Book reviews

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BOOK REVIEWS

R. Abłamowicz, B. Fauser (eds.) + J. Ryan, W. Sprössing (eds.): CLIFFORD ALGEBRAS AND THEIR APPLICATIONS IN MATHEMATICAL PHYSICS, VOLS. $1+2$. Birkhäuser, Progress in Physics, vol. 18–19, ISBN 0-8176-4182-3 (vol. 18), ISBN 0-8176-4183-1 (vol. 19), $488+344$ pages, DM 158.– + 138.–.

Given a vector space V with a quadratic form Q, Clifford algebra of the pair (V, Q) is the free associative unital algebra generated by V, modulo the relation $Q(v) = v^2 \cdot 1$, for $v \in V$. Clifford algebras turned out to be profound generalizations of various 'classical' algebras (real or complex numbers, quaternions) with close relations to matrix and spin groups. The theory of Clifford algebras has numerous applications, both in mathematics and physics, as illustrated by the present book in two volumes which stems from contributions to the series of conferences "Clifford Algebras and their Applications in Mathematical Physics."

The first volume, devoted to mathematical aspects of Clifford algebras and applications in physics, is divided into five parts: (1) Physics—Applications and Models, (2) Physics—Structures, (3) Geometry and Logic, (4) Mathematics—Deformations and (5) Mathematics—Structures. The second volume is focused to Clifford analysis and applications. The names of the sections are: (1) Partial Differential Equations and Boundary Value Problems, (2) Singular Integral Operators, (3) Applications in Geometry and Physics and (4) Möbius Transformations and Monogenic Functions. All contributions were refereed and further developed after the conference.

The book provides a broad perspective of the quickly developing field of Clifford algebras. It should be accessible to graduate students as well as to anybody interested in this important part of mathematics.

Martin Markl, Praha

H. Bass, A. Lubotzky: TREE LATTICES. With Appendices by H. Bass, L. Carbone, A. Lubotzky, G. Rosenberg, and J. Tits. Birkhäuser, Basel, 2000, 248 pages, DM 108.–.

The present monograph is a modern, systematic and self-contained treatment of tree lattices, structures which are derived from automorphism groups of locally finite trees. It offers the reader a detailed investigation of existence, structure and properties of tree lattices. The methods used are based on the notion of a graph of groups which was developed by Serre in his famous book *Trees*.

One of the characteristic features of this book is that the exposition of the material emphasises the comparison of the situation in tree lattices with the classical case of the lattices in Lie groups. Nevertheless, the book is independent of the theory of Lie groups and thus essentially elementary. Much of the space throughout the text is devoted to the construction of various examples; this is another notable feature of the book.

Many of the results presented appear for the first time in print. In particular this concerns several recently proved fundamental existence theorems.

The book has eleven chapters: 0. Introduction; 1. Lattices and volumes; 2. Graphs of groups and edge-indexed graphs; 3. Tree lattices; 4. Arbitrary real volumes, cusps, and homology; 5. Length functions, minimality; 6. Centralizers, normalizers, and commensurators; 7. Existence of tree lattices; 8. Non-uniform lattices on uniform trees; 9. Parabolic actions, lattices, and trees; 10. Lattices of Nagao type.

In addition, there are three appendices: I. The existence theorems for tree lattices, by H. Bass, L. Carbone, and G. Rosenberg; II. Discreteness criteria for tree automorphism groups, by H. Bass and J. Tits; III. The P. Neumann groups, by H. Bass and A. Lubotzky.

In all, the book is a valuable, interesting and readable successor of Serre's *Trees*. It will be useful to any researcher or graduate student in the field as well as in the adjacent fields, such as geometric methods in group theory, Lie groups, number theory, and even combinatorics.

Martin Škoviera, Bratislava

D. Beltita, M. Sabac: LIE ALGEBRAS OF BOUNDED OPERATORS. Operator Theory: Advances and Applications, vol. 120. Birkhäuser, Basel, 2001, ISBN 3-7643-6404-1, hardcover, 232 pages, DM 180.–.

This is a very nice book on an very interesting subject: namely, the authors apply operator theory to extend to the infinite-dimensional case some results on finite-dimensional Lie algebras, and, conversely, show that Lie algebra theory can be used to get interesting operator-theoretic results on spectral theory of non-commuting tuples of bounded linear operators. The first of the five chapters, occupying about one third of the book, collects all the necessary preliminaries from both areas in a fairly concise way (Lie algebras theory; operator complexes and their homology; spectral theory in complex Banach spaces, including the generalized scalar operators and the decomposable operators of Colojoara and Foias). The next chapter presents results about nilpotence and quasi-nilpotence in infinite-dimensional Lie algebras. These are then applied (among others) in the third chapter to obtain infinite-dimensional generalizations of the classical Lie and Engel theorems. The fourth chapter treats a non-commutative variant of the Taylor joint spectrum for commuting operator tuples, namely the spectral theory for systems of operators generating a nilpotent or locally solvable Lie algebra. The final chapter discusses Lie algebras with involution and normal elements in them (generalizations of the normal operators on a Hilbert or Banach space) and semisimple Lie algebras of operators. The book is both pretty self-contained and well written and should be accessible starting from the level of a graduate student. Anyone wishing to learn about the fascinating interaction between the infinite-dimensional operator theory and Lie algebras will be well rewarded for his efforts if he reads the book.

Miroslav Engliš, Praha

A. N. Kolmogorov, A. P. Yushkevich (eds.): MATHEMATICS OF THE 19TH CENTURY. Mathematical Logic-Algebra-Number Theory-Probability Theory. 2nd revised edition. Birkhäuser, Basel, 2001, xiv+308 pages, DM 76.–.

This is the second edition of the English translation of the first volume of a multi-authored survey of mathematics in the 19th century. The topics included in this volume are apparent from the subtitle of the book.

A concise description of the development of mathematics as a whole since the beginning of the 19th century is a difficult task because this is the time when mathematics was splitted into a great number of distinct fields, all of them highly specialized. The authors of the respective parts of the book have done a very good work pointing out the most important moments of the development. Since the authors are Russian specialists in history of science and mathematics considerable attention is paid also to mathematics in Russia. This gives another dimension of this valuable historical survey.

Štefan Schwabik, Praha

R.-D. Reiss, M. Thomas: STATISTICAL ANALYSIS OF EXTREME VALUES. With Applications to Insurance, Finance, Hydrology and Other Fields. Birkhäuser, Basel, 2001, 296 pages, DM 116.–.

This is the second edition of the successful monograph extending the first edition in a substantial way (by about 160 pages). The book deals with the statistical modeling and analysis of extremes. The problems of extreme values are presented in the framework of various statistical approaches such as data analysis, nonparametric estimation, survival analysis, time series analysis, regression analysis, robust statistics and parametric inference. In addition to theory the book contains a lot of extreme value applications (especially in insurance, finance and hydrology), and five case studies demonstrate the presented methods numerically for real data (e.g. for ozone or pollution data). An interactive menu-driven statistical software Academic Xtremes on the enclosed CD forms a substantial part of the book (it runs under Window 95, 98, 2000, NT and is enhanced by the integrated programming language StatPascal).

The book is divided to five parts: Part I (Modelling and Data Analysis) introduces the statistical background required in extreme values analysis. Part II (Statistical Inference in Parametric Models) deals with parametric extreme value models (e.g. with generalized Pareto model). Part III (Elements of Multivariate Analysis) supplements multivatiare results for Part II. Part IV (Topics in Insurance, Finance and Hydrology) and Part V (Case Studies in Extreme Value Analysis) present the applications mentioned above.

Tomáš Cipra, Praha

H.-O. Kreiss, H. Ulmer Busenhart: TIME-DEPENDENT PARTIAL DIFFERENTIAL EQUATIONS AND THEIR NUMERICAL SOLUTION. Birkhäuser, Basel, 2001, 96 pages, DM 44.–.

These notes collect some basic material and highlight the main ideas of the theory of time dependent partial differential equations and their numerical solution by difference approximations.

The analytic and numerical theory are developed in parallel, the well-posedness of both linear and nonlinear problems as well as stability of difference approximations are studied.

A rather general theory is developed to treat the boundary-value problems and their time discretization. The basic are the energy estimates and the Laplace transform technique.

The presented results are important for the mathematical and numerical treatment of a large class of applications like Newtonian or non-Newtonian flows, two-phase flows, and geophysical problems.

Eduard Feireisl, Praha

Reinhard Siegmund-Schultze: ROCKEFELLER AND THE INTERNATIONALIZATION OF MATHEMATICS BETWEEN THE TWO WORLD WARS. Documents and Studies for the Social History of Mathematics in the 20th Century. Birkhäuser, Basel, 2001, ISBN 3-7643-6468-8, 341 pages, DM 170.–.

This book is the first detailed study of the brief but very influential role of the Rockefeller family in the field of mathematics. This book is based on a huge research which was done in the Rockefeller Archive Center, in the archives of Harvard University, University of New Hampshire, Göttingen and New York City.

In the first chapter the author is concerned with the process of internationalization and modernization of mathematics and the conditions for international scientific collaboration between the First World War and the Nazi dictatorship after 1933. The most important trends and events which influenced the birth of the modern kind of collaboration are described and discussed.

The second chapter shows the beginnings of the International Education Board and the central role of the Fellowship Program in careers of modern generation of mathematicians like S. Banach, A. Weil, B. L. van der Waerden etc. The support of Rockefeller Foundation for new mathematical publications and foundation of new mathematical journals is mentioned.

The third chapter is devoted to general ideological and political positions in USA and Europe. The relation between "advanced" and "backward" countries, between "saving" and "developing" scientific cultures, the American ideals for communication and collaboration are described. So this chapter also analyses and compares the development of mathematics in Europe and USA in the twenties and thirties of the 20th century.

The fourth chapter explains the practice of the Fellowship Programs of the International Education Board between 1923–1928 and the Rockefeller Foundation after 1928. The author describes the criteria for the Selection of Fellows, the problems of meetings, the Fellowship programs, some Fellowship lists, the selected social problems, national and American political and scientific interests, scientific communication in the 1920s and 1930s, etc. The rise of the Soviet-Russian mathematics and the problems of its collaboration with the Rockefeller Foundation and its political self-isolation after 1917 are discussed.

The fifth chapter analyses the role of Rockefeller's Help in the foundation of new scientific institutes in Europe. The foundation of New Mathematics Institute in Göttingen, Institute Henri Poincar in Paris, The Mathematical Institute in Djursholm in Sweden and the School of Mathematics of the Institute for Advanced Study in Princenton are described and their role in the communication, collaboration and cooperative work in science, especially in the thirties of the 20th century. The competition and cooperation between various European countries and the USA after World War I are discussed in this chapter.

The sixth chapter shows the changes of programs of the Rockefeller Foundation after 1933 and during the first years of the Second World War. The reactions by Rockefeller Philantropy and Rockefeller Emergency Programs which were created after 1933 when the nazis seized the power in Germany are analysed. The author shows their consequences for science and especially for mathematics and applied mathematics. Thanks to these programs more European ideas, new meaning and scientific profits were embraced in USA and international mathematical communication were started thanks dismissed scholars, many of whom were Jews from Germany.

The seventh chapter makes a very short review of the Rockefeller support for mathematics between 1945 and 1950.

At the end of the book there are seventeen appendices. The first fourteen of them are letters and reports of great mathematicians, workers or officers of the Rockefeller Foundation which illustrate the role of Rockeffeller's contribution to science and mathematics. The next three appendices contain the list of Fellows in mathematics until 1945, the list of Guggenheim Fellows in mathematics until 1945 and the list of dismissed mathematicians from Europe who were supported by the Rockefeller Foundation Emergency Fund.

This book can be recommended to everybody who wants to know more about the birth of modern and international mathematics in the first half of the 20th century, to everybody who loves mathematics.

Martina Bečvářová (Němcová), Praha

O. E. Barndorff-Nielsen, T. Mikosch, S. I. Resnick (eds.): LÉVY PROCESSES. Theory and Applications. Birkhäuser, Boston, 2001, x+415 pages, ISBN 0-8176-4167-X, DM 196.–.

A d-dimensional Lévy process is a time-homogeneous $\mathbb{R}^d$-valued stochastic process (X_t) with independent increments, $X_0 = 0$, continuous in probability and with càdlàg paths almost surely. Since the law of X_t is infinitely divisible for any $t \geqslant 0$, its Fourier transform possesses the famous Lévy-Khinchin representation,

$$\mathbf{E}\exp(\mathrm{i}\langle z, X_t\rangle) = \exp\Big[t\Big(-\frac{1}{2}\langle z, Az\rangle + \int_{\mathbb{R}^d}\Big(\mathrm{e}^{i\langle z,x\rangle} - 1 - \mathrm{i}\langle z,x\rangle \mathbf{1}_{\{|x|\leqslant 1\}}(x)\Big)\,\mathrm{d}\nu(x) + \mathrm{i}\langle \gamma, z\rangle\Big)\Big],$$

A being a symmetric nonnegative definite $d \times d$-matrix, $\gamma \in \mathbb{R}^d$, and ν a measure on $\mathbb{R}^d$ such that $\nu(\{0\}) = 0$ and $|x|^2 \wedge 1 \in L^1(\nu)$. The foundations of the theory of Lévy processes were laid down already in the thirties of the last century, however, two special cases, the Brownian motion ($\nu = 0$, $\gamma = 0$, $A =$ the identity matrix) and the Poisson process ($A = 0$, $\nu = c\delta_1$, $c > 0$, δ_1 the unit point mass siting at 1), have played a prominent role in both the theory and in modelling until recently. Over the last ten or fifteen years we have been facing renewed interest in general, jump Lévy processes which turned to arise in both natural and social sciences and have been tied with many advanced methods of the modern probability theory. In particular, at least two monographs on Lévy processes (by J. Bertoin and K. Sato) have appeared since 1996.

The book under review aims at providing a reader with an overview of recent progress in both the theory of Lévy processes and their applications to physics and finance. It comprises eighteen survey papers, written by leading specialists in the field (e.g. S. Albeverio, D. Applebaum, J. Bertoin, J. Rosen, T. Watanabe, to name just a few). Besides serving as a survey, some of the papers also contain new results with full proofs, if these simplify or generalize previous ones.

The book is divided into six parts, the first of them being a tutorial on Lévy processes, written by Ken-iti Sato. The other parts are entitled Distributional, pathwise and structural results, Extensions and generalizations of Lévy processes, Applications in physics, Applications in finance, and Numerical and statistical aspects.

For those interested in Lévy processes and their applications, this book collects a very useful information.

Bohdan Maslowski, Praha

Tian-Xiao He: DIMENSIONALITY REDUCING EXPANSION OF MULTIVARIATE INTEGRATION. Birkhäuser, Boston, 2001, 184 pages, hardcover, ISBN 0-8176-4170-X, DM 156.–.

The book under review is a monograph dealing with advanced methods of numerical multivariate integration.

The author focuses on dimensionality reducing expansions (DREs), i.e., techniques reducing a higher dimensional integral to a lower dimensional one. Among the applications of DREs, the main attention is paid to boundary type quadrature formulas (BTQFs) approximating the domain integral by means of evaluation points lying on the boundary of the domain of integration.

The book is arranged in the following way.

Chapter 1 introduces DREs with algebraic precision whose construction is based on the Green formula. Relevant reminders are estimated in different norms. Boundary type quadrature formulas derived from DREs are the subject of Chapter 2. Various BTQF constructions, including a wavelet approach, are presented. A substantial part of the chapter consists of applications of DREs and BTQFs to rational approximations, eigenvalue problems, integrals over axially symmetric regions, etc. Chapters 3 and 4 dwell on the integration of rapidly oscillating functions. These functions emerge if the integral of a continuous function over a $2n$-dimensional cube or sphere is approximated by an integral over, let us say, an n-dimensional domain. An example of this is a two-dimensional integral over a square approximated by a one-dimensional integral over a set of parallel line segments densely arranged in the square. DREs for integrals of functions of the complex variable over complex domains are the topic of Chapter 5. The final chapter concentrates on exact DREs associated with solutions of differential equations as well as on related BTQFs and applications to the boundary element method.

The monograph is rich in information and inspiration. It is fairly self-contained though the reader desirous of some details is occasionally referred to the bibliography. The expected readership is on no account limited to advanced readers but, in my opinion, new entrants into the field might find the exposition somewhat dense.

Jan Chleboun, Praha

W. Arendt, C. J. K. Batty, M. Hieber, F. Neubrander: VECTOR-VALUED LAPLACE TRANSFORM AND CAUCHY PROBLEMS. Monographs in Mathematics, vol. 96. Birkhäuser, Boston, 2001, ISBN 3-7643-6549-8, hardcover, 536 pages, DM 196.–.

This is a systematic study of the theory of the vector-valued Laplace transform applied to the semigroup theory and abstract Cauchy problems. Existence, regularity, asymptotic behaviour and other qualitative properties of solutions are extensively studied.

The main issues include: A survey of the basic properties of the Bochner integral, Laplace and Fourier transform, Cauchy problems in the abstract setting, Hille-Yosida theory, holomorphic semigroups, fractional powers etc. An essential part of the book is devoted to Tauberain theorems and asymptotics for solutions to abstract Cauchy problems, positive semigroups, inhomogeneous problems.

The last part contains some applications, the wave and heat equations, the translation operators on $L^p(\mathbb{R}^N)$.

The book is an excellent tool for students as well as researchers interested in the abstract theory of evolution equations.

Eduard Feireisl, Praha

L. Conlon: DIFFERENTIABLE MANIFOLDS. Second Edition. Birkhäuser Advanced Texts, Birkhäuser, Boston, 2001, ISBN 0-8176-4134-3, 432 pages, DM 130.–.

This textbook is an introduction to smooth manifolds and related topics.

The first three chapters span fundamental notions of differential geometry, such as (smooth) manifolds and functions on these manifolds, submanifolds, tangent bundles, but also covering spaces and fundamental groups. In Chapter 4, flows, distributions and foliations are introduced. The main result of this section is the Frobenius theorem characterizing the integrability of distributions. This theorem resurfaces again in Chapter 9 where a formulation in terms of differential forms is given.

In Chapter 5, the basics of Lie groups and Lie algebras are explained. Chapters 6–8 are devoted to multilinear calculus on the tangent bundle, differential forms and integration. Stokes' theorem is proved there and de Rham cohomology is introduced. Chapter 8 culminates in the de Rham theorem on the isomorphism of de Rham and singular (co)homology of a smooth manifold.

Chapter 10 contains an introduction to Riemannian geometry. Among other things, connections, Cartan structure equations and Gauss curvature are introduced there. In the last chapter, principal bundles and their reductions are treated.

The main body of the book is followed by four appendixes with proofs of some auxiliary results used in the text. In the last appendix, the de Rham theorem for Čech cohomology is proved.

I warmly recommend this book, which assumes only basic knowledge of topology, algebra and differential calculus, as a first introduction to differential geometry. Besides standard notions presented in a concise and systematic form, the author gives also samples of more advanced topics which go beyond the standard course, e.g. Lie groups and algebras, foliations and principal bundles.

Martin Markl, Praha

E. Bayro Corrochano, G. Sobczyk (eds.): ADVANCES IN GEOMETRIC ALGEBRA WITH APPLICATIONS IN SCIENCE AND ENGINEERING. Birkhäuser, Basel, 2001, 624 pages, DM 196.–.

The book mainly consists of contributions to a special session at the 5th International Conference on Clifford Algebras and their Applications in Mathematical Physics (Ixtapa-Zihuatanejo, Mexico, 1999).

Geometric algebras first appeared in the paper 'Applications of Grassmann's extensive algebra', Amer. J. Math. *1* (1878), 350–358, by W. K. Clifford. Now, more than 120 years later, Geometric Algebra may be described as the branch of mathematics that investigates algebraic structures suitable for meaningful geometrical reasoning, both Euclidean and non-Euclidean, with Clifford algebras still occupying a prominent position.

The message of the present book is that Geometric Algebra has been already successfully employed in applied sciences and engineering, such as robotics, computer vision, neural and quantum computing. Even though aimed at advanced readership, the proceedings appear to have a potential of recruiting new followers of Geometric Algebra.

Michal Marvan, Opava

E. M. Alfsen, F. W. Shultz: STATE SPACES OF OPERATOR ALGEBRAS. Basic Theory, Orientations, and C^*-products. Birkhäuser, Boston, 2001, ISBN 0-8176-3890-3, hardcover, 368 pages, DM 170.–.

This is a nice monograph on the theory of state spaces of operator algebras and their geometry. The book begins with a synopsis of diverse reference material on topological vector spaces, lattices, order algebras, spectral theory, etc., which is needed later on (Chapter 1), followed by a fairly detailed and self-contained introduction to C^*-algebras and von Neumann algebras (Chapter 2). Then the ideal structure of these algebras and the facial structure of their state spaces is investigated, including a section on compressions in von Neumann algebras (Chapter 3). Chapter 4 contains a detailed study of the special case of the algebra $\mathcal{B}(H)$ of all bounded linear operators on a Hilbert space. State spaces of general C^*-algebras, their relationship with representations, and orientations of state spaces are discussed in Chapter 5, and the exposition culminates by establishing a 1-1 correspondence between global orientations of the state space and Jordan compatible associative products of the algebra. Finally, the same result is obtained also for von Neumann algebras in Chapter 7, after recalling some necessary technical prerequisites in Chapter 6. The exposition is very readable and lucid throughout and, with the exception of a survey of structure theory of von Neumann algebras in the last-mentioned chapter (where the reader is referred to Kadison's and Ringrose's *Fundamentals of the theory of operator algebras I–II* for the proofs) also quite self-contained. The required prerequisites do not go beyond standard courses in real and complex variables, measure theory and elementary functional analysis, thus making the material well accessible even to non-specialists or graduate students. The reviewer can therefore only endorse the information on the back cover that the book will be of interest not only for specialists in operator algebras, but also for anyone seeking an overview of the field or a quick introduction to the basic theory of C^*- and von Neumann algebras.

Miroslav Engliš, Praha

S. Leader: THE KURZWEIL-HENSTOCK INTEGRAL AND ITS DIFFERENTIALS. A Unified Theory of Integration on $\mathbb{R}$ and $\mathbb{R}^n$. Marcel Dekker, New York, 2001, viii+355 pages, USD 150.–.

Solomon Leader introduced and developed a new concept of differential based on the integration of "summands" in his extensive previous work. The present book represents a survey of this general (in fact axiomatic) approach to gauge-based sum integration of Kurzweil-Henstock type.

The general approach of Leader includes Stieltjes type integrals as well as the Lebesgue integral and of course the well known calculus (Newton) integral. The contribution of S. Leader is of interest also for specialists in the Kurzweil-Henstock integration because of its universality and of a slightly different viewpoint at the problem.

The main requirements for understanding the book are especially the concept of filter-bases and Riesz spaces. The essential facts about the necessary mathematical background are presented in the last chapter of the book. It has to be pointed out that Leader's approach allows to easily get integration on arbitrary (unbounded) intervals in $\mathbb{R}$; some results on integration over intervals in $\mathbb{R}^n, n > 1$ are also given (Fubini's Theorem, Green's Theorem).

The book of Leader is another contribution to the rehabilitation of the classical sum integration of Riemann which in the last century was considered for a long time a "useless anachronism". J. Kurzweil and R. Henstock changed this and nowadays we are facing a still growing new theory with all its advantages and disadvantages.

Štefan Schwabik, Praha

M. Kohlmann, S. Tang (eds.): MATHEMATICAL FINANCE. Birkhäuser, Basel, 2001, 376 pages, DM 196.–.

The publication presents 35 contributions of the participants of the Workshop of the Mathematical Finance Research Project, Konstanz, Germany, October 5–7, 2000 (the Workshop took place in the centenary year of the publication of Bachelier thesis which is considered as the starting point of modern finance as a mathematical discipline). The participation of experts from practice and their contributions reflect the exchange of theoretical and applied results in this field. More and more real world problems of financial institutions are analysed and solved in mathematical laboratories. Therefore the publication can be also evaluated as a contribution to the interdisciplinary work in mathematical finance.

In spite of the fact that particular contributions concern various problems of mathematical finance one can stress e.g. the following subjects:

- description of a market by fractional Brownian motion models (in addition to the known description by discrete Brownian and Lévy process models);
- adaptations of stochastic filtering or control methods and techniques to the classical financial problems in portfolio selection, irreversible investment, risk sensitive asset

allocation, capital asset pricing, hedging contingent claims, option pricing, interest rate theory.

Tomáš Cipra, Praha

Cristian E. Gutiérrez: THE MONGE-AMPÈRE EQUATION. Progress in Nonlinear Differential Equations and Their Applications. Birkhäuser, Boston, 2001, xi+127 pages, ISBN 0-8176-4177-7, CHF 118.–.

The objective of the book is the Monge-Ampère equation det $D^2u = 0$, which has its application especially in geometry and convex analysis. It reflects recent advances, the results of present considerable interest in this equation and its applications. In particular, the theory of weak solutions and regularity is tackled. The book is divided into six chapters, a concise preface, rich bibliography, an index and a list of notation. In Chapter 1, generalized and viscosity solutions are introduced, maximum principles established, the Dirichlet problem solved and ellipsoids of minimum volume analyzed. In Chapter 2 the Harnack inequality for non-divergence elliptic operators is presented in connection with some ideas used in studying the linearized Monge-Ampère equation. Chapter 3 presents the theory of cross sections of weak solutions to the Monge-Ampère equation. Several geometrical properties needed in subsequent chapters are proved. The main result of Chapter 4 concerns the characterization of global solutions of the equation $Mu = 1$, where Mu is a measure with the density det D^2u, u a convex function. Chapter 5 contains Caffarelli's $C^{1,\alpha}$ estimates for weak solutions and a result about the extremal points of the set where a solution u equals the supporting hyperplane. Finally, in Chapter 6 the $W^{2,p}$ estimates for the Monge-Ampère equation, recently developed by Caffarelli, are presented. Bibliographical notes are included at the end of each chapter. The book is rigorously mathematical, accessible mostly to mathematicians with university course of analysis. Prevailing audience will be specialists in partial differential equations and equations of mathematical physics. The book is recommendable to specialized mathematical-physical libraries.

Ivan Straškraba, Praha

Samuel S. Kotz, Tomasz J. Kozubowski, Krzystof Podgórski: THE LAPLACE DISTRIBUTION AND GENERALIZATIONS. A Revisit with Applications to Communications, Economics, Engineering, and Finance. Birkhäuser, Boston, 2001, xviii+350 pages, 37 Fig's, ISBN 3-7643-4166-1, DM 210.–.

The authors of the monograph hardly need any recommendation and their own words will perhaps characterize its main features at best.

The aim: "It attempts to be a systematic exposition of all that appeared in the literature and was known to us by the end of the 20th century about the Laplace distribution and its numerous generalizations and extensions. …in our literature search we tried to left no stone unturned (we collected over 400 references)."

The character: "…we view this monograph as a textbook, the exposition in earlier chapters proceeds at a rather pedestrian pace and each part of the book presupposes all earlier developments. …more advanced approach is taken in the second part of the book, where quite a few of our results appear in print for the first time."

The prerequisites: "…calculus, matrix algebra, and familiarity with the basic concepts of probability theory and statistical inference."

The style: "…to make sufficiently precise statements while striving to keep the mathematical level of the book appealing to the widest possible readership…"

The goal: "to provide an alternative to the dominance of the normal law... that reigned almost without opposition... for almost two centuries. ...to demonstrate that it is natural and sometimes superior alternative to the normal law."

Two thirds of the book are occupied by the chapter "Univariate distributions" with an extensive description of properties, order statistics and statistical inference for both the classical symmetric and the generalized asymmetric Laplace distribution with a particular attention paid to the maximum likelihood estimation. Several related distributions are mentioned and compared, namely Bessel function distribution, Laplace motion (an analog of the Brownian motion in the Laplace domain), Linnik distribution etc.

The second chapter discusses "a relatively unexplored and somewhat fragmented field" of multivariate Laplace distributions; the majority of results was obtained in the last decade of the 20th century.

The overview of applications presented in the third chapter is rather impressive, encompassing image and speech recognition, ocean and aircraft engineering, biological and environmental sciences, inventory management, quality control and finance, shortly, all cases where data with heavier than Gaussian tails.

Many sections are supplemented by exercises, frequently excerpted from the recent literature; their total number is 136. The book will certainly be an exciting and inspiring reading for anybody interested in probability and statistics occur.

Ivan Saxl, Praha

Frédéric Hélein: CONSTANT MEAN CURVATURE SURFACES, HARMONIC MAPS AND INTEGRABLE SYSTEMS. Birkhäuser, Basel, 2001, 128 pages, DM 49.–.

The reviewed book originated from lectures the author gave at the Eidgenossische Technische Hochschule in Zurich during Spring 1999. It is an introduction to constant mean curvature surfaces and harmonic maps from surfaces to symmetric spaces. The author finds many interesting links between various branches of modern differential geometry as well as between the classical and modern geometry. It connects the classical problem of constant mean curvature surfaces treated in the 19th century to the twistor theory, harmonic maps, loop groups theory and related fields. The exposition starts with Euclidean immersions and shows the connection between harmonic maps and constant mean curvature surfaces with the surprising possibility to integrate the corresponding system of equations. The author shows how this feature extends to much more general situations. The book starts rather elementary, but as the material develops, it becomes more involved. Fortunately the author gives many examples which help the reader in understanding the subject, also the extensive list of references gives the possibility to consult other authors. The book is clearly written and could be recommended not only to differential geometers but to all who are interested in modern mathematics. It shows a unified approach to different branches of mathematics.

Adolf Karger, Praha