Book Reviews

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BOOK REVIEWS

S. Bagdasarov: CHEBYSHEV SPLINES AND KOLMOGOROV INEQUALITIES. Operator Theory: Advances and Applications, Vol. 105. Birkhäuser, Basel-Boston, 1998, xiii+205 pages, ISBN 3-7643-5984-6 (Basel), ISBN 0-8176-5984-6 (Boston), hardcover DM 178,–.

The monograph deals with extremal problems in certain classes of real functions of one variable. It can be used mainly by scientists interested in the theory of extremal problems and in the approximation theory. A numerical analyst expecting common splines and their applications in numerical analysis, geometry, or computer aided design may find the subject outlying.

The book is fairly well organized as the reader can infer even from Preface, where the contents of all chapters are briefly commented.

Topics treated in the book are defined in Chapter 0 (Introduction):

(1) Classes W^rH^ω of functions of one variable defined on a segment, a half-line, or a line. A class W^rH^ω comprises a subset of r-times continuously differentiable functions. The subset is defined via a majorizing concave modulus of continuity ω.

(2) The Kolmogorov-Landau problem. It concerns the best estimates of L^p-norm of the mth derivative of a function $f \in W^rH^\omega$, $m \leqslant r$, by means of constants and proper norms of other derivatives of f. Under some conditions, as $p = \infty$, the problem is referred to as the Kolmogorov problem.

(3) Functions realizing the supremum of a norm of f over a bounded set of functions from W^rH^ω. This problem underlies (2).

(4) Functions realizing extrema in the problem $\sup f^{(m)}(0)$ over a bounded set of functions from W^rH^ω. The problem is connected to (2) and (3).

(5) A characterization of extremal functions of a linear functional defined through an integral with a kernel function changing its sign finitely or countably many times. This goal arises in the process of solving (4).

(6) The approximation of classes W^rH^ω by n-dimensional spaces. Its quality can be expressed by the n-width which says how close approximation is possible if all n-dimensional subspaces of a normed space are taken into account.

After listing basic technical results in Chapter 1, the author focuses on problem (5) with various kernels and introduces its extremal functions called perfect ω-splines (Chapters 2 and 3). Then he pays attention to a problem of type (4) the extremal functions of which belong to the Chebyshev perfect splines.

In the next chapter, numerical differentation formulae for $f^{(m)}(0)$ are derived and sufficient conditions of extremality in the Kolmogorov-Landau inequality are presented.

The existence of functions fulfilling the sufficient conditions is proved in Chapter 6, the core of the book.

Chapters 7–12 deal with functions extremal in problems (3), (4) and (5), where various m, norms, and intervals of definition are considered.

The next part focuses on estimating the n-widths from below and above with the aid of Chebyshev ω-splines (Chapters 13–16).

Two Kolomogorov problems are studied in Appendices A and B.

The bibliography comprises 93 items and is followed by a brief index.

Due to the complex nature of the methods and results exposed, the symbolization was inevitably complicated and proofreading extremly difficult. That is why some misprints have been overlooked as, e.g., the meaningless integral on page 62 or wrong references to Definitions 1.2.1, 1.2.2. on page 111 (they should be referred to as 2.2.1 and 2.2.2. instead). Also, the reader could be slightly confused by the fact that besides ω-splines and n-width he/she can find Ω-splines and N-width on many pages.

Ther reader would probably welcome more illuminative comments on hard parts of the monograph as, to give an example, Chapter 2 and Chapter 9, where complex definitions and statements are presented in a condensed form.

This and other aspects lead me to the conclusion that the book aims at advanced readers who are familiar with the techniques used to study the subject.

Jan Chleboun

A. A. Melikyan: GENERALIZED CHARACTERISTIC OF FIRST ORDER PDES. Applications in Optimal Control and Differential Games. Birkhäuser, Boston, 1998, xiv+310 pages, ISBN 0-8176-3984-5, price DM 178,–.

During the last 15 years, a general approach, based on the notion of generalized viscosity solutions to fully nonlinear first and second order PDEs, has been developed. Existence and uniqueness theorems were formulated and proved, the boundary conditions being also understood in a generalized sense.

Nonlinear first-order PDEs are considered in the book, which either have nonsmoooth generalized (viscosity) solution and/or nonsmooth left-hand side function of PDE (the Hamiltonian). A new notion of singular characteristics (SC) of a first-order PDE is introduced which are the solutions of a certain ODE-system.

First-order PDEs possess two types of characteristics: regular (classical) and singular. It is proved that singular surfaces, generally, can be constructed using singular characteristics. Combining regular and singular characteristics one can construct the (nonsmooth) viscosity solutions to a vast class of nonlinear problems.

Chapter 1 deals with the basic theory of characteristics for problems possessing smooth solutions.

In Chapter 2, generalized solutions and singular characteristics are introduced for the first order nonlinear partial differential equations.

Chapter 3 is devoted to concrete examples of equations arising in variational calculus, optimal control theory and differential games. The differential games theory is further developed in Chapters 4, 5.

In Chapter 6, the problems with nonsmooth Hamiltonians are attacked while Chapter 7 is devoted to the study of shock waves developed by solutions to the first order equations.

In Chapter 8, some second order nonlinear equations in variational form are dealt with.

Each chapter is completed by concrete examples and exercises. The book will be certainly useful for researchers as well as students and post-graduate students in mathematics, physics and engineering.

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