Book reviews

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BOOK REVIEWS

Frank Pacard, Tristan Rivière: LINEAR AND NONLINEAR ASPECTS OF VORTICES: THE GINZBURG-LANDAU MODEL. Birkhäuser, Boston, 2000, ISBN 0-8176-4133-5, 352 pages, DM 178.–.

The book is concerned with the singular limit, as $\varepsilon \to 0$, of the critical points of the Ginzburg-Landau energy functional

$$E_\varepsilon(u) = \int_\Omega |\nabla u|^2 d\,x + \frac{1}{2\varepsilon^2}\int_\Omega (1-|u|^2)^2\,\mathrm{d}x,$$

where $u\colon \Omega \subset \mathbb{R}^2 \to \mathbb{R}^2 \simeq \mathbb{C}$ satisfies the Dirichlet boundary condition with a prescribed $g\colon \partial\Omega \to \mathbb{S}^1$ of degree d. The corresponding Euler-Lagrange equations

$$\Delta u + \frac{1}{\varepsilon^2}u(1-|u|^2) = 0$$

are known as the Ginzburg-Landau (GL) equations. The functional E_ε and the GL equations arise as a simple model of superconductivity or superfluidity where u is the wavefunction of the superconducting electrons or the condensate wavefunction of the superfluids.

The phenomenology of superconductivity and superfluidity suggests that the solutions $u = u_\varepsilon$ of the GL equations should exhibit vortices, i.e., a number of isolated zeroes at $a_1(\varepsilon), \dots, a_d(\varepsilon) \in \Omega$. In a three dimensional picture, Ω is a cross section of a three dimensional domain and the points $a_j(\varepsilon)$ are the intersections of the vortex lines with Ω. In a seminal work, F. Bethuel, H. Brezis and F. Helein (*Ginzburg-Landau vortices,* Progress in Nonlinear Differential Equations & Their Appl., vol. 13, Birkhäuser, 1994) showed that the limits u_* of minimizers of E_ε are harmonic maps with exactly d singularities at $a_1, \dots, a_d \in \Omega$, and with values in $\mathbb{S}^1$. We note here that the second term of E_ε can be interpreted as a penalization of deviations of $|u|$ from the unitary values. Near each a_j the singularity of u_* is of the type $z - a_j/|z-a_j|$, and recall that d is the degree of the boundary condition g, which then acts as a topological obstruction. Moreover, the position $(a_1, \dots, a_d)$ is a minimum of some renormalized energy, a function $W_g(a_1, \dots, a_d)$ related to the functional E_ε. The work of F. Bethuel, H. Brezis and F. Helein stimulated an intensive research which, among other things, showed that every u_* corresponding to the critical point of W_g is a limit of critical points of E_ε as $\varepsilon \to 0$. Moreover, if the sequence consists of minimizers, then for all ε sufficiently small, u_ε has exactly d zeroes $a_j(\varepsilon)$ which converge to a_j. In fact, for small ε one generally expects that solutions of the GL equations are largely characterized by their zero sets.

The book under review further elaborates on the relationship between the critical points of E_ε and the critical points of W_g. It is shown that there exists a solution u_ε of the GL equations corresponding to any nondegenerate critical point $(a_1, \dots, a_d)$ of W_g which possesses exactly d vortices which converge to $(a_1, \dots, a_d)$. The main result is that, locally in some sense, the vortices of the solutions of the GL equations determine the solution uniquely. Some new techniques are employed to achieve these results.

The approach is based on PDE and nonlinear functional analysis tools rather than on the variational and topological arguments. To prove the existence of vortex solutions, the authors first construct approximate solutions by gluing axially symmetric solutions located at a_j (which are known "explicitly") with the limit function u_*. Next, the mapping properties of the linearized GL equations about the approximate solutions are examined in detail; a certain local implicit function theorem in weighted Hölder spaces then leads to the result. This method gives a very precise description of the solution and in particular of its zero set. To prove the local uniqueness result, this implicit function theorem approach is combined with the methods based on conservation laws and on a generalized Pohozaev formula.

The original results of the monograph substantially enlarge the knowledge of the mathematical aspects of the theory of the GL vortices. Undoubtedly, it will stimulate further research and as such it will serve as a valuable resource for mathematicians; also, it will be interesting to physicists, engineers, and advanced graduate students.

Miroslav Šilhavý, Praha

Rodolfo Salvi: THE NAVIER-STOKES EQUATIONS: THEORY AND NUMERICAL METHODS. Lecture Notes in Pure and Applied Mathematics Series, Marcel Dekker, New York, 2001, 308 pages, USD 150.–.

The volume contains the proceedings of the International Conference on the Navier-Stokes Equations: Theory and Numerical Methods, held in Villa Monastero in Varenna, Lecco, Italy. More than 80 invited specialists from all around the world participated. The volume collects 23 selected articles. They deal with the mathematical theory of fluid mechanics: compressible and incompressible fluids, nonnewtonian fluids, the free-boundary problem, qualitative properties of solutions, hydrodynamic potential theory and numerical experiments. Precisely, the first part of the volume contains contributions on the flow in a bounded and unbounded domain (micropolar fluids, Lyapunov functions for the Navier-Stokes equations, magnetic Bernard problem, stability of the Navier-Stokes flows through permeable boundaries, steady solutions of the Kuramoto-Sivashinsky equation, classical solutions to the stationary Navier-Stokes system, the stationary Navier-Stokes flow in a two-dimensional Y-shape channel under a general outflow condition, regularity of solutions to the Stokes equations under a certain nonlinear boundary condition, viscous incompressible flows in unbounded domains, life span and global existence of 2-D compressible fluids, the theory of nonstationary hydrodynamic potentials, the blow up criterion for the inviscid 2-D Boussinesq equations, weak solutions to viscous heat-conducting gas 1-D equations with discontinuous data).

The second part deals with the general qualitative theory (regularity criteria of the axisymmetric Navier-Stokes equations, boundary singular sets for Stokes equations, feedback stabilization for the 2-D Navier-Stokes equations, approximation of weak limits via the averaging method with applications to Navier-Stokes equations, asymptotic profiles of nonstationary incompressible Navier-Stokes flows in R^n and R^n_+, L^2 decay for the weak solution to equations of nonnewtonian incompressible fluids in the whole space) while the last part contains numerical methods and experiments (Navier-Stokes simulations of vortex flows, numerical results for the CGBI methods to the viscous channel flow, isothermal drop-wall interactions, convergence of the interface in the finite element approximation for two-fluid flows). The contributions present original results and surveys on recent developments.

Šárka Nečasová, Praha

J. Cronin: DIFFERENTIAL EQUATIONS. INTRODUCTION AND QUALITATIVE THEORY. 2nd edition, revised and expanded. Marcel Dekker, New York, 2001, xi + 370 pages, USD 150.–.

Jane Cronin presents her point of view how to teach elements of the theory of ordinary differential equations in this nice book.

Existence theorems, linear systems, autonomous systems, stability, Lyapunov methods, periodic solutions, bifurcation and branching of periodic solutions are explained in the book.

The book is well written, no doubt it can be used for teaching ordinary differential equations with preliminaries to "qualitative theory" as is usual in our times. J. Cronin explains the topic having in mind all the needs of contemporary applications of the qualitative theory in biology, chemistry, etc. The approach is formally rather classical but it has to be mentioned that modern approaches are involved, too.

In the preface to the second edition the author mentions the existence of a solutions manual for the instructor. This is not available to the reporter and it is also not noted where this manual is available.

The book is a well written introductory text for first courses on ordinary differential equations and can be warmly recommended to teachers and students starting their education in this field.

Štefan Schwabik, Praha

L. Krchy, C. Foias, I. Gohberg, H. Langer (eds.): RECENT ADVANCES IN OPERATOR THEORY AND RELATED TOPICS. The Bla Szökefalvi-Nagy Memorial Volume. Birkhäuser, Basel, 2001, l + 669 pages, EUR 165.–.

This volume contains 35 articles on results in various areas of operator theory and connected fields presented mostly at a conference in Szeged (Hungary) commemorating and honouring in 1999 the late famous Hungarian mathematician Béla Szökefalvi-Nagy.

The volume is introduced by an overview of the work of Szökefalvi-Nagy.

The book will be of use for people interested in the personality of Szökefalvi-Nagy as well as in the development of mathematics in the fields at the beginnings of which this great man was present.

Štefan Schwabik, Praha

Elliott H. Lieb, Michael Loss: ANALYSIS. 2nd edition. Graduate Studies in Mathematics 14. American Mathematical Society, Providence, RI, 2001, ISBN 0-8218-2783-9, xxii+346 pages.

Contents. Chapter 1. Measure and Integration, Chapter 2. L^p-spaces, Chapter 3. Rearrangement Inequalities, Chapter 4. Integral Inequalities, Chapter 5. The Fourier Transform, Chapter 6. Distributions, Chapter 7. The Sobolev spaces H^1 and $H^{1/2}$, Chapter 8. Sobolev Inequalities, Chapter 9. Potential Theory and Coulomb Energies, Chapter 10. Regularity of Poisson's Equation, Chapter 11. Introduction to the Calculus of Variations, Chapter 12. More about Eigenvalues; Exercises, List of Symbols, References, Index.

This is definitely a beautiful book. Starting with some important preliminaries from real and functional analysis the authors eventually arrive at topical applications of Sobolev spaces. The book might serve as a shortcut to them, giving thus a strong motivation to those who have to go through a vast area of the theoretical background; this concerns both students and specialists from other areas, who need the complicated machinery of the contemporary analysis. The introduction to various basic areas as the theory of the

Lebesgue integral, the Fourier transform, the function spaces (L_p spaces and Sobolev spaces) is naturally rather brief, the authors often consider special cases, and they do not go too far with the abstract setting in the functional analysis spirit. On the other hand, there is a lot of other specialized literature on the subject. It should be also observed that some parts of the book will be a useful reference even for specialists since the authors present some of the basic tools in a very rigorous way, e.g. inequalities with exact constants, and they show clever methods how to calculate, which is equally useful for beginners as well as advanced specialists. Needless to say, the knowledge of the best constant is of increasing importance in many current considerations. Authors' intentions are well illustrated by their sentence from the introduction: "In this way we hope that relative beginners can get some flavour of research mathematics and the feeling that the subject is open-ended". Especially the last fact easily disappears when one has to study theoretical background thoroughly, in all generality and at the contemporary level of our knowledge, which is usually a result of a long evolution of the subject.

Chapter 1 is a brief introduction to the Lebesgue integral, including the Carathéodory construction of a measure from an outer measure and with a special regard to convergence theorems.

Basic properties of L^p spaces (inequalities for the norms, uniform convexity, uniform boundedness principle, the Banach-Alaoglu theorem) are presented in Chapter 2.

Chapters 3 and 4 deal with the extremely useful concept of rearrangement of functions. The major application of convolution inequalities for rearrangements in Lebesgue and weak Lebesgue spaces, stated with sharp constants, are imbedding theorems for Sobolev spaces, for the moment in disguise of variants of Young's inequality, when the target space is rearrangement invariant. Sobolev spaces are dealt later in Chapters 6 and 7.

Chapter 5 is a basic exposition of the properties of the Fourier transform (Plancherel's theorem, the inversion formula, the sharp Hausdorff-Young inequality). Chapter 6 briefly introduces distributions with the goal to define the weak derivatives and to define the Sobolev spaces.

The special case of Sobolev spaces, namely, H^1 and $H^{1/2}$ are considered in Chapter 7. Among other it is shown that they permit a rather simple approach based on the Fourier analysis approach, giving an elegant way to spaces of non-integer order. Connections with the semigroup approach to Sobolev spaces is discussed, too.

Chapter 8 is devoted to Sobolev and Poincaré inequalities, to compactness theorems, also the remarkable logarithmic inequality due to Gross is presented. It contains an interesting material on the heat kernel, connected with the Gross inequality.

Chapter 9 is an introduction to the potential theory. Basic properties of harmonic, subharmonic, superharmonic functions, and the Coulomb energies are presented, followed by the strong maximum principle, Newton's theorem, properties of the Coulomb energy, and lower bounds on Schrödinger's wave function.

Chapter 10 is devoted to another important area of the PDEs theory, namely, to the regularity problems, which are demonstrated by regularity results for the weak solutions of the Poisson equation. Continuity, Hölder continuity and (classical) differentiability of weak solutions is briefly handled here in dependence on the "quality" of the right hand sides of the equation.

Chapter 11 is a display of interesting and topical applications of the theory developed so far in the book to various problems in the calculus of variations. This includes the problem of the lowest energy of a hydrogen atom, the Thomas-Fermi problem on atoms with many electrons, on the capacitor problems, throwing thus light on the formal mathematical treatment of capacities in the Lebesgue spaces. The concluding Chapter 12 developes further some ideas about the eigenvalues encountered in the preceding chapter and it surveys some methods for estimates of eigenvalues for the Schrödinger equation and for the Laplace operator in a domain.

The chapters are amended by well chosen exercises.

Miroslav Krbec, Praha

Javier Duoandikoetxea: FOURIER ANALYSIS. Translated from Spanish and revised by David Cruz-Uribe. Graduate Studies in Mathematics, vol. 29. American Mathematical Society, Providence, RI, 2001. xviii + 222 pages, USD 35.–.

This book is one of the many different introductions into Fourier Analysis, a large branch of mathematics, and its goal is to study the real variable methods introduced into this topic by A. P. Calderón and A. Zygmund in the 1950's.

The (standard) material of the book comes from a graduate course taught by the author at the Universidad Autónoma de Madrid during the academic year 1988–89 (and partially from a course taught at the same university in fall 1985 by José Luis Rubio de Francia to whom the book is dedicated).

The book was first published in Spanish in 1991 and republished in 1995. What makes the English translation more up-to-date are mainly the sections "Notes and further results" added to each chapter and containing new topics, results which are not discussed in the text and references for the interested reader. The nine chapters are entitled: Fourier Series and Integrals; The Hardy-Littlewood Maximal Function; The Hilbert Transform; Singular Integrals I and II; H^1 and BMO; Weighted Inequalities; Littlewood-Paley Theory and Multipliers; The T1 Theorem.

Alois Kufner, Praha

H. W. Braden, I. M. Krichever (eds.): INTEGRABILITY: THE SEIBERG-WITTEN AND WHITHAM EQUATIONS. Gordon and Breach Science Publishers, Amsterdam, 2000, ISBN 90-5699-281-3, ix+276 pages, USD 70.–.

This collection of papers concerns integrable systems arising in different fields of mathematical analysis, physics, geometry and other applied areas. It is the proceedings of the meeting *Integrability: The Seiberg-Witten and Whitham Equations* which took place in Edinburgh in 1998.

The following issues are included: Baker-Akhiezer functions and integrable systems, Algebraic geometry, integrable systems and Seiberg-Witten theory, Seiberg-Witten theory and integrable systems, Seiberg-Witten curves and integrable systems, Integrability in Seiberg-Witten theory, Witten-Dijkgraaf-Verlinde-Verlinde (WDVV) equations and Seiberg-Witten theory, Geometry of a special class of solutions to generalized WDVV equations, Picard-Fuchs equations, Hauptmoduls and integrable systems, Painlevé type equations and Hitchin systems, World-sheet instantons and Virasoro algebra, Dispersionless integrable systems and their solutions, N-component integrable systems and geometric asymptotics, systems of hydrodynamic type from Poisson commuting Hamiltonians and Integrability of equations of hydrodynamic type from the end of the 19th to the end of the 20th century. List of participants and index are included.

The book will interest specialists in the above mentioned fields and analysts dealing with the theory of integrable systems.

Ivan Straškraba, Praha

J. Eells, B. Fuglede: HARMONIC MAPS BETWEEN RIEMANNIAN POLYHEDRA. Cambridge Tracts in Mathematics, vol. 142. Cambridge University Press, 2001, ISBN 0-521-77311-3, GBP 40.–.

This monograph studies harmonic maps between admissible Riemannian polyhedra. Chapter 1 contains a brief summary of the theory of smooth harmonic maps between Riemannian manifolds. Further, a description of Riemannian polyhedra as harmonic and geodesic spaces is given. Chapter 2 summarizes briefly the topics concerning harmonic spaces, Dirichlet spaces and geodesic spaces. Chapter 3 contains some geometric examples of harmonic, Dirichlet and geodesic spaces. Chapter 4 presents in detail the necessary background concerning polyhedra. Chapter 5 deals with the Sobolev space $W^{1,2}(X)$ on an admissible Riemannian polyhedron X. Weakly harmonic and weakly subharmonic functions on X are defined as weak solutions/subsolutions of class $W^{1,2}(X)$ to the Laplace equation on X. Chapter 6 is devoted to the study of Harnack inequality and Hölder continuity for weakly harmonic functions. In Chapter 7 the authors develop the potential theory on Riemannian polyhedra. Chapter 8 offers a variety of examples of Riemannian polyhedra. In Chapter 9 a concept of energy is defined and investigated for maps from an admissible Riemannian polyhedron into an arbitrary metric space. In Chapter 10 and Chapter 11 the authors establish the existence and regularity of the energy minimizing maps between certain Riemannian polyhedra. In Chapter 12 the authors introduce the notion of a harmonic map from an admissible Riemannian polyhedron to some metric space. Chapter 13 is devoted to the study of harmonic morphisms and their relation to harmonic maps.

Dagmar Medková, Praha

Tanton, James: SOLVE THIS. Math activities for students and clubs. Classroom resource, materials. The Mathematical Association of America, Boston, 2001, 218 pages, ISBN 0-88385-717-0.

The subtitle of this book, as the author himself notes, might create an incorrect perception—namely, that the book is devoted to students with a deep interest in mathematics and to their teachers or instructors. This, in fact, is not the case—the book is conceived in a way that enables a wide range of readers of various age- and knowledge-levels to discover mathematics as an attractive, interesting, and meaningful activity. The main aim of the activities, which the author promotes, is "... to foster original inquiry, to transform the notion of "solved" from one of completion and closure to one of a new opportunity for continued exploration and creative endeavour."

The book is divided into three parts:

1. Activities, Discussions and Problem Statements
2. Hints, Some Solutions and Further Thoughts
3. Solutions and Discussions

The first part consists of thirty chapters, which describe the subjects of activities and formulate the problems. Each of these thirty chapters is devoted to a "main" topic. At first glance, it may seem that a large number of chapters is devoted to geometry, especially to topology. However, in the course of solving the problems the solvers need a wide spectrum of knowledge; a list of mathematical topics and pieces of knowledge is included at the end of the book. In the majority of chapters, a section titled "Taking It Further" outlines further possible ways of investigating the problem or the given situation.

The content and orientation of the second and third parts is apparent from their titles. These chapters further explore the author's interpretations and enable a deeper identification with the topic. In addition to answers to Taking It Further, there are also Open Problems, Challenges, Hard Challenges, and Notes to the subject of the chapter, which facilitate deeper grasping of the topic. The reader is motivated not only to the solving of certain problems and to critical thinking through the suggested solution, but also to the formulation of questions and to problem posing.

Going through the suggested activities and solving the problems requires a creative approach. It will contribute to students' capabilities and skills to solve problems, and to their overall mathematical literacy. The book is based on collaboration and thus contributes to the development of abilities to communicate, debate, etc. It also contributes to teachers' competence. It brings further impulses to the improvement of constructive character of mathematical education of students. Working with this book will enrich both the students and the teachers.

Marie Tichá, Praha

KOLMOGOROV IN PERSPECTIVE. Translated from the Russian by Harold H. McFaden. History of Mathematics, vol. 20. American Mathematical Society, London Mathematical Society, Providence, RI, 2000, ix + 230 pages, ISBN 0-8218-0872-9, USD 49.–.

The book presents a selection of writings from two volumes published in Russian (1991, 1993) and dedicated to A. N. Kolmogorov. The opening "Biographical Sketch of A. N. K. Life and Creative Paths" written by A. N. Shiryaev covers nearly half of the volume and is an excellent account of the mathematician's life and achievements framed by the preceding and subsequent history of mathematics. It is followed by several pieces written by Kolmogorov's students and colleagues. A special position is taken up by a short article prepared by P. S. Alexandrov shortly before his death for the occasion of the eightieth birthday of Andreĭ Nikolaevich accompanied by long "Memories of P. S. Alexandrov" written by Kolmogorov and describing the close lifelong friendship of the two mathematicians; included are also numerous quotations from their correspondence. The second Kolmogorov's article honors the tricentennial of Newton's birth and discusses fundamental concepts of analysis. The book is closed by 50 pages of bibliography including the main publications of A. N. K. (about 700 items) and a list of selected publications concerning him.

The impact of Kolmogorov on mathematics is so great and lasting that the book hardly needs any recommendation to mathematicians. However, a sketch of life covering nearly the whole century (1903–1987) and remembrances of the contemporaries cannot avoid the dark shadows of this disputable period. The book is full of them, explicit as well as only implicit ones, and they deserve and would perhaps arouse an interest of non-mathematicians as well.

Ivan Saxl, Praha

James F. Davis, Paul Kirk: LECTURE NOTES IN ALGEBRAIC TOPOLOGY. Graduate Studies in Mathematics, vol. 35, American Mathematical Society, Providence, RI, 2001, ISBN 0-8218-2160-1, USD 55.–.

As noted by the authors, the book, which grew from lecture notes written while teaching second-year algebraic topology at Indiana University, might have as well been titled 'What Every Young Topologist Should Know.' It indeed presents, in a self-contained and clear manner, all classical constituents of algebraic topology—homological algebra, differential topology, CW-complexes, fiber spaces, spectral sequences and homotopy theory.

The first chapter is devoted to various chain complexes associated to a space, namely to the singular chain complex of a topological space, to the cellular chain complex of a CW-complex, and to the simplicial chain complex of a simplicial space. A definition of an (ordinary) homology theory based on the Eilenberg-Steenrod axioms is also given.

The second chapter serves as an introduction to homological algebra. The reader can find there the concept of resolutions leading up to definitions of derived functors *Tor* and

Ext. The following chapter is devoted to tensor products, Eilenberg-Zilber maps, diagonal approximations, and cup and cap products. Chapter 4 explains the fundamental notions of the theory of locally trivial fiber bundles: free actions, principal and associated bundles, reduction of structure groups and pullbacks. In Chapter 5, homology with local coefficients is introduced.

Homotopy theory in the category of compactly generated spaces is the theme of Chapter 6. The exposition is based on notions of fibrations, cofibrations and related homotopy sequences. Homotopy groups are introduced there and the Hurewicz and Whitehead theorems are proved. Obstruction theory, both for extension and lifting problems, including characteristic classes, is treated in Chapter 7. Chapter 8 is devoted to generalized homology and spectra. Various versions of bordism theories are discussed there in detail.

Chapter 9 is dedicated to spectral sequences and Chapter 10 to their applications, such as Serre classes of abelian groups, cohomology operations, mod 2 Steenrod algebra and localization. The final chapter contains fundamental definitions and results of simple homotopy theory—the Whitehead torsion, the Reidemeister torsion and the s-cobordism theorem.

We recommend this book as a valuable tool for everybody teaching graduate courses of algebraic topology, as well as a self-contained introduction to the above mentioned topics for independent reading.

Martin Markl, Praha

J. C. McConnell, J. C. Robson: NONCOMMUTATIVE NOETHERIAN RINGS. Graduate Studies in Mathematics, vol. 30. American Mathematical Society, Providence, Rhode Island, ISBN 0-8218-2169-5, 636+xx pages, USD 72.–.

The book starts with chapter 0 "Preliminaries" and then it is divided into four parts. The first, "Basic Theory", consists of five chapters, namely "Some Noetherian Rings", "Quotient Rings and Goldie's Theorem", "Structure of Semiprime Goldie Rings", "Semiprime ideals in Noetherian Rings" and "Some Dedekind-like Rings". The second part "Dimensions" has three chapters, "Krull Dimension", "Global Dimension" and "Gelfand-Kirillov Dimension". The chapters "The Nullstellensatz" , "Prime Ideals in Extension Rings", "Stability" and "K_0 and Extension Rings" are collected in the third part "Extensions" while the last one, "Examples", studies "Polynomial Identity Rings", "Enveloping Algebras of Lie Algebras" and "Rings of Differential Operators on Algebraic Varieties". At the end of the book a comprehensive list of references (containing more than 500 items) is included.

Ladislav Bican, Praha

José M. Garcia-Bondía, Joseph C. Várilly, H. Figueroa: ELEMENTS OF NONCOMMUTATIVE GEOMETRY. Birkhäuser, Boston, 2001, 685 pages.

The history of Noncommutative Geometry, which deals with the unification of mathematics under the aegis of the quantum apparatus, is not longer than 15 years. During this time an enormous number of articles has been published, but as for books, there was only one, written by A. Connes, the founder of Noncommutative Geometry. "Elements of Noncommutative Geometry" fills an important gap in the literature.

The subject of this monograph is an algebraic and operatorial reworking of geometry, which traces its roots to quantum physics.

The book is divided into four parts, comprising 14 chapters.

Part I deals mainly with the task of building a corridor between ordinary topology and algebra. In this part the authors prove the Tannaka-Krein theorem, the Serre-Swan categorical equivalence between vector bundles and projective modules over unital commutative algebras, Bott's periodicity, and they discuss why are Hopf algebras expected to play an important role in noncommutative geometry.

Part II—Calculus and linear algebra—may provide some relief, by focusing first on the linear algebra underpinnings and then on the real analysis foundation of noncommutative geometry.

Chapter 5 contains the Clifford algebra and spin representations.

Chapter 6 extends the Clifford algebra to the infinite dimensional context, with an eye towards the quantum field theory.

In Chapter 7 the authors introduce the noncommutative integral showing that in the commutative context it reduces to the ordinary integral by means of the Connes trace theorem.

Part III deals with the constructions of noncommutative geometries. First it includes an account of cyclic cohomology, which is the receptacle of the noncommutative Chern character.

Authors include here Connes' proof, heretofore unpublished, of the formula which gives the Hochschild class of the character as a "local" expression involving the noncommutative integral.

Part IV contains applications of noncommutative geometry. In Chapters 12, 13 the authors give applications of this type in the standard model of particle physics, string theory, quantum field theory and renormalization theory.

Rich in proofs, examples and exercises, the book is an introduction to the language and techniques of noncommutative geometry at a level suitable for graduate students, and also provides sufficient detail to be useful for physicists and mathematicians wishing to enter this rapidly growing field.

Alexander Elashvili, Praha

Sigurdur Helgason: DIFFERENTIAL GEOMETRY, LIE GROUPS, AND SYMMETRIC SPACES. American Mathematical Society, Providence, RI, 2001, 641 pages, USD 69.–.

In 1962 S. Helgason published his book Differential Geometry and Symmetric Spaces. This book has been the basic textbook for several generations of mathematicians working in differential geometry, Lie groups and algebras, and symmetric spaces.

The present book, published by American Math. Society, is an extensive revision of the old version of Differential Geometry and Symmetric Spaces. Apart from numerous minor changes, the following material has been added: in Chapter I—Topology and mappings of constant rank; in Chapter II—Invariant differential forms and perspectives; in Chapter III—Symmetric spaces of rank one and shortest geodesics and minimal totally geodesic spheres.

Many new exercises have been added, and solutions to the old and new exercises are now included at the end of the book. We are firmly convinced that important improvements in the new edition of S. Helgason's book will turn it into a deskbook for many following generations.

Alexander Elashvili, Praha

Armand Borel: ESSAYS IN THE HISTORY OF LIE GROUPS AND ALGEBRAIC GROUPS. American Mathematical Society and London Mathematical Society, Providence, RI, 2001, 184 pages, USD 39.–.

One of the founders of the theory of algebraic groups, A. Borel, gives in his present book a historical review of the development of algebraic groups theory from the point of view of the theory of Lie algebras and groups.

Chapter I of the book is dedicated to the theory of Lie groups and algebras.

Chapter II is devoted mostly to full reducibility and invariants for $SL_2(C)$. On sixteen pages the author gives geometric proofs (which use purely geometric ideas of S. Lie and E. Study) and algebraic proofs of full reducibility which use integration in a localy compact Lie group (A. Hurwitz, H. Weyl) and some techniques of homological algebra (E. Cartan, H. Casimir, B. L. van der Warden).

In Chapter III the author describes H. Weyl's contribution to the development of the Lie groups theory, the representation theory of semisimple Lie groups and algebras, and harmonical analysis.

The famous French mathematician E. Cartan is the founder of the theory of symmetric spaces, and Chapter IV is concerned with his work on this theme.

In Chapters V, VI, VII the history of development of linear algebraic groups from L. Maurer and E. Cartan to M. Freudenthal and C. Chevalley is considered.

In Chapter VIII the development of the Galois theory of differential equations in E. Kolchin's papers is discussed.

The exposition in this book is given in a fascinating and understandable language, therfore the book is useful for specialists as well as for people interested in the history of the development of the theory of algebraic groups and Lie groups.

Alexander Elashvili, Praha

Rolf Berndt: AN INTRODUCTION TO SYMPLECTIC GEOMETRY. American Mathematical Society, Providence, Rhode Island, 2001, 195 pages, USD 36.–.

The goals of this book are to present the idea of the formalism of symplectic forms, introduce the symplectic group, describe symplectic manifolds and demonstrate connections between mathematical objects and the formalism of theoretical mechanics.

Chapter 0 contains a brief introduction into a few topics from theoretical mechanics needed later in the text. In Chapter 1 symplectic vector spaces and several specific important subspaces are considered: the isotropic, coisotropic and Lagrangian subspaces, and it is proved that the symplectic subspaces of a given dimension and rank are fixed up to symplectic isomorphism.

Chapter 2 is dedicated to the central object of the book, namely symplectic manifolds. It takes a glance at new research by considering the assignment of invariants to symplectic manifolds: the symplectic capacities and pseudoholomorphic curves are given. Chapter 3 introduces the standard concepts of the Hamiltonian vector field, the Poisson bracket and the contact manifold.

In Chapters 4 and 5 the author considers mathematical constructions of the moment map, symplectic reduction and quantization and their physical interpretations.

The text is written for graduate students who have fairly good training in analysis and linear algebra.

Alexander Elashvili, Praha