Applications of Mathematics

Book Reviews

BOOK REVIEWS

Heinz Bauer: PROBABILITY THEORY. De Gruyter Studies in Mathematics 23, Walter de Gruyter, Berlin-New York 1995, xv + 523 pp., ISBN 3-11-013935-9, price DM 148,–.

A precursor of the book under review, Professor Bauer's *Wahrscheinlichkeitstheorie und Grundzüge der Maßtheorie* (which appeared in 1968 and was reissued in revised versions several times, both in German and in English) has deserved the fame of one of the best standard textbooks in the field. Its part devoted to the measure theory, completely rewritten, was published as a separate book in the year 1990 and now it is followed by a new text on the probability theory. Let us sketch its contents: Chapter I introduces the basic concepts, while the next chapter in devoted to the concept of independence (including the construction of infinite products of probability spaces). In the third chapter, zero-one laws, strong laws of large numbers and almost sure convergence of infinite series are treated. Martingales are thoroughly investigated in Chapter IV. Fourier analysis is dealt with in the fifth chapter; the next two chapters are devoted to limit theorems (the central limit theorem, including a multi-dimensional version, and Strassen's law of the iterated logarithm, respectively). Stochastic processes are treated in the last two chapters (projective limits of probability spaces and the construction of stochastic processes with prescribed finite-dimensional distributions, Markov, Gauss and Poisson processes, ...), with a particular emphasis on the properties of the Brownian motion. Even this brief list of covered topics indicates that the book is intended for a reader who looks for an in-depth introduction to the probability theory. At the same time, *Probability theory* remains a textbook: carefully organized, essentially self-contained, with complete detailed proofs, the text being accompanied with many examples and exercises.

To conclude, Professors Bauer's new book can be strongly recommended to everybody who wants to get acquainted with the probability theory in a profound and systematic manner.

Jan Seidler

DIRICHLET FORMS AND STOCHASTIC PROCESSES. Z. M. Ma, M. Röckner and J. A. Yan, Editors, Walter de Gruyter, Berlin-New York 1995, xi + 443 pp., ISBN 3-11-014284-8.

The theory of Dirichlet forms has grown recently as an independent and important field of study on the borderline between functional analysis and probability theory, providing a unified approach to many topics in the Markov processes theory, in both the analytical and probabilistic potential theory, in the theory of differential and pseudo-differential operators, etc. Its applications to the construction of "highly singular" stochastic processes (solutions to stochastic differential equations in infinite-dimensional spaces and/or with a very irregular drift term) is of particular interest.

The proceedings under review comprise 38 papers related to the lectures delivered by the authors at the "School on Dirichlet forms" and at the subsequent "International conference on Dirichlet forms and stochastic processes" which were held in Beijing in October 1993 and were attended by many of the leading specialist in the field (including, e.g., M. Fukushima,

S. Albeverio, M. Röckner or T. Hida). Most of the contributions are full-length papers with complete proofs, and a wide spectrum of problems concerning Dirichlet forms and Markov processes is considered, which corresponds to the still growing number of applications of the theory.

Jan Seidler

Mark Freidlin: MARKOV PROCESSES AND DIFFERENTIAL EQUATIONS: ASYMPTOTIC PROBLEMS. Lectures in Mathematics, Birkhäuser Verlag, Basel 1996, 153 pages, price DM 44,–, ISBN 3-7643-5392-9.

Solutions to linear or semilinear elliptic or parabolic problems can be represented in terms of diffusion processes, that is, solutions of stochastic differential equations; an analogous representation is available for systems of equations, provided diffusion processes with random switching are employed. This interplay between analytical and probabilistic approaches brings new methods and results in both the partial differential equations theory and stochastic analysis, the investigation of asymptotic behaviour of equations depending on a small parameter $\varepsilon \downarrow 0$ being one of the most prominent examples. Professor Freidlin has contributed in a substantial way to the last topic and the booklet under review can be viewed as an introduction into his recent papers (joint, in part, with A. N. Borodin, A. Eizenberg and A. D. Wentzell).

Let us mention briefly several problems treated in the book. First, the Dirichlet problem

$$\frac{\varepsilon^2}{2} \sum_{k,l=1}^{r} a^{kl}(x) \frac{\partial^2 u^\varepsilon}{\partial x_k \partial x_l} + \sum_{k=1}^{r} b^k(x) \frac{\partial u^\varepsilon}{\partial x_k} = 0 \quad \text{in } G, \quad u^\varepsilon = 0 \quad \text{on } \partial G,$$

is considered. The behaviour of the corresponding diffusion process depends on the properties of the dynamical system $\dot{X} = b(X)$ and either large deviations theory or averaging methods are to be used; this topic is dealt with in Chapters 2–6. The apparatus of diffusion processes on graphs developed there is then applied to the study of partial differential equations and Markov processes in narrow branching tubes (Chapter 7). Chapters 8–10 are devoted to probabilistic investigation of propagation of the wave front in reaction diffusion equations of the Kolmogorov-Petrovskii-Piskunov type. The last (eleventh) chapter contains homogenization procedures.

The book is based on lectures delivered by the author in 1994 at the Bonn University and at ETH Zürich, and its vivid and intuitive style bears a clear trace of them. Results and underlying ideas are presented in a lucid way, whilst proofs are sketched only if they contribute to a better understanding of the problem. To enjoy the book the reader is presupposed to be familiar with a rather advanced background material from both stochastic analysis and PDE's. On the other hand, these lecture notes offer a quick and nice way to get acquainted with the deep achievements of the recent research in the field.

Jan Seidler