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GOODWYN'S THEOREM FOR SEQUENTIAL ENTROPY ON PSEUDOCOMPACT SPACES

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The notion of measure theoretic sequential entropy was introduced by Kushnirenko (cf. [7]), who showed that this invariant can be more sensitive than the Kolmogorov—Sinai entropy.

The new invariant was studied by Newton in [9]. He introduced a function $K(A)$ as a measure of asymptotic expansion of a subsequence A of the sequence Z^+ of nonnegative integers.

He proved that for the sequential entropy $h_{A,\mu}(T)$ and the standard entropy $h_\mu(T)$ of an automorphism T of probability measure space $(X, \mathcal{B}, \mu)$ the equality

$$h_{A,\mu}(T) = K(A) \cdot h_\mu(T) \tag{1}$$

holds except for the case

$$(K(A), h(T)) = \begin{cases} (0, \infty) \\ \text{or} \\ (\infty, 0) \end{cases} \tag{2}$$

Moreover he proved that

$$K(A) = 0 \quad \text{implies} \quad h_{A,\mu}(T) = 0. \tag{3}$$

In a other words, the sequential entropy can be more sensitive only if $K(A) = \infty$ and $h_\mu(T) = 0$.

The topological sequence entropy $h_A(T)$ of a continuous transformation T of a compact X was introduced by Godman in [4]. He obtained the following analogy of Newton's result. The inequality

$$h_A(T) \leq K(A) \cdot h(T) \tag{4}$$

holds except for the case

$$(K(A), h(T)) = \begin{cases} (0, \infty) \\ \text{or} \\ (\infty, 0) \end{cases} \tag{2'}$$

Moreover he proved that if X has a finite covering dimension

$$(5) \qquad h_A(T)=\sup\{h_{A,\mu}(T):\mu\in\mathcal{M}_T(X)\}$$

except for the case

$$(2'') \qquad (K(A),\, h(T))=(\infty,0)$$

where $\mathcal{M}_T(X)$ is the system of all regular invariant probability measures on X.

We recall that a topological space X is pseudo-compact (cf. [2]) if any real continuous function on X is bounded.

We say that a probability measure μ on the σ-algebra $\mathcal{B}(X)$ generated by open sets is regular if for any $B\in\mathcal{B}(X)$

$$(6) \qquad \mu(B)=\inf\{\mu(U):B\subset U,\ U\ \text{open}\}.$$

Let us consider the topological entropy defined by means of open coverings in [1]. The following generalization of Goodwyn's theorem was presented in [6].

Let T be a continuous transformation of a Hausdorff normal pseudo-compact space X. Then

$$(7) \qquad h(T)=\sup\{h_\mu(T):\mu\in\mathcal{M}_T(X)\}.$$

The aim of this paper is to complete the above results.

Theorem 1. *Let T be any measure preserving transformation of a probability space $(X,\mathcal{B},\mu)$. Then the equality*

$$(1) \qquad h_{A,\mu}(T)=K(A)h_\mu(T)$$

holds except for the case (2).

Proof. We only need to prove the inequality

$$(3') \qquad h_{A,\mu}(T)\leq K(A)\cdot h_\mu(T).$$

This can be done by the methods used in the proof of the inequality (4) given in [4].

Theorem 2. *Let X be a Hausdorff normal and pseudo-compact topological space and T a continuous transformation of X. Then the following equalities hold:*

$$(8) \qquad h_A(T)=\sup\{h_{A,\mu}(T):\mu\in\mathcal{M}_T(X)\}$$

except for the case (2″) *and*

$$(8') \qquad h_A(T)=K(A)\cdot h(T)$$

except for the case (2′).

Proof. The inequality (4) can be obtained by the same way as for compact X (cf. [4]). Suppose that (2′) holds. Combining the relations (4), (7) and (1) we get

$$h_A(T)\leq K(A)\cdot h(T)=K(A)\cdot\sup\{h_\mu(T):\mu\in\mathcal{M}_T(X)\}=$$
$$=\sup\{h_{A,\mu}(T):\mu\in\mathcal{M}_T(X)\}.$$

If $K(A)=0$ then we have

$$h_A(T)=\sup\{h_{A,\mu}(T):\mu\in\mathcal{M}_T(X)\}=0$$

(cf. [4], [7]).

By the same argumente as in [5] or [6] we can show that

$$h_{A,\mu}(T)=\sup\{H_A(T,P):P\in\mathcal{P}^0_\mu\}$$

where $\mathcal{P}^0_\mu$ is the system of all finite partition of X consisting of closed G_δ sets having intersections of μ-measure zero. The function $H_A(T, P)$ is defined as in [8] or [7].

Let $P=\{C_1, \ldots, C_m\}$. We construct a compact metrizable space

$$Y=\prod_{n=0}^{\infty} Y_n$$

where

$$Y_n=\langle 0, 1\rangle^m$$

and the continuous mapping Φ: $X\to Y$ defined by

$$[\Phi(x)]_{n,i}=\varphi_i\cdot T^n(x), \qquad n=0, 1, \ldots, \quad i=1, \ldots, m$$

where φ_i are real continuous functions on X such that

$$0\leqslant\varphi_i\leqslant 1 \text{ and } C_i=\varphi_i^{-1}(0) \text{ for } i=1, \ldots, m.$$

The subspace $K=\Phi(X)$ is metrizable and compact (cf. [6]). We have $\Phi\cdot T=\tau\cdot\Phi$ where the shift τ: $K\to K$ is defined by

$$[\tau(y)]_{n,i}=[y]_{n+1,i}, \qquad n=0, 1, \ldots, \quad i=1, \ldots, m.$$

Put

$$B_i=\{y\in K:[y]_{0,i}=0\} \text{ for } i=1, \ldots, m$$

and

$$Q=\{B_1, \ldots, B_m\}.$$

Then we have

$$C_i=\Phi^{-1}(B_i), \qquad i=1, \ldots, m$$

hence (cf. [6])

$$H_{A,\mu}(T, P)=H_{A,\mu\cdot\Phi^{-1}}(\tau, Q)\leqslant h_{A,\mu\cdot\Phi^{-1}}(\tau)\leqslant h_A(\tau).$$

The last inequality follows from the compactness of K. The mapping Φ is a flow homomorphism from (X, T) onto K, hence (cf. [4], [6])

$$h_A(T)\geqslant h_A(\tau)\geqslant H_{A,\mu}(T, P).$$

REFERENCES

[1] ADLER R. L., KONHEIM A. G., MacANDREW M. H.: Topological entropy, Trans. AMS 119, 1965, 309—319.

[2] BAGLEY R. W., CONNEL E. H., McKNIGHT J. D.: On properties characterising pseudo-compact spaces. Proc. Amer. Math. Soc. 9, 1958, 500—506.

[3] EBERLEIN E.: On topological entropy of semigroups of commuting transformations, Ásterisque 40, 1976, 17—62.

[4] GOODMAN T. N. T.: Topological sequence entropy. Proc. London Math. Soc. (3) 29, 1974, 331—350.

[5] GOODWYN L.: Comparing topological entropy with measure theoretic entropy, Amer. J. Math. 94, 1972, 366—388.

[6] KOMORNÍKOVÁ M., KOMORNÍK J.: Regular measures and entropy on pseudo-compact spaces, Math. Slovaca 31, 1981, 297—309.

[7] KUŠNIRENKO A. G.: O metričeskich invarijantoch tipa entropiji, Usp. mat. nauk SSSR (5) 137, 1967, 57—65.

[8] NEWTON D.: On sequential entropy I, II. Math. Systems Theory 4, 1970, 119—125, 126—128.

[9] NEWTON D., KRUG E.: On sequence automorphismus of a Lebesgue spaces. Z. Wahrscheinlichtkeits-theorie und Ver. 24, 1972 211—214.

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ТЕОРЕМА ГУДВИНА ДЛЯ ЭНТРОПИИ ПОСЛЕДОВАТЕЛЬНОСТЕЙ НА ПСЕВДО-КОМПАКТНЫХ ПРОСТРАНСТВАХ

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Резюме

Доказывается теорема о сравнении для топологической и вероятностной энтропии последовательностей на нормальных псевдо-компактных пространствах Гаусдорффа.