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SPR0 SUBSTITUTIONS AND FAMILIES OF ALGEBRAIC RICCATI EQUATIONS

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We study in this paper Algebraic Riccati Equations associated with single-input single-output linear time-invariant systems bounded in H_∞-norm. Our study is focused in the characterization of families of Algebraic Riccati Equations in terms of strictly positive real (of zero relative degree) substitutions applied to the associated H_∞-norm bounded system, each substitution characterizing then a particular member of the family. We also consider here Algebraic Riccati Equations associated with systems characterized by both an H_∞-norm constraint and an upper bound on their corresponding McMillan degree.

Keywords: linear time invariant systems, positive real substitutions, properties preservation, algebraic Riccati equations, H_∞-norm bounded systems

AMS Subject Classification: 93B25, 93B36, 15A24

1. INTRODUCTION

As is pointed out in [14] and [15], the concept of *positive realness* of a transfer function plays a central role in *Stability Theory.* The definition of rational Positive Real functions (PR functions) arose in the context of *Circuit Theory.* In fact, the driving point impedance of a passive network is rational and positive real. If the network is *dissipative* (due to the presence of resistors), the driving point impedance of the network is a Strictly Positive Real transfer function (SPR function). Thus, positive real systems, also called *passive systems*, are systems that do not generate energy. The celebrated Kalman–Yakubovich–Popov (KYP) lemma (see for instance the Lefschetz–Kalman–Yakubovich version of this result in [14]) established the key role that strict positivity realness plays in the obtension of Lyapunov functions associated with the stability analysis of a particular class of nonlinear systems, i. e., Linear Time Invariant systems (LTI systems) with a single memoryless nonlinearity. In fact, positive realness has been extensively studied by the Automatic Control community, see for instance the studies concerning: absolute stability [11]; characterization and construction of robust strictly positive real systems [3]; relationship between time domain and frequency domain conditions for strict positive realness [19]; stability of adaptive control schemes based on parameter adaptation algorithms [1]; passive filters [2].

As far as the frequency-described continuous LTI systems are concerned, the study of control-oriented properties (like stability) resulting from the substitution of the complex Laplace variable s by rational transfer functions have been little studied by the Automatic Control community. However, some interesting results have recently been published. Concerning the study of the so-called *uniform* systems, i. e., LTI systems consisting of identical components and amplifiers, it was established in [16] a general criterion for robust stability for functions of the form $D\left(f\left(s\right)\right)$, where $D\left(s\right)$ is a polynomial and $f\left(s\right)$ is a real rational transfer function. The application of such a criterion has lead to a generalisation of the celebrated Kharitonov's theorem [12], as well as to some robust stability criteria under H_∞-uncertainty. As far as robust stability of polynomial families is concerned, some Kharitonov's like results [12] are given in [18] (for a particular class of polynomials), when interpreting substitutions as nonlinearly correlated perturbations of the coefficients. More recently, in [5], some results for proper and stable real rational Single-Input Single-Output (SISO) functions and coprime factorizations were proved, by making substitutions with $\alpha\left(s\right) = \left(as+b\right)/\left(cs+d\right)$, where a, b, c, and d are strictly positive real numbers, and with $ad-bc \neq 0$. But these results are limited to the bilinear transforms, which are very restricted. The preservation of some interesting H_∞-robustness properties in LTI SISO systems is studied in [7] when applying SPR (of zero relative degree) substitutions, including the preservation of both robust stability and weighted robust performance properties of controlled LTI SISO systems, as well as the preservation of controller optimality in both the weighted robust performance problem and the model-matching problem. Some similar results corresponding to LTI Multi-Input Multi-Output (MIMO) systems are presented in [9].

Concerning the celebrated Riccati Equation, it is well known the key role that this famous equation plays in some significative applications, such as linear quadratic optimal control, stability theory, stochastic filtering and stochastic control, synthesis of of linear passive networks, differential games and H_∞-control and robust stabilization (see for instance [4] and the references therein). Since some studies concerning important robust control problems (like the so-called (regular) H_∞-control problem, see for instance [20]), characterize the controller solution in terms of (stabilizing) solutions of particular Algebraic Riccati Equations (e. g., the central controller which solves the regular H_∞-control problem depends on the solution of two Algebraic Riccati Equations, see for instance [20] and the references therein), it is natural to study the changes that substitutions produce in the solvability properties of Algebraic Riccati Equations associated with the transfer functions in which the substitutions are performed. In this paper we are concerned by Algebraic Riccati Equations associated with SISO LTI systems bounded in H_∞-norm. Our results are mainly based on some well known existing connections between the H_∞-norms of a LTI system and the stabilizing solutions of a particular Algebraic Riccati Equation completely characterized by the parameters of the system (see for instance [20]). Our aim is to characterize families of Algebraic Riccati Equations in terms of the substitutions applied to the associated H_∞-norm bounded system, each substitution characterizing then a particular member of the family. We also consider here Algebraic Riccati Equations associated with SISO LTI systems characterized by both an H_∞-norm constraint and an upper bound on their corresponding McMillan degree.

The paper is organized as follows:

Section 2 is dedicated to the notation and some preliminary results, mainly the preservation of an H_∞-norm constraint on a LTI system, when applying substitutions of the Laplace variable s by a particular class of SPR functions in the corresponding transfer functions (strictly positive real functions of zero relative degree). We tackle in Section 3 the preservation of solvability conditions of Algebraic Riccati Equations when applying SPR substitutions (of the specified class) in the corresponding transfer functions. In particular, we study the preservation of solvability conditions associated with the existence of stabilizing solutions characterized by an H_∞-norm constraint on a particular SISO LTI system. The results obtained then give rise to the characterization of families of Algebraic Riccati Equations parametrized by the coefficients of the SPR functions which replace the Laplace variable s in the associated LTI systems. We also give in Section 3 the characterization of families of Algebraic Riccati Equations associated with SISO LTI systems characterized by both an H_∞-norm constraint and an upper bound on their corresponding McMillan degree. Section 4 is dedicated to some concluding remarks.

2. NOTATION AND PRELIMINARY RESULTS

2.1. Notation

- $\mathbb{C}$, field of complex numbers (the complex plane);
- $\mathbb{R} := (-\infty, \infty)$, field of real numbers;
- $\mathbb{R}^+ := (0, \infty)$, open interval of strictly positive real numbers;
- $\mathrm{Re}[z]$, real part of $z \in \mathbb{C}$;
- $\mathrm{Im}\,\mathbb{C} := \{z \in C \ : \ \mathrm{Re}(z) = 0\}$, imaginary axis of the complex plane;
- $\mathbb{C}^+ := \{\sigma + j\omega \in \mathbb{C} \ : \ \sigma > 0\}$, open right-half complex plane;
- $\mathbb{C}^- := \{\sigma + j\omega \in \mathbb{C} \ : \ \sigma < 0\}$, open left-half complex plane;
- $\mathbb{C}_e^+ := \mathbb{C}^+ \cup \{\infty\}$, extended open right-half complex plane;
- $\overline{\mathbb{C}}^+ := \mathbb{C}^+ \cup \mathrm{Im}\,\mathbb{C}$, closed right-half complex plane;
- $\overline{\mathbb{C}}_e^+ := \mathbb{C}^+ \cup \{\infty\} \cup \mathrm{Im}\,\mathbb{C}$, extended closed right-half complex plane;
- s, complex Laplace variable;
- $R\,[s]$, ring of real polynomials;
- $R(s)$, field of real rational functions;
- $G^{\sim}(s) := G^T(-s)$;
- $R_P(s)$, ring of real rational and proper functions;
- $X_-(H)$, spectral subspace corresponding to eigenvalues of H in $\mathbb{C}^-$;
- $\mathrm{Im}\,(A)$ stands for the image of the linear map A.

2.2. Preliminaries

At this point, we introduce the definition of RH_∞ transfer functions:

Definition 1. Let RH_∞ be the Euclidean domain of proper, stable and rational real transfer functions. This set with the norm:

$$\|P(s)\|_\infty := \sup_\omega |P(j\omega)|$$

is a subspace of H_∞, the space of open right-half plane analytic and bounded transfer functions, with the same norm. The real number $\|P(s)\|_\infty$ is the H_∞*-norm of* $P(s)$.

Now, consider a rational proper transfer function, say $P(s) = N_p(s)/D_p(s)$, with $N_p(s)$ and $D_p(s)$ being real polynomials. We say that $P(s)$ is of zero relative degree if $\deg(N_p(s)) = \deg(D_p(s))$, where $\deg(\cdot)$ stands for the degree of the polynomial.

We introduce at this level the formal definition of Strictly Positive Real SPR functions of zero relative degree:

Definition 2. (Goodwin and Sin [10], Narendra and Taylor [15]) A real rational transfer function $P(s)$ of zero relative degree is SPR (SPR0 function) if and only if:

1. $P(s)$ is analytic in $\mathrm{Re}[s] \geq 0$.
2. $\mathrm{Re}[P(j\omega)] > 0$ for all $\omega \in R$.

The set of SPR0 functions is just denoted by SPR0.

When joining the rational function s to SPR0 we have the following extended set:

Definition 3. $\mathrm{SPR0}^* := \mathrm{SPR0} \cup \{s\}$.

Remark 4. The rational function $s \in \mathrm{SPR0}^*$ can be interpreted as the limit of a sequence of SPR0 functions. Indeed:

$$s = \lim_{a \to 0} \frac{s+a}{as+1} \quad \text{with} \quad a > 0 \quad \text{and} \quad a^2 \neq 1.$$

In what follows we shall recall the formal definition of *Strictly Bounded Real* (SBR) rational transfer functions. This definition will be useful in the result concerning H_∞-norm preservation of functions in RH_∞ when performing SPR0 substitutions.

Definition 5. (Goodwin and Sin [10]) A rational transfer function $P(s)$ is SBR if:

1. $P(s)$ is analytic in $\mathrm{Re}[s] \geq 0$ and:
2. $\|P(s)\|_\infty < 1$.

The set of SBR functions is just denoted by SBR.

The following lemma establishes the closedness of RH_∞, SPR0, and the set of Hurwitz polynomials when performing the substitution of the Laplace variable s by SPR0 functions:

Lemma 6. (Fernández [6]) The following statements are true:

1. If $P(s) \in RH_\infty$ with $Q(s)$ any function belonging to SPR0, then $P(Q(s)) \in RH_\infty$.
2. If $P(s)$, $Q(s) \in$ SPR0, then $P(Q(s))$, $Q(P(s)) \in$ SPR0.
3. If the function $q(s) \in$ SPR0, then $q(\overline{C}_e^+) \subseteq C^+$.

Remark 7. The statements concerning SPR0 in Lemma 6 are still true when changing this set by the extended set SPR0*.

The following result will be useful in the sequel:

Lemma 8. The function $\Xi_Q : RH_\infty \to RH_\infty$ defined by $\Xi_Q[P(s)] := P(Q(s))$, for any $P(s) \in RH_\infty$ and for a fixed $Q(s) \in$ SPR0*, is an homomorphism non-surjective if:

$$\deg\left(D_P(s)\right) \geq 2,$$

where:

$$P(s) = \frac{N_P(s)}{D_P(s)}.$$

P r o o f o f L e m m a 8. The substitution, of the complex Laplace variable s in a function belonging to RH_∞, by a fixed $Q(s) \in$ SPR0*, preserves sums, products, quotients, units and constants in RH_∞ (see [5] for the details). Now, by item 1 in Lemma 6 we conclude that Ξ_Q is an homomorphism. It is clear that Ξ_Q can not be a surjective function if the denominator of $P(s)$ has degree two or larger. □

Remark 9. For a fixed $Q(s) \in$ SPR0*, the function $\Xi_Q[P(s)] := P(Q(s))$ is also an homomorphism in $R_P(s)$, i. e., $P(s) \in R_P(s)$ implies that $P(Q(s)) \in R_P(s)$.

We shall proceed now to present our first result concerning preservation of H_∞-norm properties, when performing the substitution of the Laplace variable s by SPR0 functions.

Lemma 10. (H_∞-norm preservation) Let $P(s)$ be a real proper rational transfer function in RH_∞ with

$$\|P(s)\|_\infty < \gamma.$$

If $P(s)$ is not constant and analytic in $\mathrm{Re}[s] \geq 0$, then for all s such that $\mathrm{Re}[s] \geq 0$

$$\|P(Q(s))\|_\infty < \gamma,$$

for each $Q(s) \in \mathrm{SPR0}^*$.

Proof of Lemma 10. First of all we define $P_\gamma(s) := \gamma^{-1}P(s)$. Note that $\|P_\gamma(s)\|_\infty = \gamma^{-1}\|P(s)\|_\infty$. Thus, $P_\gamma(s) \neq 1$ is analytic in $\mathrm{Re}[s] \geq 0$ and satisfies $\|P_\gamma(s)\|_\infty < 1$, i.e., $P_\gamma(s) \in \mathrm{SBR}$ (see Definition 5). Now, by Theorem 2.9 in [13], if $P_\gamma(s) \neq 1$ for all $\mathrm{Re}[s] \geq 0$, then $H(s) = \frac{1+P_\gamma(s)}{1-P_\gamma(s)}$ is SPR if and only if $P_\gamma(s)$ is SBR. By item 2 in Lemma 6 and Lemma 8, $H(Q(s)) = \frac{1+P_\gamma(Q(s))}{1-P_\gamma(Q(s))}$ is SPR for each $Q(s) \in \mathrm{SPR0}^*$. Then, by Theorem 2.9 in [13], $P_\gamma(Q(s))$ is SBR for each $Q(s) \in \mathrm{SPR0}^*$ and therefore $\|P(Q(s))\|_\infty < \gamma$ for each $Q(s) \in \mathrm{SPR0}^*$. □

Remark 11. The previous result (originally presented in [7] and repeated here for the convenience of the reader) establishes that the input-output gain of a SISO proper and stable transfer function (when the input signal is a square integrable and measurable function defined on $\mathbb{R}$), does not increase when the s variable is substituted by a function belonging to $\mathrm{SPR0}^*$, assuming some real boundedness conditions on the transfer function.

3. ALGEBRAIC RICCATI EQUATIONS AND PRESERVATION OF THE H_∞–NORM

In this section we consider the preservation of solvability conditions in Algebraic Riccati Equations associated with an H_∞-norm constraint on a LTI system.

Let a Single-Input Single-Output (SISO) transfer function $G(s) = C(sI - A)^{-1}B + D$ be given. Consider the algebraic Riccati equation:

$$0 = F^T X + XF + XRX + Q, \tag{1}$$

where F, R and Q are known matrices (R and Q are symmetric), and X is a unknown symmetric matrix.

At this point, we introduce some necessary definitions:

Definition 12. X is called a stabilizing solution of (1) if $A + RX$ is a stability matrix, i.e. if all the eigenvalues of $A + RX$ are in $\mathbb{C}^-$.

Definition 13. We say that a hamiltonian matrix[1] H defined as:

$$H := \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}$$

[1] H is called a hamiltonian matrix if it satisfies:

$$\begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix} H \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix} = -H^T,$$

which implies that H and $-H^T$ are similar.

belongs to the the so-called domain of Riccati, denoted $dom(Ric)$, if there exists a matrix X being a stabilizing solution to the algebraic Riccati equation (1), with $F := H_{11}$, $R := H_{12}$, $Q := -H_{21}$, and $F := -H_{22}^T$. It is convenient to denote $Ric(H) := XH_{11} - H_{22}X + XH_{12}X - H_{21}$.

Remark 14. (Zhou, Doyle, and Glover [20]) $dom(Ric)$ consists of the hamiltonian matrices H with two properties: H has no eigenvalues on the imaginary axis and the subspaces $X_-(H)$ and $\mathrm{Im}\left(\begin{bmatrix} 0 & I \end{bmatrix}^T\right)$ are complementary (I stands for the identity map). Moreover, X is a real and symmetric unique matrix. Note that $Ric : H \to X$.

We proceed now to present our:

Proposition 15. Let a real constant $\gamma > 0$ and a SISO transfer function $G(s) = C(sI - A)^{-1}B + D \in RH_\infty$ be given. We also assume that $(C,\, A)$ is observable. Consider the following hamiltonian matrix:

$$H := \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix},$$

with:

$$\begin{aligned} H_{11} &:= A + BR^{-1}D^TC, \\ H_{12} &:= BR^{-1}B^T, \\ H_{21} &:= -C^T\left(I + DR^{-1}D\right)C, \\ H_{22} &:= -\left(A + BR^{-1}D^TC\right)^T, \end{aligned}$$

where $R := \gamma^2 I - D^2$.

Suppose that $\|G(s)\|_\infty < \gamma$. For each $Q(s) \in \mathrm{SPR0}^*$ we have a state realization $(A_Q,\, B_Q,\, C_Q,\, D_Q)$ of $G(Q(s))$, i.e. $G(Q(s)) = C_Q(sI - A_Q)^{-1}B_Q + D_Q$. Then, the following conditions are equivalent:

i') $\|G(Q(s))\|_\infty < \gamma$;

ii') $D_Q < \gamma$ and H_Q has no eigenvalues on $\mathrm{Im}\,\mathbb{C}$;

iii') $D_Q < \gamma$ and $H_Q \in dom(Ric)$;

iv') $D_Q < \gamma$ and $H_Q \in dom(Ric)$ and $Ric(H_Q) \geq 0$,

where

$$H_Q := \begin{bmatrix} H_{Q11} & H_{Q12} \\ H_{Q21} & H_{Q22} \end{bmatrix},$$

with:

$$\begin{aligned} H_{Q11} &:= A_Q + B_Q R_Q^{-1} D_Q^T C_Q, \\ H_{Q12} &:= B_Q R_Q^{-1} B_Q^T, \\ H_{Q21} &:= -C_Q^T \left(I + D_Q R_Q^{-1} D_Q^T \right) C_Q, \\ H_{Q22} &:= -\left(A_Q + B_Q R_Q^{-1} D_Q^T C_Q \right)^T, \end{aligned}$$

and where

$$R_Q = \gamma^2 I - D_Q^2.$$

Proof of Proposition 15. By Corollary 13.24 in [20] we have that the following conditions are equivalent:

i) $\|G(s)\|_\infty < \gamma$;

ii) $D < \gamma$ and H has no eigenvalues on $\mathrm{Im}\,\mathbb{C}$;

iii) $D < \gamma$ and $H \in dom(Ric)$;

iv) $D < \gamma$ and $H \in dom(Ric)$ and $Ric(H) \geq 0$.

By item 1 in Lemma 6 we have that $G(Q(s)) \in RH_\infty$ for each $Q(s) \in \mathrm{SPR0}^*$. Now by Lemma 10 we have that $\|G(Q(s))\|_\infty < \gamma$, which lets us conclude the proof. □

In what follows we consider stabilizing solutions of a particular class of Algebraic Riccati Equations.

Corollary 16. Let a SISO transfer function $G(s) = C(sI - A)^{-1}B + D \in \mathrm{SPR0}^*$ and $R_0 = 2D > 0$ be given. Then:

i) there is at most one stabilizing solution:

$$X_Q \left(A_Q - B_Q R_{0Q}^{-1} C_Q \right) + \left(A_Q - B_Q R_{0Q}^{-1} C_Q \right)^T X_Q + X_Q B_Q R_{0Q}^{-1} B_Q^T X_Q + C_Q^T R_{0Q}^{-1} C_Q = 0;$$

ii) the system:

$$M(Q(s)) = R_{0Q}^{-\frac{1}{2}} \left(C_Q - B_Q^T X_Q \right) \left[sI - A_Q \right]^{-1} B_Q + R_{0Q}^{\frac{1}{2}}$$

is minimal phase and:

$$G(Q(s)) + G^{\sim}(Q(s)) = M^{\sim}(Q(s)) M(Q(s)),$$

for each $Q(s) \in \mathrm{SPR0}^*$, where:

$$G(Q(s)) = C_Q (sI - A_Q)^{-1} B_Q + D_Q$$

and $R_{0Q} := 2D_Q$.

Proof of Corollary 16. By Corollary 13.27 in [20], $G(s)$ belongs to SPR0 (even to SPR0*) if and only if there exists a stabilizing solution to the following Riccati equation:

$$X_Q\left(A - BR_0^{-1}C\right) + \left(A - BR_0^{-1}C\right)^T X_Q + X_Q B R_0^{-1} B^T X_Q + C^T R_0^{-1} C = 0.$$

Moreover:

$$M(s) = R_0^{-\frac{1}{2}}\left(C - B^T X_Q\right)[sI - A]^{-1} B + R_0^{\frac{1}{2}}$$

is minimal phase and:

$$G(s) + G^{\sim}(s) = M^{\sim}(s) M(s).$$

Since $G(s) \in$ SPR0* (and by Lemma 6) we have that $G(Q(s)) \in$ SPR0* for each $Q(s) \in$ SPR0*. The result is then a consequence of Corollary 13.27 in [20]. □

Remark 17. The results shown in Proposition 15 and its corresponding corollary are mainly based on Lemma 10, which concerns H_∞-norm preservation. The H_∞-norm constraint on $G(s)$ (Proposition 15) characterizes a family of Algebraic Riccati Equations (Corollary 16) with stabilizing solutions. We should note that the parameters of the SPR0 functions ($Q(s) \in$ SPR0*) can vary continuously in given real intervals and the McMillan degrees of the SPR0 functions ($Q(s) \in$ SPR0*) can be different; in this way, we have families of Algebraic Riccati Equations with different solutions.

Let us now tackle the closedness of a set of LTI invariant systems characterized by an H_∞-norm constraint and an upper bound on their corresponding McMillan degree.

Proposition 18. Let $\boldsymbol{P}(n,\gamma) := \{G(s) : G \in RH_\infty, \deg_{\text{McMillan}}(G) \leq n, \|G\|_\infty < \gamma\}$ be the set of all stable, real rational, proper SISO systems with McMillan degree of at most n, with H_∞-norm less than γ. Now define a SISO system $G_0(s) := C(sI - A_0)^{-1}B + D$ such that the real constant matrix

$$A_0 := A_{sk} - \frac{1}{2}\left(B + C^T D\right) R^{-1}\left(B + C^T D\right)^T - \frac{1}{2} C^T C,$$

where $R := \gamma^2 - D^2$, $D < \gamma$ has no eigenvalues on the imaginary axis. Matrix A_{sk} is defined as:

$$A_{sk} := \begin{bmatrix} 0 & -a_1 & 0 & \cdots & 0 \\ a_1 & 0 & -a_2 & \ddots & \vdots \\ 0 & a_2 & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & 0 & -a_{n-1} \\ 0 & \cdots & 0 & a_{n-1} & 0 \end{bmatrix}$$

with $a_i \geq 0$, and the first row of matrix $B = [b_{ij}]$ satisfies $b_{11} \geq 0$, $b_{1j} = 0$, $\forall j > 1$. Then $G_0(Q(s)) \in \boldsymbol{P}(m,\gamma)$, where $m \geq \deg_{\text{McMillan}}(G(s)) \deg_{\text{McMillan}}(Q(s))$, for each $Q(s) \in$ SPR0*.

Conversely, for each $G_0(Q(s))$ there exists a pseudo-canonical parametrization:

$$G_0(Q(s)) = C_Q(sI - A_{0Q})^{-1}B_Q + D_Q$$

where:

$$A_{0Q} := A_{skQ} - \frac{1}{2}\left(B_Q + C_Q^T D_Q\right) R_Q^{-1} \left(B_Q + C_Q^T D_Q\right)^T - \frac{1}{2} C_Q^T C_Q$$

and $R_Q := \gamma^2 - D_Q^2$, such that the real constant matrix A_{0Q} has no eigenvalues on the imaginary axis and $D_Q < \gamma$. Matrix A_{skQ} is defined as:

$$A_{skQ} := \begin{bmatrix} 0 & -\bar{a}_1 & 0 & \cdots & 0 \\ \bar{a}_1 & 0 & -\bar{a}_2 & \ddots & \vdots \\ 0 & \bar{a}_2 & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & 0 & -\bar{a}_{m-1} \\ 0 & \cdots & 0 & \bar{a}_{m-1} & 0 \end{bmatrix}$$

with $\bar{a}_i \geq 0$, and the first row of matrix $B_Q = [\bar{b}_{ij}]$ satisfies $\bar{b}_{11} \geq 0$, $\bar{b}_{1j} = 0$, $\forall j > 1$.

Proof of Proposition 18. This result is a direct consequence of item 1 in Lemma 6, Lemma 8, and Lemma 10:

$$G_0(Q(s)) \in \boldsymbol{P}(m, \gamma);$$

by Theorem 1 in [17], there exists a pseudo-canonical parametrization of $G_0(Q(s)) \in \boldsymbol{P}(m, \gamma)$. □

Remark 19. The parametrization discussed in Proposition 18 is illustrated by Example 1 in [17]. Indeed, the cited example shows that all strictly proper second order systems of the form:

$$G_1(s) = \frac{b_1\left(c_1 s + a_1 c_2\right)}{s^2 + \frac{1}{2}\left(c_1^2 + c_2^2 + b_1^2/\gamma^2\right)s + a_1^2 + b_1^2 c_2^2/4\gamma^2}$$

(with $a_1 \geq 0$, $b_1 \geq 0$, $a_1 \neq b_1 |c_2|/2\gamma$, and $a_1 > b_1 |c_2|/2\gamma$ if $c_1^2 + c_2^2 = b_1^2/\gamma^2$) are stable and have an H_∞-norm less than γ. Now, Lemma 6 and Lemma 10 let us affirm that $G_1(Q(s))$ is a stable system of H_∞-norm less than γ. Thus, Proposition 18 lets us affirm that there always exists a pseudo-canonical parametrization for $G_1(Q(s))$, for each $Q(s) \in \text{SPR0}^*$.

4. CONCLUDING REMARKS

We have boarded in this paper a study concerning Algebraic Riccati Equations and the preservation of solvability conditions when performing strictly proper and real substitutions (on the related single-input and single-output linear time-invariant system). Our study was mainly focused on the preservation of the existence of stabilizing solutions in Algebraic Riccati Equations associated with linear time-invariant systems constrained in terms of a fixed H_∞-norm bound. In particular, we established the closedness (when considering strictly positive real substitutions of zero relative degree) of sets of Algebraic Riccati Equations defined in terms of particular H_∞-norm bounded linear time-invariant systems.

The results reported here have been applied to the study of robust sliding hyperplanes. In [8] is presented an application showing the preservation of closed-loop quadratical stability of a family of linear time-invariant systems affected by time-varying uncertainty, and controlled via sliding modes control. The family of uncertain systems is characterized by a set of strictly positive real functions of zero relative degree.

The study reported in this paper has as a natural perspective in some extensions concerning Algebraic Riccati Equations linked to Multi-Input Multi-Output systems.

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