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# DECISION-MAKING FOR LONG MEMORY DATA IN TECHNICAL-ECONOMIC DESIGN, FRACTALS AND DECISION AREA BUBBLES*

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*Abstract.* Economic and management theories are very often based in their applications on the perception of homogeneity of the application space. The purpose of this article is to query such a conviction and indicate new possible directions of discipline development. The article deals with symbiosis of process and his steering model as a process of management. It is possible that in relative near future it will be necessary to accept approaches and changes in interpretations of decision-making. Applications of fractals can be one of the interesting stimulations.

*Keywords*: dispersion of innovations, investment, decision making, management, cellular automata, fractals, market bubbles, technical economic design, simulations, process and regulation, steering process and management

*MSC 2000*: 62C12, 28A80

Existing economic models operate mostly on the basis of analytic concepts of description, *quantitative* or selected *qualitative* relations. But most of them do not contain build-in *decision-making* mechanisms or connections to possible *management or steering interventions* feasible to be used in a model that enabled the design.

Economic solutions are events running in *time* and *space* (factual space of opportunities, see Fig. 1). Models of technical-economic events are in their classical analytical form based on description of equations systems, matrix models, or models

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based on quantitative formulas and on the use of differential or difference calculus if qualitative descriptions are needed. There exist also other models based on the theory of graphs [10], symbolical logic and sets [7] but also on cellular automats [9], or symbolic verbal oriented models.

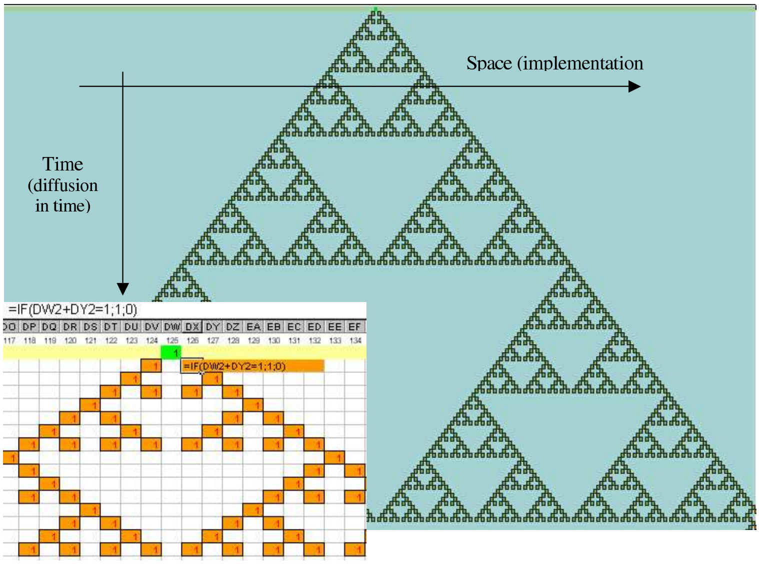


Figure 1. Criterion sum$[(N-1)+(N+1)]$ for investment or innovation, graphical implementation in time and space.

Design, however, means to create new possibilities and new spaces and functions. The economics of design of new technical solutions should solve and anticipate. It should be oriented toward possible future solutions and this is an activity *ex ante* [11]. Modern design has to create values the *Homo economicus* is ready to accept and finally pay for as a purchase price. Modern design is not a question of contemporary trends but a question of ability to create new (*added*) *values*. Any reproduced and repeated solutions that are available in many variations and are matter of mass production have only a decreasing ability to create some *added values* and in its final implication a profit.

On the other hand, the *Homo technicus* is generally aiming at presenting his technical brilliance and skills. The result that might be called *optimal* depends for the most projects on a wide range of asserted *decisions in time and technical-economic space* [13]. Every particular decision (and their sequence) should be chosen optimally. It is a so called necessary condition of optimality in the time ongoing process.

Applied mathematics offers its own ways in terms of model ranking (e.g. linear, non-linear, static, dynamic, ...) [1]. Economics and organization of production processes are using their own instruments. Just an overall look at applications of the theory of graphs and applications of mathematical models from elementary linear models to complicated dynamical behaviour of non-linear economic processes (with cycles of ballance and deterministic chaotic states [3]) reveals interesting applications.

The model that is the right one presents a true picture of functions, opportunities and, by means of the project solution, the designed *added values*. We are in fact seeking for models that represent and refer to those qualities that have an ability to create innovative design, create an added value over the common technical-economic (standard) design and solutions. Economics, unlike technical science, presents values that are partly impermanent (unsustainable). Values are changing in time, vanish and other come into view. Economics requires a description of universal attributes on the one hand and changes its involvements and objectives in time on the other. In its nature it is the technical piece of work that creates long-term *sustainable* values of economic life-cycle of every region, enterprise, and city.

Under conditions for sustainability it is desirable to define not only sets of activities (sets of processes) that operate as a substance (material) transforming the *controlled models* ($\mathbf{P}$), but also comprehensive (derivative) structures of controlling character, i.e., a set of *controlling models* (*steering models*) ($\mathbf{L}$).

Let's delineate the synergetic symbiosis of $\mathbf{P}$ and $\mathbf{L}$ as the *process of management* ($\mathbf{M}$) (see [7], [5] and [6]). To simplify the situation the operating model will be described only in the space of quantitatively derived components created on the basis of the so called networked processes $P_i \implies N_i\langle\mathbf{A},\mathbf{K}\rangle$, where $\mathbf{A}$ represents a set of components with their physical descriptions $\mathbf{U}$ (text, proportions, drawing, plan), dependences in time $\mathcal{D}$ and sets of dependences of quantitative character $\mathcal{Q}$ (see in (1) the second line). $\mathbf{K}$ is the interconnection set (causalities) between components with their set of physical descriptions $\mathbf{V}$ (like the process organisation, technology), construction of connections $\Delta$ (determination of the time sequences, ISO standards etc.) and the starter of connections $\varepsilon$ (the time dependent starting points [like dates of technical, organisational revisions, time dependent cycles of deliveries of materials, spare parts, etc.], volume dependent starting points [the limits in quantity, minima in financial means, etc.]).

In the symbolical notation we deal with a record of *management* like:

$$\mathcal{M}_N = \begin{cases} M_i^N \big| \mathbf{M} = [\langle\varphi(t,\mathbf{P},\mathbf{L})\big|\mathbf{D}^{\sim}\rangle\big|\mathbf{K}^{\sim}], \;\; \mathbf{P} = \langle\mathbf{A},\mathbf{K}\rangle, \\ \mathbf{A} = \langle\mathbf{U},\mathcal{D},\mathcal{Q}\rangle, \\ \mathbf{K} = \langle\mathbf{V},\Delta,\varepsilon\rangle, \\ \mathbf{L} = \langle\cdot\rangle, \\ \mathbf{K}^{\sim} = \langle\mathbf{V}^{\sim},\Delta^{\sim},\varepsilon^{\sim}\rangle, \\ \mathbf{D}^{\sim} = \langle\mathbf{F}^{\sim},\dim(\mathbf{h})\rangle. \end{cases} \tag{1}$$

The operating process built on the class of networking processes $M_i^N$ (the author subdivides processes into the *elementary, networking* and *planning* classes) is executed if and only if *decision making procedures* for selection of the *variant* and *alternative* solutions of possible operating (steering) management interventions created by means of $\varphi(t, P_i, L_i)$, selected by decision-making mechanisms $\mathbf{D}^\sim$ (necessary condition) are separately defined. *Deciding* (about variants of $\varphi(t, \mathbf{P}, \mathbf{L})$) is necessary for reaching the targets. Without it an operational leading process might only use regulative rules for decision-making procedures $\mathbf{D}'$ inside steering processes $\mathbf{L}_{\langle\cdot\rangle}$ provided that the operational steering process was based on a sufficiently efficient level, i.e. it would be on the level $\mathbf{L}_S$ (planning level). A lower *elementary* level $\mathbf{L}_A$ and causal process $\mathbf{L}_K$ (on the basis of network casual relations) does not offer any chances to prevent the operational process from transitioning to degenerative states. The network shaped (designed) process is quite arduous for the scheduling (steering) reason. It requires that the steering (leading) process should substitute (virtually describe) elements of the reality ($\mathbf{P}$-process), should be able to change (recommend to the management body to change) their structure (influence the structure) and at the same time to control the model of the regulative decision part $\mathbf{D}'$. The decision component of the controll block $\mathbf{D}^\sim$ is part of the steering (management) model and in its frame it was also created (designed) for this function. We suppose that it is stable in the development scheme (1) and that a component of information transmission $\mathbf{K}^\sim$ is able to implement in time all necessary communications about managerial information and interventions (decisions, instructions about changers, restructuring of $\mathbf{P}$, that is in reality $\mathbf{A}$, $\mathbf{K}$, and $\mathbf{D}$). The $\mathbf{K}^\sim$ has to secure the transfer of all necessary information for needs of the steering model $\mathbf{L}_{\langle\cdot\rangle}$.

Let us attempt to answer briefly a few partial questions coherent to entry (1) from the time-oriented viewpoint on the decision descriptor $\mathbf{D}^\sim$. There would be certainly interesting to know whether in designing a technical work the *decision processes* have in their use certain specific *properties*. It is important to comprehend this, especially the moment when we are looking for explanation and causes of an unexpected project development. The successful or unsuccessful faties of a number of engineering projects from hands of *Homo economicus* or *technicus* may have its realistic causation out of rational vision, commonly seen. The very character of steering space, its structure in time, may be so specific and inhomogeneous for applications in engineering design that unexpected disturbances of the *technical work sustainability life-cycle* occur. A sequence of decision-making in steering processes (more simply said, management) may have other rules and place for implementation than they are commonly visible at the time. The reasons are mainly economic limits and indicators; however, their duration is mostly only a fragment of the total technical life-cycle of a designed project. Application of designing (formation) might have proceeded in another way by using better rules for selection of their solutions (decisions) inside the steering process ($\mathbf{L}$), at least better than we are using when thinking about these $\mathbf{D}$-rules, which is the main and real reason why this article was written.

In fact, here are some important problems requiring solution:

1. Is the steering area for decision-making homogeneous?
2. Does inhomogeneous decision-making implementation of environment (space, area) influence the decision steering process $\mathbf{D}^{\sim}$?
3. To what extent is the decision process $\mathbf{D}^{\sim}$ in time period $t_x$ interfered (or even determined and programmed) by the previous decisions in the former time layer? Does decision process $\mathbf{D}^{\sim}$ depend on the memory of the decision already made?
4. To what extent will the decision processes $\mathbf{D}^{\sim}$-in time period $t_{x+n}$ (further future) be influenced by the decision in time nearer to the period $t_x$ (see Fig. 1).

Before we try to answer the questions, it is desirable to elucidate the importance and utility of such search of answers. Homo economicus or *technicus* along many generations have worked with (mainly) intuitively or only economically censored technical solutions.

Today the entire contrivance of public competitions in EU, US and the whole series of other countries is already quite consciously based more or less on mechanisms oriented to the assumption about the (purely) homogeneous decision area. The basis of Bayes probability theory [13] was extensively applied for technical-economic decision-making in the second half of the past century [12].

Intentionally searching new productive technique how to protect decisions from unfavourable judgements of an on-coming reviewer, working with better data or facts, we may be critical about the existing methodological status. The course and usage of methods practiced for the technical-economic project design is necessarily going to change. On the basis of *knowledge* of the model a technician and an economist use or can create *steering measures* (interventions) freely in the sense of $\varphi(t, \mathbf{P}, \mathbf{L})$ in (1). More sophisticated methods are in progress (simulation, goal parametrization, optimization, construction of scenarios, etc.).

For further explanation there is no need to distinguish among particular phases in description of a real process ($\mathbf{P}$) or steering processes ($\mathbf{L}$). Completion to a model that is able to generate interventions carrying (propagating) changes from the steering level to the realisation level was denoted in (1) by $\mathbf{M}$. Tools for formation of steering interference $\varphi(t, \mathbf{P}, \mathbf{L})$ may, without requirements for determination of theirs robustness or suitability, be illustrated as follows:

- search for satisfaction of *goals* by means of solutions regardless of the restrictive conditions,

- search for solutions on the basis of *simulations*,
- search for an *optimal solution*,
- search for solutions with regard to the factual conception of future solution along *scenarios.*

Therefore, it is not possible to speak about a single management model class. A number of approaches exist, being different in evidence and efficiency.

Practical modelling of steering intervention for management has not a uniform character and does not rely on a single theoretic or practical pattern.

The submitted paper is an attempt at generalization. The author considers as a *process* every technical-economic presentation of reality. He considers as *steering models* every abstract description of reality employable for elaboration of management interventions (1). In this sense he certainly associates himself with the management school that exploits modelling as a certain instrument for generation of steering proposals. *The assumption of homogeneous application fields providing production resources and application of continuous space and time can be of practical importance for decision making.*

The decision process $\mathbf{D} = \langle \mathbf{F}, \dim(\mathbf{h})\rangle$ or $\mathbf{D}^{\sim} = \langle \mathbf{F}^{\sim}, \dim(\mathbf{h})\rangle$ mentioned already in (1) in the course of $\mathbf{L}$ applications for steering processes requires completion of the corresponding area of homogeneity within which the solution can be created and implemented.

It will be argued later that a decision is not only dependent on homogeneity, but also on memory (Mem). Relevant decisions made in the *past* influence decisions at the *time being.* The symbolic notation given in (1) might be completed for memory phenomenon as (1a).

It would not be, however, correct to creat impression that it is feasible to break away from the existing mathematical theoretic rudiment. Even in modelling of $\mathbf{P}$ and $\mathbf{L}$, the main model streams practised are simplified models. However, implementation and propagation of decisions (implementation of decision interventions) may be done more carefully, if we are aware of the complicated Mem space.

$$\text{(1a)} \qquad \mathcal{M}_N = \begin{cases} M_i^N \big| \mathbf{M} = [\langle \varphi(t, \mathbf{P}, \mathbf{L})(\mathbf{D}^{\sim} \big| \mathrm{Mem})\rangle] \big| \mathbf{K}, \;\; \mathbf{P} = \langle \mathbf{A}, \mathbf{K}\rangle, \\ \mathbf{A} = \langle \mathbf{U}, \mathcal{D}, \mathcal{Q}\rangle, \;\; \mathbf{K} = \langle \mathbf{V}, \Delta, \varepsilon\rangle, \;\; \mathbf{L} = \langle \cdot \rangle, \\ \mathbf{K}^{\sim} = \langle \mathbf{V}^{\sim}, \Delta^{\sim}, \varepsilon^{\sim}\rangle, \;\; \mathbf{D}^{\sim} = \langle \mathbf{F}^{\sim}, \dim(\mathbf{h})\rangle, \\ \mathrm{Mem} = (\text{dBase, time series, } \dots) \end{cases}$$

**Fixed decision rule $\mathbf{D}^{\sim}$ and long memory dependent decision space.** The memory dependent decision space $(\mathbf{D}^{\sim} \big| \mathrm{Mem})$ was written in (1a) as

$$\text{(1b)} \qquad [(\mathbf{D}^{\sim} \big| \mathrm{Mem}), \;\; \mathrm{Mem} = \text{dBase, time series, } \dots)].$$

It is worth noting that any decision rule influences the structure (location in time and placement of implementation) of $\mathbf{P}$. We speak about

- dispersion in time,
- value change of implemented action (activities) in time,
- present value change according to profit rate (discounted value),
- changed value in time according to construction of decision rule,
- changed value in time according to stick to memory of input data.

In the sense of (1) there exists a real manufacturing or investment process $\mathbf{P}$ which capitalizes on the existing market and produces a volume parameter $\mathcal{Q}$ (here filling the delimitated market area by products). So the investments, whose purposes are to make use of demand in an economic area, for example, dwelling (house) investment in time periods $t = 1, 2, \ldots$ and separate areas $A$, $B$, $C$, …

Existing demands for goods at a time $t_0$ is filled by volumes of initial investments (houses and lands) in $A$, $B$, $C$, … However, the filling in (implementation) is limited by resources (capital) that are available. For simplicity we can suppose that the market space can be, from the voluminous point of view ($\mathcal{Q}$), filled and will have value 1, or unfilled and will have value 0. We can write that management decision will be for example $A^{\mathcal{Q}} = 1$, or $A^{\mathcal{Q}} = 0$, further $B^{\mathcal{Q}} = 1$, or $B^{\mathcal{Q}} = 0$. In that situation we can write down the vector of the starting action state for $P_{t=0}(A, B, C, \ldots)$ as the vector $(1, 1, 1, 1, 1, 1, 1, \ldots)$ when it denotes the fully *filled-out space of potential needs.* The vector initialising state processes $\mathbf{P}_{t=0} = (0, 0, 0, 0, 0, 0, 0, \ldots)$ expresses an *empty space of possible needs* (for example of housing in the initial time period and sites $A$, $B$, $C$, … ). Supposed that the steering process evaluate by 1 indicate for example realization of new capital assets or innovations into industrial areas $A$, $B$, $C$, … Implementation areas are reordered (sorted) into the chain of the respective importance for the decision-maker (for example, according to his investment interests). New ordered areas are $M$, $N$, $O$ ($M$ as the neighbouring left, $N$ is the position of the decision maker, $O$ the neighbouring right).

The decision rules $\mathbf{D}^{\sim}$ might be constructed on the basis of very diversified classes of comparisons.

The practicing economy uses mainly criteria on the basis of *summation* or on *maximization* of advantages (max of interest (ROI), max rate of return, etc.). We might speak about $\mathbf{D}_{\text{sum}}$, $\mathbf{D}_{\text{max}}$, $\mathbf{D}_{\text{min}}$, but also about decision conditioned by risk, uncertainty or long memory[1] ($\mathbf{D}_{\text{sum}} \middle| \text{risk}$), ($\mathbf{D}_{\text{sum}} \middle| \text{LMem}$). However, there are other approaches in many applications available. There are criteria as security, ecology, ethics, aesthetics, prestige, domination, long duration, sustainability, robustness, etc. If we apply different criteria, different results are obtained. Illustrative examples are

[1] Memory aspects are in economic applications very frequently substituted by uncertainty (discount, decay multiplier, etc.).

given in Fig. 1 and 2. The simulation in Fig. 1 is based on a criterion that might be commented as follows:

**if** for actual sector $N$ and given time interval $t_n$ to $t_{n-1}$ there was free *space* for expansion in sector $(N-1)$ or in sector $(N+1)$

**and**

at $t_{n-1}$ there was positive *experience* for expansion in $(N-1)$ or in $(N+1)$ (read innovation or investment opportunity exist)

**then** further investment (innovation dispersion) is obtainable.

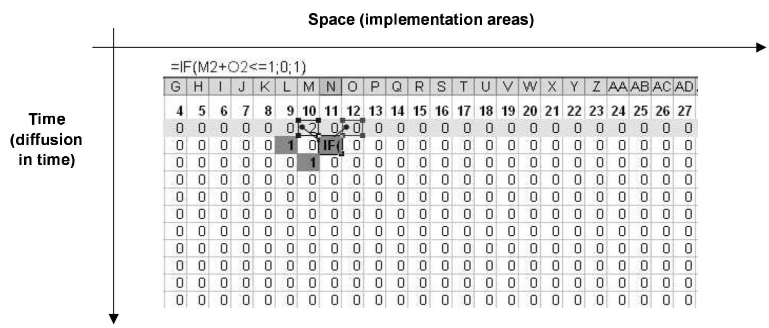


Figure 2. Criteria sum$[(N-1)+(N+1)]$ for investment or innovation, graphical implementation in time and space.

We will write this criterion as $\mathbf{D}^{\sim}_{\text{sum}=\alpha} = (\mathbf{F}^{\sim}, \dim(\mathbf{h}))$. Graphical trace (simulation) of this criterion is given in Fig. 1. This and all further mentioned criterion simulations start expansion processes from the single kernel cell $\mathbf{P}_{t=0} = (0, 0, 0, \ldots 0, 1, 0, \ldots)$. The construction of a criterion means to develop formulas controlling the volume of gains that were available in the past and to anticipate future implementation development. Such a criterion is constructed and simulated in Fig. 1. Fig. 2 shows that a slightly different criterion might result in a totally different result. However this criterion looks like an economic one it is not much friendly to the dispersion of activities in time. The stop of dispersion activity in the second time step is the result.

If we observe Fig. 1 in more detail it is obvious that the kind of criteria builds up inhomogeneous implementation space. Significant are empty application areas, let us call them *market bubbles*. An interesting information about the application space (structure) is that the implementation frequency in time (sum of filled horizontal fields) is not steadily increasing but is oscillating in time. In Fig. 3, the middle curve in the chart shows the sum of filled fields (say successful applications, frequency of successful applications) that were achieved in time period $t_x$. This characteristic was assigned as the first virtual characteristic $A^{sQ}$, change of this characteristic was assigned as the second virtual characteristic $A^{s2Q}$ and is shown in the upper

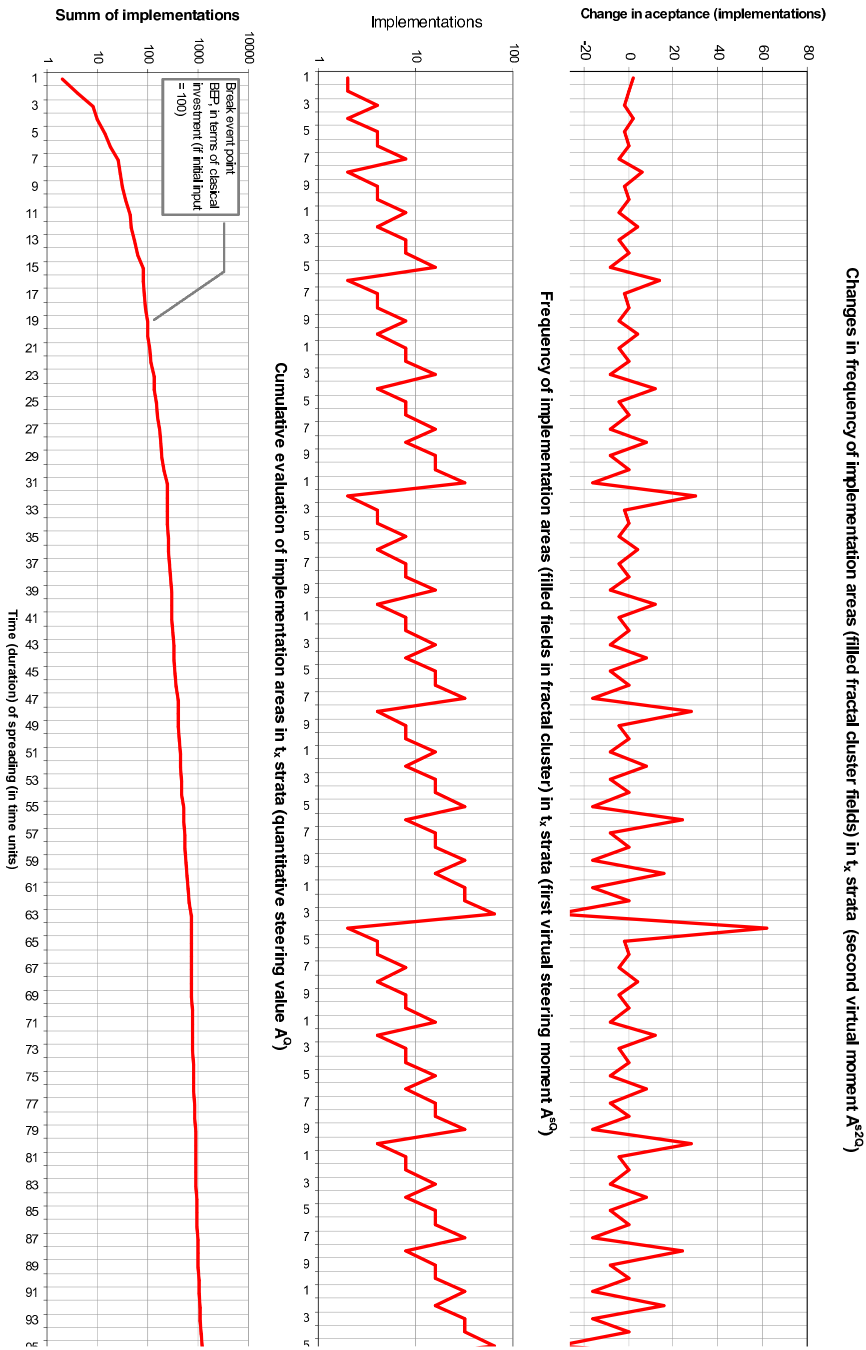


Figure 3. Changes in frequency of implementation areas (filled fractal cluster fields) in $t_x$ strata (second virtual moment $A^{s2Q}$).

time series in Fig. 3. The quantitative judgement (the one most frequently used in economy) is described in the lower time series in Fig. 3. Cumulative curve of advantages gained in time (characteristic described as quantitative result $A^Q$).

Other results on the basis of the criterion $\mathbf{D}^{\sim}_{\mathrm{sum}=\alpha}$ is shown in Fig. 4 and $\mathbf{D}^{\sim}_{\mathrm{sum}<\alpha}$ in Fig. 5.

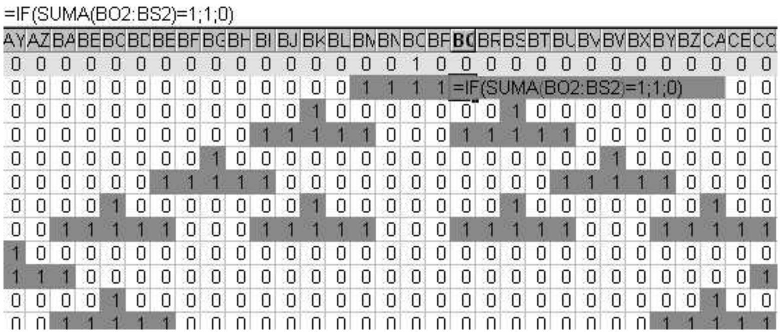


Figure 4. Criterion sum$[(N-2)+(N-1)+N+(N+1)+(N+2)]$ for investment or inovation, graphical implementation in time and space.

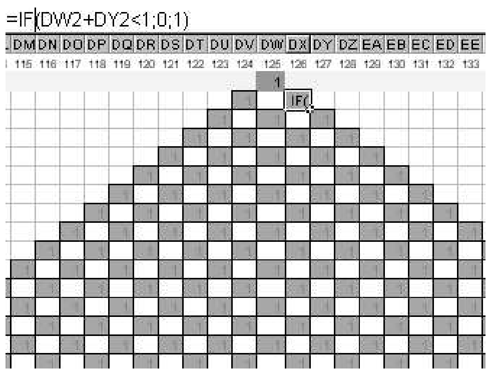


Figure 5. Criterion sum$[(N-1)+(N+1)]$ for investment or innovation, graphical implementation in time and space.

Let us suppose that the solution for which a decision-maker decides is a new solution in a quite vacant demand market (situation described in Figs. 1, 2, 4 and 5). Say we have a fully new product like a handy phone on a totally wire framed phone market or a PC on a mainframe computer market a few decades ago. Moreover, the market area will be limited in the short future neither by any boundary conditions for demands nor even by market exhaustion. A diagrammatic calculation according to (1a) is shown in Fig. 1 (a segment from a more extensive calculation how the market will be filled up in time (vertical axis) and in the space of steering decisions for management shows demand opportunities (horizontal axis)). The applied decision criterion $\mathbf{D}^{\sim}_{\text{sum}} = (\mathbf{F}^{\sim}, \dim(\mathbf{h}))$ in Figs. 1, 2 and 5 is a criterion of two dimensions. The *sum* criterion tests $F_1$ and $F_2$ ($F_1$ tests whether during the last time period the action on the left of the decision makers area $M$ occurred and $F_2$ tests whether during the past time period the action on the right from the area of the decision maker occurred). The metric of evaluation $d$ and the set of evaluations $\mathbf{h}$ are given in this case as 1 and 0 (see expression (1)).

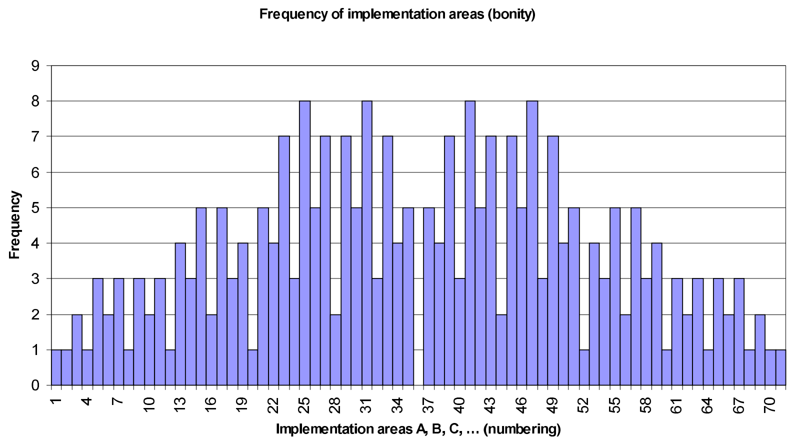


Figure 6. Dispersion of an innovation (investment), measured in frequency of applications for 35 time units.

If we are looking for indicators how the dispersion of a solution is able to spread, it is necessary to analyse information given in Fig. 3. The first available information might be the frequency of applications for application areas $(A, B, C, \ldots, M, N, O, \ldots)$. Fig. 6 displays these frequencies for the sequence from $t = 1$ till $t = 35$. It is interesting that dispersion occurs in waves. Dispersion space and destination of implementations is not homogeneous. This result has important consequences for decision-making.

**Interpretation.** Some interesting facts:

- Dispersion area (time and space) on the basis of the decision-making criterion $D^{\sim}_{\text{sum}}$ is not homogeneous.
- In the area of dispersion there are spaces with no change of implementation (*dispersion holes and dispersion bubbles*).
- Every area of implementation for the (capital, innovation) activity has other *life-cycle* in time.
- Lifetime of activity in area $X$ is not given only by the life-cycle of the process $\mathbf{P}(\cdot)$, or by means of the standards of the steering process $\mathbf{L}(\cdot)$; it is condi-

tioned by the *decision criterion* $\mathbf{D}^{\sim}$ and its mechanism accepted for the steering area and *vice versa* for the management of the task. It is not difficult to prove that almost every *criterion forms* another *fractal space* in time and place.

- Dispersion in time and space creates dispersion waves (changes in intensity), see Fig. 7.

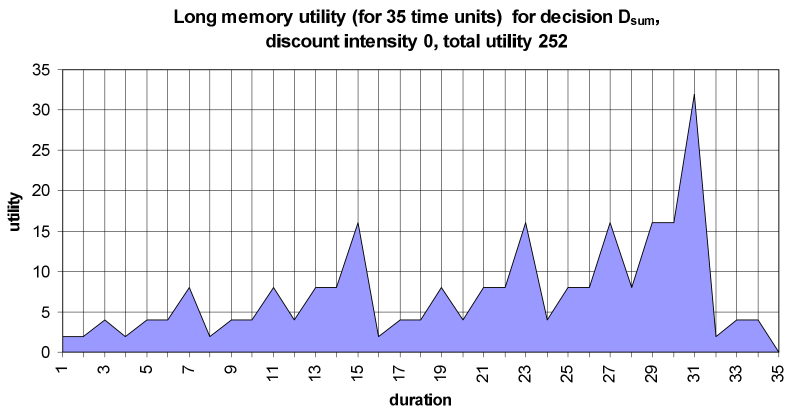


Figure 7. Utility for the sequence of 35 time units (compare with Fig. 3).

**Conclusion.** Economic and management theories of technical-economic processes are very often based in their applications on the perception of homogeneity of the application space. A number of technical and economic projects are outlined under the condition that all areas of implementation of management decisions are homogeneous and the rules that were valid in analogous cases in the past would be valid also in the possible future designs and projects [14], [15]. The purpose of this article is to query such a conviction and indicate new possible directions of discipline development. It intends nothing else than to iniciate new orientation for consideration and thinking in new directions. After the model interpretation of the deterministic chaos in management it was necessary to see the whole series and passages of areas of the technical-economic management in other highlights [3]. Competition evaluation, crisis management decisions in a complicated technical-economic project bring new and new in practice justifiable doubts. It is, however, possible that also in the relatively near future it will be necessary to accept approaches and changes in interpretations of more powerful implementation of interferences between the decision-making and implementation areas. Applications of fractals can be one of the interesting stimulations [2].

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