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Applications of Mathematics, Vol. 49 (2004), No. 1, 39–55

Persistent URL: `http://dml.cz/dmlcz/134557`

A MATHEMATICAL MODEL OF SUSPENSION BRIDGES

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(Received September 29, 2001)

Abstract. We prove the existence of weak T-periodic solutions for a nonlinear mathematical model associated with suspension bridges. Under further assumptions a regularity result is also given.

Keywords: suspension bridges, periodic solution, Galerkin approximation, Leray-Schauder principle

MSC 2000: 35B10, 70K30, 74K10

1. INTRODUCTION

The purpose of this paper is the investigation of a nonlinear mathematical model associated with suspension bridges.

In the following we consider the model of a one-dimensional suspension bridge which consists of a vibrating beam (the roadbed) with hinged ends, coupled with a vibrating string (the main cable) by the stays treated as nonlinear springs.

The beam is subject to three separate forces: the one due to the stays, which hold it up as nonlinear springs, the weight per unit length of the beam which pushes it down, and the external forcing term which we will assume to be periodic. The main cable is subject to the action of the stays which pull it down as nonlinear springs, to the weight per unit length of the cable which pushes it down, and to some oscillatory forcing term which might be due to the wind or to motions in the towers or side-spans (see Fig. 1).

We do not take into account the other two dimensions because the proportions of the bridge in these dimensions are very small in comparison with its length and so can be omitted.

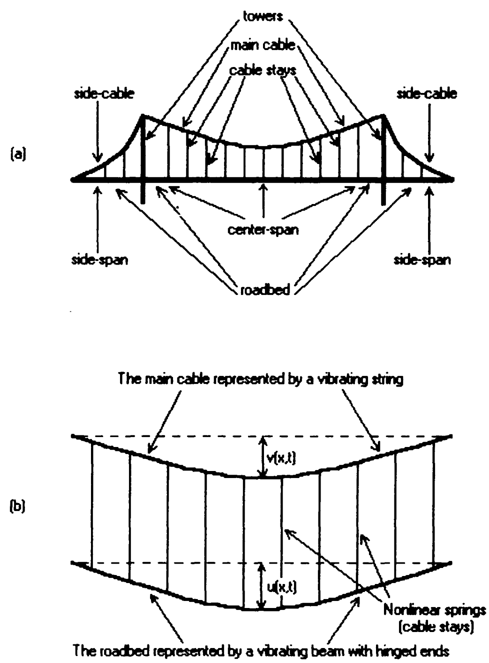

Figure 1. (a) The main ingredients in a one-dimensional suspension bridge
(b) A model of the one-dimensional bridge represented by the coupling of the cable (a vibrating string) and the roadbed (a vibrating beam) by the stays, treated as nonlinear springs.

Before formulating exactly the problem we will work on, we would like to present a model which was introduced firstly in the work of A. C. Lazer and P. J. McKenna [6], but has been studied under rather restrictive assumptions. So, if $v(x,t)$ measures the displacement from equilibrium of the vibrating string and $u(x,t)$ denotes the displacement of the beam in the downward direction at position x and time t, then a damped model of a suspension bridge is given by the system of two connected equations in the form

$$m_1 v_{tt} - a_1 v_{xx} + b_1 v_t - k(u-v)^+ = W_1(x) + \varepsilon f_1(x,t), \tag{1.1}$$

$$m_2 u_{tt} + a_2 u_{xxxx} + b_2 u_t + k(u-v)^+ = W_2(x) + \varepsilon f_2(x,t) \tag{1.2}$$

with the boundary conditions

$$u(0,t) = u(L,t) = u_{xx}(0,t) = u_{xx}(L,t) = v(0,t) = v(L,t) = 0 \tag{1.3}$$

and the periodicity condition

$$v(x,t+T) = v(x,t), \quad u(x,t+T) = u(x,t), \quad x \in (0,L), \ t \in \mathbb{R}, \ T > 0. \tag{1.4}$$

The parameters used in the equations (1.1), (1.2), (1.3) are the following:

m_1 = mass per unit length of the main cable;

m_2 = mass per unit length of the roadbed;

a_1 = the tension in the main cable;

$a_2 = EI$ where E is Young's modulus and I is the moment of inertia of the cross section;

b_1, b_2 = damping coefficients;

k = stiffness of the cable stays (spring constant);

W_1 = weight of the main cable per unit length;

W_2 = weight of the roadbed per unit length;

εf_1, εf_2 = external time-periodic forcing terms;

L = length of the center-span of the bridge.

The mass per unit length of the main cable, m_1, is much less than the mass per unit length of the roadbed, m_2.

The nonlinear cable stays connecting the beam and the string can be taken as one-sided springs obeying Hooke's law, with a restoring force proportional to the displacement if they are stretched, and with no restoring force if they are compressed. This fact is described by the nonlinear term $k(u-v)^+$.

The nonlinear stays pull the cable down, hence we have the minus sign at $k(u-v)^+$ in the equation (1.1), and hold the roadbed up, therefore we consider the plus sign at the same term in the equation (1.2).

Using a previous result due to P. Drábek [3], G. Tajčová [10] proved the existence of a unique solution of the problem (1.1)–(1.4) (with $T = 2\pi$) by using the Banach contraction principle. The disadvantage of this principle consists in the fact that its application requires a rather restrictive assumption on the parameters k, m_1, m_2, b_1, b_2, E, I, T and the conditions obtained are too restrictive and are not satisfied by the real values of the bridge parameters. P. Drábek, H. Leinfelder and G. Tajčová [4] have established the existence of a unique time-periodic solution near stationary equilibrium under rather general assumptions on the above mentioned parameters, provided the external time-periodic forcing terms are small in a certain sense.

For a good survey of the literature dedicated to various mathematical models of suspension bridges we mention the papers of P. J. McKenna and W. Walter [7], [8], Q.-H. Choi and T. Jung [2], J. M. Ball [1], N. Krylová [5].

2. Preliminaries

We now introduce some well known function spaces and some notation.

Let $Q = (0, L) \times (0, T)$, where $L > 0$ is the natural length of the beam and $T > 0$ is fixed. We denote x-derivatives by primes or upper indices, e.g.

$$\frac{\partial}{\partial x}(\ ) = (\ )', \quad \frac{\partial^4}{\partial x^4}(\ ) = (\ )^{(4)} \quad \text{etc.}$$

and for t-derivatives we use dots

$$\frac{\partial}{\partial t}(\ ) = (\dot{\ }).$$

If a function f is of the type $f = [f_1, f_2, \dots, f_n]^t$, then we denote $f = (f_j)$, without indicating the range of j's if no confusion is possible.

Constants are denoted generically by C_i $(i = 1, 2, \dots)$.

For simplicity, the spaces of scalar functions defined on the interval $(0, L)$ will be denoted by dropping the reference to the interval, e.g.

$$L^2 = L^2(0, L); \quad H_0^1 = H_0^1(0, L); \quad H^2 \cap H_0^1 = H^2(0, L) \cap H_0^1(0, L).$$

The duality in L^2 will be denoted by $(\cdot, \cdot)$. Let $\breve{H} = \{u \in H^4;\ u, u'' \in H^2 \cap H_0^1\}$. Note that $H^2 \cap H_0^1$ is a Hilbert space isomorphic to L^2 with the scalar product $(f, g)_{H^2 \cap H_0^1} = (f'', g'')_{L^2}$. For general information on Sobolev spaces see [11].

Let X be a Banach space with a norm $\| \cdot \|_X$. A function f defined a.e. on $\mathbb{R}$, with values in X, is called T-periodic if the following holds: if f is defined for $t \in \mathbb{R}$, it is also defined for $t + kT$, $k \in \mathbb{Z}$, and $f(t) = f(t + kT)$.

In the usual way, if X and Y are Banach spaces, then let $X \times Y$ be the Banach space with the norm $\|w\|_{X \times Y} = (\|v\|_X^2 + \|u\|_Y^2)^{1/2}$ where $w = [v, u]^t$ and $\| \cdot \|_X$, $\| \cdot \|_Y$ denote the X-norm and Y-norm, respectively.

If X and Y are Hilbert spaces, then the scalar product of two elements $w = [v, u]^t$ and $\tilde{w} = [\tilde{v}, \tilde{u}]^t$ in $X \times Y$ is written as $(w, \tilde{w})_{X \times Y} = (v, \tilde{v})_X + (u, \tilde{u})_Y$ where $(\cdot, \cdot)_X$, $(\cdot, \cdot)_Y$ denote the scalar products on X and Y, respectively.

Lemma 1. *Let X and Y be Banach spaces, $M\colon X \to Y$ an isomorphism and $S\colon X \to Y$ a compact mapping. If the operator $L\colon X \to Y$, $L = M + S$ is an injection, then L is an isomorphism.*

Proof. Let us introduce the operator $\Gamma\colon X \to X$, $\Gamma = M^{-1} \circ S$. Since Γ is compact and the operators $L = M + S = M \circ (I + \Gamma)$ and M^{-1} are injective, we have that $I + \Gamma$ is an isomorphism. $\square$

3. Main results

We consider a more general model of a suspension bridge

$$m_1 v_{tt} - a_1 v_{xx} + b_1 v_t - \varphi(u-v) = W_1(x) + \varepsilon f_1(x,t), \tag{3.1}$$

$$m_2 u_{tt} + a_2 u_{xxxx} + b_2 u_t + \varphi(u-v) = W_2(x) + \varepsilon f_2(x,t) \tag{3.2}$$

with the boundary conditions

$$u(0,t) = u(L,t) = u_{xx}(0,t) = u_{xx}(L,t) = v(0,t) = v(L,t) = 0 \tag{3.3}$$

and the periodicity condition

$$v(x,t+T) = v(x,t), \quad u(x,t+T) = u(x,t), \quad x \in (0,L), \ t \in \mathbb{R}, \ T > 0. \tag{3.4}$$

The time T is a positive number, bearing no relation to the length of the x-interval. The present paper is based on the technique of L. Sanchez's work [9].

Under suitable hypothesis on W_1, W_2, f_1 and f_2 we can show that weak solutions of (3.1)–(3.4) exist, by using a combination of Galerkin's method and a compactness argument with a version of the Leray-Schauder principle.

Introducing the vector functions

$$W(x) = \begin{bmatrix} W_1(x) \\ W_2(x) \end{bmatrix}, \quad f(x,t) = \begin{bmatrix} f_1(x,t) \\ f_2(x,t) \end{bmatrix}$$

we consider in this section the problem (3.1)–(3.4) with the functions φ, W and f satisfying the following hypotheses:

(H1) φ is a continuous function on the real line such that

$$\varphi(u)u \geqslant 0, \quad \forall u \in \mathbb{R}$$

and φ has polynomial growth at infinity, i.e. there exist two numbers $C > 0$ and $p \geqslant 1$ such that

$$|\varphi(u)| \leqslant C(|u|^p + 1), \quad \forall u \in \mathbb{R};$$

(H2) $W \in L^2 \times L^2$;

(H3) $f \in L^2(T; L^2) \times L^2(T; L^2) \simeq L^2(Q) \times L^2(Q), \quad Q = (0,L) \times (0,T)$.

Remark 1. Of course, the condition (H1) is satisfied for $\varphi\colon \mathbb{R} \to \mathbb{R}_+$, $\varphi(u) = ku^+$, $k > 0$.

Definition 1. A weak solution of (3.1)–(3.4) is a mapping

$$w = [v,u]^t \in L^2(T;H_0^1) \times L^2(T;H^2 \cap H_0^1)$$

having a derivative $\dot{w} = [\dot{v},\dot{u}]^t \in L^2(T;L^2) \times L^2(T;L^2)$ such that, for any function $\theta = [\varrho,\psi]^t$ satisfying the same requirements, we have

$$\begin{aligned}
(3.5)\quad -m_1\int_0^T(\dot{v},\dot{\varrho})\,\mathrm{d}t + a_1\int_0^T(v',\varrho')\,\mathrm{d}t + b_1\int_0^T(\dot{v},\varrho)\,\mathrm{d}t - \iint_Q \varphi(u-v)\varrho\,\mathrm{d}x\,\mathrm{d}t \\
= \iint_Q W_1\varrho\,\mathrm{d}x\,\mathrm{d}t + \varepsilon\iint_Q f_1\varrho\,\mathrm{d}x\,\mathrm{d}t, \\
(3.6)\quad -m_2\int_0^T(\dot{u},\dot{\psi})\,\mathrm{d}t + a_2\int_0^T(u'',\psi'')\,\mathrm{d}t + b_2\int_0^T(\dot{u},\psi)\,\mathrm{d}t + \iint_Q \varphi(u-v)\psi\,\mathrm{d}x\,\mathrm{d}t \\
= \iint_Q W_2\psi\,\mathrm{d}x\,\mathrm{d}t + \varepsilon\iint_Q f_2\psi\,\mathrm{d}x\,\mathrm{d}t.
\end{aligned}$$

R e m a r k 2. We note that the fourth term in (3.5) makes sense because $u,v \in H^1(Q)$ and the imbedding $H^1(Q) \hookrightarrow L^q(Q)$ is compact for all $1 \leqslant q < \infty$. The same holds for the fourth term in (3.6).

Theorem 2. *Under the assumptions* (H1), (H2) *and* (H3), *the problem* (3.1)–(3.4) *has a weak solution* $w = [v,u]^t \in L^\infty(T;H_0^1) \times L^\infty(T;H^2 \cap H_0^1)$ *such that* $\dot{w} = [\dot{v},\dot{u}]^t \in L^\infty(T;L^2) \times L^\infty(T;L^2)$.

P r o o f. *Step 1.* We use the Galerkin approximative procedure. Let

$$\omega_n = \sin\frac{n\pi}{L}x \quad (n = 1,2,\ldots)$$

be an orthogonal basis for H_0^1, $H^2 \cap H_0^1$ and L^2 ([1], Lemma 2).

We search the n-th approximation

$$w_n(t) = [v_n(t),u_n(t)]^t \quad (n = 1,2,\ldots)$$

which satisfies the system of equations

$$\begin{aligned}
(3.7)\qquad & m_1(\ddot{v}_n,\omega_j) + a_1(v_n',\omega_j') + b_1(\dot{v}_n,\omega_j) - (\varphi(u_n - v_n),\omega_j), \\
& = (W_1,\omega_j) + \varepsilon(f_1,\omega_j), \quad 1 \leqslant j \leqslant n, \\
(3.8)\qquad & m_2(\ddot{u}_n,\omega_j) + a_2(u_n'',\omega_j'') + b_2(\dot{u}_n,\omega_j) + (\varphi(u_n - v_n),\omega_j) \\
& = (W_2,\omega_j) + \varepsilon(f_2,\omega_j), \quad 1 \leqslant j \leqslant n
\end{aligned}$$

and the periodicity conditions

$$v_n(x,0) = v_n(x,T), \quad \dot{v}_n(x,0) = \dot{v}_n(x,T), \tag{3.9}$$

$$u_n(x,0) = u_n(x,T), \quad \dot{u}_n(x,0) = \dot{u}_n(x,T). \tag{3.10}$$

Some estimates of these approximations $w_n(t)$ enable us to establish the convergence (in a certain sense) of a convenient subsequence of $\{w_n(t)\}_{n=1}^{\infty}$ to a weak solution of our problem (3.1)–(3.4).

We define approximations $w_n(t) = [v_n(t), u_n(t)]^t$ of the form

$$v_n(t) = \sum_{i=1}^{n} y_{in}(t)\omega_i, \tag{3.11}$$

$$u_n(t) = \sum_{i=1}^{n} z_{in}(t)\omega_i \tag{3.12}$$

where $y_{in}(t)$, $z_{in}(t)$ are real-valued functions. Inserting the expression (3.11) in (3.7) and (3.12) in (3.8), we obtain a system of ordinary differential equations

$$\begin{aligned} m_1\ddot{y}_{jn}(t) + a_1\frac{\pi^2}{L^2}j^2 y_{jn}(t) + b_1\dot{y}_{jn}(t) - (\varphi(u_n - v_n), \omega_j) \\ = \xi_{1j} + \varepsilon f_{1j}(t), \quad 1 \leqslant j \leqslant n, \end{aligned} \tag{3.13}$$

$$\begin{aligned} m_2\ddot{z}_{jn}(t) + a_2\frac{\pi^4}{L^4}j^4 z_{jn}(t) + b_2\dot{z}_{jn}(t) + (\varphi(u_n - v_n), \omega_j) \\ = \xi_{2j} + \varepsilon f_{2j}(t), \quad 1 \leqslant j \leqslant n \end{aligned} \tag{3.14}$$

where $\xi_{ij} = (W_i, \omega_j) \in \mathbb{R}$ and $f_{ij}(t) = (f_i(t), \omega_j) \in L^2(0,T)$, $i = 1,2$.

Next, we show that the system of nonlinear ordinary differential equations (3.13), (3.14) has a solution

$$x_n(t) = [(y_{jn}(t)), (z_{jn}(t))]^t$$

such that

$$y_{jn}(0) = y_{jn}(T), \quad \dot{y}_{jn}(0) = \dot{y}_{jn}(T), \tag{3.15}$$

$$z_{jn}(0) = z_{jn}(T), \quad \dot{z}_{jn}(0) = \dot{z}_{jn}(T) \tag{3.16}$$

for all $j = 1, \dots, n$.

Step 2. Existence of the Galerkin approximations.

Let H_T^2 be the closed subspace of $H^2(0,T)$ consisting of functions f which satisfy the boundary conditions

$$f(0) = f(T), \quad \dot{f}(0) = \dot{f}(T).$$

We consider the linear and bounded mapping

$$L\colon (H_T^2)^n \times (H_T^2)^n \to (L^2(0,T))^n \times (L^2(0,T))^n$$

defined by

$$Lx_n = \begin{bmatrix} \left(m_1\ddot{y}_{jn} + a_1\frac{\pi^2}{L^2}j^2 y_{jn} + b_1\dot{y}_{jn}\right) \\ \left(m_2\ddot{z}_{jn} + a_2\frac{\pi^4}{L^4}j^4 z_{jn} + b_2\dot{z}_{jn}\right) \end{bmatrix}.$$

Computing the scalar product of $Lx_n = 0$ with x_n, we get

$$(3.17)\qquad -m_1\sum_{j=1}^{n}\int_0^T \dot{y}_{jn}^2\,\mathrm{d}t + a_1\frac{\pi^2}{L^2}\sum_{j=1}^{n}\int_0^T j^2 y_{jn}^2\,\mathrm{d}t - m_2\sum_{j=1}^{n}\int_0^T \dot{z}_{jn}^2\,\mathrm{d}t + a_2\frac{\pi^4}{L^4}\sum_{j=1}^{n}\int_0^T j^4 z_{jn}^2\,\mathrm{d}t = 0.$$

Now, we multiply $Lx_n = 0$ by $\dot{x}_n$ and integrate over $(0,T)$. We obtain

$$(3.18)\qquad b_1\sum_{j=1}^{n}\int_0^T \dot{y}_{jn}^2\,\mathrm{d}t + b_2\sum_{j=1}^{n}\int_0^T \dot{z}_{jn}^2\,\mathrm{d}t = 0.$$

By (3.17) and (3.18) we obtain $x_n = 0$.

Thus L is an injection and we conclude, by virtue of Lemma 1, that L is an isomorphism from $(H_T^2)^n \times (H_T^2)^n$ onto $(L^2(0,T))^n \times (L^2(0,T))^n$ because it is the sum of an isomorphism between $(H_T^2)^n \times (H_T^2)^n$ and $(L^2(0,T))^n \times (L^2(0,T))^n$ with a compact linear mapping.

Next, we consider the mapping

$$H\colon (H_T^2)^n \times (H_T^2)^n \to (L^2(0,T))^n \times (L^2(0,T))^n$$

defined by

$$Hx_n = \begin{bmatrix} \left(-\left(\varphi\left(\sum_{i=1}^{n}(z_{in}(t) - y_{in}(t))\omega_i\right),\omega_j\right)\right) \\ \left(\left(\varphi\left(\sum_{i=1}^{n}(z_{in}(t) - y_{in}(t))\omega_i\right),\omega_j\right)\right) \end{bmatrix}.$$

Because the mapping φ is continuous and the imbedding $H^1(Q) \to L^q(Q)$ $(1 \leqslant q < \infty)$ is compact, H is a continuous, compact mapping from $(H_T^2)^n \times (H_T^2)^n$ to $(L^2(0,T))^n \times (L^2(0,T))^n$.

Now, writing $\xi_n = [(\xi_{1j}),(\xi_{2j})]^t$ and $f_n(t) = [(f_{1j}(t)),(f_{2j}(t))]^t$, the problem (3.13), (3.14), (3.15), (3.16) coincides with

$$(3.19)\qquad Lx_n + Hx_n = \xi_n + \varepsilon f_n.$$

By a version of the Leray-Schauder principle the existence of solutions for (3.19) will be a consequence of the boundedness in $(H_T^2)^n \times (H_T^2)^n$ of all possible solutions of

$$Lx_n + \lambda H x_n = \xi_n + \varepsilon f_n \tag{3.20}$$

with $0 \leqslant \lambda < 1$, which we now prove.

We compute the scalar product of (3.20) with $\dot{x}_n$ which yields

$$\begin{aligned}
(3.21)\quad & b_1 \sum_{j=1}^{n} \int_0^T \dot{y}_{jn}^2 \,\mathrm{d}t + b_2 \sum_{j=1}^{n} \int_0^T \dot{z}_{jn}^2 \,\mathrm{d}t + \lambda \int_0^T (\varphi(u_n - v_n), \dot{u}_n - \dot{v}_n)\,\mathrm{d}t \\
&= \sum_{j=1}^{n} \xi_{1j} \int_0^T \dot{y}_{jn}\,\mathrm{d}t + \sum_{j=1}^{n} \xi_{2j} \int_0^T \dot{z}_{jn}\,\mathrm{d}t \\
&\quad + \varepsilon \sum_{j=1}^{n} \int_0^T f_{1j}\dot{y}_{jn}\,\mathrm{d}t + \varepsilon \sum_{j=1}^{n} \int_0^T f_{2j}\dot{z}_{jn}\,\mathrm{d}t.
\end{aligned}$$

If we denote by G a primitive of φ, then we have

$$(\varphi(u_n - v_n), \dot{u}_n - \dot{v}_n)) = \frac{\mathrm{d}}{\mathrm{d}t} \int_0^L G(u_n - v_n)\,\mathrm{d}x.$$

Since

$$\left(\sum_{j=1}^{n} \int_0^T f_{ij}^2(t)\,\mathrm{d}t \right)^{1/2} \leqslant \sqrt{\frac{2}{L}} \|f_i\|_{L^2(Q)}, \quad i = 1, 2,$$

we have by (3.21):

$$\begin{aligned}
(3.22)\quad & b_1 \sum_{j=1}^{n} \int_0^T \dot{y}_{jn}^2\,\mathrm{d}t + b_2 \sum_{j=1}^{n} \int_0^T \dot{z}_{jn}^2\,\mathrm{d}t \\
&\leqslant \sqrt{\frac{2}{L}} \Bigg[\sqrt{T} \|W_1\|_{L^2} \left(\sum_{j=1}^{n} \int_0^T \dot{y}_{jn}^2\,\mathrm{d}t \right)^{1/2} + \sqrt{T} \|W_2\|_{L^2} \left(\sum_{j=1}^{n} \int_0^T \dot{z}_{jn}^2\,\mathrm{d}t \right)^{1/2} \\
&\quad + \varepsilon \|f_1\|_{L^2(Q)} \left(\sum_{j=1}^{n} \int_0^T \dot{y}_{jn}^2\,\mathrm{d}t \right)^{1/2} + \varepsilon \|f_2\|_{L^2(Q)} \left(\sum_{j=1}^{n} \int_0^T \dot{z}_{jn}^2\,\mathrm{d}t \right)^{1/2} \Bigg].
\end{aligned}$$

Then, we have by (3.22):

$$\begin{aligned}
(3.23)\quad & \left(\sum_{j=1}^{n} \int_0^T \dot{y}_{jn}^2\,\mathrm{d}t \right)^{1/2} + \left(\sum_{j=1}^{n} \int_0^T \dot{z}_{jn}^2\,\mathrm{d}t \right)^{1/2} \\
&\leqslant C_1 \big(\sqrt{T} \|W\|_{L^2 \times L^2} + \varepsilon \|f\|_{L^2(Q) \times L^2(Q)} \big), \\
(3.24)\quad & \|\dot{x}_n\|^2_{(L^2(0,T))^n \times (L^2(0,T))^n} \leqslant C_2 \big(\sqrt{T} \|W\|_{L^2 \times L^2} + \varepsilon \|f\|_{L^2(Q) \times L^2(Q)} \big)^2.
\end{aligned}$$

From (3.20) multiplied by x_n we obtain

$$\begin{aligned}(3.25)\quad & a_1\frac{\pi^2}{L^2}\sum_{j=1}^{n}\int_0^T j^2y_{jn}^2\,\mathrm{d}t + a_2\frac{\pi^4}{L^4}\sum_{j=1}^{n}\int_0^T j^4z_{jn}^2\,\mathrm{d}t \\ &\quad + \lambda\int_0^T(\varphi(u_n-v_n),u_n-v_n)\,\mathrm{d}t \\ &\quad = m_1\sum_{j=1}^{n}\int_0^T \dot y_{jn}^2\,\mathrm{d}t + m_2\sum_{j=1}^{n}\int_0^T \dot z_{jn}^2\,\mathrm{d}t + \sum_{j=1}^{n}\xi_{1j}\int_0^T y_{jn}\,\mathrm{d}t \\ &\quad + \sum_{j=1}^{n}\xi_{2j}\int_0^T z_{jn}\,\mathrm{d}t + \varepsilon\sum_{j=1}^{n}\int_0^T f_{1j}y_{jn}\,\mathrm{d}t + \varepsilon\sum_{j=1}^{n}\int_0^T f_{2j}z_{jn}\,\mathrm{d}t\end{aligned}$$

If we denote $\alpha = \min\{a_1\frac{\pi^2}{L^2}, a_2\frac{\pi^4}{L^4}\}$, we have by (3.24) and (3.25):

$$\begin{aligned}&\sum_{j=1}^{n}\int_0^T j^2y_{jn}^2\,\mathrm{d}t + \sum_{j=1}^{n}\int_0^T j^4z_{jn}^2\,\mathrm{d}t \\ &\qquad \leqslant \frac{m_2C_2}{\alpha}\big(\sqrt{T}\|W\|_{L^2\times L^2} + \varepsilon\|f\|_{L^2(Q)\times L^2(Q)}\big)^2 \\ &\qquad + \frac{1}{a}\sqrt{\frac{2}{L}}\big(\sqrt{T}\|W\|_{L^2\times L^2} + \varepsilon\|f\|_{L^2(Q)\times L^2(Q)}\big) \\ &\qquad \times\left[\Big(\sum_{j=1}^{n}\int_0^T y_{jn}^2\,\mathrm{d}t\Big)^{1/2} + \Big(\sum_{j=1}^{n}\int_0^T z_{jn}^2\,\mathrm{d}t\Big)^{1/2}\right],\end{aligned}$$

$$\begin{aligned}(3.26)\quad & \Big(\sum_{j=1}^{n}\int_0^T j^2y_{jn}^2\,\mathrm{d}t\Big)^{1/2} + \Big(\sum_{j=1}^{n}\int_0^T j^4z_{jn}^2\,\mathrm{d}t\Big)^{1/2} \\ &\qquad \leqslant C_3\big(\sqrt{T}\|W\|_{L^2\times L^2} + \varepsilon\|f\|_{L^2(Q)\times L^2(Q)}\big).\end{aligned}$$

In particular,

$$\begin{aligned}(3.27)\quad & \Big(\sum_{j=1}^{n}\int_0^T y_{jn}^2\,\mathrm{d}t\Big)^{1/2} + \Big(\sum_{j=1}^{n}\int_0^T z_{jn}^2\,\mathrm{d}t\Big)^{1/2} \\ &\qquad \leqslant C_3\big(\sqrt{T}\|W\|_{L^2\times L^2} + \varepsilon\|f\|_{L^2(Q)\times L^2(Q)}\big),\end{aligned}$$

$$(3.28)\quad \|x_n\|^2_{(L^2(0,T))^n\times(L^2(0,T))^n} \leqslant C_3^2\big(\sqrt{T}\|W\|_{L^2\times L^2} + \varepsilon\|f\|_{L^2(Q)\times L^2(Q)}\big)^2$$

$$\begin{aligned}(3.29)\quad & \sum_{j=1}^{n}\int_0^T j^2y_{jn}^2\,\mathrm{d}t + \sum_{j=1}^{n}\int_0^T j^4z_{jn}^2\,\mathrm{d}t \\ &\qquad \leqslant C_4\big(\sqrt{T}\|W\|_{L^2\times L^2} + \varepsilon\|f\|_{L^2(Q)\times L^2(Q)}\big)^2.\end{aligned}$$

By (3.24) and (3.28) we have

$$\|w_n\|_{H^1(Q)\times H^1(Q)} \leqslant C_5\big(\sqrt{T}\|W\|_{L^2\times L^2} + \varepsilon\|f\|_{L^2(Q)\times L^2(Q)}\big) \tag{3.30}$$

Computing the scalar product of (3.20) with $\ddot{x}_n$, we get

$$\begin{aligned} m_1\sum_{j=1}^{n}\int_0^T \ddot{y}_{jn}^2\,\mathrm{d}t + m_2\sum_{j=1}^{n}\int_0^T \ddot{z}_{jn}^2\,\mathrm{d}t + \lambda\int_0^T (\varphi(u_n-v_n),\ddot{u}_n-\ddot{v}_n)\,\mathrm{d}t \\ = a_1\frac{\pi^2}{L^2}\sum_{j=1}^{n}\int_0^T j^2\dot{y}_{jn}^2\,\mathrm{d}t + a_2\frac{\pi^4}{L^4}\sum_{j=1}^{n}\int_0^T j^4\dot{z}_{jn}^2\,\mathrm{d}t + \sum_{j=1}^{n}\xi_{1j}\int_0^T \ddot{y}_{jn}\,\mathrm{d}t \\ + \sum_{j=1}^{n}\xi_{2j}\int_0^T \ddot{z}_{jn}\,\mathrm{d}t + \varepsilon\sum_{j=1}^{n}\int_0^T f_{1j}\ddot{y}_{jn}\,\mathrm{d}t + \varepsilon\sum_{j=1}^{n}\int_0^T f_{2j}\ddot{z}_{jn}\,\mathrm{d}t. \end{aligned} \tag{3.31}$$

Let

$$\Phi_j(t) = (\varphi(u_n-v_n),\omega_j), \quad 1\leqslant j\leqslant n.$$

Using (3.30) and the imbedding $H^1(Q)\hookrightarrow L^2(Q)$ we have

$$\|\Phi_j\|_{L^2(0,T)} \leqslant C_6\big(\sqrt{T}\|W\|_{L^2\times L^2} + \varepsilon\|f\|_{L^2(Q)\times L^2(Q)}\big)^p.$$

Then we obtain by (3.31):

$$\begin{aligned} \Big(\sum_{j=1}^{n}\int_0^T \ddot{y}_{jn}^2\,\mathrm{d}t\Big)^{1/2} + \Big(\sum_{j=1}^{n}\int_0^T \ddot{z}_{jn}^2\,\mathrm{d}t\Big)^{1/2} \\ \leqslant C_7(n)\big(\sqrt{T}\|W\|_{L^2\times L^2} + \varepsilon\|f\|_{L^2(Q)\times L^2(Q)}\big)^p \end{aligned} \tag{3.32}$$

and

$$\|\ddot{x}_n\|^2_{(L^2(0,T))^n\times(L^2(0,T))^n} \leqslant C_7^2(n)\big(\sqrt{T}\|W\|_{L^2\times L^2} + \varepsilon\|f\|_{L^2(Q)\times L^2(Q)}\big)^{2p} \tag{3.33}$$

where $C_7(n)$ is a constant depending on n.

The estimates (3.24), (3.28) and (3.33) show that

$$\begin{aligned} \|x_n\|_{(H_T^2)^n\times(H_T^2)^n} = (\|x_n\|^2_{(L^2(0,T))^n\times(L^2(0,T))^n} + \|\dot{x}_n\|^2_{(L^2(0,T))^n\times(L^2(0,T))^n} \\ + \|\ddot{x}_n\|^2_{(L^2(0,T))^n\times(L^2(0,T))^n})^{1/2} \end{aligned}$$

is bounded for all possible solutions of (3.20) and this implies that (3.19) has a solution.

Step 3. Estimates on the Galerkin approximations.

Let

$$w_n(x,t) = \begin{bmatrix} \sum_{j=1}^{n} y_{jn}(t)\omega_j \\ \sum_{j=1}^{n} z_{jn}(t)\omega_j \end{bmatrix}$$

be the solution of the system (3.7)–(3.10) where the $y_{jn}(t)$, $z_{jn}(t)$, $1 \leqslant j \leqslant n$ satisfy (3.13)–(3.16) and the estimates we have just found.

We obtain from (3.24), (3.29):

$$\begin{aligned}(3.34)\qquad & \|w_n\|^2_{L^2(T;H_0^1)\times L^2(T;H^2\cap H_0^1)} + \|\dot{w}_n\|^2_{L^2(T;L^2)\times L^2(T;L^2)} \\ & \leqslant C_8\big(\sqrt{T}\|W\|_{L^2\times L^2} + \varepsilon\|f\|_{L^2(Q)\times L^2(Q)}\big)^2.\end{aligned}$$

From (3.34) it follows that there exists $t_n \in [0,T]$, such that

$$\begin{aligned}(3.35)\qquad & a_1\|v_n(t_n)\|^2_{H_0^1} + m_1\|\dot{v}_n(t_n)\|^2_{L^2} + a_2\|u_n(t_n)\|^2_{H^2\cap H_0^1} \\ & + m_2\|\dot{u}_n(t_n)\|^2_{L^2} \leqslant \frac{C_9}{T}.\end{aligned}$$

On the other hand, as u_n, v_n satisfy (3.7), (3.8), respectively, we have

$$\begin{aligned}(3.36)\qquad & \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}(m_1\|\dot{v}_n\|^2_{L^2} + a_1\|v_n\|^2_{H_0^1}) + b_1\|\dot{v}_n\|^2_{L^2} - (\varphi(u_n - v_n), \dot{v}_n) \\ & = (W_1, \dot{v}_n) + \varepsilon(f_1, \dot{v}_n), \\ (3.37)\qquad & \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}(m_2\|\dot{u}_n\|^2_{L^2} + a_2\|u_n\|^2_{H^2\cap H_0^1}) + b_2\|\dot{u}_n\|^2_{L^2} + (\varphi(u_n - v_n), \dot{u}_n) \\ & = (W_2, \dot{u}_n) + \varepsilon(f_2, \dot{u}_n).\end{aligned}$$

Integrating the equalities (3.36) and (3.37) over any subinterval (t_n, t) or (t, t_n) of $(0,T)$ and using (3.34) and (3.35) we get

$$(3.38)\qquad m_1\|\dot{v}_n(t)\|^2_{L^2} + a_1\|v_n(t)\|^2_{H_0^1} \leqslant C_{10},$$

$$(3.39)\qquad m_2\|\dot{u}_n(t)\|^2_{L^2} + a_2\|u_n(t)\|^2_{H^2\cap H_0^1} \leqslant C_{11}.$$

Then, by (3.38), (3.39), we have

$$(3.40)\qquad \|w_n\|^2_{L^\infty(T;H_0^1)\times L^\infty(T;H^2\cap H_0^1)} + \|\dot{w}_n\|^2_{L^\infty(T;L^2)\times L^\infty(T;L^2)} \leqslant C_{12}.$$

Step 4. Convergence of the Galerkin approximations.

The estimates (3.40), (3.30) imply that

$\{w_n\}$ is bounded in $L^\infty(T;H_0^1) \times L^\infty(T;H^2 \cap H_0^1)$;

$\{\dot{w}_n\}$ is bounded in $L^\infty(T; L^2) \times L^\infty(T; L^2)$;
$\{w_n\}$ is bounded in $H^1(Q) \times H^1(Q)$.

Thus, by virtue of Lemma 4 of [1] and using the classical diagonal procedure, we can extract a subsequence $\{w_m\}$ of $\{w_n\}$ with the properties

$w_m \to w = [v, u]^T$ in $L^\infty(T; H_0^1) \times L^\infty(T; H^2 \cap H_0^1)$ weak∗;
$\dot{w}_m \to \dot{w} = [\dot{v}, \dot{u}]^T$ in $L^\infty(T; L^2) \times L^\infty(T; L^2)$ weak∗.

Furthermore, the injection $H^1(Q) \hookrightarrow L^2(Q)$ is compact and as $w_m \to w$ in $L^2(Q) \times L^2(Q)$ weak∗, it follows that

$$w_m \to w = [v, u]^t \text{ in } L^2(Q) \times L^2(Q) \quad \text{strongly and a.e.}$$

Now, let $\theta = [\varrho, \psi]^t$ be any function of the form

$$\theta(x,t) = \begin{bmatrix} \varrho(x,t) \\ \psi(x,t) \end{bmatrix} = \begin{bmatrix} \sum\limits_{i=1}^{N} \varrho_i(t)\omega_i(x) \\ \sum\limits_{i=1}^{N} \psi_i(t)\omega_i(x) \end{bmatrix}$$

where $\varrho_i(t)$, $\psi_i(t)$, $1 \leqslant i \leqslant N$, are continous, T-periodic functions with $L^2(0,T)$-derivatives $\dot{\varrho}_i(t)$, $\dot{\psi}_i(t)$, respectively.

By construction, for $m \geqslant N$ we have

$$\begin{aligned} -m_1\int_0^T (\dot{v}_m, \dot{\varrho})\,dt + a_1\int_0^T (v_m', \varrho')\,dt + b_1\int_0^T (\dot{v}_m, \varrho)\,dt - \iint_Q \varphi(u_m - v_m)\varrho\,dx\,dt \\ = \iint_Q W_1\varrho\,dx\,dt + \varepsilon \iint_Q f_1\varrho\,dx\,dt, \\ -m_2\int_0^T (\dot{u}_m, \dot{\psi})\,dt + a_2\int_0^T (u_m'', \psi'')\,dt + b_2\int_0^T (\dot{u}_m, \psi)\,dt + \iint_Q \varphi(u_m - v_m)\psi\,dx\,dt \\ = \iint_Q W_2\psi\,dx\,dt + \varepsilon \iint_Q f_2\psi\,dx\,dt \end{aligned}$$

and this implies that the equations (3.5), (3.6) hold for $w = [v, u]^t$ and $\theta = [\varrho, \psi]^t$ as above.

By a density argument the last equalities imply that the equations (3.5), (3.6) hold for those θ's mentioned in Definition 1 as well. □

Theorem 3. *In addition to the assumptions of Theorem 2, assume that f has a derivative $\dot{f} \in L^\infty(T; L^2) \times L^\infty(T; L^2)$ and that $\varphi \in C^1(\mathbb{R}, \mathbb{R})$, its derivative being nonnegative and bounded. Then the problem (3.1)–(3.4) has a solution*

$$w = [v, u]^t \in L^\infty(T; H^2 \cap H_0^1) \times L^\infty(T; \overset{\frown}{H}),$$

with

$$\dot{w} = [\dot{v}, \dot{u}]^t \in L^\infty(T; H_0^1) \times L^\infty(T; H^2 \cap H_0^1), \ \ddot{w} = [\ddot{v}, \ddot{u}]^t \in L^\infty(T; L^2) \times L^\infty(T; L^2)$$

and (3.1)–(3.2) hold a.e. on Q.

Proof. We use the approximations $w_n(t) = [v_n(t), u_n(t)]^t$ $(n = 1, 2, \ldots)$ whose existence has been shown in the preceding theorem. Now we differentiate (3.7) and (3.8) with respect to t to obtain

$$\begin{aligned}
(3.41) \quad & m_1(\dddot{v}_n, \omega_j) + a_1(\dot{v}_n', \omega_j') + b_1(\ddot{v}_n, \omega_j) - (\varphi'(u_n - v_n)(\dot{u}_n - \dot{v}_n), \omega_j) \\
& \quad = \varepsilon(\dot{f}_1, \omega_j), \quad 1 \leqslant j \leqslant n, \\
(3.42) \quad & m_2(\dddot{u}_n, \omega_j) + a_2(\dot{u}_n'', \omega_j'') + b_2(\ddot{u}_n, \omega_j) + (\varphi'(u_n - v_n)(\dot{u}_n - \dot{v}_n), \omega_j) \\
& \quad = \varepsilon(\dot{f}_2, \omega_j), \quad 1 \leqslant j \leqslant n
\end{aligned}$$

and it follows that

$$\begin{aligned}
(3.43) \quad & m_1 \dddot{y}_{jn}(t) + a_1 \frac{\pi^2}{L^2} j^2 \dot{y}_{jn}(t) + b_1 \ddot{y}_{jn}(t) - (\varphi'(u_n - v_n)(\dot{u}_n - \dot{v}_n), \omega_j) \\
& \quad = \varepsilon \dot{f}_{1j}(t), \quad 1 \leqslant j \leqslant n, \\
(3.44) \quad & m_2 \dddot{z}_{jn}(t) + a_2 \frac{\pi^4}{L^4} j^4 \dot{z}_{jn}(t) + b_2 \ddot{z}_{jn}(t) + (\varphi'(u_n - v_n)(\dot{u}_n - \dot{v}_n), \omega_j) \\
& \quad = \varepsilon \dot{f}_{2j}(t), \quad 1 \leqslant j \leqslant n,
\end{aligned}$$

where $\dot{f}_{ij}(t) = (\dot{f}_i(t), \omega_j) \in L^\infty(0, T)$, $i = 1, 2$.

From (3.43) and (3.44) and the hypotheses of the theorem we deduce that

$$x_n(t) = [(y_{jn}(t)), (z_{jn}(t))]^t \in (H^3(0, T))^n \times (H^3(0, T))^n.$$

Furthermore,

$$\begin{aligned}
(3.45) \quad & \ddot{y}_{jn}(0) = \ddot{y}_{jn}(T), \quad 1 \leqslant j \leqslant n, \\
(3.46) \quad & \ddot{z}_{jn}(0) = \ddot{z}_{jn}(T), \quad 1 \leqslant j \leqslant n
\end{aligned}$$

because of the periodicity of f and by (3.13) and (3.14).

Multiplying (3.41) by $\ddot{y}_{jn}$ and (3.42) by $\ddot{z}_{jn}$, adding for $j = 1, \ldots, n$ and integrating over $(0, T)$, we have

$$b_1 \iint_Q \ddot{v}_n^2 \, dx \, dt = \iint_Q \varphi'(u_n - v_n)(\dot{u}_n - \dot{v}_n)\ddot{v}_n \, dx \, dt + \varepsilon \iint_Q \dot{f}_1 \ddot{v}_n \, dx \, dt,$$

$$b_2 \iint_Q \ddot{u}_n^2 \, dx \, dt = - \iint_Q \varphi'(u_n - v_n)(\dot{u}_n - \dot{v}_n)\ddot{u}_n \, dx \, dt + \varepsilon \iint_Q \dot{f}_2 \ddot{u}_n \, dx \, dt.$$

Now, (3.34) and the hypothesis on φ and f imply

$$\|\ddot{w}_n\|^2_{L^2(T;L^2)\times L^2(T;L^2)} \leqslant C_{13}, \tag{3.47}$$

where C_{13} denotes a constant which does not depend on n.

Multiplying now (3.41) by $\dot{y}_{jn}$ and (3.42) by $\dot{z}_{jn}$, adding for $j = 1, \ldots, n$ and integrating over $(0, T)$, we get

$$\begin{aligned} -m_1 \iint_Q \ddot{v}_n^2 \, dx \, dt + a_1 \iint_Q \dot{v}_n'^2 \, dx \, dt &= \iint_Q \varphi'(u_n - v_n)(\dot{u}_n - \dot{v}_n)\dot{v}_n \, dx \, dt \\ &\quad + \varepsilon \iint_Q \dot{f}_1 \dot{v}_n \, dx \, dt, \\ -m_2 \iint_Q \ddot{u}_n^2 \, dx \, dt + a_2 \iint_Q \dot{u}_n''^2 \, dx \, dt &= - \iint_Q \varphi'(u_n - v_n)(\dot{u}_n - \dot{v}_n)\dot{u}_n \, dx \, dt \\ &\quad + \varepsilon \iint_Q \dot{f}_2 \dot{u}_n \, dx \, dt. \end{aligned}$$

By the hypotheses on φ and f and the estimations (3.34) and (3.47), we deduce that

$$\|\dot{w}_n\|^2_{L^2(T;H_0^1)\times L^2(T;H^2\cap H_0^1)} \leqslant C_{14}, \tag{3.48}$$

where C_{14} denotes a constant which does not depend on n.

From (3.47) and (3.48) it follows that there exists $t_n \in [0, T]$ such that

$$a_1 \|\dot{v}_n(t_n)\|^2_{H_0^1} + m_1 \|\ddot{v}_n(t_n)\|^2_{L^2} \leqslant \frac{C_{15}}{T}, \tag{3.49}$$

$$a_2 \|\dot{u}_n(t_n)\|^2_{H^2\cap H_0^1} + m_2 \|\ddot{u}_n(t_n)\|^2_{L^2} \leqslant \frac{C_{16}}{T}. \tag{3.50}$$

Multiply (3.41) by $\ddot{y}_{jn}$ and sum for $j = 1, \ldots, n$. It follows that

$$\begin{aligned} &\frac{1}{2}\frac{d}{dt}(m_1 \|\dot{v}_n\|^2_{L^2} + a_1 \|v_n\|^2_{H_0^1}) + b_1 \|\dot{v}_n\|^2_{L^2} \\ &\qquad - (\varphi'(u_n - v_n)(\dot{u}_n - \dot{v}_n), \ddot{v}_n) = \varepsilon(\dot{f}_1, \ddot{v}_n). \end{aligned} \tag{3.51}$$

Now, multiply (3.42) by $\ddot{z}_{jn}$ and sum for $j = 1, \ldots, n$. We have

$$\tag{3.52} \begin{aligned} \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}(m_2\|\ddot{u}_n\|^2_{L^2} + a_2\|\dot{u}_n\|^2_{H^2\cap H^1_0}) + b_2\|\ddot{u}_n\|^2_{L^2} \\ + (\varphi'(u_n - v_n)(\dot{u}_n - \dot{v}_n), \ddot{u}_n) = \varepsilon(\dot{f}_2, \ddot{u}_n). \end{aligned}$$

Integrating the equalities (3.51) and (3.52) over any subinterval with endpoints t_n, $t \in (0,T)$ (regardless of their order), and arguing as in the step 3 of the proof of Theorem 2, we obtain the estimate

$$\tag{3.53} \|\dot{w}_n\|^2_{L^\infty(T;H^1_0)\times L^\infty(T;H^2\cap H^1_0)} + \|\ddot{w}_n\|^2_{L^\infty(T;L^2)\times L^\infty(T;L^2)} \leqslant C_{17}.$$

On the other hand, the approximations $w_n(t) = [v_n(t), u_n(t)]^t$ $(n = 1,2,\ldots)$ satisfy (3.7) and (3.8), so that

$$\tag{3.54} m_1\ddot{v}_n - a_1 v_n^{(2)} + b_1\dot{v}_n - P_n[\varphi(u_n(t) - v_n(t))] = P_n[W_1 + \varepsilon f_1(t)],$$

$$\tag{3.55} m_2\ddot{u}_n + a_2 u_n^{(4)} + b_2\dot{u}_n + P_n[\varphi(u_n(t) - v_n(t))] = P_n[W_2 + \varepsilon f_2(t)]$$

where P_n denotes the orthogonal projection of L^2 onto $\mathrm{sp}\{\omega_1, \ldots, \omega_n\}$.

Since by our hypotheses we have $f \in C([0,T]; L^2) \times C([0,T]; L^2)$, the relations (3.53), (3.54) and (3.55) imply

$$\tag{3.56} \|w_n\|_{L^\infty(T;H^2\cap H^1_0)\times L^\infty(T;\widehat{H})} \leqslant C_{18}$$

where C_{18} denotes a constant which does not depend on n.

Now, on the basis of (3.53) and (3.56) we may extract from $\{w_n\}$ a subsequence converging to a function w as stated in the theorem. The last assertion is an immediate consequence of passing to the limit in (3.54) and (3.55). □

References

[1] *J. M. Ball*: Initial-boundary value problems for an extensible beam. J. Math. Anal. Appl. *42* (1973), 61–90.

[2] *Q. H. Choi, T. Jung*: On periodic solutions of the nonlinear suspension bridge equation. Differ. Integral Equ. *4* (1991), 383–396.

[3] *P. Drábek*: Jumping nonlinearities and mathematical models of suspension bridges. Acta Math. et Inf. Univ. Ostraviensis *2* (1994), 9–18.

[4] *P. Drábek, H. Leinfelder, and G. Tajčová*: Coupled string-beam equations as a model of suspension bridges. Appl. Math. *44* (1999), 97–142.

[5] *N. Krylová*: Periodic solutions of hyperbolic partial differential equation with quadratic dissipative term. Czechoslovak Math. J. *20(95)* (1970), 375–405.

[6] *A. C. Lazer, P. J. McKenna*: Large amplitude periodic oscillations in suspension bridges: some new connections with nonlinear analysis. SIAM Rev. *32* (1990), 537–578.

[7] *P. J. McKenna, W. Walter*: Nonlinear oscillations in a suspension bridge. Arch. Ration. Mech. Anal. *98* (1987), 167–177.
[8] *P. J. McKenna, W. Walter*: Travelling waves in a suspension bridge. SIAM J. Appl. Math. *50* (1990), 703–715.
[9] *L. Sanchez*: Periodic solutions of a nonlinear evolution equation with a linear dissipative term. Rend. Sem. Mat. Univ. Politec. Torino *37* (1980), 183–191.
[10] *G. Tajčová*: Mathematical models of suspension bridges. Appl. Math. *42* (1997), 451–480.
[11] *E. Zeidler*: Nonlinear Functional Analysis and Its Applications, Vol. I–III. Springer-Verlag, New York, 1985–1990.

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