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ON GRAPHS CONTAINING MANY SUBGRAPHS WITH THE SAME NUMBER OF EDGES

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J. Bosák (oral communication) proposed to study the following question: Given a graph G with n vertices such that each two induced subgraphs of G with k vertices have the same number of edges, does it necessarily imply that G or $\bar{G}$ (the complement of G) is complete?

The theorem below shows that the answer to this question is positive for $n \geqq 4$ and each k satisfying $2 \leqq k \leqq n-2$.

All graphs discussed in this note are finite, undirected, without loops and multiple edges. When speaking about a subgraph of a graph G we always mean the subgraph induced by a subset of the set of all vertices of G. All other terms are used in the usual sense (cf. [1]).

Theorem. *Let G be a graph with n vertices, $n \geqq 4$. If there exists an integer k, $2 \leqq k \leqq \ -2$ such that each two subgraphs of G with k vertices have the same number of edges, then G or $\bar{G}$ is complete.*

Proof. Suppose that each subgraph of G with k vertices has q edges. Choose an arbitrary subgraph H of G with $k+1$ vertices and h edges. Obviously H is regular, since all its subgraphs with k vertices have the same number of edges. The sum of the numbers of edges of all $k+1$ such subgraphs of H is equal to $(k+1)q$. On the other hand, each edge of H is contained in exactly $k-1$ point-deleted subgraphs of H, and we immediately obtain the equality $(k+1)q=(k-1)h$. Thus, we have proved that each two subgraphs of G with $k+1$ vertices have the same number of edges. Now we may assume (by induction) that each two subgraphs of G with $n-1$ vertices are regular and have the same number of edges. Then G is regular, too. Let d denote the degree of each vertex of G. If $1 \leqq d \leqq n-2$, there would exist three vertices u, v, w such that uv is an edge of G and uw is an edge of $\bar{G}$. But in this case the degree of the vertex v in $G-u$ would be $d-1$, whereas the degree of w in $G-u$ is d. This is a contradiction because $G-u$ is regular. Thus $d=0$ or $d=n-1$ and the proof is finished.

Note that putting in our theorem $k=n-1$ we can only claim that G is an arbitrary regular graph. The cases $k=1$ or $k=n$ are trivial.

REFERENCES

[1] HARARY, F.: Graph Theory, Addison-Wesley, Reading, Mass., 1069.

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ГРАФЫ СОДЕРЖАЩИЕ МНОГО ПОДГРАФОВ С ОДИНАКОВЫМ ЧИСЛОМ РЕБЕР

Йосеф Ширань

Резюме

В статье доказана следующая теорема: Пусть G-граф с n вершинами, $n \geqq 4$. Если существует натуральное k, $2 \leqq k \leqq n-2$ такое, что любые два k-вершинных подграфа графа G имеют одинаковое число ребер, то G или $\bar{G}$ является полным графом.