BOOK REVIEWS

Werner O. Amrein, Andreas M. Hinz, David B. Pearson (ed.): STURM-LIOUVILLE THEORY—PAST AND PRESENT. Birkhäuser, Basel, 2005, 335 pages, EUR 68.–/CHF 108.–.

Twelve articles represent the contributions to the second, purely mathematical, part of a conference commemorating the 200th anniversary of the birth of C. F. Sturm.

This part is devoted to the scientific contributions and consists of the following: David Pearson, *Introduction* (xiii–xx); Don Hinton, *Sturm's 1836 oscillation results: evolution of the theory* (1–27); Barry Simon, *Sturm oscillation and comparison theorems* (29–43); W. Norrie Everitt, *Charles Sturm and the development of Sturm-Liouville theory in the years 1900 to 1950* (45–74); Joachim Weidmann, *Spectral theory of Sturm-Liouville operators: approximation by regular problems* (75–98); Yoram Last, *Spectral theory of Sturm-Liouville operators on infinite intervals: a review of recent developments* (99–120); Daphne Gilbert, *Asymptotic methods in the spectral analysis of Sturm-Liouville operators* (121–136); Christer Bennewitz and W. Norrie Everitt, *The Titchmarsh-Weyl eigenfunction expansion theorem for Sturm-Liouville differential equations* (137–171); Victor A. Galaktionov and Petra J. Harwin, *Sturm's theorems on zero sets in nonlinear parabolic equations* (173–199); Chao-Nien Chen, *A survey of nonlinear Sturm-Liouville equations* (201–216); Rafael del Río, *Boundary conditions and spectra of Sturm-Liouville operators* (217–235); Mark M. Malamud, *Uniqueness of the matrix Sturm-Liouville equation given a part of the monodromy matrix, and Borg type results* (237–270); W. Norrie Everitt, *A catalogue of Sturm-Liouville differential equations* (271–331).

A separate volume containing the collected works of Sturm with historical commentary is in preparation.

The development of Sturm oscillation theory until about the middle of the twentieth century with some indication of its more recent extensions to systems, left-definite problems, parameter dependent boundary conditions, and higher order equations is discussed in Hinton's paper. Simon presents a recent approach to Sturm oscillation theory which includes difference equations and is related to spectral theory and orthogonal polynomials. Everitt describes the development of Sturm-Liouville theory in 1900–1950 (a sophisticated and instructive account of the history) and provides a catalogue of 50 problems (a nice zoology-like account). In the paper with Bennewitz, Everitt deals with the eigenfunction expansion theorem. Recent results on spectral theory are discussed by Weidmann, Last, Gilbert, and del Río. The other contributions deal with extensions of some particular aspects of Sturm-Liouville theory to parabolic equations, systems and nonlinear problems.

Even if the presentation of the contributions is not uniform in notation and form, the volume is instructive and of high value for all specialists in field.

Štefan Schwabik, Praha

A. Bermúdez de Castro: CONTINUUM THERMOMECHANICS. Birkhäuser, Basel, 2005, 209 pages, EUR 58.–.

The purpose of this book is to give a modern and unified approach to continuum mechanics and thermomechanics in a rigorous mathematical framework.

The first chapter revisits the conservation principles of continuum thermomechanics while writing these local equations in Lagrangian coordinates is the subject of Chapter 2.

Chapter 3 and Chapter 4 deal with the constitutive laws of continuum thermomechanics and the principle of material frame indifference. In Chapter 5, the partial differential equations governing a thermodynamic process are written with entropy being replaced by temperature.

Chapter 6 is devoted to isotropy, the simple forms for the response functions of Coleman-Noll materials. In Chapter 7, the equations satisfied by each thermodynamic process of these materials are written in Lagrangian coordinates. In Chapter 8 linear approximations of these equations about static reference state are deduced. Quasi-static thermoelasticity can be found in Chapter 9. Chapters 10, 11, 12, 13, 14 are devoted to the fluids, acoustics and gases. In Chapters 15, 16, 17, 18 we can find a description of mixtures, chemical reactions and chemical equilibrium. Chapter 19 is devoted to the method of mixture fractions and the last one to turbulent flows with mixtures. Appendices include some mathematical background in tensor algebra and analysis and also the formulation of ALE configuration.

Šárka Nečasová, Praha

Steven G. Krantz: GEOMETRIC FUNCTION THEORY—EXPLORATIONS IN COMPLEX ANALYSIS. Birkhäuser, Boston, 2006, 314 pages, CHF 88.–/EUR 62,–.

This book shows many different aspects of complex analysis. The reader will get acquainted with rich interactions that exist among the various topics. With this book, the student as well as the advanced worker are introduced to a rich tapestry of fuction theory as it interacts with other parts of mathematics.

The first part of the book concentrates on the classical function theory and geometry. The Poincaré-Bergman metric, the Bergman space and the Bergman kernel are studied in Chapter 1. Chapter 2 explores the Schwarz lemma and its variants. As a byproduct, an elegant new proof of Picard's theorems is given. Chapter 3 is devoted to the study of normal families of meromorphic functions. Chapter 4 provides a detailed exposition of the Riemann mapping theorem and its generalizations. Chapter 5 studies the boundary behavior of conformal mappings. Chapter 6 contains a brief summary of Hardy spaces. The second part of the book provides a panorama of ideas coming from partial differential equations, harmonic analysis, singular integral operators and Banach algebras. It gives a strong sense of the synergy among different parts of modern mathematics, and points to research directions that are of current interest. The inhomogeneous Cauchy-Riemann equation is studied in Chapter 7. Chapter 8 provides an introduction to the Laplacian, the Green function and the Poisson kernel. Chapter 9 is devoted to the study of harmonic measures. Chapter 10 establishes the relation between the convergence of Fourier series in L^p and the behavior of the Hilbert transform on this space. Wolff's proof of the Corona theorem is presented in Chapter 11. The last part of the book illustrates the use of algebraic topics in complex analysis. Chapter 12 studies the automorphism groups of domains in the plane. Chapter 13 introduces the Cousin problems and shows how they can be applied to some problems that arise in classical function theory. It also includes a brief introduction to sheaves and sheaf cohomology.

Dagmar Medková, Praha

A. Ambrosetti, A. Malchiodi: PERTURBATION METHODS AND SEMILINEAR ELLIPTIC PROBLEMS ON $\mathbb{R}^n$. Progress in Mathematics, Birkhäuser, Basel 2005, ix+183 pages, hardcover, ISBN 3-7643-7321-0, CHF 68.–/EUR 45.–.

The monograph focuses on elliptic problems of variational and perturbative nature that cannot be solved in general by the technique of nonlinear functional analysis based on compactness arguments. The abstract tools take advantage of the perturbative setting of the problems and the critical point theory. These abstract methods are discussed and applied to several perturbation problems whose common feature is involving semilinear elliptic equations with a variational structure. The applications involve bifurcation from the essential spectrum, elliptic problems with subcritical growth and the critical exponent, the Yamabe problem and other problems in conformal geometry, nonlinear Schrödinger equations, and singularly perturbed Neumann problems on bounded domains.

The monograph provides a self-contained presentation of the theory in a systematic and unified way. It is addressed to researches and graduate students interested in the theory of partial differential equations. The book was awarded the Ferran Sunyer i Balaguer 2005 prize.

Hana Petzeltová, Praha

T. Cazenave, D. Costa, O. Lopes, R. Manásevich, P. Rabinowitz, B. Ruf, C. Tomei (eds): CONTRIBUTIONS TO NONLINEAR ANALYSIS. A Tribute to D. G. de Figueiredo on the Occasion of his 70th Birthday. Progress in Nonlinear Differential Equations and Their Applications, Vol. 66, Birkhäuser, Basel, 2006, xii+518 pages, hardcover, ISBN 3-7643-7149-8, CHF 248.–/EUR 148.–.

The volume under review is based on the material presented at the workshop held in Campinas, June 7–11, 2004 on the occasion of the 70th birthday of the prominent Brasilian mathematician D. G. de Figueiredo. It collects 34 research and survey articles reflecting the wide range of his interests and written by distinguished mathematicians. The papers cover new developments and offer up to date surveys in nonlinear functional analysis and various types of nonlinear partial differential equations, including questions of existence, uniqueness and multiplicity of solutions, qualitative properties, symmetry, regularity and shape of solutions, *a priori* estimates and asymptotic behavior, phenomena connected with the critical Sobolev growth, applications to models such as asymptotic membranes, nonlinear plates and inhomogeneous fluids, among others.

The book should be useful for graduate students and researchers in nonlinear problems.

Hana Petzeltová, Praha

V. A. Marchenko, E. Y. Khruslov: HOMOGENIZATION OF PARTIAL DIFFERENTIAL EQUATIONS. Birkhäuser, Basel, 2006, 398 pages, EUR 116.–.

The authors are interested in the problems of homogenization for partial differential equations describing various physical phenomena in microinhomogeneous media.

In the first chapter the authors introduce the simplest homogenized model, the nonlocal homogenized model, the two-component homogenized model, the homogenized model with memory and the homogenization of boundary value problems in strongly perforated domains.

The second chapter deals with the Dirichlet boundary value problem in strongly perforated domains with fine-grained boundary. In the third chapter the authors describe the Dirichlet boundary value problem in strongly perforated domains with complex boundary. In the fourth chapter we can find a description of problems in strongly connected domains.

Chapter five is devoted to the Neumann boundary value problems in strongly perforated domains. The next chapters deal with nonstationary problems, spectral problems and with differential equations with rapidly oscillating coefficients. Finally, the last chapter is devoted to the homogenized conjugation conditions.

Graduate students, applied mathematicians, physicists and engineers will benefit from this monograph, which may be used in the classroom or as a comprehensive reference text.

Šárka Nečasová, Praha

Karl-Georg Steffens: THE HISTORY OF APPROXIMATION THEORY—FROM EULER TO BERNSTEIN. Birkhäuser, Basel, 2006, 219 pages, EUR 73.–.

A typical problem of approximation theory is to find a polynomial p_n of order n that provides the best approximation to a given continuous function $f\colon [a,b] \to \mathbb{R}$, i.e. a polynomial that minimizes the number

$$(1) \qquad \|f - p_n\|_\infty = \max_{x\in[a,b]} |f(x) - p_n(x)|.$$

The introductory chapter of the monograph is concerned with the analysis of two problems of a similar nature. The first one was solved by Euler in 1777 when analyzing the Delislian cartographic projection used to construct a map of Russia. The second problem was discussed in Laplace's work on celestial mechanics from 1843 and consisted in finding an ellipsoid that provides the best approximation to the Earth's surface.

The roots of approximation theory as an independent discipline lie in the work of P. L. Chebyshev. The visit to Western Europe in 1852 aroused his interest in the so-called parallelogram mechanisms, which were used in steam engines to transform circular motion to a linear one. Since the transformation isn't perfect, Chebyshev was led to the problem of determining the parallelogram dimensions so that the deviation from linear motion would be as small as possible. As he noticed, there was no suitable mathematical theory to handle such problems except for the approximation formulae derived by Poncelet in 1835. Chebyshev was also aware that the general problem (1) is too difficult to solve; apart from that, the definition of a continuous function was not well established at that time. He thus restricted himself to analytic functions $f(x) = \sum\limits_{i=0}^{\infty} a_i x^i$ defined in interval $[-1,1]$; then, after neglecting the terms of order x^{n+1} and higher, the problem is reduced to finding a polynomial of order n least deviating from the zero function and whose coefficient σ_n at x^n is given. The solution is the polynomial $(\sigma_n/2^{n-1}) \cdot T_n(x)$, where $T_n(x) = \cos(n \arccos x)$ is now known as the Chebyshev polynomial of order n.

Chebyshev was also interested in the approximation by rational functions, he wrote on orthogonal polynomials, approximation in the sense of least squares, and other topics. His work had a great influence on Russian mathematics. Approximation theory flourished especially at the St. Petersburg mathematical school under the leadership of Chebyshev himself. The third chapter of the present book describes the life and work of his successors, namely A. N. Korkin, E. I. Zolotarev, A. A. Markov, V. A. Markov, J. K. Sochocki and K. A. Posse.

Until the beginning of the 20th century, the results of approximation theory were almost unknown outside Russia. Felix Klein became acquainted with these results through the correspondence with A. A. Markov; in his talk, given before the Göttingen Mathematical Society in 1895, he summarized the contributions of Russian mathematicians. Shortly after, in 1899, Paul Kirchberger finished his thesis on approximation theory under the supervision of David Hilbert. Apart from presenting some new results, the work had a good theoretical background: Whereas the Russians were interested especially in applications and didn't care too much for rigour, Kirchberger gave the proofs of existence and uniqueness of the best approximation (including the proof of the so-called alternation theorem). Small deficiencies in his proofs were later fixed by E. Borel, J. W. Young and M. R. Fréchet.

It is a simple consequence of the Weierstrass approximation theorem that the polynomials p_n converge uniformly to the given function f as the degree n increases. The corresponding result for trigonometric polynomials has been obtained by L. Fejér in 1900: Given a 2π-periodic continuous function f, the partial sums of its Fourier series converge uniformly to f in case the Fejér summation is used. Quantitative approximation theory, which is interested in estimating the speed of convergence of the approximating polynomials, originated in the works of H. Lebesgue and C. de la Vallée Poussin in 1908, and in the thesis of D. Jackson awarded by a prize in Göttingen in 1911.

The last chapter of the monograph discusses the development of approximation theory in Kharkov, the mathematical centre of Ukraine, and is concerned with the work of A. P. Psheborski and especially S. N. Bernstein. The latter mathematician is probably best known due to the Bernstein polynomials introduced in a constructive proof of Weierstrass theorem, but he also made significant contributions to quantitative approximation theory.

The publication will be best appreciated by experts in approximation theory, but may be interesting to historians of mathematics as well. Detailed biographies of Russian mathematicians are especially valuable as it is difficult to find them elsewhere.

Antonín Slavík, Praha

Aghate Keller: EXPOUNDING THE MATHEMATICAL SEED. Vol. 1, 2. Birkhäuser, Basel, 2006, ISBN 10: 3-7643-7291-5, 13: 978-3-7643-7291-0, ISBN 10: 3-7643-7292-3, 13: 978-3-7643-7292-7, 528 pages, EUR 128.–.

The book is an English translation of Bhāskara Ist's commentary on the mathematical part of Āryabhaṭa's *Āryabhaṭīya*. An English translation was last published in 1976 (K. S. Shukla). According to the author of the present book Shukla's translation was based only on a single faulty source without giving any dating of it. Problematic places were not thoroughly explained there, so Bhāskara's commentary seems to be senseles in several places. These were probably the main reasons for writing a new translation, which is nevertheless based on Shukla's one. To give another reason for writing the new translation let us remark that Bhāskara's work represents probably the oldest extant commentary on *Āryabhaṭīya*. It came to life after a long period of about five hundred years, after which no mathematical text in Sanskrit has been preserved.

The translation and notes to it are very well arranged into a two-volume book. The first volume contains the literal translation of the commentary—*Āryabhaṭīyabhāṣya*. The original Āryabhaṭa's verses are in bold typeface, while Bhāskara's own commentary is in the standard typeface. This differentiation makes the orientation in the book easier especially for those readers who look only for specific parts of *Āryabhaṭīya* and its commentary. In the second volume, the author gives explanations (linguistic as well as mathematical) to verses and modern mathematical notation of given rules. In addition to the translation proper the first volume contains a historical introduction, information about the underlying manuscripts, structure of the commentary and remarks on Bhāskara's mathematics.

Through the given historical introduction the reader is dragged into Hindu mathematics of the 7th century. Thanks to numerous references to other sources the book is useful also for the reader who looks for more detailed information on Hindu mathematics.

Martina Ernestová, Plzeň

M. Haase: THE FUNCTIONAL CALCULUS FOR SECTORIAL OPERATORS. Operator Theory: Advances and Applications, Vol. 169, Birkhäuser, Basel, 2006, xiv+392 pages, hardcover, ISBN 3-7643-7697-X, EUR 78.–.

The book focuses on the theory of holomorphic functional calculus for sectorial operators. This elementary calculus based on Cauchy integral is then extended to a larger class of functions, which makes it possible to construct functional calculi also for more general (so-called strip-type) unbounded operators. Fractional powers, logarithms and holomorphic semigroups generated by sectorial operators are treated as an elegant application of the general theory. Relation between bounded numerical calculus for operators in a Hilbert spaces, their numerical range and similarity problems are studied. An important part of the book is devoted to the questions of boundedness of the H^∞ functional calculus and to connecting the perturbation and the interpolation theory. The last two chapters account for the applications to elliptic operators with constant coefficients, relation of the calculus to the Fourier multipliers theory, functional calculus approach to a problem from numeric analysis and a maximal regularity problem.

The monograph is provided with six Appendices, which makes the book self-contained and optimally organized. It will be useful for graduate students as well as for researchers in operator theory and evolution equations.

Hana Petzeltová, Praha

T. Andreescu, D. Andrica, Z. Feng: 104 NUMBER THEORY PROBLEMS—FROM THE TRAINING OF THE USA IMO TEAM. Birkhäuser, Boston, 2007, 204 pages, EUR 38.–.

The publication 104 Number Theory Problems is a collection of problems from number theory which are topically divided into several smaller sets, for example divisibility, prime numbers, g.c.d., linear Diophantine equations, Fermat and Mersenne numbers, etc. In each subset, definitions of notions and some theorems and their proofs are given. Then a set of problems follows.

As for the level of difficulty, the problems belong to problems intended for mathematical olympiads, even for International mathematical olympiads (for that matter, it is written in the subtitle of the book). There are problems in the book which were already at some point used in various similar competitions.

The publication is an excellent material for solvers of mathematical olympiads who prepare for the national or international competition. It is also useful for interspersing of lessons in classes with talented pupils in mathematics. It is also a material for inspiration for authors of similar problems. And last but not least, it is an interesting material for all enthusiasts of nontraditional mathematical problems.

Jaroslav Zhouf, Praha