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THE ABSOLUTE CONTINUITY OF FUNCTIONS DEFINED ON THE σ-RING GENERATED BY A RING

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In the measure theory the following theorem is known: "Let μ, v be two measures on the σ-ring $\mathcal{S}$ generated by a ring $\mathcal{R}$ and let μ be finite and v be σ-finite. Let μ_1 and v_1 be measures on $\mathcal{R}$ such that $\mu_1 = \mu/\mathcal{R}$, $v_1 = v/\mathcal{R}$. Then $\mu \ll v$ if and only if $\mu_1(\ll)v_1$" [1].

The aim of this paper is to generalize the mentioned theorem for vector measures, signed measures and subadditive measures. A common formulation of all the cases considered will be given in terms of small systems [2], [3]. We shall show further this theorem to be valid even if the ring $\mathcal{R}$ is replaced by the semi-ring.

Recall that a vector measure is a σ-additive set function defined on a ring with values in a normed vector space [5]. A subadditive measure is a non-negative non-decreasing subadditive and continuous set function.

If φ is a set function (real or vector) on $\mathcal{S}$, then $|\varphi|$ denotes the variation of φ in the sense of [5], i. e.

$$|\varphi|(A) = \sup\left\{\sum_{i\in I} |\varphi(A_i)|, \quad A_i \subset A, A_i \cap A_j = \emptyset, \qquad i \neq j \text{ and } A_i \in \mathcal{S}\right\}.$$

It can easily be proved that the variation of a signed measure defined on $\mathcal{S}$ coincides with the total variation of the Jordan decomposition of the signed measure. Therefore it is a positive measure. Also the variation of a vector measure and variation of a non-negative σ-subadditive function (specially the variation of a subadditive measure) is a positive measure.

Throught this paper, the symbol P is used for the set of non-negative integers and $\emptyset$ for the empty set.

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Let X be an abstract set and $\mathcal{S}$ a σ-ring of subsets of X and $\{\mathcal{N}_n\}_{n=0}^{\infty}$ a sequence of subclasses of $\mathcal{S}$ such that

(1) For each $n \in P$, $\emptyset \in \mathcal{N}_n$.

(2) For each $n \in P$, there exists an increasing sequence $\{k_i\}_{i=1}^{\infty}$ of positive integers such that $E_i \in \mathcal{N}_{k_i}$ $(i=1, 2, \ldots)$ implies $\bigcup_{i=1}^{\infty} E_i \in \mathcal{N}_n$.

(3) Let $\{E_i\}_{i=1}^{\infty}$ be an arbitrary non-increasing sequence of sets of $\mathcal{N}_0$ and $\bigcap_{i=1}^{\infty} E_i = \emptyset$, then for each $n \in P$ there is $m \in P$ such that $E_m \in \mathcal{N}_n$.

(4) For each $n \in P$, if $E \subset F$, $F \in \mathcal{N}_n$, then $E \in \mathcal{N}_n$.

(5) $\mathcal{N}_{n+1} \subset \mathcal{N}_n$ for all $n \in P$.

(6) For each $n \in P$ we have: If $E \in \mathcal{N}_n$ and $F \in \bigcap_{n=0}^{\infty} \mathcal{N}_n$, then $E \cup F \in \mathcal{N}_n$.

A sequence $\{\mathcal{N}_n\}_{n=0}^{\infty}$ satisfying all the properties will be called *a small system on $\mathcal{S}$*.

Example. Let $(X, \mathcal{S})$ be a measurable space and let μ be a positive or subadditive measure defined on $\mathcal{S}$. Then the sequence $\{\mathcal{N}_n\}_{n=0}^{\infty}$ defined by

$$\mathcal{N}_0 = \{E \in \mathcal{S}: \mu(E) < \infty\},$$
$$\mathcal{N}_n = \{E \in \mathcal{S}: \mu(E) < 1/n\} \qquad (n=1, 2, \ldots)$$

satisfies the properties (1)—(6).

Since the variation $|v|$ of a vector measure v or a signed measure v defined on $\mathcal{S}$ is a positive measure, the sequence $\{\mathcal{N}_n\}_{n=0}^{\infty}$ generated by $|v|$, i. e.

$$\mathcal{N}_0 = \{E \in \mathcal{S}: |v|(E) < \infty\}, \quad \mathcal{N}_n = \{E \in \mathcal{S}: |v|(E) < 1/n\},$$

satisfies the properties (1)—(6), too.

Now we formulate two lemmas whose proofs are actually contained in the proof of Theorem 8 in [2]. In the following, let $\mathcal{N}$ stand for $\bigcap_{n=0}^{\infty} \mathcal{N}_n$.

Lemma 1. *Let $\{\mathcal{N}_n\}_{n=0}^{\infty}$ satisfy the properties* (3), (5), (6), *then $\bigcap_{i=1}^{\infty} E_i \notin \mathcal{N}$ for any non-increasing sequence of sets in $\mathcal{N}_0$ such that $E_i \notin \mathcal{N}_m$ $(i=1, 2, \ldots)$.*

Lemma 2. *Let $\{\mathcal{N}_n\}_{n=0}^{\infty}$ satisfy the properties* (1), (2), (5), (6). *There is a sequence $\{k_i\}_{i=1}^{\infty}$ of positive integers such that for any $n \in P$ there is $r(n) \in P$ such that*

$$\bigcup_{i \geq r(n)} E_i \in \mathcal{N}_n \text{ whenever } E_i \in \mathcal{N}_{k_i} \; (i=1, 2, \ldots).$$

Definition. *Let $\{\mathcal{N}_n\}_{n=0}^{\infty}$ and $\{\mathcal{M}_n\}_{n=0}^{\infty}$ be two small systems. The system $\{\mathcal{N}_n\}_{n=0}^{\infty}$ is said to be absolutely continuous with respect to $\{\mathcal{M}_n\}_{n=0}^{\infty}$* (notation $\{\mathcal{N}_n\} \ll \{\mathcal{M}_n\}$) *iff $\mathcal{M} \subset \mathcal{N}$.*

The system $\{\mathcal{N}_n\}_{n=0}^{\infty}$ is said to be strongly absolutely continuous with respect to $\{\mathcal{M}_n\}_{n=0}^{\infty}$ (notation $\{\mathcal{N}_n\}$ $(\ll)$ $\{\mathcal{M}_n\}$) *iff, for each $m \in P$, there is $n \in P$ such that $\mathcal{M}_n \subset \mathcal{N}_m$.*

Lemma 3. *Let $(X, \mathcal{S})$ be a measurable space and $\mathcal{A} \neq \emptyset$, $\mathcal{A} \subset \mathcal{S}$. Let μ and v be vector or signed measures on $\mathcal{S}$ and $\{\mathcal{N}_n\}$, $\{\mathcal{M}_n\}$ be small systems generated by the variations $|\mu|$, $|v|$, respectively. If μ, v are subadditive measures, let $\{\mathcal{N}_n\}$, $\{\mathcal{M}_n\}$ be small systems generated by μ, v, respectively. Put $\mathcal{N}'_n = \mathcal{N}_n \cap \mathcal{A}$, $\mathcal{M}'_n = \mathcal{M}_n \cap \mathcal{A}$ $(n = 0, 1, 2, \ldots)$ and $\mu_1 = \mu / \mathcal{A}$, $v_1 = v / \mathcal{A}$.*

Then a) $\mu \ll v$ *if and only if* $\{\mathcal{N}'_n\} \ll \{\mathcal{M}'_n\}$,
b) $\mu_1 (\ll) v_1$ *if and only if* $\{\mathcal{N}'_n\} (\ll) \{\mathcal{M}'_n\}$.

The proof is evident.

Theorem 1. *Let $(X, \mathcal{S})$ be a measurable space and $\{\mathcal{N}_n\}_{n=0}^{\infty}$ and $\{\mathcal{M}_n\}_{n=0}^{\infty}$ be two small systems on $\mathcal{S}$ and $\mathcal{N}_0 = \mathcal{S}$.*
Then $\{\mathcal{N}_n\} \ll \{\mathcal{M}_n\}$ *if and only if* $\{\mathcal{N}_n\} (\ll) \{\mathcal{M}_n\}$.

Proof. Let $\{\mathcal{N}_n\} (\ll) \{\mathcal{M}_n\}$, then $\bigcap_{m=0}^{\infty} \mathcal{M}_{n_m} \subset \bigcap_{n=0}^{\infty} \mathcal{N}_n = \mathcal{N}$. Since $\mathcal{M} \subset \bigcap_{m=0}^{\infty} \mathcal{M}_{n_m}$, we get $\mathcal{M} \subset \mathcal{N}$, i. e. $\{\mathcal{N}_n\} \ll \{\mathcal{M}_n\}$.

Let now $\mathcal{M} \subset \mathcal{N}$. Assume that there is $m \in P$ such that, for each $n \in P$, we have $\mathcal{M}_n \not\subset \mathcal{N}_m$. Therefore $\mathcal{M}_{k_i} \not\subset \mathcal{N}_m$ $(i = 1, 2, \ldots)$, where $\{k_i\}_{i=1}^{\infty}$ is the sequence of positive integers from Lemma 2. Hence we get a sequence $\{E_i\}_{i=1}^{\infty}$ of sets such that $E_i \in \mathcal{M}_{k_i}$ and $E_i \notin \mathcal{N}_m$ $(i = 1, 2, \ldots)$.

Put $E = \limsup E_i = \bigcap_{k=1}^{\infty} F_k$, where $F_k = \bigcup_{i=k}^{\infty} E_i$. Evidently $E \subset F_k$ $(k = 1, 2, \ldots)$.

Let $\{n_j\}_{j=1}^{\infty}$ be an increasing sequence of positive integers. In view of Lemma 2, we have the increasing sequence $\{r_j\}_{j=1}^{\infty}$ such that, for each $j = 1, 2, \ldots$, we get $\bigcup_{i \geq r_j} E_i \subset \mathcal{M}_{n_j}$, whenever $E_i \in \mathcal{M}_{k_i}$ $(i = 1, 2, \ldots)$.

Hence $F_{r_j} \in \mathcal{M}_{k_j}$ $(j = 1, 2, \ldots)$ and by the property (5) we get $E \in \mathcal{M}$.

Now we shall prove that $E \notin \mathcal{N}$, which contradicts the assumption $\mathcal{M} \subset \mathcal{N}$. Clearly $F_k \searrow E$, $F_k \in \mathcal{N}_0$ and $F_k \notin \mathcal{N}_m$ $(k = 1, 2, \ldots)$. In view of Lemma 1, we get $E \notin \mathcal{N}$. Therefore $\{\mathcal{N}_n\} (\ll) \{\mathcal{M}_n\}$. Theorem 1 is proved.

Corollary 1. *Let $(X, \mathcal{S})$ be a measurable space and let $\varphi, \psi : \mathcal{S} \to A$ be set functions, where A is either $\bar{R}$ or a normed vector space. Let the variations $|\varphi|$, $|\psi|$ be positive or subadditive measures and let $|\varphi|$ be finite. Then $\varphi \ll \psi$ if and only if $\varphi (\ll) \psi$.*

Proof. Let $\{\mathcal{N}_n\}_{n=0}^{\infty}$ and $\{\mathcal{M}_n\}_{n=0}^{\infty}$ be small systems generated by the measures $|\varphi|$, $|\psi|$. Evidently they satisfy the properties (1)—(6). Since $|\varphi|$ is finite, the assumptions of Theorem 1 are satisfied. Therefore $\{\mathcal{N}_n\} \ll \{\mathcal{M}_n\}$ if and only if $\{\mathcal{N}_n\} (\ll) \{\mathcal{M}_n\}$. Now we apply Lemma 3 and the proof is completed.

Corollary 2. *Let $(X, \mathcal{S})$ be a measurable space.*
a) *Let μ, v be signed measures on $\mathcal{S}$ and let $|\mu|$ be finite. Then $\mu \ll v$ if and only if*

$\mu\ (\ll)\ v$.

c) *Let* μ, v *be subadditive measures on* $\mathscr{S}$ *and* μ *be finite. Then* $\mu \ll v$ *if and only if* $\mu\ (\ll)\ v$.

c) *Let* μ, v *be two vector measures on* $\mathscr{S}$ *and* $|\mu|$ *be finite. Then* $\mu \ll v$ *if and only if* $\mu\ (\ll)\ v$.

Remark. The proof for signed measures μ, v is in [4]. In this paper we define the absolute continuity of subadditive measures μ, v as follows: $\mu \ll v$ iff $v(E)=0$ implies $\mu(E)=0$ $(E\in\mathscr{S})$.

If the absolute continuity of subadditive measures is defined by means of a variation, then Corollary b) remains true.

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Now we shall generalize the theorem mentioned in the introduction. Let $\mathscr{S} = \mathscr{S}(\mathscr{R})$ be a σ-ring generated by a ring $\mathscr{R}$ and let $\mathscr{R}^* = \{R : R = \limsup R_n\ ;\ R_n \in \mathscr{R}\}$ Let $\{\mathscr{N}_n\}_{n=0}^{\infty}$, $\{\mathscr{M}_n\}_{n=0}^{\infty}$ be two small systems on $\mathscr{S}$. Denote $\mathscr{N}'_n = \mathscr{N}_n \cap \mathscr{R}$, $\mathscr{M}'_n = \mathscr{M}_n \cap \mathscr{R}$ and $\mathscr{N}^*_n = \mathscr{N}_n \cap \mathscr{R}^*$, $\mathscr{M}^*_n = \mathscr{M}_n \cap \mathscr{R}^*$ $n = 0, 1, 2, \ldots$).

Theorem 2. *Let* $(X, \mathscr{S})$ *be a measurable space and* $\mathscr{S} = \mathscr{S}(\mathscr{R})$. *Let* $\{\mathscr{N}_n\}_{n=0}^{\infty}$ *and* $\{\mathscr{M}_n\}_{n=0}^{\infty}$ *be two small systems on* $\mathscr{S}$ *and* $\mathscr{N}_0 = \mathscr{S}$. *Let there be for each* $E \in \mathscr{R}$ *and for each* $i \in P$ *a set* $F_i \in \mathscr{R} \cap \mathscr{M}_0$ *such that* $E \subset \bigcup_{i=1}^{\infty} F_i$.

Then $\{\mathscr{N}_n\} \ll \{\mathscr{M}_n\}$ *if and only if* $\{\mathscr{N}'_n\}\ (\ll)\ \{\mathscr{M}'_n\}$.

Proof. By Theorem 1 $\{\mathscr{N}_n\} \ll \{\mathscr{M}_n\}$ implies $\{\mathscr{N}'_n\}\ (\ll)\ \{\mathscr{M}'_n\}$. We shall prove the inverse implication. First we can see that the sequence $\{\mathscr{L}_n\}_{n=0}^{\infty} = \{\mathscr{M}_n \cap \mathscr{N}_n\}_{n=0}^{\infty}$ satisfies the properties (1)—(4). Then the following assertion holds: For each $n \in P$ and for each $E \in \mathscr{L}_0$, there is $F \in \mathscr{R}$ such that $E \triangle F \in \mathscr{L}_n$ (see Theorem 3 in [3]).

Now we prove that $\{\mathscr{N}'_n\}\ (\ll)\ \{\mathscr{M}'_n\}$ implies $\mathscr{M} \subset \mathscr{N}$. Let $E \in \mathscr{M} \subset \mathscr{M}_0 \cap \mathscr{S} = \mathscr{L}_0$. Let n be an arbitrary positive integer. There exists a set $F \in \mathscr{R}$ such that $E \triangle F \in \mathscr{L}_n$. Since $(F—E) \subset F \triangle E \in \mathscr{M}_n$ and $E \in \mathscr{M}$, it follows by the property (6) that $(F—E) \cup E \in \mathscr{M}_n$. Since $F \subset (F—E) \cup E$ and $F \in \mathscr{R}$, we obtain $F \in \mathscr{M}'_n$.

Let m be an arbitrary positive integer. Choose $p, q \in P$ such that $A \cup B \in \mathscr{N}_m$, whenever $A \in \mathscr{N}_p$ and $B \in \mathscr{N}_q$ by the property (2). By the assumption there is n_q such that $\mathscr{M}'_{n_q} \subset \mathscr{N}'_q$. Put $n = \max\{n_q, p\}$. We have $\mathscr{M}_n \subset \mathscr{M}'_{n_q} \subset \mathscr{N}'_q \subset \mathscr{N}_q$, hence $F \in \mathscr{N}_q$. Since $(E—F) \subset E \triangle F \in \mathscr{N}_n \subset \mathscr{N}_p$, we get $E \subset (E—F) \cup F \in \mathscr{N}_m$. It is true for each $m \in P$. Therefore $E \in \mathscr{N}$. Theorem 2 is proved.

Corollary 1. *Let* $(X, \mathscr{S})$ *be a measurable space and* $\mathscr{S} = \mathscr{S}(\mathscr{R})$. *Let* φ, ψ *be two set function, real or vector. Let the variations* $|\varphi|$, $|\psi|$ *be positive or* *subadditive measures and $|\varphi|$ be finite and $|\psi|$ be σ-finite. Then $\varphi \ll \psi$ if and only if φ $(\ll)$ ψ on $\mathcal{R}$.*

Corollary 2. *Let $(X, \mathcal{S})$ be a measurable space and $\mathcal{S} = \mathcal{S}(\mathcal{R})$.*

a) *Let μ, v be signed measures on $\mathcal{S}$ and let μ be finite and v be σ-finite. Then $\mu \ll v$ if and only if μ $(\ll)$ v on $\mathcal{R}$.*
b) *Let μ, v be subadditive measures on $\mathcal{S}$ and μ be finite and v be σ-finite. Then $\mu \ll v$ if and only if μ $(\ll)$ v on $\mathcal{R}$.*
c) *Let μ, v be two vector measures on $\mathcal{S}$. Let $|\mu|$ be finite and $|v|$ be σ-finite. Then $\mu \ll v$ if and only if μ $(\ll)$ v on $\mathcal{R}$.*

Remark. For the case of signed measures this corollary is in [1].

Theorem 3. *Let the assumptions of Theorem 2 be satisfied. Then $\{\mathcal{N}_n\} \ll \{\mathcal{M}_n\}$ if and only if $\{\mathcal{N}_n^*\} \ll \{\mathcal{M}_n^*\}$.*

Proof. Evidently $\mathcal{M} \subset \mathcal{N}$ implies $\mathcal{M}^* \subset \mathcal{N}^*$. Now we shall prove the converse. Let m be an arbitrary positive integer. Construct the system $\bar{\mathcal{N}}_m = \{E \in \mathcal{S} : E \notin \mathcal{N}_m\}$.

Put $n_0 = \sup\{n :$ there is A such that $A \in \mathcal{M}_n \cap \bar{\mathcal{N}}_m\}$. If $n_0 < \infty$, then take $n_m = n_0 + 1$. Clearly $\mathcal{M}_{n_m} \subset \mathcal{N}_m$, hence $\{\mathcal{N}_n\}$ $(\ll)$ $\{\mathcal{M}_n\}$ and, by Theorem 2, $\{\mathcal{N}_n\} \ll \{\mathcal{M}_n\}$. We prove that $n_0 = \infty$. Assume $n_0 = \infty$. Then there is an increasing sequence $\{n_i\}_{i=1}^{\infty}$ of positive integers such that $n_i \geq k_i$ (where $\{k_i\}_{i=1}^{\infty}$ is the sequence from Lemma 2) and there is a sequence $\{A_i\}_{i=1}^{\infty}$ of sets such that $A_i \in \mathcal{M}_{n_i} \cap \bar{\mathcal{N}}_m$ $(i = 1, 2, \ldots)$. Put $A = \limsup A_i = \bigcap_{s=1}^{\infty} E_s$, where $E_s = \bigcup_{i=s}^{\infty} A_i$. Evidently $E_s \notin \mathcal{N}_m$ $(s = 1, 2, \ldots)$, therefore $A \notin \mathcal{N}$ by Lemma 1. Since $\mathcal{N}^* \subset \mathcal{N}$, we have $A \notin \mathcal{N}^*$.

But $A \subset E_s$ $(s = 1, 2, \ldots)$. Since, by Lemma 2, we have $E_{r(p)} \in \mathcal{M}_p$ $(p = 1, 2, \ldots)$, we get $A \in \mathcal{M}_p$ $(p = 1, 2, \ldots)$. Therefore $A \in \mathcal{M}^*$. This contradicts the assumption $\mathcal{M}^* \subset \mathcal{N}^*$. Hence $n_0 < \infty$ and now the proof of Theorem 3 follows immediately.

Corollary 1. *Let the assumption of Corollary 1 of Theorem 2 be satisfied. Then $\varphi \ll \psi$ if and only if $\varphi \ll \psi$ on $\mathcal{R}^*$.*

Corollary 2. *Let μ, v be signed measures or vector measures on a σ-ring $\mathcal{S} = \mathcal{S}(\mathcal{R})$. Let the variation $|\mu|$ be finite and $|v|$ be σ-finite. Then $\mu \ll v$ if and only if $\mu \ll v$ on $\mathcal{R}$.*

If μ, v are subadditive measures on $\mathcal{S} = \mathcal{S}(\mathcal{R})$ and μ is finite and v is σ-finite, then $\mu \ll v$ if and only if $\mu \ll v$ on $\mathcal{R}$.

Remark. Corollary 2 is proved for the case of signed measures in [1].

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In Theorem 2 and Theorem 3, the ring $\mathcal{R}$ can be replaced by the semi-ring $\mathcal{P}$

satisfying appropriate conditions for $\mathscr{R}$. It follows immediately from the following theorem:

Theorem 4. *Let $\mathscr{R}$ be generated by a semi-ring $\mathscr{P}$. Let $\{\mathscr{N}_n\}_{n=0}^{\infty}$, $\{\mathscr{M}_n\}_{n=0}^{\infty}$ be small systems on $\mathscr{R}$ and $\{\mathscr{N}'_n\}_{n=0}^{\infty} = \{\mathscr{N}_n \cap \mathscr{P}\}_{n=0}^{\infty}$, $\{\mathscr{M}'_n\}_{n=0} = \{\mathscr{M}_n \cap \mathscr{P}\}_{n=0}^{\infty}$. Then $\{\mathscr{N}'_n\}$ $(\leqslant)$ $\{\mathscr{M}'_n\}$ implies $\{\mathscr{N}_n\}$ $(\leqslant)$ $\{\mathscr{M}_n\}$.*

Proof. Suppose that $\{\mathscr{N}_n\}$ $(\leqslant)$ $\{\mathscr{M}_n\}$ is not true. Then there is $n_0 \in P$ having the following property: for each $n \in P$, there is $E \in \mathscr{R}$ such that $E \in \mathscr{M}_n$ and $E \notin \mathscr{M}_{n_0}$. Denote the system $\{E \in \mathscr{M}_n : E \notin \mathscr{N}_{n_0}\}$ by $\mathscr{L}_n$. We shall show $\bigcap_{n=0}^{\infty} \mathscr{L}_n \neq \emptyset$. *Let us consider the topological space* $(\mathscr{X}, \mathscr{T})$, where $\mathscr{X} = \mathscr{R}$ and $\mathscr{T} = \{\emptyset, \mathscr{R}, \mathscr{M}_1, \mathscr{M}_2, \ldots\}$. It is clear that $\mathscr{L}_1, \mathscr{L}_2, \ldots$ are compact sets of $\mathscr{X}$. Further, $\mathscr{L}_1 \supset \mathscr{L}_2 \supset \mathscr{L}_3 \ldots$ and therefore $\bigcap_{n=1}^{\infty} \mathscr{L}_n \neq \emptyset$. We have used the known theorem of topology.

It means that $E_0 \in \mathscr{M}$ and $E \notin \mathscr{N}_{n_0}$ for some E_0. Since $E_0 \in \mathscr{R}$ and $\mathscr{R}$ is generated by $\mathscr{P}$, there are $F_i \in \mathscr{P}$ $(i = 1, 2, \ldots, p)$ such that $E = \bigcup_{i=1}^{p} F_i$. There is a sequence $\{k_i\}_{i=1}^{\infty}$ such that $E_i \in \mathscr{N}_{k_i}$ $(i = 1, 2, \ldots)$ implies $\bigcup_{i=1} E_i \in \mathscr{N}_{n_0}$. We have used the property (2) of a small system. Since $\{\mathscr{N}'_n\}$ $(\leqslant)$ $\{\mathscr{M}'_n\}$, for each k_i, there is l_i such that $\mathscr{M}_{l_i} \subset \mathscr{N}_{k_i}$. Evidently, $F_i \in \mathscr{M}_{l_i}$ $(i = 1, 2, \ldots, p)$. Hence $F_i \in \mathscr{N}_{k_i}$. But it leads to the relation $E_0 = \bigcup_{i=1}^{p} F_i \in \mathscr{N}_{n_0}$, which is a contradiction with our relation $E_0 \in \bigcap_{n=1}^{\infty} \mathscr{L}_n$. Hence Theorem 4 is proved.

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АБСОЛЮТНАЯ НЕПРЕРЫВНОСТЬ ФУНКЦИЙ ОПРЕДЕЛЕННЫХ НА σ-КОЛЬЦЕ ПОРОЖДЕННОМ КОЛЬЦОМ

Мариа Пасторова

Резюме

Пусть μ, v меры определенные на σ-кольце $\mathscr{S}$ прожденном кольцом $\mathscr{R}$, мера μ конечна и мера v σ-конечна. Пусть μ_1, v_1 — меры определенные на $\mathscr{R}$ таким образом, что $\mu_1 = \mu/\mathscr{R}$, $v_1 = v/\mathscr{R}$. Potom $\mu \ll v$ тогда и только тогда, если μ_1 $(\ll)$ v_1.

В работе обобщена предыдущая теорема для векторных и полуадитивных мер. В третьей части работы доказывается теорема для σ-кольца порожденного полукольцим.