Book Reviews

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BOOK REVIEWS

M. G. Nadkarni: BASIC ERGODIC THEORY. Second edition. Birkhäuser Advanced Texts, Birkhäuser-Verlag, Basel, 1998, ix+149 pages, ISBN 3-7643-5816-5, price DM 68,–.

M. G. Nadkarni: SPECTRAL THEORY OF DYNAMICAL SYSTEMS. Birkhäuser Advanced Texts, Birkhäuser-Verlag, Basel, 1998, ix+182 pages, ISBN 3-7643-5817-3, price DM 88,–.

Not only the covers but also the style and aims of these two books by M. G. Nadkarni are so similar that we find it appropriate to review them together.

Basic Ergodic Theory moves from fundamental definitions and theorems to rather advanced topics that are not treated in standard textbooks. The readers are not supposed to have any preliminary knowledge of ergodic theory, but a sound knowledge of measure theory and some familiarity with general topology are necessary. The results are given complete and reasonably detailed proofs, which to achieve in a slim book was possible due to a careful selection of the material, topics that can be easily found elsewhere (as the whole entropic theory of dynamical systems) being omitted. In accordance with the author's research interests, considerable attention is paid to the descriptive set-theoretic approach to ergodic theory. Let us indicate what it means in the simplest case by quoting the famous Poincaré recurrence theorem as it is treated in the first chapter. Let τ be an automorphism of a measurable space $(X,\mathfrak{B})$ (i.e., τ is a measurable bijection such that τ^{-1} is also measurable). A set $W\in\mathfrak{B}$ is called wandering if τ^nW, $n\in\mathbb{Z}$, are pairwise disjoint; denote by $\mathfrak{W}$ the σ-ideal generated by all wandering sets. In this setting, the Poincaré theorem reads as follows: Given $A\in\mathfrak{B}$ there exists $N\in\mathfrak{W}$ such that the set $\{n\geqslant 0;\ \tau^nx\in A\}$ is infinite for each $x\in A\setminus N$. Assuming further that τ preserves a finite measure m and noting that the m-null sets form a σ-ideal that contains $\mathfrak{W}$ we recover the usual measure theoretic version of the recurrence theorem; a Baire category version and a version for conservative automorphisms follow analogously.

To show the scope of the book, it may be useful to describe briefly the contents of the other chapters. In Chapter 2, two proofs of Birkhoff's pointwise ergodic theorem are given and von Neumann's ergodic theorem is discussed (again, two different proofs are provided). The next two chapters introduce the notion of ergodicity and mixing conditions, respectively. The fifth chapter is about Bernoulli shifts. The theorem of Halmos and von Neumann stating that any two ergodic measure preserving automorphisms with discrete spectrum having the same eigenvalues are metrically isomorphic is the main result of Chapter 6. Chapter 7 is devoted to induced automorphisms and related concepts, in particular, the rank of an automorphism is discussed. In the next chapter, the following theorem is established: Let τ be an automorphism of a standard Borel space $(X,\mathfrak{B})$, then there exists a totally disconnected Polish topology t on X such that τ is a homeomorphism of (X,t) and $\mathfrak{B}$ coincides with the Borel sets over (X,t). The Glimm-Effros theorem is proved in Chapter 9. The tenth chapter is devoted to E. Hopf's theorem: an automorphism of X has a finite invariant measure provided X is not compressible. In the course of the proof of Hopf's theorem a third, measure free proof of the pointwise ergodic theorem is given. H. Dye's theorem (any two free ergodic measure preserving automorphims on a standard probability space (X,m) are orbit equivalent mod m) is proven in Chapter 11. The last chapter is devoted to a descriptive version of the Ambrose-Kakutani theorem on representation of flows by flows built under a function.

Notwithstanding that *Spectral Theory of Dynamical Systems* is more a monograph on spectral properties of non-singular automorphisms of measure spaces than a textbook, its general structure remains the same as that of the preceding book. The author leads the reader to some topics of current research trying to choose subjects that have not been treated in the book form yet and to provide relative self-contained and accessible proofs of them. Although the book opens with the Hahn-Hellinger theorem on the complete set of invariants of unitary equivalence of spectral measures on a Hilbert space and with the spectral theorem for unitary operators, the requirements on the reader are higher than in the case of *Basic Ergodic Theory* and the book aims at advanced students or researchers. In his Preface, the author singles out two theorems "which are not on the surface": the one due to H. Helson and W. Parry (if τ is an aperiodic measure preserving automorphism of a standard probability space, then there exists a measurable function φ taking values $+1$, -1 only such that the maximal spectral type of the unitary operator $f \mapsto \varphi(\cdot)f(\tau\cdot)$ is Lebesgue) which is treated in Chapter 6, and the other by B. Host on independent joinings Chapter 10 is devoted to. But the contents of the book is much richer, let us mention at least the in-depth discussion of Riesz products that provide a description of maximal spectral types of rank one automorphisms (Chapter 15).

To conclude, both books by M. G. Nadkarni are a welcome addition to the literature on ergodic theory and are useful to everybody who wants to get acquainted with some recent trends in the field; moreover, an introductory course may be based on the first of them.

Jan Seidler

K. Bichteler: INTEGRATION—A FUNCTIONAL APPROACH. Birkhäuser Advanced Texts, Birkhäuser-Verlag, Basel, 1998, viii+193 pages, ISBN 3-7643-5936-6, price DM 78,–.

The book under review is an introduction to a Daniell type integration theory. A version of this theory developed by the author is based, roughly speaking, on completing the space of elementary integrands with respect to a suitable seminorm (the particular seminorms that appear in this context are called means in the book). To fix up the main ideas, the author constructs the Riemann integral on the real line $\mathbb{R}$ by an analogous procedure in the first chapter. It might be illuminating to sketch the basic steps here: Let $\mathcal{E}$ be the space of all step functions of the form $\varphi = \sum\limits_{1}^{n} r_i \mathbf{1}_{A_i}$, where $r_i \in \mathbb{R}$ and $A_i =]a_i, b_i]$ are disjoint bounded intervals on $\mathbb{R}$. The Riemann integral $\int \varphi$ of a step function is defined in the obvious way. Note that the upper Riemann (or Darboux) integral $\int^{\natural} f$ of a bounded real function f with a bounded support may be defined alternatively by $\int^{\natural} f = \inf\{\int \varphi;\ \varphi \geqslant f,\ \varphi \in \mathcal{E}\}$. A bounded function g with a bounded support is then Riemann integrable if and only if a sequence $\{\varphi_n\}$ of step functions may be found such that $\int^{\natural} |g - \varphi_n| \to 0$ as $n \to \infty$; in this case the Riemann integral $\int g = \lim\limits_{n\to\infty} \int \varphi_n$. Therefore, the Riemann integrable functions may be viewed as the completion of $\mathcal{E}$ with respect to the seminorm $\|f\|^{\natural} = \int^{\natural} |f|$.

To obtain "better" integrals, one has to replace the seminorm $\|\cdot\|^{\natural}$ by a smaller mean, e.g. by that given by the upper Daniell integral. The corresponding theory is developed in Chapter 2. The next chapter is devoted to measurability (in the approach of the book, measurable functions are introduced as those which are sufficiently close to elementary ones). In Chapter 4, the spaces $\mathcal{L}^p$ are introduced (again as a closure of elementary integrands under an appropriate mean) and their basic properties are established, including the characterization of the dual space in the case $p \in]1, \infty[$. In the last chapter several different topics are addressed: the Fubini theorem is proved and, as its application, convolutions in L^1 are treated and a version of the Marcinkiewicz interpolation theorem is given. Then signed measures are discussed in some detail and the Radon-Nikodym theorem is proved. Finally, it is shown that a function of bounded variation on $\mathbb{R}$ is Lebesgue almost everywhere differentiable; in this connection, Vitali's covering theorem and the Hardy-Littlewood maximal operator appear.

The book is amended with numerous exercises. Many of them are used later in the main body of the text, these are marked by an asterisk. In Appendix, solutions (or hints) are given to selected exercises.

Almost thirty years ago, the author published another book on the same subject (*Integration theory*, Lecture Notes in Mathematics 315, Springer-Verlag 1973). It was a research monograph, aimed at applications to vector measures, while the present book is an introductory textbook: the results are well motivated and proofs are reasonably detailed. The only prerequisite for reading the book is a working knowledge of the basic ε-δ calculus; the few sections using deeper results from general topology may be omitted on first reading.

The style is very informal and sometimes it leads to small flaws. A rather extensive list of corrections may be found on the author's webpage (`http://www.ma.utexas.edu/users/kbi`). Nevertheless, *Integration—a functional approach* is a very interesting book and may be recommended to everybody willing to get acquainted with the Daniell integral.

Jan Seidler

G. Wimmer, G. Altmann: THESAURUS OF UNIVARIATE DISCRETE PROBABILITY DISTRIBUTIONS. Stamm, Essen, 1999, xxxviii+838 pages, ISBN 3-87773-025-6, price DM 198,–

The book is not a monograph but a true thesaurus, i.e. a list of subjects with the cross-reference system used in the organization of collections of items for reference and retrieval (as the Webster dictionary). The total number of existing distributions of this type is estimated to one thousand and 750 of them are listed here. Anybody buying such a book needs to know the information which can be learned from each entry. For this reason, a commented example (from page 143 of the book) follows.

Ehrenfest d. (a, b) *has two parameters* $a \in \mathbb{N}_0$, $b \in \mathbb{N}$ *and another name "Ehrenfest heat exchange model"*. The authors claim that beside the name in the title also all other names are listed under which the distribution has been ever known (however, also *"Ehrenfest urn model"* appears in the references). The total number of names of all distributions included is about 1500, the "richest" of all being the negative hypergeometric distribution with nine names.

Next, the probability (mass) function P_x *and the probability generating function* $G(t)$ *are given*; if various equivalent formulae appear in literature, they are also presented.

Then interrelations follow: Ehrenfest d. belongs to the Kapur-hypergeometric family and, on the other hand, the special choice $b = 1$ *gives a more special* 1*-shifted Naor's urn distribution. Two other interrelations of similar types are also shown, namely to the Kemp-Dacey-hypergeometric family and to the deterministic distribution. Also convergences of other distributions to the distribution in the title or the convergences of the distribution in the title to other distributions are mentioned and, at last, interrelations like convolutions, n-fold convolutions and mixtures of distributions are included.* Certain basic distributions have an extreme number of interrelations: the negative binomial and the Poisson distributions have more than 100 and 250 of them, respectively.

Nine references listed for Ehrenfest d. specify its properties, e.g. the mean transition time, fluctuations and relations to the associated differential equations and to the passage problem for stationary Markov chains. The number of references varies extremely in particular entries; from one to nearly one thousand for the Poisson d. The complete list of references has about 4 000 entries (150 pages).

Glossary of symbols, list of definitions and important formulas and list of names of the distributions represent the remaining parts of the book the precursors of which (in particular Patil and Joshi, 1968, and several versions of the book by Johnson, Kotz and recently also Kemp) are mentioned in the introduction. This Thesaurus hardly needs any recommendation; its value is proportional to the hardly imaginable amount of work hidden behind each page.

Ivan Saxl

F. Pacard, T. Rivière: LINEAR AND NONLINEAR ASPECTS OF VORTICES. THE GINZBURG-LANDAU MODEL. Birkhäuser-Verlag, Basel-Berlin-Boston, 2000, 480 pages, ISBN 3-7643-5887-4, hardcover, price DM 198,–.

Ginzburg-Landau equations model a great number of physical phenomena such as phase transition in superconductors, Yang-Mills-Higgs fields, superfluids and many others.

The sets where the "wave function" vanishes are commonly called vortices. The present book addresses a number of important questions concerning the location of vortices in space, the shape of solutions near vortices, and the relation between vortices and solutions. One of the main achievements is the description of a one-to-one correspondence between the admissible configurations of vortices and the space of solutions of the Ginzburg-Landau equations.

The main topics discussed include: qualitative aspects of Ginzburg-Landau equations, elliptic operators in weighted Hölder and Sobolev spaces, families of approximate solutions with prescribed zero set, the existence of Ginzburg-Landau vortices, generalized Pohozaev formula for p-conformal fields, and a discussion of Jaffe and Taubes conjectures.

The book will be useful in a number of contexts in the study of nonlinear phenomena arising in geometry and mathematical physiscs. The material covers recent and original results and will serve as an excellent texbook or a valuable self-study resource.

Eduard Feireisl