)} + Qy' + Q'y = 0$$

*(i) If* $Q(x) \geq \left[\frac{2\sqrt{3}}{9} + \varepsilon_1(x)\right] \frac{1}{x^3}$, *where* $\varepsilon_1(x) \geq 0$ $\int^{\infty} \frac{\varepsilon_1(x)}{x}\,\mathrm{d}x = \infty$ $Q'(x) \geq 0$ $(=0$ *does not hold in any subinterval of* $[a, \infty)$), *then there exists a fundamental system of solutions of* $(\mathrm{a}_1)$ *such that three solutions are oscillatory and one is nonoscillatory and approaches zero monotonically as* $x \to \infty$.

*(ii) If* $Q(x) \leq \left[-\frac{2\sqrt{3}}{9} - \varepsilon_2(x)\right] \frac{1}{x^3}$, $\varepsilon_2(x) \geq 0$, $\int^{\infty} \frac{\varepsilon_2(x)}{x}\,\mathrm{d}x = \infty$, *then there exists a fundamental system of solutions of* $(\mathrm{a}_1)$ *such that two solutions are oscillatory and the other two solutions are nonoscillatory and diverge to* $\infty$ *monotonically as* $x \to \infty$.

If $Q(x) > 0$, then Theorem 1 gives another sufficient condition so that the equation $(\mathrm{a}_1)$ may have a fundamental system of solutions which consists of two oscillatory and two nonoscillatory solutions. The nonoscillatory solutions converge to 0 and 1. From this it evidently follows that there also exists a fundamental system which consists of three oscillatory and one nonoscillatory solution that approaches zero.

Similarly if $Q(x) < 0$, then the differential equation $(\mathrm{a}_1)$ has a fundamental system such that two solutions are oscillatory, the third diverges to $\infty$ and the fourth converges to 1. From this it easily follows that there also exists a fundamental system which consists of two oscillatory and two nonoscillatory solutions. The nonoscillatory solutions diverge to $\infty$.

In paper [3] Mamrilla proved a sufficient condition for all solutions of (1) to be oscillatory. Then there exists a fundamental system of oscillatory solutions. He requires $q(t)$ to be of one sign. Therefore we cannot apply his theorem to the differential equation

$$y^{(iv)} + \sin t\ y' + t^2 y = 0. \tag{9}$$

However, the conditions of Theorem 2 are satisfied. Indeed:

$$\frac{q(t)}{r(t)}=\frac{\sin t}{t^2}\in L[1, \infty)$$

and

$$\frac{r''(t)}{r^{5/4}(t)}=\frac{2}{t^{5/2}}\in L[1, \infty).$$

Hence equation (9) has a fundamental system of oscillatory solutions.

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## КОЛЕБАТЕЛЬНЫЕ СВОЙСТВА ДИФФЕРЕНЦИАЛЬНОГО УРАВНЕНИЯ ЧЕТВЕРТОГО ПОРЯДКА

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### Резюме


В работе рассматриваются колебательные свойства фундаментальной системы решений уравнения (1), если несобственные интегралы из некоторых дробей функции $q$ и $r$ являются конечными.