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Jin Akiyama; Geoffrey Exoo; Frank Harary

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COVERING AND PACKING IN GRAPHS III: CYCLIC AND ACYCLIC INVARIANTS

JIN AKIYAMA[1]—GEOFFREY EXOO—FRANK HARARY

Dedicated to Crispin St. John Alvah Nash-Williams

Abstract

It is possible to define many variations of packing and covering invariants for graphs which involve paths and cycles. These can be given terminology which is sufficiently intuitive that one can remember the definitions, e.g., arboricity, linear arboricity, point arboricity, point linear arboricity, anarboricity, path number, unpath number, point anarboricity, and cyclicity. Most of these concepts are fundamental but it is not easy to determine the value of these invariants for general graphs. We investigate these concepts and relations among them for specific families of graphs. In particular, we determine them for complete graphs, complete bipartite graphs, and their line graphs.

1. Cyclicity

The *cyclicity* of a block G, $o\ (G)$, as introduced in [3] is the minimum number of cycles, not necessarily line-disjoint, needed to cover all the lines of G. Let $[x]$ be the greatest integer $n \leqq x$, and $\{x\} = -\{-x]$. All other terminology and notation used here can be found in [2].

Theorem 1. *For the complete graph K_p with $p \geqslant 3$, the cyclicity is given by:* $o\ (K_p) = [p/2]$.

Proof. Since the degree of each point in a cycle is 2 and since K_p is regular of degree $p-1$, we need at least $\{(p-1)/2\}$ cycles to cover G. When p is odd, the graph K_p is the sum of $(p-1)/2$ spanning cycles [2, p. 89] which meets the lower bound. When p is even, the line set of K_p can be covered by $p/2$ spanning paths as

[1]Visiting Scholar, 1978—1979, from Nippon Ika University, Kawasaki, Japan.

determined in [4, Theorem 4] or in [7]. We thus obtain $p/2$ cycles whose union is G, by adding a line joining two endpoints of each spanning path. □

Theorem 2. *For the complete bipartite graph $K_{m,n}$ with $m \geqq n > 1$, the cyclicity is given by:*

$$o\,(K_{m,n}) = \begin{cases} \{m/2\} & n \text{ even}, \\ \{m(n+1)/2n\} & n \text{ odd}. \end{cases}$$

Sketch of proof. Let the point set of $K_{m,n}$ be $W = \{w_1, \ldots, w_m\}$ and $U = \{u_1, \ldots, u_n\}$. If n is even, $o\,(K_{m,n}) \geqq \{mn/2n\} = \{m/2\}$ holds, since the length of any cycle in $K_{m,n}$ is at most $2n$.

We now construct $\{m/2\}$ cycles whose union is $K_{m,n}$.

Case 1. m even, n even. As illustrated in Figure 1, define $m/2$ cycles C_i, $i = 1, \ldots, m/2$, by:

$$C_i = (w_{2i-1}u_1w_{2i}u_2 \ldots w_{2i-2+j}u_j \ldots w_{2i+n-2}u_nw_{2i-1}),$$

where $j = 1, \ldots, n$ and subscripts are taken modulo m.

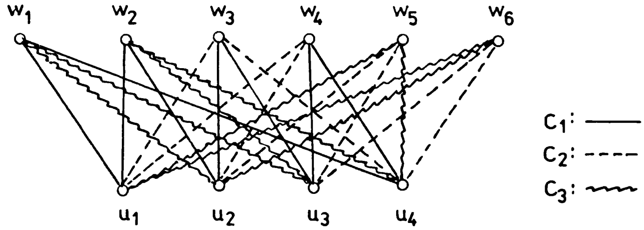

Figure 1. Cycles in a decomposition of $K_{6,4}$.

Then we see that the union of $m/2$ line-disjoint cycles C_i of length $2n$ is $K_{m,n}$.

Case 2. m odd, n even. We add a new point w_{m+1} to W so that the cycle construction in Case 1 can be applied. We now alter these $(m+1)/2$ cycles C_i whose union is $K_{m+1,n}$ to construct cycles C'_i such that none of them cover the lines incident to w_{m+1}. For each C_i containing w_{m+1}, there is a point w_j not covered by C_i since $m > n$. By replacing w_{m+1} with w_j in C_i, we have a new cycle C'_i, that is,

$$C'_i = \ldots u_kw_ju_{k+1} \ldots \text{ if } C_i = \ldots u_kw_{m+1}u_{k+1} \ldots .$$

When this procedure is repeated for every cycle containing w_{m+1}, this point becomes isolated and then we may omit it without destroying cycles.

It remains to handle the proof when n is odd. In this case, at least one line incident to each point w_i must be covered by two distinct cycles, because deg w_i is odd. Hence we may conclude that $o\,(K_{m,n}) \geqq \{m(n+1)/2n\}$, since the length of a longest cycle in $K_{m,n}$ is $2n$.

The brutal construction which establishes equality is omitted. □

2. Linear arboricity

In a *linear forest*, each component is a path. The *linear arboricity* $\Xi(G)$ of a graph G as defined in [3] is the minimum number of linear forests whose union is G. Note that the Greek letter, capital xi, looks like three paths!

Theorem 3. *If T is a tree with maximum degree ΔT, then*

$$\Xi(T)=\{\Delta T/2\}. \tag{1}$$

Proof. The lower bound $\Xi(T)\geqq\{\Delta T/2\}$ is obvious. Since tree T has maximum degree ΔT, its line chromatic number $\chi'(T)$ is equal to ΔT. Each subgraph induced by subsets of lines with two colors is a linear forest. Thus we obtain the upper bound $\Xi(T)\leqq\{\chi'(T)/2\}=\{\Delta T/2\}$. □

The linear arboricity of the complete graph coincides with its path number which was determined by Stanton, Cowan and James [7]. We also calculate this for complete bipartite graphs.

Theorem 4. *For the complete graph K_p, $\Xi(K_p)=\{p/2\}$.*

The notation $\delta(m, n)$ is the conventional Kronecker delta.

Theorem 5. *For the complete bipartite graph $K_{m,n}$ with $m\geqq n$, the linear arboricity is given by:*

$$\Xi(K_{m,n})=\{(m+\delta(m,n))/2\}. \tag{2}$$

Proof. For a star $K_{m,1}$, it is obvious that $\Xi(K_{m,1})=\{m/2\}$. Before handling the general complete bipartite graph $K_{m,n}$, let us determine $\Xi(K_{m,m})$ first. We write $V(K_{m,m}) = U\cup W$, where $U=\{u_1, \dots, u_m\}$ and $W=\{w_1, \dots, w_m\}$. As $m^2=q(K_{m,m})$ and the number of lines in a spanning tree of $K_{m,m}$ is $2m-1$, it follows at once that

$$\Xi(K_{m,m})\geqq\{m^2/(2m-1)\}=\{(m+1)/2\}. \tag{3}$$

We will now show the converse inequality.

Case 1. m even.

The line-set of $K_{m,m}$ can be partitioned into $m/2$ line-disjoint spanning cycles C_i, $i=1, \dots, m/2$, which can be written as:

$$C_i=(u_1w_{2i+1}u_2w_{2i+2}\dots u_jw_{2i+j}\dots u_mw_{2i+m}u_1),\quad j=1,\dots m,$$

where subscripts are taken modulo m.

Let $[u, v]$ denote the line joining u and v. As illustrated in Figure 2, define $m/2$ paths P_i, $i=1, \dots, m/2$, and one linear forest F by:

$$P_i=C_i-[u_{m/2-i+1}, w_{m/2+i}],\quad i=1,\dots,m/2,$$

$$F=\bigcup_{i=1}^{m/2}[u_{m/2-i+1},\ w_{m/2+i}].$$

Thus we have $\Xi(K_{m,m})=\{(m+1)/2\}$.

Case 2. m odd.

The line-set of $K_{m,m}$ can be partitioned into $(m-1)/2$ cycles C_i and one linear forest F as follows: For each $i=1, \ldots, (m-1)/2$,

$$C_i=(u_1w_{2i+1}u_2w_{2i+2}\ \ldots\ u_jw_{2i+j}\ \ldots\ u_mw_{2i+m}u_1),\quad j=1, \ldots, m,$$

where subscripts are taken modulo m, and $F=\bigcup_{i=1}^{m}[u_i,\ w_i]$. Of course F is just a 1-factor of $K_{m,m}$.

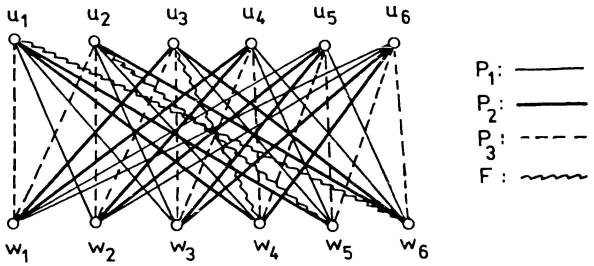

Figure 2. Decomposition of $K_{6,6}$ into paths.

Now we construct $(m-1)/2$ paths P_i, $i=1, \ldots, (m-1)/2$, and one linear forest F_0, whose union is $K_{m,m}$ as illustrated in Figure 3:

$$P_i=C_i-[u_{(m-1)/2-i+1},\ w_{(m-1)/2+i}],\quad \mathrm{i}=1, \ldots, (\mathrm{m}-1)/2,$$

and

$$F_0=\bigcup_{i=1}^{(m-1)/2}[u_{(m-1)/2-i+1},\ w_{(m-1)/2+i}]\cup F.$$

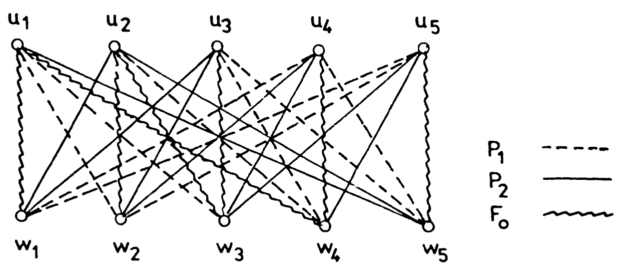

Figure 3. Decomposition of $K_{5,5}$ into linear forests.

Thus we have $\Xi(K_{m,m})=\{(m+1)/2\}$. Now we will use this result for $K_{m,m}$ to derive the more general equation (2).

By the hypothesis that $m \geqq n$, we have the inequalities:

$$\text{(4)} \qquad \{m/2\}=\Xi(K_{m,1}) \leqq \Xi(K_{m,n}) \leqq \Xi(K_{m,m})=\{(m+1)/2\}.$$

Here again we divide the proof into two cases, depending on the parity of m. If m is odd, then $\Xi(K_{m,m}) = \{(m+1)/2\}$ follows from (4).

If m is even, the line set of $K_{m,m}$ can be partitioned into the $m/2$ line-disjoint cycles C_i of Case 1 above. To complete the proof, we shall construct $m/2$ paths P_i, $i=1, \ldots, m/2$, whose union is $K_{m,m-1}$. For each $i=1, \ldots, m/2$, let

$$P_i' = C_i - [u_{m\ 2i+1}, w_1] - [u_{m\ 2i+2}, w_1].$$

Therefore we have $\Xi(K_{m,m-1}) \leqq \{m/2\}$. This equality together with (4) implies for all $n<m$:

$$\{m/2\} \leqq \Xi(K_{m,1}) \leqq \Xi(K_{m,n}) \leqq \Xi(K_{m,m-1}) \leqq \{m/2\}.$$

That is, $\Xi(K_{m,n})=\{m/2\}$ if m is even and $n<m$.

Combining the odd and even cases, we obtain precisely the equation (2). □

Having determined the linear arboricity of a complete graph trivially, of a tree easily, and of a complete bigraph more tediously, we now turn our attention to cubic graphs G and find that a rather pictorial proof serves to show that the linear arboricity of G is 2.

Given two families of graphs, **N** and **H**, we say that **N** is a set of *necessary subgraphs* for **H** if every graph in **H** has a subgraph in **N**. For example, the family of all subdivisions of $K_{3,3}$ and K_5 is a set of necessary subgraphs for the nonplanar graphs. In the study of the linear arboricity of the family **C** of cubic graphs, we found it necessary to consider a family of necessary subgraphs of three subfamilies of **C**, namely, $\mathbf{C}_3$ containing cubic graphs of girth 3, $\mathbf{C}_4$ for girth 4 and $\mathbf{C}' = \mathbf{C}-\mathbf{C}_3-\mathbf{C}_4$ of girth at least 5.

If H is a subgraph of G, then the *points of attachment* of H in G are those points of H which are adjacent to points not in H. A *shrinking* of a graph G at a subgraph H is obtained on replacing H by a smaller graph H' such that H' and H have precisely the same points of attachment in G, with the proviso that for each point of attachment v, $\deg(v, H) = \deg(v, H')$. For example, in Figure 4 we show a graph G' obtained by shrinking G at H, replacing it by H'. In Theorem 6 and the preparatory lemmas we can assume G is connected with no loss in generality.

Lemma 6a. *A necessary subgraph for the family* $\mathbf{C}'$ *of cubic graphs having girth* $g \geqq 5$ *is the tree* T *of Figure 8a.*

Proof. Let $G \in \mathbf{C}'$. Since the girth of G is at least 5, we can find a smallest cycle of length $n \geqq 5$ in G. Let v_1 to v_5 be consecutive points on this cycle. Then as G is cubic, v_2, v_3 and v_4 are adjacent to points u_2, u_3 and u_4 not on this cycle, for otherwise a smaller cycle would result. Since $g \geqq 5$ the u_i are distinct and so T is a subgraph of G with these 8 points as its point set. □

Lemma 6b. *The four graphs of Figure 7a are a family of necessary subgraphs for* $\mathbf{C}_4$, *the family of cubic graphs with girth* $g = 4$.

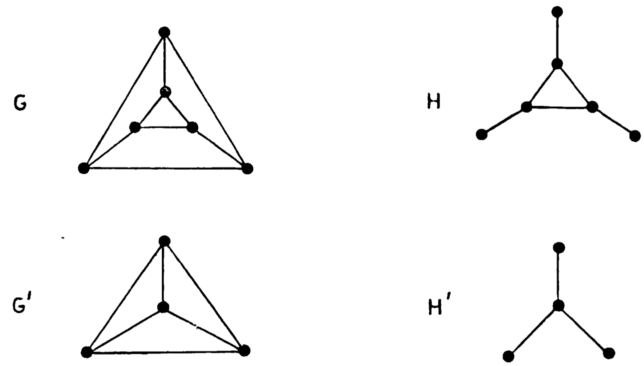

Figure 4. G' is a shrinking of G at H.

Proof. Let v_1, v_2, v_3, v_4 be consecutive points of a 4-cycle in G. Then each v_i is adjacent to exactly one point in G, not on this 4-cycle. Let u_i be the point adjacent to v_i, for $i = 1$ to 4. Then $u_i \neq u_{i+1}$, for $i = 1, 2, 3, 4$, where addition of subscripts, is modulo 4, since $g > 3$. So the possibilities are: (1) the u_i are all distinct, (2) $u_1 = u_3$ and u_2, u_4 are distinct nonadjacent points, (3) $u_1 = u_3$ and $u_2 = u_4$, and (4) $u_1 = u_3$ and u_2, u_4 are adjacent.

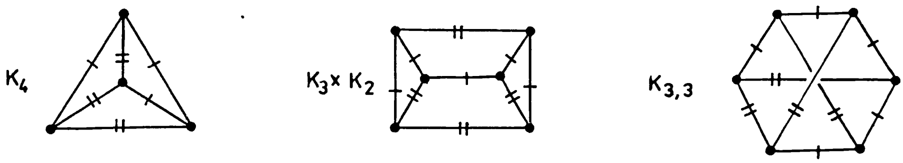

Figure 5. The cubic graphs with $p \leqq 6$, colored to exhibit the linear arboricity.

These four possibilities determine the graphs I_1, I_2, I_3 and I_4 of Figure 7a, as the labeling of the points v_i in the figure shows. □

Lemma 6c. *The three graphs of Figure 6a are a family of necessary subgraphs for* $\mathbf{C}_3$, *the family of cubic graphs with girth* $g = 3$ *and* $p \geqq 6$.

Proof. Let v_1, v_2 and v_3 be the points of a 3-cycle in G. Each v_i is adjacent to a point u_i not on the cycle. There are three possibilities for the distinctness of the points u_i. If $u_1 = u_2 = u_3$, then G has K_4 as a component, contrary to the convention that G is connected. If u_1, u_2 and u_3 are distinct, then H_1 (see Figure 6a) must be a subgraph of G. The only remaining possibility is that $u_1 \neq u_2 = u_3$. If this holds then there is a point w, different from the v_i, adjacent to u_2. If $w \neq u_1$ then we must find H_2 in G; but if $w = u_1$ we can find H_3, as can be verified with the help of the labeled points of Figure 6a. □

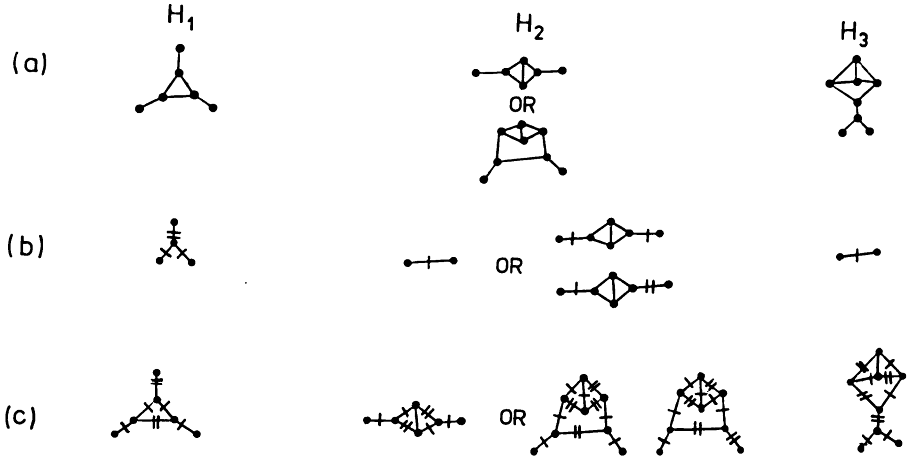

Figure 6. The necessary subgraphs for $p \geq 8$ and $g = 3$.

Theorem 6. *Every cubic graph has linear arboricity 2.*

Proof. Let g be the girth of G. We divide the proof into three cases: Case 3, $g = 3$; Case 4, $g = 4$; and Case 5, $g \geqq 5$. We use induction on even p, because all cubic graphs have even order. The induction begins easily for $p = 4$ and $p = 6$; see Figure 5 in which lines with one slash are red while those with two are green, noting that the lines of each color form a linear forest.

For each of the cases we proceed along the same lines. We have already identified a set of necessary subgraphs for each of Cases 3, 4 and 5. Figure 6a shows $\mathbf{N}_3$ consisting of the three subgraphs H_1, H_2, H_3 for Case 3, $g = 3$, Figure 7a gives I_1, I_2 and I_3 for Case 4; Figure 8a has the necessary subtree T for Case 5.

Then we show in Figures 6b, 7b and 8b how to shrink G to a smaller cubic graph $\hat{G}$ by replacing the necessary subgraphs with smaller subgraphs.

We take as the hypothesis of complete induction that any cubic graph with at most $p - 2$ points has linear arboricity 2. So in each of the three cases $\hat{G}$ can be line colored to exhibit $\Xi(\hat{G}) = 2$. We show in Figures 6b, 7b, 8b all the essentially different ways in which the smaller subgraphs can be colored. Of course it is sufficient to show the coloring of the lines of the smaller subgraph incident with points of attachment.

Finally we indicate in Figures 6c, 7c, 8c how to color the original subgraph, consistent with the coloring of the rest of G given by the coloring of G.□

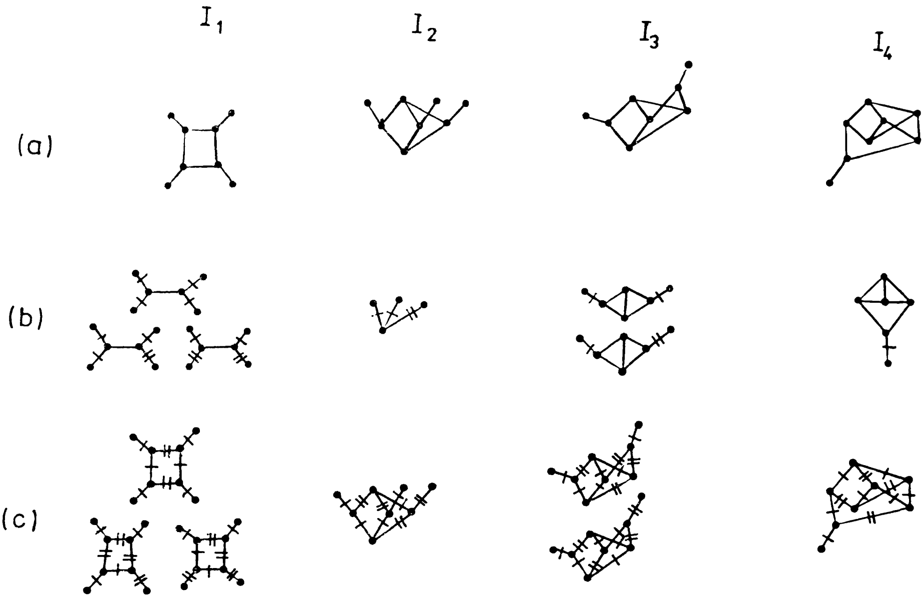

Figure 7. The necessary subgraphs for $p \geq 8$ and $g = 4$.

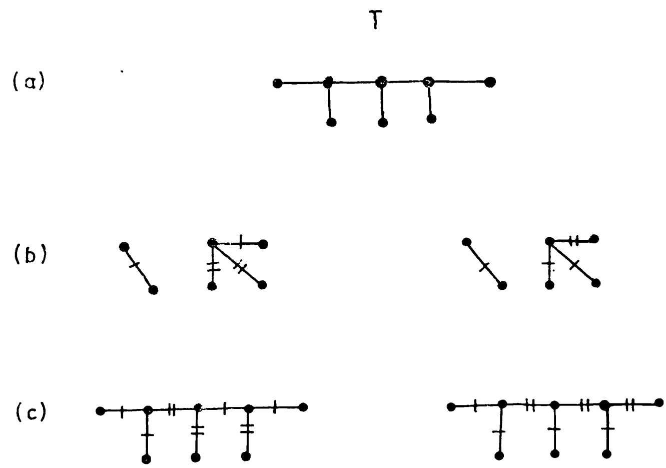

Figure 8. The necessary subgraph in a cubic graph with $p \geqq 8$ and $g \geqq 5$.

3. Path numbers

Related to the linear arboricity, there are two covering invariants involving paths: the path number and the overlapping path number. We shall compare these three acyclic invariants for a tree, a complete bipartite graph and a cubic graph.

Nash-Wiliams [6] derives a criterion for a graph G to have a partition of $E(G)$ into a prescribed class of open trails. In this sense, he anticipated the concept of the path number of a graph.

The *path number* of G, $\pi(G)$, is the smallest number of line-disjoint paths which cover all the lines of G. Similarly, the *overlapping path number* of G, $\tilde{\pi}(G)$, is the minimum number of paths, not necessarily line-disjoint (so that overlapping is admitted), needed to cover the lines. These two invariants were introduced in [3] and studied by Stanton, Cowan, James [7], [8] and Harary, Schwenk [4].

We require some results due to Lovász [5].

Theorem L. *Every graph G of order p has a partition of $E(G)$ into $[p/2]$ paths and cycles.*

Corollary L. *If every point of G has odd degree, then $\pi(G)=p/2$.*

It follows at once that the path number of a tree T with p_0 points of odd degree is given by $\pi(T)=p_0/2$. This little result was independently discovered later in [4] and also by others.

We will also need two results from [4] which we now call Theorems 5HS and 7HS.

Theorem 5HS. *If $m \geqq n$, the path number of $K_{m,n}$ is given by:*

$$\pi(K_{m,n}) = \begin{cases} (m+n)/2, & mn \text{ odd}, \\ \{mn/(2n-\delta(m,n))\}, & mn \text{ even}. \end{cases}$$

Theorem 7HS. *The overlapping path number of a tree T with e endpoints is given by: $\tilde{\pi}(T)=\{e/2\}$.*

A *starlike tree* T is homeomorphic to a star $K_{1,n}$. Thus T has one point of degree n and all its other points have degree 1 or 2. For any tree T, we denote by T' the subtree obtained on deleting the endpoints of T, [2, p. 35]. A *double star* is a tree T such that $T'=K_2$; it is denoted by $S(m,n)$ when m endpoints are adjacent to one point of this K_2 and n to the other.

By applying Corollary L and Theorem 7HS to Theorem 3, we easily obtain the following results.

Corollary 3a *For a tree,*

(a) $\pi(T)=\Xi(T)$ *if and only if T is starlike,*

(b) $\tilde{\pi}(T)=\Xi(T)$ *if and only if T is starlike or homeomorphic to a double star of the form $S(2,2n)$.*

It is easy to see that $\pi(G)=\tilde{\pi}(G)=\Xi(G)$ when G is a complete graph K_p and that their common value is $\{p/2\}$.

By applying Theorem 5HS to Theorem 2, we get the next result at once.

Corollary 3b. *For a complete bipartite graph $K_{m,n}$ with $m \geqq n$, the path number $\pi(K_{m,n})$ coincides with the linear arboricity $\Xi(K_{m,n})$ if neither m nor n is odd and $m \geqq n \geqq 3$.*

4. Point arboricity

Next we shall note that the linear arboricity of G is precisely the point arboricity of the line graph of G.

The *point arboricity* $\varrho(G)$, as defined by Chartrand, Geller and Hedetniemi [1], is the minimum number of subsets V_i into which the point set V of G can be partitioned so that the subgraph $\langle V_i \rangle$ induced by each subset is a forest. Analogously, *the point linear arboricity* $\varrho_0(G)$ is the minimum number of subsets V_i with every $\langle V_i \rangle$ a linear forest.

Theorem 7. *For any graph G, the linear arboricity of G equals both the point arboricity and the point linear arboricity of its line graph:*

$$\Xi(G)=\varrho(L(G))=\varrho_0(L(G)). \tag{5}$$

Proof. If one colors all the lines of G so that the subgraph induced by lines of each color is a linear forest in G, then the points of $L(G)$ corresponding to lines in G of one color induce a forest in $L(G)$, in fact, a linear forest. Hence

$$\varrho(L(G)) \leqq \varrho_0(L(G)) \leqq \Xi(G).$$

We now show the opposite inequality.

Let $\varrho(L(G))=r$, so that by definition there exists a partition of the point set of $L(G)$ into r subsets U_i, $i=1, \ldots, r$, in such a way that each induced subgraph $\langle U_i \rangle$ of $L(G)$ is a forest. The maximum degree of every induced subgraph $\langle U_i \rangle$ is at most 2, since otherwise there would be a point of $\langle U_i \rangle$ whose degree is at least 3, contradictiong the fact that $K_{1,3}$ is a forbidden induced subgraph for line graphs. Thus, each component of $\langle U_i \rangle$ is a path. Since the lines of G corresponding to the points of $\langle U_i \rangle$ induce a linear forest, we see that $r=\varrho(L(G)) \geqq \Xi(G)$. Furthermore, exactly the same discussion can be applied to the point linear arboricity $\varrho_0(L(G))$.

5. Unpath number

To indicate that a graph (or digraph) has no cycles (or dicycles), we call it *acyclic.* The opposite concept to arboricity (which is the minimum number of forests needed to *cover* the lines of G) has been humorously called the *anarboricity,* $\tilde{T}$, (meaning the maximum number of nonforests which can be packed into the lines of G) by the great wit Ronald Read. Similarly in [3], the packing invariants opposite to the path number $\pi(G)$ were introduced. Thus this is the maximum number of line-disjoint connected non-paths which can be packed into G; it was called the "apathy" of G in [3] but we now call it the *unpath number* and write it $Y(G)$, as the letter Y looks like $K_{1,3}$ which is not a path.

It is quite easy to prove the following results.

Theorem 8. *The unpath number of the complete graph K_p and of the complete bipartite graph $K_{m,n}$ are given by:*

$$Y(K_p) = [p(p-1)/6], \tag{6}$$

$$Y(K_{m,n}) = [mn/3]. \tag{7}$$

The *point anarboricity,* $\tilde{T}_0(G)$, as introduced by Chartrand, Geller and Hedetniemi [1] is the maximum number of disjoint subsets in G so that the subgraph induced by each subset is a nonforest.

Theorem 9. *The unpath number of a graph G is equal to the point anarboricity of its line graph $L(G)$:*

$$Y(G) = \tilde{T}_0(L(G)). \tag{8}$$

Proof. This follows immediately from the fact that either a cycle or a point of degree at least 3 in G will give a nonforest in $L(G)$. □

6. Unsolved problems

We have obtained the cyclicity, the linear arboricity and the unpath number for some specific families of graphs.

I. It follows from Corollary L above that the path number of a cubic graph with p points is $p/2$. We showed in Theorem 6 that if G is cubic, then the linear arboricity of G is 2. As a generalization of this theorem, a stronger statement can be formulated.

Conjecture. *The linear arboricity of an r-regular graph G is* $\{(r+1)/2\}$.

This is obvious for 2-regular graphs and we have seen that it also holds for complete graphs (Theorem 4) and regular complete bipartite graphs (Theorem 5). We have recently proved this conjecture for $r = 4$, but do not know whether it holds for $r \geqslant 5$.

II. The path number of a complete tripartite graph and more generally of a complete n-partite graph were studied in [8]. The linear arboricity, the unpath number and the cyclicity of these graphs have not yet been determined.

III. It is known that $\pi(G)=\Xi(G)$ for those trees specified in Corollary 3a, for all complete graphs K_p, and for those complete bipartite graphs $K_{m,n}$ specified in Corollary 3b. Characterize the graphs for which $\pi(G)=\Xi(G)$.

IV. Some relations among cyclic and acyclic invariants are known, e.g., those in Theorems 7 and 9. Are there other relations among them?

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Department of Mathematics
Nippon Ika University
2-297-2, Kosugi, Nakahara-ku
Kawasaki, 211,
JAPAN

Department of Mathematics
University of Michigan
Ann Arbor, Michigan 48109
U.S.A.

ПОКРЫТИЕ И УПАКОВКА В ГРАФАХ III: ЦИКЛИЧЕСКИЕ И АЦИКЛИЧЕСКИЕ ИНВАРИАНТЫ

Джин Акияма, Джефри Эксу, Фрэнк Харари

Резюме

Существует несколько инвариантов графов, касающихся простых цепей и циклов, связанных с упаковками и покрытиями графов, напр. древесность, линейная древесность, вершинная древесность, вершинная линейная древесность, антидревесность, цепное число, антицепное число, вершинная антидревесность и цикличность. Эти понятия по большей части являются фундаментальными, однако найти значения этих инвариянтов вообще не легко, В статье это сделано для специальных классов графов, именно для полных графов, полных двудольных графов и их реберных графов.