Book Reviews

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BOOK REVIEWS

Jean-François Le Gall: SPATIAL BRANCHING PROCESSES, RANDOM SNAKES AND PARTIAL DIFFERENTIAL EQUATIONS. Lectures in Mathematics. ETH Zürich, Birkhäuser Verlag, Basel 1999, x+163 pages. ISBN 3-7643-6126-3, price DM 44,–.

Measure-valued branching processes (or superprocesses) have recently attracted a considerable attention, due both to the beauty and richness of their theory and their relations to various problems in probability and analysis, notably to partial differential equations. J.-F. Le Gall has made many important contributions to the theory of superprocesses. In particular, a construction of a superprocess using a snake—a path-valued process which, intuitively, describes the "history of all particles" in the superprocess—was proposed by him. In the slim book under review, this approach is developed systematically and in a self-contained manner, so that it is accessible to a reader with no preliminary knowledge of measure-valued processes. Confining himself mainly to a quadratic branching mechanism, the author presents the basic ideas in a very lucid way.

Chapter I offers an informal overview of the topics discussed in the book and of the relations of the subject to other parts of probability theory. In the second chapter, continuous-state branching processes and superprocesses are introduced, their existence is proven via approximations by branching particle systems, and their basic properties are studied. In Chapter III it is shown that the genealogical structure of the superprocess may be coded by Brownian excursions, this result being used in the next chapter where the construction of a measure-valued branching process based on the Brownian snake is carried out. Chapters V–VII are devoted to a thorough study of the interplay between the super-Brownian motion and the elliptic problem $\Delta u = 4u^2$. Finally, the last chapter treats, from a similar point of view, superprocesses with a more general branching mechanism.

We may recommend the book to everybody desiring to get acquainted with the rapidly advancing theory of measure-valued processes and their applications.

Ivo Vrkoč

Michael Demuth, Jan A. van Casteren: STOCHASTIC SPECTRAL THEORY FOR SELFADJOINT FELLER OPERATORS. A functional integration approach. Probability and its Applications. Birkhäuser Verlag, Basel 2000, xii+463 pages. ISBN 3-7643-5887-4, price DM 198,–.

Let E be a locally compact separable metrizable space and μ a Radon measure on E. Consider an operator $-K_0$ in $L^2(\mu)$ generating a self-adjoint semigroup on $L^2(\mu)$ which is Feller (maps the space of continuous functions on E vanishing at infinity into itself) and possesses an integral representation. The main aim of the book under review is an in-depth investigation of the spectral theory of zeroth-order perturbations $-(K_0 \dot{+} V)$ of $-K_0$ by using probabilistic tools, in particular Markov processes theory. Two types of perturbations are studied, regular potentials V that belong to the Kato-Feller class, and singular ones which may be infinite on some sets. The operators $-(K_0 \dot{+} V)$ are introduced as infinitesimal generators of semigroups which are defined via Feynman-Kac formulas involving the Markov process with the generator $-K_0$.

In Chapter 1 of the book, the basic assumptions on the operator K_0 are listed and it is shown that many important particular models satisfy these hypotheses. The next two chapters are devoted to the construction of the perturbed operators $-(K_0 \dot{+} V)$. The results on spectra and spectral data of these operators are based on estimates of the corresponding resolvent and semigroup differences in operator, Hilbert-Schmidt or nuclear norm, these estimates being developed in Chapters 4–7. In the final, eighth chapter the spectral and scattering properties of the operators $-(K_0 \dot{+} V)$ are analysed. Five appendices, almost one hundred pages long, survey part of the results from functional analysis and probability theory which are applied in the book and contribute to the fact that it is reasonably self-contained. The book is written in a lucid manner and sufficiently detailed, nevertheless, as the authors fairly state in their Preface, the reader is presupposed to have some mathematical maturity in spectral theory, operator semigroups, martingale and Markov process theory.

The monograph by M. Demuth and J. van Casteren stemmed from the joint research the authors have pursued during the last ten or more years and it is another beautiful illustration of the strength of stochastic methods in analysis of partial differential operators.

Ivo Vrkoč

A. B. Cruzeiro, J.-C. Zambrini (eds.): STOCHASTIC ANALYSIS AND MATHEMATICAL PHYSICS. Progress in Probability Vol. 50. Birkhäuser Verlag, Boston 2001, 158 pages. ISBN 0-8176-4246-3, price DM 210,–.

The proceedings under review comprise nine papers which are, according to Preface, related to a meeting organized in Lisbon by the Group of Mathematical Physics. (Surprisingly, no precise information about the meeting is given in the book.) Six of the papers are full-length papers with proofs. A paper by R. Rebolledo is a brief introduction to noncommutative stochastic analysis, the remaining two papers being announcements of results that will appear elsewhere. The articles are devoted to diverse topics in stochastic analysis, in part reaching over to mathematical physics. The covered subjects include stochastic Volterra equations with singular kernels, measure-preserving shifts on abstract Wiener spaces, stochastic cohomology theories, infinite-dimensional Nelson diffusions, and stochastic wave equations. Let us list the contributors: H. Airault, L. Coutin, L. Decreusefond, R. Léandre, C. Léonard, P. Lescot, P. Malliavin, M. Oberguggenberger, R. Rebolledo, F. Russo, A. S Üstünel and L. M. Wu.

We may only regret that because of the high price the papers will be hardly accessible to interested readers.

Bohdan Maslowski

George A. Anastassiou, Sorin G. Gal: APPROXIMATION THEORY. MODULI OF CONTINUITY AND GLOBAL SMOOTHNESS PRESERVATION. Birkhäuser Verlag, Boston 2000, xi+525 pages. ISBN 0-8176-4151-3, DM 188,–.

For more than a century the approximation theory has attracted attention of mathematicians, being permanently developed and applied in mathematical, functional and numerical analysis. The monograph concentrates on extensive and comprehensive investigation of two topics in the theory: computational aspects of the moduli of smoothness and the Global Smoothness Preservation Property.

The Introduction presents necessary definitions, surveys some of the main results and shows a motivation.

Part I is devoted to the calculus of the moduli of smoothness in various classes of functions including uniform moduli of smoothness, L^p-moduli, weighted moduli and various moduli of special types. While using convexity properties of the moduli the authors systematically avoid the K-functional method because it provides constants in non-explicit forms.

The notion of the Global Smoothness Preservation Property (GSPP) has the following meaning: Let $U \subseteq \mathbb{R}^n$ and let $\theta(U)$ be tbe space of real functions on U. Let $L_n\colon \theta(U) \to \theta(U)$, $n \in \mathbb{N}$, be a sequence of linear operators that converge in a certain sense to the unit operator. It is said that the operators L_n possess the GSPP if this approximation is nice in the sense that $L_n(f)$ do not ripple more than f, i.e. the graphs of $L_n(f)$ come close to the graph of f while oscillating no more than the graph of f. Part II concentrates on the investigation of GSPP for a large variety of linear approximation operators including trigonometric and algebraic interpolation operators of Lagrange, Hermite-Fejér and Shepard type, operators of stochastic or convolution type, wavelet type integral operators, singular integral operators, etc. The authors present many applications to the approximation theory, probability theory, numerical and functional analysis, and computer-aided geometric design. Methods of Part I allow exact calculation of the error of global smoothness preservation.

It seems that such a comprehensive survey of calculus of moduli of continuity and of GSSP was presented for the first time. The monograph contains the research of both authors over the past ten years as well as numerous references of other results in these areas. Many of them appear in the book form for the first time.

Most of the twenty sections are amended by applications, bibliographical remarks and open problems. The text is accessible to graduate students.

Jiří Rákosník

Kai Borre: PLANE NETWORKS AND THEIR APPLICATIONS. Birkhäuser Verlag, Basel-Boston, 2001, 184 pages. ISBN 3-7643-4193-9 (Basel), ISBN 0-8176-4193-9 (Boston), price DM 108,–.

The book deals with geodetic networks, namely with tools for error propagation analysis under various network boundary conditions. Another goal is to show that mathematical means used in discrete network models, i.e., networks with a few hundred points, have their natural counterparts in continuous network models, where the number of points is so large that a differential operator based approach becomes more appropriate.

This programme is well illustrated in Introduction (Chapter 1). Also, fundamental notions as weighted least squares, normal equations, singular value decomposition, pseudoinverse, differential equations, and Green's function are introduced there. Chapter 2 focuses on discrete networks and, in particular, on eigenvalues and eigenvectors of the normal equations matrix. One and two dimensional models are considered. The transformation of discrete two dimensional networks into continuous analogues forms the contents of Chapter 3. The result for leveling networks, i.e., networks where height differences between points are observed, is the Poisson equation. To perform error analysis, Green's function is constructed for various regions. Distance and azimuth networks are also treated. The network with relative observations, the corresponding differential equation problem, and its fundamental as well as finite element solutions are the subject of Chapter 4. As the condition number derived from the spectrum of the normal equations matrix is used as a measure of a good or a bad network design, it is important to investigate the eigenvalue distribution. This is briefly done in Chapter 5. Finally, Chapter 6 deals with specific examples illustrating the theory. A separate section comprises sixteen exercises which can be solved by MATLAB M-files downloadable from the author's web site. Bibliography (62 items), author index, and subject index follow.

Though each chapter is carefully directed from simple problems to more complex situations, the book under review is not a comprehensive textbook on geodetic network theory. It does not explain all the relevant details from the realms of geodesy and mathematics. Instead of that, the author frequently summarizes only the necessary starting facts and refers to other sources for particulars. Thus the exposition is made terse and straightforward but somewhat fast or fuzzy at some places, where a reader not familiar with a geodesic or a mathematical background might appreciate a few explanatory lines to get a better insight. The use of mathematics is limited to a graduate student level and to a rather classical or engineering approach.

Making a transition from discrete networks to continuous networks and to the corresponding boundary value problems, the book opens a way to numerous applications of the finite element method which is well established in solving such problems. The book will certainly inspire both the geodesists interested in mathematical tools applicable in geodesy and the mathematicians, notably numerical analysts, interested in geodesy.

Jan Chleboun

H. Freistühler, A. Szepessy, eds.: ADVANCES IN THE THEORY OF SHOCK WAVES. Birkhäuaser Verlag, Basel-Boston-Berlin 2001, 528 pages. ISBN 0-8176-4187-4, price EUR 105,–.

This book provides a comprehensive introduction as well as a sample of the most recent results in the theory of nonlinear conservation laws and shock waves.

The main topics include.

1. *Well-posedness theory for hyperbolic systems of conservation laws* written by Tai-Ping Liu. The author—one of the leading scientists in the field of conservation laws—provides an excellent survey of recent methods for proving existence and uniqueness for systems of nonlinear conservation laws. Starting with the scalar case, he introduces the Glimm scheme, and the study is completed by the most recent approach of wave tracing.

2. *Stability of multidimensional shocks* by Guy Métivier. The author addresses several stability conditions, well posedness of the linearized shock front equations, the existence of multidimensional shocks as well as stability of weak shocks.

3. *Shock wave solutions of the Einstein equations* by Joel Smoller and Blake Temple. The recent applications of the theory of shock waves in cosmology is presented.

4. *Basic aspects of hyperbolic relaxation systems* written by Wen-An Yong. The main topics include the Chapman-Enskog expansion, admissible boundary conditions, discrete kinetic velocity models, and shock structure problems.

5. *Multidimensional stability of planar viscous shock waves* by Kevin Zumbrun. The author introduces the Evans function, discusses and compares several conditions for stability, pointwise bounds for scalar equations as well as several open problems.

The book provides basic reading for students of mathematics, physics, and theoretical engineeiring, as well as for specialists in the theory of conservation laws.

Eduard Feireisl