Mathematica Slovaca

Ľudovít Niepel

On decompositions of complete graphs into factors with given diameters and radii

Mathematica Slovaca, Vol. 30 (1980), No. 1, 3--11

Persistent URL: http://dml.cz/dmlcz/136225

Math. Slovaca 30, 1980, No. 1, 3—11

ON DECOMPOSITIONS OF COMPLETE GRAPHS INTO FACTORS WITH GIVEN DIAMETERS AND RADII

ĽUDOVÍT NIEPEL

1. Introduction

In the present paper we study the existence of a decomposition of the complete graph into factors with given diameters and radii. The cases where the diameters and the radii of the factors are investigated separately are studied in [1] and [2], respectively. The first part of the present paper deals with the general case. There is shown the existence of a decomposition of the complete graph and its hereditary property. In the second part the case of two factors is completely solved.

2. General case.

All graphs considered in this paper are finite and undirected, without loops or multiple edges. The distance $\varrho(u, v)$ between vertices u and v in a graph is defined as the length of a shortest path joining these vertices. If such a path does not exist, we put $\varrho(u, v)=\infty$. By the eccentricity of a vertex v in the graph G we understand the maximum $e_G(v)$ of distances from v to each vertex in G. The maximal eccentricity in G is called the diameter of G and denoted by d_G. The minimal eccentricity in G is called the radius of G and denoted by r_G. In a disconnected graph we put $r_G=d_G=\infty$. For the diameter and the radius of a connected graph G the following inequalities hold:

$$2r \leqq n, \tag{1}$$

$$d+1 \leqq n, \tag{2}$$

$$r \leqq d \leqq 2r, \tag{3}$$

where n is the number of vertices of G, r and d are the radius and the diameter of G, respectively.

The first inequality is proved in [2]. A path of the length d contains $d+1$ distinct vertices and this fact implies the second inequality. The third inequality follows from the triangle inequality for distances in a connected graph.

A relation between the diameter, the radius and the number of vertices in a connected graph is given by the following lemma:

Lemma 1. *Let r and d denote the radius and the diameter of a connected graph G, respectively. Then for the number n of vertices of G the following inequality holds:*

$$n \geqq d+1+s(r-1), \quad \text{where} \quad s=\begin{cases} 1 & \text{if } 2r \geqq d+2, \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

Proof: A. Let $2r \leqq d+1$. Then $s=0$. The graph G contains at least one path of the length d and hence at least $d+1$ distinct vertices.

B. Let $2r \geqq d+2$. Then $s=1$. In the graph G we consider a path of the length d as an induced subgraph G' of the graph G. As the inequality $d \leqq 2(r-1)$ holds, there exists in the graph G' at least one vertex u with eccentricity $e_{G'}(u)=r-1$ and a vertex u' with the eccentricity $e_{G'}(u')=r$ adjacent to u. We denote by u'' the second vertex adjacent to u in the graph G'. Moreover, we denote the end-vertices of G' by w and w' in such a way that $\varrho_{G'}(w, u')=r$.

In the graph G there holds the inequality $e_G(u) \geqq r$ so that exists a vertex v such that $\varrho_G(u, v) \geqq r$. We show that there exists a $u-v$ path containing the edge (u, u''). In the graph G there exists a $v-w$ path of the length $\leqq d$. Denote by u_1 the first vertex of this path contained in the graph G'. There hold the inequalities:

$$\varrho_G(v, u_1)+\varrho_G(u_1, u)+\varrho_G(u, w) \geqq r+r-1>d.$$

That means the vertex u_1 belongs to the $u-w$ path contained in the graph G' and $u_1 \neq u$. Now it is easy to construct a $u-v$ path using the vertex u_1 and containing the edge (u, u'').

Similarly we show that there exists a $u-v$ path in G containing the edge (u, u'). Consider an arbitrary $v-w'$ path of the length $\leqq d$. Denote by u_2 the first vertex of this path contained in the graph G'. There hold the following inequalities:

$$\varrho_G(v, u_2)+\varrho_G(u_2, u)+\varrho_G(u, w') \geqq \varrho_G(v, u)+$$
$$+\varrho_G(u, w')>\varrho_G(w, u)+\varrho_G(u, w')=d.$$

That means the vertex u_2 belongs to the $u-w'$ path contained in the graph G' and $u_2 \neq u$. It is easy to construct a $u-v$ path using the vertex u_2 and containing the edge (u, u').

Let p_1 denote a $u-u_1-v$ path such that the $u-u_1$ path belongs to G' and the u_1-v path is disjoint with G' except for the vertex u_1. Let p_2 denote a $u-u_2-v$ path such that the $u-u_2$ path belongs to G' and the u_2-v path is disjoint with G' except for the vertex u_2. Moreover, let $(p_1 \cap p_2)-\{u\}$ be a path or a vertex.

Let $i = \varrho(u, u_1)$, $j = \varrho(u, u_2)$. Denote by u_3 the first common vertex of p_1 and p_2 different from u. Let $q = \varrho_{p_1}(u_1, u_3)$, $t = \varrho_{p_2}(u_2, u_3)$ and $p = \varrho_{p_1}(u_3, v)$. Then the following inequalities hold:

$$i + q + p \geqq r \tag{5}$$

$$j + t + p \geqq r \tag{6}$$

$$q + t \geqq i + j. \tag{7}$$

The first and the second inequalities follow from the condition that the length of p_1 and p_2 is at least r. A subgraph of G' joining the vertices u_1 and u_2 is a shortest path joining the vertices u_1 and u_2 in the graph G. Using the vertices of p_1 and p_2 we construct a $u_1 - u_3 - u_2$ path. As the length of this path is $q + t$, the third inequality follows. By a short calculation one can obtain the following inequality:

$$p + q + t \geqq r. \tag{8}$$

Thus the path p_1 and p_2 contains at least $r - 1$ vertices not contained in G' and the proof follows.

A subgraph of G containing all vertices of G is called a factor of G. The complete graph with n vertices will be denoted by $\langle n \rangle$. Under a decomposition of a complete graph into factors we mean a system of its factors such that every edge of this graph is contained in exactly one factor of that system.

Now we try to find conditions for the existence of a number n such that the complete graph $\langle n \rangle$ can be decomposed into m factors $F_1, \ldots, F_m$ with diameters $d_1, \ldots, d_m$ and radii $r_1, \ldots, r_m$, respectively.

Denote by $H(d_1, \ldots, d_m, r_1, \ldots, r_m)$ the smallest number n such that the graph $\langle n \rangle$ is decomposable into m factors with diameters $d_1, \ldots, d_m$ and radii $r_1, \ldots, r_m$. If such a number does not exist, we put $H(d_1, \ldots, d_m, r_1, \ldots, r_m) = \infty$.

Theorem 1. *Let $m \geqq 2$, $N \geqq n \geqq 2$, $d_i \geqq r_i \geqq 1$ for $i = 1, \ldots, m$ be natural numbers. If the complete graph with n vertices is decomposable into m factors with given diameters and radii, then for all $N \geqq n$ the complete graph $\langle N \rangle$ is decomposable into m factors with the same diameters d_i and r_i.*

Proof. We can suppose that $2 \leqq r_i \leqq d_i$ for every $i = 1, \ldots, m$. The case $2 \geqq d_i \geqq r_i = 1$ for some index i is trivial. Set $G = \langle N \rangle$, let K be a clique of the graph G such that K has n vertices. Denote by A the set of its vertices and by B the set of all remaining vertices of G. Let us choose an arbitrary vertex u of A. If $F_1, \ldots, F_m$ is a decomposition of the graph K into factors with diameters d_i and radii r_i, we can construct factors F'_i of G in the following way.

1. F'_i contains all the edges contained in F_i.

2. If $v \neq u$, $v \in A$, $w \in B$, then the factor F'_i contains the edge (v, w) if and only if the edge (v, u) is contained in F_i. All the remaining edges of G are contained in F'_m.

Now we are going to relate the eccentricities of the vertices of $F'_1, \ldots, F'_m$ to those of $F_1, \ldots, F_m$. The eccentricity of any vertex of B in F'_i is the same as the eccentricity of the vertex u in F_i. It remains to consider vertices of A. Let $v \in A$ and $v \neq u$, then the distances $\varrho_{F'_i}(v, w)$ and $\varrho_{F_i}(u, v)$ are equal for any $w \in B$. Now let $w \in A$. It is evident that the inequality $\varrho_{F'_i}(v, w) \leqq \varrho_{F_i}(v, w)$ holds. Therefore it is sufficient to prove that $\varrho_{F'_i}(v, w) \geqq \varrho_{F_i}(v, w)$. Assume that $\varrho_{F'_i}(v, w) < \varrho_{F_i}(v, w)$. Then in F'_i there exists a $v - w$ path of the length less than $\varrho_{F_i}(v, w)$. This path obviously contains vertices from the set B and has the following form: $v_0 v_1 \ldots v_p$, where $p = \varrho_{F'_i}(v, w)$ and $v_0 = v$, $v_p = w$. Let k be the smallest integer such that $v_{k+1} \in B$. Similarly let s be the greatest integer with $v_{s-1} \in B$. Then the $v - v_k u v_s - w$ path is of a length less than $\varrho_{F_i}(v, w)$, which is a contradiction. The case $r_i = \infty$ is obvious. Therefore our construction does not change eccentricities of vertices in factors F_i. This completes the proof of the theorem.

From Theorem 1 it follows that the graph $\langle n \rangle$ is decomposable into factors with diameters $d_1, \ldots, d_m$ and radii $r_1, \ldots, r_m$ if and only if $n \geqq H(d_1, \ldots, d_m, r_1, \ldots, r_m)$.

Lemma 2. *Let m, d_i, r_i be natural numbers such that $r_i \leqq d_i \leqq 2r_i$ for every $i = 1, \ldots, m$. Then $H(d_1, \ldots, d_m, r_1, \ldots, r_m) \geqq \max\,(2m, \max\,(d_i + 1 + s_i(r_i - 1)))$, where $s_i = 1$ if $2r_i \geqq d_i + 2$ and $s_i = 0$, otherwise $(i = 1, \ldots, m)$.*

Proof. The proof follows immediately from Lemma 1, Theorem 2 of [1] and from Theorem 1.

For $H(d_1, \ldots, d_m, r_1, \ldots, r_m)$, where $m \geqq 3$, $d_i \geqq 3$, we can find also an upper bound. Results concerning the case of diameters $d_i = 2$ are the same as in [1]. In that case we can suppose $r_i = 2$, as the case $r_i = 1$ is trivial.

Theorem 2. *Let m, $d_1, \ldots, d_m$, $r_1, \ldots, r_m$ be integers, where $m \geqq 3$, $d_i \geqq 3$ and $r_i \leqq d_i \leqq 2r_i$ for every $i = 1, \ldots, m$. Then $H(d_1, \ldots, d_m, r_1, \ldots, r_m)$ is finite and we have:*

$$H(d_1, \ldots, d_m, r_1, \ldots, r_m) \leqq \sum_{i=1}^{m} (d_i + 1 + s_i(r_i - 1)),$$

where the symbol s_i has the same meaning as in Lemma 2.

Proof. The proof of this theorem is based on a construction of a decomposition of $\langle n \rangle$ into factors with diameters $d_1, \ldots, d_m$ and radii $r_1, \ldots, r_m$. Denote the vertices of $\langle n \rangle$ by the symbols $v_{i,j}$ where $1 \leqq i \leqq m$ and $1 \leqq j \leqq d_i + 1 + s_i(r_i - 1)$. Let us form the factors $F_1, \ldots, F_m$ of $\langle n \rangle$ in the following way. In the factor F_i we have:

1) The vertex $v_{i,1}$ is adjacent to all vertices of $\langle n \rangle$ except for the vertices $v_{x,1}$, $v_{x,3}$, where $x > i$, except for the vertices $v_{x,2}$, $v_{x,4}$, where $x < i$ and also except for all the vertices $v_{i,x}$, where $x \neq 2$ and if $s_i = 1$, then also $x \neq 2r_i$.
2) The vertex $v_{i,2}$ is adjacent to all the vertices of the graph $\langle n \rangle$, except for all the vertices $v_{x,1}$, $v_{x,3}$, where $x < i$, except for the vertices $v_{x,2}$, $v_{x,4}$, where $x > i$, and except for the vertices $v_{i,x}$, where $x \neq 1$, $x \neq 3$.

3) The vertex $v_{i,3}$ is adjacent to all the vertices $v_{x,1}$, $v_{x,3}$, where $x<i$, the vertices $v_{x,2}$, $v_{x,4}$, where $x>i$ and the vertices $v_{i,2}$, $v_{i,4}$.
4) The vertex $v_{i,4}$ is adjacent to all the vertices $v_{x,1}$, $v_{x,3}$, where $x>i$, the vertices $v_{x,2}$, $v_{x,4}$, where $x<i$ and the vertices $v_{i,3}$, $v_{i,5}$ (if any).
5) The factor F_i contains all the edges $(v_{i,x}, v_{i,x+1})$ and in the case $s_i=1$ also the edge $(v_{i,1}, v_{i,2r_i})$.
6) The factor F_i contains all the edges $(v_{i+1,x}\ v_{i+1,y})$, where $x-y\geqq 2$, except for the edge $(v_{i+1,1}, v_{i+1,2r_{i+1}})$ in the case $s_{i+1}=1$.
7) If $i\leqq m-2$, then the vertices $v_{i+1,3}$, $v_{i+1,4}$ are adjacent to all the vertices $v_{y,x}$ where $x>4$, $y>i+1$.
8) If $i\leqq m-2$, then the vertices $v_{i+1,x} (x>4)$ are adjacent to all the vertices $v_{y,z}$ where $y>i+1$, $z\geqq 3$.
9) If $i=m-1$, then the vertices $v_{m,x} (x>4)$ are adjacent to the vertices $v_{1,z} (z\geqq 3)$. The vertices $v_{m,3}$, $v_{m,4}$ are adjacent to the vertices $v_{2,x}$ where $x>4$.
10) All the edges not included in 1) to 9) are contained in the factor F_m. We have constructed a decomposition of the complete graph $\langle n\rangle$ into the factors $F_1, \ldots, F_m$. The radius of F_i is r_i because the vertex $v_{i,p}$, where $p=r_i+1+s_i(r_i-1)$, has eccentricity r_i, which is the minimal eccentricity in the factor F_i. The maximal eccentricity of F_i is d_i. The vertex $v_{i,q}$, $q=d_i+1+s_i(r_i-1)$ has eccentricity d_i, which is the maximal eccentricity in F_i this completes the proof.

This theorem gives us an answer to the question of the existence of a decomposition of $\langle n\rangle$ into m connected factors $(m\geqq 3)$. In the following theorems we formulate conditions for the existence of a decomposition of a complete graph into factors which may be disconnected.

Theorem 3. *Let* $m\geqq 3$, $d_2=r_2=\ldots=d_m=r_m=\infty$. *Then we have:*

$$H(d_1, \ldots, d_m, r_1, \ldots, r_m)=\begin{cases} 3 \text{ if } d_1=r_1=\infty, \\ d_1+1+s_1(r_1-1), \text{ if } 1\leqq r_1\leqq d_1\leqq 2r_1<\infty, \\ \infty, \text{ otherwise} \end{cases}$$

Proof. The proof follows from Lemma 1, Theorem 1 and the following construction.

We construct a decomposition of $\langle k\rangle$, $k=d_1+1+s_1(r_1-1)$ into factors with prescribed diameters and radii. The factor F_1 contains the edges (v_i, v_{i+1}) $i=1, \ldots, k-1$, in the case $s_1=1$ also the edge (v_1, v_{2r_1}). The factor F_2 contains all the edges (v_i, v_j) $i\neq 1, j\neq 1$ not contained in F_1. All the remaining edges of $\langle k\rangle$ can be divided in an arbitrary way into factors $F_3, \ldots, F_m$. As the decomposition of $\langle 3\rangle$ into disconnected factors is trivial, the proof follows.

Theorem 4. *Let* $m\geqq 3$, $d_1\leqq\ldots\leqq d_m$, $r_1, \ldots, r_m$ *be natural numbers such that* $r_i\leqq d_i\leqq 2r_i$, $d_i\geqq 3$ *or* d_i *(and also* r_i*) be symbols* ∞, $i=1, \ldots, m$. *Then* $H(d_1, \ldots, d_m, r_1, \ldots, r_m)$ *is finite.*

Proof. a) If $d_3 \neq \infty$, then we can use the construction from the proof of Theorem 2. The following inequality holds:

$$H(d_1, \ldots, d_m, r_1, \ldots, r_m) \leqq H(d_1, \ldots, d_k, r_1, \ldots, r_k),$$

where d_k is the last finite diameter in the sequence of diameters.

b) Let $d_2 \neq \infty$, $d_3 = \infty$. Now we shall prove the inequality:

$$H(d_1, \ldots, d_m, r_1, \ldots, r_m) \leqq$$
$$\leqq d_1 + d_2 + 3 + s_1(r_1 - 1) + s_2(r_2 - 1) = k.$$

It is sufficient to construct a decomposition of the graph $\langle k \rangle$ into the factors F_i with diameters d_i and radii r_i for $i = 1, \ldots, m$. Choose an arbitrary vertex v of the graph $\langle k \rangle$. Divide all the remaining vertices into two groups: In the first group there are $d_1 + 1 + s_1(r_1 - 1)$ vertices and in the second group $d_2 + 1 + s_2(r_2 - 1)$ vertices of $\langle k \rangle$. Denote the vertices of the i-th group by $v_{i,j}$ where $1 \leqq j \leqq d_i + 1 + s_i (r_i - 1)$, $i = 1, 2$.

The factor F_1 contains edges $(v_{1,x}, v_{1,x+1})$ where $1 \leqq x \leqq d_1 + s_1(r_1 - 1)$, in the case $s_1 = 1$ it contains also the edge $(v_{1,1}, v_{1,2r_1})$. The factor F_1 contains also the edges $(v_{1,3}, v)$, $(v_{1,4}, v)$, $(v_{1,3}, v_{2,1})$, $(v_{1,3}, v_{2,2})$, $(v_{1,4}, v_{2,1})$, $(v_{1,4}, v_{2,2})$ and all the edges $(v_{1,1}, v_{2,x})$, $(v_{1,2}, v_{2,x})$, where $x \geqq 3$.

The factor F_2 contains the edges $(v_{2,x}, v_{2,x+1})$, where $1 \leqq x \leqq d_2 + s_2(r_2 - 1)$ and if $s_2 = 1$ also the edge $(v_{2,1}, v_{2,2r_2})$. The factor F_2 contains also the edges $(v_{2,1}, v_{1,x})$, $(v_{2,2}, v_{1,x})$, $(v, v_{1,x})$, where $x \neq 3$, $x \neq 4$, the edges $(v_{2,3}, v_{1,3})$, $(v_{2,3}, v_{1,4})$, $(v_{2,4}, v_{1,3})$, $(v_{2,4}, v_{1,4})$, and the edges $(v, v_{2,1})$, $(v, v_{2,2})$.

All the remaining edges of the graph $\langle k \rangle$ can be divided in an arbitrary way into the factors $F_3, \ldots, F_m$.

The radii of F_1 and F_2 are r_1 and r_2, respectively, because the eccentricities of the vertices $v_{2,r_1+1+s_1(r_1-1)}$ and $v_{2,r_2+1+s_2(r_2-1)}$ are r_1 and r_2, respectively. Analogously F_1 and F_2 have diameters d_1 and d_2, respectively. The factors $F_3, \ldots, F_m$ are disconnected.

c) In the case $d_2 = \infty$ the assertion follows from Theorem 3. This completes the proof.

3. The case $m = 2$

In this part we shall consider 4-tuples d_1, d_2, r_1, r_2 of numbers or symbols ∞, such that $d_1 \leqq d_2$ and moreover if $d_1 = d_2$, then $r_1 \leqq r_2$.

Lemma 3. *Let $1 \leqq r_i \leqq d_i \leqq 2r_i$, $i = 1, 2$, $d_2 \geqq 4$, $d_2 \neq \infty$. Then we have:*

$$H(d_1, d_2, r_1, r_2) = \begin{cases} d_2 + 1 + s_2(r_2 - 1) & \text{if } d_1 = r_1 = 2, \\ \infty, & \text{otherwise.} \end{cases}$$

Proof. In the case $d_1=r_1=2$, $s_2=0$ the factor F_2 consists of d_2+1 various vertices v_i and all the edges (v_i, v_{i+1}) for $i=1, \ldots, d_2$.

In the case $d_1=r_1=2, s_2=1$ the factor F_2 consists of d_2+r_2 various vertices v_i, all the edges (v_i, v_{i+1}) for $i=1, \ldots, d_2+r_2-1$ and also the edge (v_1, v_{2r_2}).

In both these cases the factor F_1 contains all the remaining edges.

If $d_2 \geqq 4$, then by Lemma 3 of [1] the complement of a graph with a diameter greater or equal to 4 has diameter 2. It follows that if $d_1 \neq 2$, then $H(d_1, d_2, r_1, r_2)=\infty$.

Lemma 4. *If G is a disconnected graph, then its complement $\bar{G}$ is connected and for its radius $r_{\bar{G}}$ and diameter $d_{\bar{G}}$ we have: $1 \leqq r_{\bar{G}} \leqq d_{\bar{G}} \leqq 2$.*

Proof. The proof follows from Lemma 2 of [1] and Lemma 2 of [2].

Lemma 5. *If r_G is the radius of a graph G, $r_G \geqq 3$, then for the diameter $d_{\bar{G}}$ of the complement $\bar{G}$ of G we have $d_{\bar{G}} \leqq 2$.*

Proof. We can assume that G is connected (Lemma 4).

a) If $d_G \geqq 4$, then from Lemma 3 of [1] we obtain $d_{\bar{G}} \leqq 2$.

b) Let $d_G=r_G=3$. We shall show that the distance between two arbitrary vertices in $\bar{G}$ is not greater than 2. Suppose that for some couple of vertices u, v we have $\varrho_{\bar{G}}(u, v) \geqq 3$. In the graph G the eccentricity of every vertex is equal to 3. It means that there exists a vertex w such that $\varrho_G(v, w)=3$. Obviously the graph G contains the edge (u, v) and the graph $\bar{G}$ the edge (v, w). The edge (u, w) is contained in one of the graphs G or $\bar{G}$. If this edge belongs to G, then $\varrho_G(v, w)=2$. If the edge (u, w) belongs to $\bar{G}$, then $\varrho_{\bar{G}}(u, v)=2$. Both cases give us a contradiction. Hence $d_{\bar{G}} \leqq 2$.

Lemma 6. *Let d_1, d_2, r_1, r_2 be natural numbers satisfying the inequalities $r_i \leqq d_i \leqq 2r_i$, $i=1, 2$. Then:*

$$H(d_1, d_2, r_1, r_2) > 4,$$

except for the case $H(3, 3, 2, 2)=4$.

Proof. The proof follows by checking all the decompositions of $\langle 4 \rangle$ into two connected factors.

Theorem 5. *For the natural numbers d_1, d_2, r_1, r_2 or the symbols ∞ satisfying the inequalities $r_i \leqq d_i \leqq 2r_i$, $i=1, 2$, we have:*

(a) $H(1, d_2, 1, r_2)=\infty$ for $d_2 \neq \infty$,
(b) $H(1, \infty, 1, \infty)=2$,
(c) $H(2, \infty, 1, \infty)=3$,
(d) $H(2, \infty, 2, \infty)=4$,
(e) $H(d_1, \infty, r_1, \infty)=\infty$ for $d_1 \geqq 3$,
(f) $H(2, 2, 2, 2)=5$,
(g) $H(2, 3, 2, 2)=6$,

(h) $H(2, 3, 2, 3)=6$,
(i) $H(3, 3, 2, 2)=4$,
(j) $H(3, 3, 2, 3)=\infty$,
(k) $H(3, 3, 3, 3)=\infty$,
(l) $H(2, d_2, 1, r_2)=\infty$ for $d_2\neq\infty$,

$$(m)\quad H(d_1, d_2, r_1, r_2)=\begin{cases} d_2+1+s_2(r_2-1), & \text{if } d_1=r_1=2,\\ & 4\leqq d_2\neq\infty,\\ \infty, & \text{if } d_1\geqq 3 \text{ and } d_2\geqq 4.\end{cases}$$

Proof. The proof follows from Theorem 1, Lemmas 1, 2, 3, 4, 5, 6 and from the decompositions drawn in Figs. 1, 2, 3.

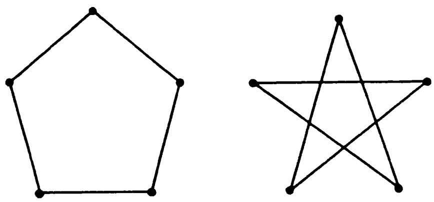
Fig. 1. The case *(f)*.

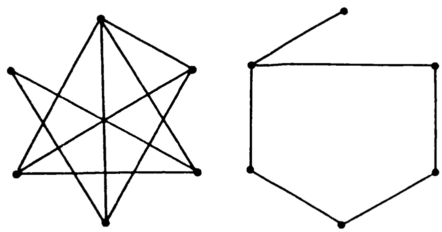
Fig. 2. The case *(g)*.

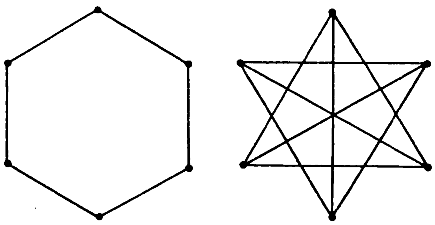
Fig. 3. The case *(h)*.

In the case *(g)* the graph $\langle 5\rangle$ is not decomposable into factors with those diameters and radii, because a graph with five vertices and diameter 3 contains at least one vertex of degree 3 and so this vertex is an endpoint in the graph $\bar{G}$. If $\bar{G}$ has diameter 2, then at least one degree of its vertices is 4, which is a contradiction with the connectedness of G. This completes the proof.

REFERENCES

[1] BOSÁK, J.—ROSA, A.—ZNÁM, Š.: On decompositions of complete graphs into factors with given diameters, In: Theory of graphs, Proc. Colloq. Tihany 1966, Akadémiai Kiadó, Budapest 1968, 37—56.

[2] PALUMBÍNY, D.—ZNÁM, Š.: On decompositions of complete graphs into factors with given radii. Mat. Čas., 23, 1973, 306—316.

Received May 13, 1976

Katedra geometrie
Prírodovedeckej fakulty UK
Mlynská dolina
816 31 Bratislava

О РАЗЛОЖЕНИИ ПОЛНОГО ГРАФА НА ФАКТОРЫ С ДАННЫМИ ДИАМЕТРАМИ И РАДИУСАМИ

Людовит Нипел

Резюме

В предлагаемой работе исследуется существование разложения полного графа на факторы с заранее заданными диаметрами и радиусами. Находятся условия для существования разложения и ограничения для числа вершин полного графа допустимого разложения.

Во второй части показано наследственное свойство разложения полного графа и дана верхняя грань для функции $H(d_1, \ldots, d_m, r_1, \ldots, r_m)$ минимального числа вершин полного графа допускающего соответствующее разложение.

В третьей части полностью решен случай разложения на два фактора. Значит, точно определены значения функции $H(d_1, d_2, r_1, r_2)$.