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LOCAL PROPERTIES OF STABLY COMPLEX G-ACTIONS[1]

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ABSTRACT. The paper studies local properties of stably complex finite group actions on disks, spheres, and Euclidean spaces whose fixed point sets are diffeomorphic, respectively, to disks, spheres, and Euclidean spaces. In particular, such smooth actions are constructed so that at different points with the same isotropy subgroup, the normal representations are inequivalent.

Introduction

For smooth G-actions on Euclidean spaces, the question of the equivalence of the slice representations at two points in orbits of the same type was raised by W.-C. Hsiang and W.-Y. Hsiang [5; Problem 16]. For smooth G-actions on spheres, in the special case when the two points are fixed under the action of G, this question was posed by P. A. Smith [18; p. 406, the footnote]. For G-actions on disks, spheres, and Euclidean spaces, the weaker question of the equality of the dimensions of any two G-fixed point set connected components goes back to G. E. Bredon [2; p. 58, the second remark].

A lot of the effort goes toward trying to answer the question posed by P. A. Smith for smooth G-actions on spheres with exactly two G-fixed points. Also, a number of articles deal with the slice representations at G-fixed points for smooth G-actions on disks and Euclidean spaces, as well as, for smooth G-actions on spheres with at least three G-fixed points (see, e.g., [3], [4], [7], [11], [13], and [14] for systematic surveys of related results).

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Key words: G-CW complex, G-vector bundle, equivariant thickening, stably complex G-action, slice representation, normal representation.

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It turns out that unlike a linear G-action, an arbitrary smooth G-action on a disk, sphere, or Euclidean space U can determine different H-actions locally around two different points with the same isotropy subgroup H. In the study of transformation groups, it is natural to impose some restrictions on general smooth G-actions on U, and ask similar questions. In this paper, we impose two restrictions for any smooth G-action on U. The first restriction says that the tangent bundle TU of U stably admits the structure of a complex G-vector bundle; we refer to such a G-action on U as to a *stably complex action.* The second restriction says that the G-fixed point set U^G is diffeomorphic to a disk (resp., sphere; resp., Euclidean space) which is always the case when the G-action on U is linear.

The goal of this paper is to show that smooth G-actions on disks, spheres, and Euclidean spaces fulfilling both restrictions may also have non-linear local behaviour. More precisely, we deal with the following problem related with the above questions. Let G be a finite group, and let H be a proper subgroup of G. Let $m_1, \dots, m_k$ and $W_1, \dots, W_k$ be lists of non-negative integers m_i and complex H-representations W_i without trivial summands. Assume G acts smoothly on a disk, sphere, or Euclidean space U so that the submanifold $U_{(H)}$ of U (consisting of all points in U with isotropy subgroup conjugate to H) contains smooth G-invariant submanifolds $M_1, \dots, M_k$ of U such that, for $i = 1, \dots, k$, the orbit space M_i/G is connected, $m_i = \dim M_i$, and W_i occurs as the normal representation over M_i. This implies that the following condition holds.

DIMENSION CONDITION. For all $1 \leq i, j \leq k$, $m_i + \dim_{\mathbb{R}}(W_i) = m_j + \dim_{\mathbb{R}}(W_j)$. In particular, the integers $m_1, \dots, m_k$ either all are even or all are odd.

It follows from the Smith Theory, for any smooth G-action on a disk or Euclidean space, as well as, for any smooth G-action on a sphere with at least three G-fixed points, the P-fixed point set is non-empty and connected for each prime power order subgroup P of G. Therefore, the next condition (usually) also holds for stably complex G-actions (cf. [10; Propositions 7.1 and 7.2]).

SMITH CONDITION. For each $P \in \mathcal{P}(H)$, the set of all prime power order subgroups P of H, and for all $1 \leq i, j \leq k$, the nontrivial summands of the restricted representations $\mathrm{Res}^H_P(W_i)$ and $\mathrm{Res}^H_P(W_j)$ are equivalent as complex P-representations.

Clearly, for a list of non-negative integers $m_1, \dots, m_k$ with $m_1 = \dots = m_k$ and a list of complex H-representations $W_1, \dots, W_k$ without trivial summands, the Dimension Condition and the Smith Condition both hold if and only if the restricted representations $\mathrm{Res}^H_P(W_i)$ and $\mathrm{Res}^H_P(W_j)$ are equivalent for each $P \in \mathcal{P}(H)$ and all $1 \leq i, j \leq k$.

PROBLEM. Are the Dimension Condition and the Smith Condition also sufficient for lists of non-negative integers $m_1, \ldots, m_k$ and complex H-representations $W_1, \ldots, W_k$ without trivial summands to occur, respectively, as the dimensions of the manifolds $M_1, \ldots, M_k$ (described above) and the normal representations over these manifolds?

Earlier, the author obtained some results related with this problem in the case of smooth S^1-actions and cyclic group actions (see [15] and [16]). In this paper, the author presents results concerned with the above problem for any finite group G with a proper subgroup H not of prime power order, such that any two Sylow subgroups of H intersect trivially (see [13] for similar results when $H = G$). As an example of H, one may take a finite nilpotent group not of prime power order, as well as, any extension of the form

$$0 \to K \to H \to L \to 0,$$

where K is a finite nontrivial nilpotent group, and L is a finite group whose order is a prime, or a product of distinct primes, not dividing the order of K. It turns out that with such G and H, the Dimension Condition and the Smith Condition are (stably) also sufficient to give the affirmative answer to the above problem. More precisely, the following two theorems hold provided G is a finite group, and H is a proper subgroup of G not of prime power order, such that any two Sylow subgroups of H intersect trivially.

THEOREM A. *Let $m_1, \ldots, m_k$ and $W_1, \ldots, W_k$ be lists of integers $m_i \geq 0$ and complex H-representations W_i without trivial summands, such that $m_i = m_j$ and the restricted representations $\mathrm{Res}^H_P(W_i)$ and $\mathrm{Res}^H_P(W_j)$ are equivalent for each $P \in \mathcal{P}(H)$, $1 \leq i, j \leq k$. Moreover, assume $k \geq 1$ and $k \equiv 0 \pmod{n_H}$, where n_H is the Oliver integer of H (resp., assume $k \geq 1$). Then there exists a stably complex G-action on a disks (resp., Euclidean space) U with G-fixed point set U^G diffeomorphic to a disk (resp., Euclidean space), such that the following two conditions hold.*

(1) *$U_{(H)}$ contains smooth G-invariant submanifolds $M_1, \ldots, M_k$ of U such that $\dim M_i = m_i + 2\ell$ for some integer $\ell \geq 0$, and the orbit space M_i/G is contractible for $i = 1, \ldots k$.*
(2) *For some complex H-representation W without trivial summand, $W_i \oplus W$ occurs as the normal representation over M_i for $i = 1, \ldots, k$.*

THEOREM B. *Let $m_1, \ldots, m_k$ and $W_1, \ldots, W_k$ be lists of integers $m_i \geq 1$ and complex H-representations W_i without trivial summands, such that the Dimension Condition and the Smith Condition both hold. Then there exists a stably complex G-action on a sphere (resp., disk; resp., Euclidean space) U with*

G-fixed point set U^G diffeomorphic to a sphere (resp., disk; resp., Euclidean space), such that the following two conditions hold.

(1) *$U_{(H)}$ contains smooth G-invariant submanifolds $M_1, \ldots, M_k$ of U such that $\dim M_i = m_i + 2\ell$ for some integer $\ell \geq 0$, M_i is closed and the orbit space M_i/G is connected for $i = 1, \ldots, k$.*

(2) *For some complex H-representation W without trivial summand, $W_i \oplus W$ occurs as the normal representation over M_i for $i = 1, \ldots, k$.*

If H is a finite group such that each element of H has prime power order, it follows from character theory that two H-representations are equivalent if and only if the same holds for their restrictions to each prime power order subgroup of H. Therefore, for such an H, lists of non-negative integers $m_1, \ldots, m_k$ and complex H-representations $W_1, \ldots, W_k$ (without trivial summands) fulfill both the Dimension Condition and the Smith Condition if and only if $m_i = m_j$ and $W_i \cong W_j$ for all $1 \leq i, j \leq k$. Consequently, for such an H in Theorems A and B, the normal representations are equivalent. On the other hand, if H is a finite group with a cyclic subgroup not of prime power order, then for a list of non-negative integers $m_1, \ldots, m_k$ either all even or all odd, one may construct complex H-representations $W_1, \ldots, W_k$ without trivial summands, so that the Dimension Condition and the Smith Condition both hold, and the H-representations W_i and W_j are not equivalent whenever $i \neq j$ (see [13; Comments (1) and (2)]; cf. [10; Example 7.5 and Remark 7.6]). As a result, Theorems A and B yield the following corollary.

COROLLARY C. *Let G and H be as in Theorems A and B, and assume further H has a cyclic subgroup not of prime power order. Then G has a stably complex action on a sphere (resp., disk; resp., Euclidean space) U with G-fixed point set U^G diffeomorphic to a sphere (resp., disk; resp., Euclidean space) and inequivalent normal representations at two points (with isotropy subgroup H) in orbits which are in different connected components of $U_{(H)}/G$. Moreover, the connected components may have different dimensions, as well as, the same dimension.*

The material of this paper is organized as follows. In Section 1, we provide some background material needed in this paper. In Section 2, for a finite group G, we restate the Equivariant Thickening Theorem presented in [12] for any compact Lie group G. In Section 3, we briefly recall some results on constructions of contractible G-CW complexes with prescribed G-fixed point sets. In Section 4, we study some equivariant function spaces which we use for obtaining of G-vector bundles with prescribed properties. In Section 5, we provide the proofs of Theorems A and B.

1. Background material

Let G be a compact Lie group acting smoothly on a smooth manifold M. Then, for any point $x \in M$, the differential of the action of the isotropy subgroup G_x at x determines a linear G_x-representation on the tangent space $T_x(M)$. This representation is orthogonal with respect to a chosen invariant Riemannian metric on M, and splits into the following two summands. The first summand is the trivial G_x-representation on the tangent space to the orbit $G(x)$ passing through x. The second summand is an orthogonal G_x-representation on the normal space of $G(x)$ at x, called the *slice representation at* x and denoted here by S_x.

According to the Slice Theorem, the G_x-action around x is uniquely determined by two local invariants. The first invariant is the isotropy orbit type of $G(x)$, that is, the conjugacy class (G_x) of G_x in G. The second invariant is the equivalence class of S_x. More specifically, the Slice Theorem asserts that there is an equivariant diffeomorphism of the twisted product $G \times_H V$ onto some invariant neighborhood of $G(x)$ which extends the natural embedding of G/H onto $G(x)$, where $H = G_x$ and $V = S_x$. For two orbits $G(x)$ and $G(y)$ in M with $(G_x) = (G_y)$, one may choose the points x and y so that $G_x = G_y$ and ask whether the slice representations S_x and S_y are equivalent. By the Slice Theorem, this amounts to asking whether the natural equivariant diffeomorphism between $G(x)$ and $G(y)$ extends to an equivariant diffeomorphism between some invariant neighborhoods of $G(x)$ and $G(y)$ in M.

For a closed subgroup H of G, consider the smooth invariant submanifold $M_{(H)}$ of M consisting of all orbits $G(x)$ of the isotropy type (H), that is, with $(G_x) = (H)$. Then, for any point $x \in M_{(H)}$, the G_x-nontrivial summand of S_x occurs as the G_x-representation on the normal space of $M_{(H)}$ in M at x, and it is called the *normal representation at* x. For two orbits $G(x)$ and $G(y)$ in $M_{(H)}$, choose the points x and y so that $G_x = G_y = H$. Now assume M is connected. Then it follows from the Slice Theorem that the slice (equivalently, normal) representations at x and y are equivalent whenever, in the orbit space M/G, the orbits $G(x)$ and $G(y)$ are in the same connected component of $M_{(H)}/G$. In other words, if M_0 is a smooth G-invariant submanifold of M such that M_0/G is a connected component of $M_{(H)}/G$, then all points $x \in M_0$ with $G_x = H$, determine (up to linear equivalence) just one normal representation. We refer to this H-representation as to the *normal representation over* M_0. Clearly, if W_0 is the normal representation over M_0, then for any $x \in M_0$ with $G_x = H$, as H-representations, $T_x(M) \cong \mathbb{R}^{m_0} \oplus W_0$, where $m_0 = \dim M_0$ and H acts trivially on $\mathbb{R}^{m_0}$.

We refer the reader to Bredon's book [2], or Kawakubo's book [6] for background material on transformation groups that we use in this paper. For a finite group G and a G-representation V, the product G-vector bundle $X \times V$ over a G-space X we denote by $\langle V \rangle$ provided the base space X is obvious from the context, and we write $\langle \mathbb{R}^n \rangle$ (resp., $\langle \mathbb{C}^n \rangle$) when V is the real (resp., complex) vector space $\mathbb{R}^n$ (resp., $\mathbb{C}^n$) with the trivial G-action.

2. Equivariant thickening

Let G be a finite group. For a smooth G-manifold M and a smooth G-vector bundle ν over M, consider the problem of constructing a smooth G-manifold U of a given homotopy type, such that the following two conditions hold.

(1) U contains M as a (closed) invariant smooth submanifold with equivariant normal bundle $\nu_{M \subset U}$ equivalent to ν.

(2) $U \smallsetminus M$ has the same isotropy subgroups as does $N \smallsetminus M$, where N is an open invariant tubular neighborhood of M in U.

According to the Equivariant Tubular Neighborhood Theorem (see, e.g., [2; Chapter VI, Theorem 2.2], or [6; Chapter 4, Theorem 4.8]), there exists an equivariant diffeomorphism from the total space $E(\nu)$ of ν onto N which coincides with the identity on M. Hence, the condition (2) amounts to saying that $U \smallsetminus M$ has the same isotropy subgroups as does the total space $S(\nu)$ of the invariant (unit) sphere bundle of ν.

Now, assume such a manifold U has been constructed. Then U has the structure of a finite dimensional, countable G-CW complex containing M as a G-invariant subcomplex, and the tangent bundle TU has the structure of a G-vector bundle over U such that, as G-vector bundles, $TU|_M \cong TM \oplus \nu$. Therefore, it follows that in order to construct such a manifold U, it is necessary to have a finite dimensional, countable G-CW complex X of the given homotopy type, containing M as a G-invariant subcomplex, and to have a G-vector bundle ξ over X such that the following two conditions hold.

Normal Bundle Condition. As G-vector bundles, $\xi|_M \cong TM \oplus \langle \mathbb{R}^\ell \rangle \oplus \nu$, $\ell \geq 0$.

Isotropy Subgroup Condition. Each isotropy subgroup occurring in $X \smallsetminus M$ occurs also in $S(\nu)$.

It turns out that the existence of such a complex X and a bundle ξ is also sufficient whenever ν fulfills the following condition.

General Position Condition. For each isotropy subgroup H occurring in $X \smallsetminus M$, and for the G_x-representation on the fibre $F_x(\nu)$ of ν over any point $x \in M$ with $G_x \supsetneqq H$,

$$\dim F_x(\nu)^H > 2 \cdot \dim(X \smallsetminus M)^H ,$$

and for any subgroup K of G_x with $H \subsetneqq K$,

$$\dim F_x(\nu)^H - \dim F_x(\nu)^K > \dim(X \smallsetminus M)^H .$$

2.1. Theorem. (cf. [12; Theorem 2.2]) *Assume G is a finite group, M is a smooth G-manifold, ν is a smooth G-vector bundle over M without trivial summand, X is a finite dimensional, countable, connected G-CW complex, ξ is a G-vector bundle over X such that the Normal Bundle Condition, the Isotropy Subgroup Condition, and the General Position Condition all hold. Assume further that, if $X \smallsetminus M$ has infinitely many cells, M is without boundary. Then there exist a smooth G-manifold U and a G-equivariant map $f\colon U \to X$ such that U contains M as a G-invariant smooth submanifold, and the following four conditions hold.*

(1) *The bundles $\nu_{M\subset U}$ and ν are equivariant as G-vector bundles.*
(2) *$U \smallsetminus M$ and $S(\nu)$ have the same isotropy subgroups.*
(3) *f is a (simple) G-homotopy equivalence coinciding with the identity on M.*
(4) *The bundles $f^*(\xi)$ and $TU \oplus \langle \mathbb{R}^\ell \rangle$ are equivalent as G-vector bundles.*

The idea of the proof is to take the total space $D(\nu)$ of the (unit) disk bundle of ν and then, by using the equivariant thickening technique, replace inductively all equivariant cells in $X \smallsetminus M$ by equivariant handles in a way prescribed by ξ to obtain both U and f (see [12; Section 2] for the details of the proof).

2.2. Remark. Under the hypotheses of Theorem 2.1, assume further that X is finite (resp., infinite) and contractible. Then, the manifold U is diffeomorphic to the disk D^n (resp., Euclidean space $\mathbb{R}^n$) with $n = \dim(\xi) - \ell$ provided $n \geq 6$ (resp., $n \geq 5$); see [12; Remark 2.5].

2.3. Remark. If X, ξ, M, and ν occurring in Theorem 2.1 fulfill only the Normal Bundle Condition, and M contains a point left fixed under the G-action, then, by adding to ξ and ν the product bundle $\langle V \rangle$ over X and M, respectively, for a suitable (real or complex) G-representation V, we may assume that the Isotropy Subgroup Condition and the General Position Condition both also hold. For example, V may be taken as a sufficiently large multiple of the nontrivial summand of the (real or complex) regular representation of G.

3. Contractible G-CW complexes

In this section, we briefly recall some results on the existence of contractible G-CW complexes due to Oliver [8] and Assadi [1]. First, recall that, according to [8], there exists an integer $n_G \geq 0$ such that a finite CW complex

F is the G-fixed point set of a finite contractible G-CW complex if and only if the Euler characteristic $\chi(F) \equiv 1 \pmod{n_G}$. We refer to n_G as to the *Oliver integer of* G (see the work of Oliver [9] for complete calculations of n_G). If $\chi(F) \equiv 1 \pmod{n_G}$, then a finite contractible G-CW complex X with $X^G = F$ is built up one orbit type at a time, adding cells $G/H \times D^j$ (for various j) until the H-fixed point set has the desired homology. More precisely, the construction is done in the following three steps.

STEP 1. Cells $G/H \times D^j$ are added so that to produce from F a finite G-CW complex with G-fixed point set F, such that for any nontrivial subgroup H of G, the H-fixed point set has the desired Euler characteristic and it is $\mathbb{Z}_p$-acyclic whenever H is a p-group for a prime p dividing the order of G.

STEP 2. Free orbits of cells $G \times D^j$ are added so that to obtain a finite 1-connected G-CW complex with all of the reduced homology concentrated in one dimension.

STEP 3. Again free orbits of cells are added so that to kill the reduced homology concentrated in one dimension and, as a result, to obtain a finite contractible G-CW complex X with $X^G = F$.

It follows from this construction that for any subgroup H of G not of prime power order, the H-fixed point set has only the desired Euler characteristic. Thus, we may assume that the following condition holds for subgroups H of G of composite (i.e., not of prime power) order.

(CC) For each proper subgroup H of G of composite order, whenever an equivariant cell $G/H \times D^j$ occurs in X, it is added by an attaching map (defined on $G/H \times S^{j-1}$) which is constant on each copy $\{gH\} \times S^{j-1}$ of the sphere S^{j-1} ($g \in G$, $j \geq 1$).

It also follows that each time the P-fixed point set is made $\mathbb{Z}_p$-acyclic for a p-subgroup P of G, the dimension of the P-fixed point set needs to be raised by at most 1. In order to perform Steps 2 and 3, the dimension of the G-CW complex needs to be raised again by at most 1. Therefore, if we denote by $\ell(P)$ the length of the longest chain $P = P_1 \subsetneqq P_2 \subsetneqq \cdots \subsetneqq P_k$ of p-subgroups P_i of G, we may assume that the following dimension condition also holds.

(DC) For each $P \in \mathcal{P}_0(G)$, the set of all *nontrivial* prime power order subgroups of G,

$$\begin{aligned} \dim X^P &\leq \max\{1, \dim F\} + \ell(P) \qquad \text{and} \\ \dim X &\leq \max\{1, \dim F\} + \max\{\ell(P) \mid P \in \mathcal{P}_0(G)\} + 1 . \end{aligned}$$

3.1. THEOREM. (Oliver [8]) *Let G be a finite group not of prime power order. Let F be a finite CW complex with $\chi(F) \equiv 1 \pmod{n_G}$, where n_G is the Oliver integer of G. Then there exists a finite contractible G-CW complex X with $X^G = F$, such that the conditions* (CC) *and* (DC) *both hold.*

By using results and techniques of Assadi [1; Chapter II, especially II.1.4 and II.7.2], we get a similar theorem on the existence of infinite contractible G-CW complexes with prescribed G-fixed point sets. Even, if not stated explicitly, each CW complex, as well as, G-CW complex is finite dimensional and consists of countably many cells.

3.2. THEOREM. (Assadi [1]) *Let G be a finite group not of prime power order. Let F be a CW complex. Then there exists an infinite contractible G-CW complex with $X^G = F$ such that the conditions* (CC) *and* (DC) *both hold.*

4. Equivariant function spaces

In this section, we study properties of some equivariant function spaces that we use to construct G-vector bundles over contractible G-CW complexes. Let G be a compact Lie group, and let $C(n)$ be one of the classical groups, either $U(n)$ or $SU(n)$, $n \geq 1$. Consider the neutral elements of G and $C(n)$ as their base points, and take the space $\mathrm{Map}_\star(G, C(n))$ of pointed maps $\theta\colon G \to C(n)$, equipped with the compact-open topology. This space admits the G-action defined so that for $g, h \in G$,

$$(g\theta)(h) = \theta(hg)\theta(g)^{-1}.$$

Now we wish to describe unitary (resp., special unitary) G-vector bundles whose underlying non-equivariant bundles are just the product bundles (cf. [2; Chapter VI, Proposition 11.1] and [10; Proposition 4.1]).

4.1. PROPOSITION. *For any G-space X, there is a natural one-one correspondence between unitary (resp., special unitary) G-vector bundle structures on $X \times \mathbb{C}^n$ over X and G-maps*

$$X \to \mathrm{Map}_\star(G, C(n)), \quad x \mapsto \theta_x,$$

with $C(n) = U(n)$ (resp., $C(n) = SU(n)$). For a given G-map $x \mapsto \theta_x$, the corresponding G-action on $X \times \mathbb{C}^n$ is defined so that for all $g \in G$, $x \in X$, $v \in \mathbb{C}^n$,

$$g(x, v) = \big(gx,\, \theta_x(g) \cdot v\big).$$

4.2. Remark. For two G-maps $X \to \mathrm{Map}_\star(G, C(n))$, $x \mapsto \theta_x$ and $x \mapsto \theta'_x$, a G-homotopy between the two G-maps yields a G-vector bundle structure on

$X \times [0,1] \times \mathbb{C}^n$ over $X \times [0,1]$, so that G-homotopic maps yield equivalent G-vector bundles. In general, the G-vector bundles given by the two G-maps are equivalent if and only if there is a map $\varphi\colon X \to U(n)$ such that for all $x \in X$ and $g \in G$,

$$\theta'_x(g) = \varphi(gx)\theta_x(g)\varphi(x)^{-1}.$$

It follows from the definition of the G-action on $\mathrm{Map}_\star(G, C(n))$ that the G-fixed point set is just the space $\mathrm{Hom}(G, C(n))$ of homomorphisms from G to $C(n)$, i.e., representations of G in $C(n)$.

4.3. Remark. The connected component of $\mathrm{Hom}(G, C(n))$ containing a representation $\rho\colon G \to C(n)$ is just the $U(n)$-equivalence class of ρ. In fact, if a representation $\rho'\colon G \to C(n)$ lies in the same connected component as does ρ, then there is a homotopy between the maps sending a point to ρ and ρ' in $\mathrm{Hom}(G, C(n))$. Hence, ρ and ρ' are equivalent (cf. Remark 4.2). Conversely, if there exists a matrix $A \in U(n)$ such that $\rho'(g) = A\rho(g)A^{-1}$ for all $g \in G$, take a path $[0,1] \to U(n)$, $t \mapsto A_t$, from the identity $(n \times n)$-matrix to A. Then, the map

$$[0,1] \to \mathrm{Hom}(G, C(n)), \quad t \mapsto \rho_t; \qquad \rho_t(g) = A_t\rho(g)A_t^{-1}, \quad g \in G,$$

is a path in $\mathrm{Hom}(G, C(n))$ from ρ to ρ'. On the other hand, the $U(n)$-equivalence class of ρ is the orbit of ρ under the $U(n)$-action on $\mathrm{Hom}(G, C(n))$ given by $(A\rho)(g) = A\rho(g)A^{-1}$. Hereafter, we denote by $U(\rho)$ the isotropy subgroup at ρ under this action. Hence, it follows that the connected component of $\mathrm{Hom}(G, C(n))$ containing $\rho\colon G \to C(n)$ is homeomorphic to the homogeneous space $U(n)/U(\rho)$. Clearly, $U(\rho)$ is just the centralizer of $\rho(G)$ in $U(n)$, and it can be identified with the group of unitary G-equivariant automorphisms of $\mathbb{C}^n(\rho)$, the space $\mathbb{C}^n$ with the linear G-action given via ρ.

Let G be a finite group. Let $\sigma_0\colon G \to U(1)$ be the trivial G-representation, and let $\sigma_i\colon G \to U(d_i)$, $i = 1, \ldots, \ell$, be a complete list of nontrivial complex irreducible G-representations. For a G-representation $\rho\colon G \to C(n)$, consider the decomposition of ρ into the irreducible summands,

$$\rho \cong \bigoplus_{i \in \mathfrak{I}} n_i\sigma_i, \qquad \sum_{i \in \mathfrak{I}} n_i d_i = n, \quad \mathfrak{I} \subset \{0, 1, \ldots, \ell\},$$

where $n_i > 0$ for each $i \in \mathfrak{I}$. We write $c(\rho)$ for the greatest common divisor of the integers d_i, $i \in \mathfrak{I}$, and we refer to $c(\rho)$ as to the *cyclic loop number of* ρ. The motivation of this notion is provided by the following proposition which is a generalization of author's result [10; Proposition 4.4].

4.4. PROPOSITION. *Let G be a finite group. For a representation $\rho\colon G \to C(n)$, the connected component of* $\mathrm{Hom}(G, C(n))$ *containing ρ has the* *fundamental group isomorphic to the cyclic group $\mathbb{Z}/c(\rho)\mathbb{Z}$. In particular, this connected component is simply connected if and only if the cyclic loop number $c(\rho) = 1$.*

P r o o f . As above, for a representation $\rho\colon G \to C(n)$, consider the decomposition of ρ into the irreducible summands,

$$\rho \cong \bigoplus_{i\in\mathfrak{I}} n_i\sigma_i\,, \qquad \sum_{i\in\mathfrak{I}} n_i d_i = n\,, \quad \mathfrak{I} \subset \{0,1,\dots,\ell\}\,,$$

where $n_i > 0$ for each $i \in \mathfrak{I}$. Since $U(\rho)$ defined in Remark 4.3 can be identified with the group of unitary G-equivariant automorphisms of $\mathbb{C}^n(\rho)$, thus

$$U(\rho) \cong \bigoplus_{i\in\mathfrak{I}} U(n_i)\,,$$

and the embedding of $U(\rho)$ into $U(n)$ upon restricting to any component $U(n_i)$ induces on the fundamental groups $\mathbb{Z} \cong \pi_1\big(U(n_i)\big) \to \pi_1(U(n)) \cong \mathbb{Z}$ multiplication by d_i. Consequently,

$$\operatorname{Image}\big(\pi_1\big(U(\rho)\big) \to \pi_1\big(U(n)\big)\big) \cong \sum_{i\in\mathfrak{I}} d_i\mathbb{Z} \subset \mathbb{Z}\,.$$

Thus, it follows from the exact sequence of the fibration $U(\rho) \to U(n) \to U(n)/U(\rho)$ that $\pi_1\big(U(n)/U(\rho)\big) \cong \mathbb{Z}/c(\rho)\mathbb{Z}$, which shows the result by Remark 4.3. □

Now, for a subgroup H of G consider the H-fixed point set of $\operatorname{Map}_\star\big(G, C(n)\big)$,

$$\operatorname{Map}_\star\big(G, C(n)\big)^H = \big\{\theta :\ G \to C(n) \mid \ \theta(gh) = \theta(g)\theta(h)\,,\ g \in G\,,\ \ h \in H\big\}\,.$$

Recall from [10; Proposition 4.2], that each choice of representatives a_{gH} of cosets gH, $g \in G$, with $a_H = e$, yields the homeomorphism

$$\begin{gathered}\operatorname{Map}_\star\big(G, C(n)\big)^H \to \operatorname{Hom}\big(H, C(n)\big) \times \operatorname{Map}_\star\big(G/H, C(n)\big)\\ \theta \mapsto \big(\theta|_H, \sigma\big)\,, \qquad \sigma(gH) = \theta(a_{gH})\,.\end{gathered}$$

Therefore, in order to study the connected components of $\operatorname{Map}_\star\big(G, C(n)\big)$, it suffices to study the connected components of $\operatorname{Hom}\big(H, C(n)\big)$ because

$$\operatorname{Map}_\star\big(G/H, C(n)\big) \cong C(n) \times \cdots \times C(n)\,, \qquad (G:H) - 1 \ \text{ times}\,.$$

In particular, the following corollary holds (cf. [10; Corollary 4.3]).

4.5. COROLLARY. *Let G be a finite group, and let H be a subgroup of G. Then, for two maps $\theta, \theta' \colon G \to C(n)$ in $\mathrm{Map}_\star(G, C(n))^H$, the representations $\theta|_H$ and $\theta'|_H$ are equivalent if and only if θ and θ' are in the same connected component of $\mathrm{Map}_\star(G, C(n))^H$.*

Since $SU(n)$ is 2-connected, so is $\mathrm{Map}_\star(G, SU(n))$. Moreover, by Proposition 4.4 and the above product decomposition of $\mathrm{Map}_\star(G, SU(n))^H$ into $\mathrm{Hom}(G, SU(n))$ and $\mathrm{Map}_\star(G/H, SU(n))$, the following corollary also holds (cf. [10; Corollary 4.5]).

4.6. COROLLARY. *Let G be a finite group, and let H be a nontrivial subgroup of G. Then, a connected component of $\mathrm{Map}_\star(G, SU(n))^H$ is simply connected if and only if it contains a map $\theta \colon G \to SU(n)$ such that the H-representation $\theta|_H$ has the cyclic loop number $c(\theta|_H) = 1$.*

Now we can prove a proposition which we will use to get G-vector bundles over G-CW complexes with prescribed G-fixed point bundles. The proposition is a generalization of author's result [10; Proposition 4.6].

4.7. PROPOSITION. *Let G be a finite group, and let X be a G-CW complex with G-fixed point set F, fulfilling the condition (CC) of Section 3, such that for any $P \in \mathcal{P}_0(G)$, each equivariant cell of type (P) in $X \smallsetminus F$ has dimension less than or equal to 2, and each free cell in $X \smallsetminus F$ has dimension less than or equal to 3. Let $F_1, \ldots, F_k$ be all connected components of F, and for $i = 1, \ldots, k$, let $\rho_i \colon G \to SU(n)$ be a representation with cyclic loop number $c(\rho_i) = 1$ such that $\rho_i|_P$ and $\rho_j|_P$ are equivalent for all $P \in \mathcal{P}(G)$, $1 \le i, j \le k$. Then there exists a complex G-vector bundle ξ over X such that $\xi|_{F_i} \cong \langle V_i \rangle$ with $V_i = \mathbb{C}^n(\rho_i)$ for $i = 1, \ldots, k$.*

P r o o f. Consider the map $F \to \mathrm{Hom}(G, SU(n)) \subset \mathit{Map}_\star(G, SU(n))$, $x_i \mapsto \rho_i$, for all $x_i \in F_i$, $i = 1, \ldots, k$. We claim that this map extends to an equivariant map $X \to \mathrm{Map}_\star(G, SU(n))$. Once such an extension has been obtained, the proof is completed by using Proposition 4.1.

First, we get an extension of this map on all equivariant 0-cells in $X \smallsetminus F$ by mapping each 0-cell G/H from $X \smallsetminus F$ into the set $\{\rho_1, \ldots, \rho_k\}$.

Now, note that X is built up from F and all equivariant 0-cells in $X \smallsetminus F$ by attaching a sequence of equivariant cells of the form $G/H \times D^j$ via equivariant maps α defined on $G/H \times S^{j-1}$, where $H \subsetneq G$, $j \ge 1$. In order to obtain an extension on a cell $G/H \times D^j$, it suffices to show that α composed with the previous extension when restricted to $\{H\} \times S^{j-1}$ is null-homotopic in $\mathrm{Map}_\star(G, SU(n))^H$.

For $H \notin \mathcal{P}(G)$, α is a constant map on $\{H\} \times S^{j-1}$ because the condition (CC) holds, and we choose the extension so that it is the constant map on

$\{H\} \times D^j$. For $H = P \in \mathcal{P}_0(G)$, an extension on cells $G/P \times D^1$ exists due to Corollary 3.5 because $\rho_i|_P \cong \rho_j|_P$, and it does on cells $G/P \times D^2$ due to Corollary 3.6 because $c(\rho_i) = 1$. Finally, an extension on free cells $G \times D^1$, $G \times D^2$, and $G \times D^3$ exists because $\mathrm{Map}_\star(G, SU(n))$ is 2-connected. □

5. Proofs of Theorems A and B

In this section, we wish to prove Theorems A and B. First, we state the following three remarks.

5.1. Remark. Let G be a finite group. For a representation $\rho\colon G \to C(n)$, where $C(n) = U(n)$ or $SU(n)$, consider the decomposition of ρ into the irreducible summands,

$$\rho \cong \bigoplus_{i\in\mathfrak{I}} n_i\sigma_i\,, \qquad \sum_{i\in\mathfrak{I}} n_i d_i = n\,, \quad \mathfrak{I} \subset \{0,1,\ldots,\ell\}\,,$$

where $n_i > 0$ for each $i \in \mathfrak{I}$. Recall that the cyclic loop number $c(\rho)$ is the greatest common divisor of all d_i's, $i \in \mathfrak{I}$. We claim that by adding to ρ, if necessary, some irreducibles $\sigma_j\colon G \to U(d_j)$, $1 \le j \le \ell$, we may assume that $c(\rho) = 1$. In fact, assume $d_i > 1$ for each $i \in \mathfrak{I}$ (otherwise, there is nothing to prove) and recall from Serre's book [17; Section 2.4, Corollary 2 and Section 6.5, Corollary 2] that ${d_1}^2 + {d_2}^2 + \cdots + {d_\ell}^2 = |G| - 1$ and $d_1 \cdot d_2 \cdot \ldots \cdot d_\ell$ divides $|G|$. Hence, the integers $d_1, d_2, \ldots, d_\ell$ are relatively prime, proving the claim. Similarly, we see that if

$$\rho_i\colon G \to C(n)\,, \qquad i = 1,\ldots,k\,,$$

is a list of representations, then by adding the same irreducibles $\sigma_j\colon G \to U(d_j)$ to each ρ_i, we may assume that each cyclic loop number $c(\rho_i) = 1$.

5.2. Remark. Let G be a finite group. Let $\rho_i\colon G \to U(n)$, $i = 1,\ldots,k$, be a list of representations such that $\rho_i|_P$ and $\rho_j|_P$ are equivalent for all $P \in \mathcal{P}(G)$, $1 \le i,j \le k$. Then

$$\det \rho_i(g) = \det \rho_j(g) \qquad \text{for all} \quad g \in G\,, \;\; 1 \le i,j \le k\,.$$

Consider the representation $\delta\colon G \to U(1)$ given by $\delta(g) = \big(\det \rho_i(g)\big)^{-1}$ for $g \in G$. Then each $\rho_i \oplus \delta$ is a special unitary representation (cf. [10; Lemma 7.3]).

5.3. Remark. Let G be a finite group, and let H be a proper subgroup of G. Let W be a (complex) H-representation without H-trivial summand. Then there exists a (complex) G-representation V without G-trivial summand such that W occurs as a direct summand of the restricted representation $\mathrm{Res}^G_H(V)$.

For example, set $V = \operatorname{Ind}_H^G(W)$, the induced G-representation. Then it follows (e.g., from [17; Section 7.3, Proposition 22]) that W occurs as a direct summand of $\operatorname{Res}_H^G(V)$, and clearly $V^G = W^H = \{0\}$. If there exists an epimorphism from G onto H, then one may also take V to be W with the linear G-action given via the given epimorphism (to deal with the effective G-actions, just add to V a faithful G-representation without trivial summand).

Proof of Theorem A. Let G and H be as in the hypotheses of Theorem A. Let $W_0, W_1, \dots, W_k$ be complex H-representations without H-trivial summands, such that $\operatorname{Res}_P^H(W_i) \cong \operatorname{Res}_P^H(W_j)$ for all $P \in \mathcal{P}(H)$ and $0 \leq i, j \leq k$. According to Remarks 5.1 and 5.2, we may assume that each $W_i = \mathbb{C}^n(\rho_i)$, where $\rho_i \colon H \to SU(n)$ has the cyclic loop number $c(\rho_i) = 1$.

Assume $k \geq 1$ and $k \equiv 0 \pmod{n_H}$, where n_H is the Oliver integer of H (resp., assume $k \geq 1$). Then it follows from Theorem 3.1 (resp., Theorem 3.2) that there exists a finite (resp., infinite) contractible H-CW complex Y with Y^H consisting of exactly $k+1$ points $b_0, b_1, \dots, b_k$, such that the conditions (CC) and (CD) both hold. Due to (CD), $\dim Y^P = 2$ for each $P \in \mathcal{P}_0(H)$, and $\dim Y = 3$ because any two Sylow subgroups of H intersect trivially. Let $N_1, \dots, N_k$ be compact contractible smooth manifolds (resp., contractible smooth manifolds without boundary) with the trivial H-action, all of the same dimension. Consider $B = \{b_1, \dots, b_k\}$ as a subset of the disjoint union $N = N_1 \sqcup \dots \sqcup N_k$ by identifying b_i with an interior point of N_i for $i = 1, \dots, k$. Since N_i contains b_i as a deformation retract, the sum $Y \cup_B N$ of Y and N along B is contractible. Clearly, it has the obvious structure of an H-CW complex with

$$(Y \cup_B N)^H = \{b_0\} \sqcup N .$$

According to Proposition 4.7, there exists a complex H-vector bundle η over $Y \cup_B N$ such that

$$F_{b_0}(\eta) \cong W_0 \qquad \text{and} \qquad \eta|_{N_i} \cong \langle W_i \rangle \quad \text{for } i = 1, \dots, k .$$

Clearly, the twisted product $G \times_H (Y \cup_B N)$ is a finite (resp., infinite) G-CW complex whose connected components all are contractible. By adding to $G \times_H (Y \cup_B N)$ the cone C_0 over the orbit $G(b_0)$ with the cone G-action, we obtain a finite (resp., infinite) G-CW complex X with $X^G = \{v_0\}$, where v_0 is the vertex of C_0. Moreover,

$$X_{(H)} = \big(C_0 \smallsetminus \{v_0\}\big) \sqcup (G/H \times N) .$$

Let V_0 be a complex G-representation without G-trivial summand, such that

$$\operatorname{Res}_H^G(V_0) \cong W_0 \oplus W \oplus \mathbb{C}^r ,$$

where W is a complex H-representation without H-trivial summand, and H acts trivially on $\mathbb{C}^r$, $r \geq 0$ (cf. Remark 5.3). Then, it follows that the G-vector bundle $G\times_H(W_0 \oplus W \oplus \mathbb{C}^r)$ over $G(b_0)$ extends to a complex G-vector bundle over C_0 with V_0 occurring as the fiber over v_0 (cf. Corollary 4.5). Hence, the G-vector bundle

$$G\times_H\big(\eta \oplus \langle W \oplus \mathbb{C}^r\rangle\big) \quad \text{over} \quad G\times_H(Y \cup_B N)$$

extends to a complex G-vector bundle ξ over X such that $F_{v_0}(\xi) = V_0$. By contracting the cone C_0 to its vertex v_0, we may assume that $X_{(H)} = G/H \times N$. Now, choose N_i so that $\dim N_i = 2r$, and set $M = \{v_0\} \sqcup (G/H \times N)$. Let ν be the complex G-vector bundle over M defined by

$$F_{v_0}(\nu) = V_0 \qquad \text{and} \qquad \nu|_{(G/H \times N_i)} = G\times_H\langle W_i \oplus W\rangle \quad \text{for } i = 1, \dots, k.$$

Then, as complex G-vector bundles, $\xi|_M \cong TM \oplus \nu$, and thus, the Normal Bundle Condition (stated in Section 2) holds for X, ξ, M, and ν. By adding to ξ and ν a suitable product bundle over X and M respectively, we may assume that the Isotropy Subgroup Condition and the General Position Condition both also hold (cf. Remark 2.3). Then, according to Theorem 2.1 and Remark 2.2, there exists a smooth action of G on a disk (resp., Euclidean space) U such that U contains M as an invariant smooth submanifold with equivariant normal bundle ν. Moreover,

$$U^G = \{v_0\} \quad \text{and} \quad U_{(H)} = M_0 \sqcup M_1 \sqcup \cdots \sqcup M_k$$

with $M_0 = D(V_0)_{(H)}$ and $M_i = G/H \times N_i$ for $i = 1, \dots, k$. Clearly, $W_i \oplus W$ occurs as the normal representation over M_i for $i = 1, \dots, k$. □

Proof of Theorem B. Let G and H be as in the hypotheses of Theorem B. Moreover, let $m_0, m_1, \dots, m_k$ and $W_0, W_1, \dots, W_k$ be lists of integers $m_i \geq 1$ and complex H-representations W_i without H-trivial summands, such that the Dimension Condition and Smith Condition both hold. For $i = 1, \dots, k$, put $n_i = [(m_i + 1)/2]$, the greatest integer in $(m_i + 1)/2$, and consider the H-representation $\mathbb{C}^{n_i} \oplus W_i$, where H acts trivially on $\mathbb{C}^{n_i}$. It follows from the Dimension Condition that the representations $\mathbb{C}^{n_i} \oplus W_i$ all have the same (complex) dimension, say n, and it follows from the Smith Condition that

$$\mathrm{Res}^H_P(\mathbb{C}^{n_i} \oplus W_i) \cong \mathrm{Res}^H_P(\mathbb{C}^{n_j} \oplus W_j)$$

for all $P \in \mathcal{P}(H)$ and $0 \leq i, j \leq k$. According to Remarks 5.1 and 5.2, we may assume that each $\mathbb{C}^{n_i} \oplus W_i = \mathbb{C}^n(\rho_i)$, where $\rho_i \colon H \to SU(n)$ has the cyclic loop number $c(\rho_i) = 1$.

Let $B_1, \dots, B_k$ be CW complexes such that each B_i is either a point or a circle, or a wedge of finitely many circles. Arrange the B_i's so that for the disjoint union $B = B_1 \sqcup \cdots \sqcup B_k$, the Euler characteristic $\chi(B) \equiv 0 \pmod{n_H}$, where n_H is the Oliver integer of H. Then, it follows from Theorem 3.1 that there exists a finite contractible H-CW complex Y with $Y^H = \{b_0\} \sqcup B$, such that the conditions (CC) and (CD) both hold. Due to (CD), $\dim Y^P = 2$ for each $P \in \mathcal{P}_0(H)$, and $\dim Y = 3$ because any two Sylow subgroups of H intersect trivially. Now, take compact smooth parallelizable manifolds $N_1, \ldots, N_k$ all with the trivial H-actions, such that N_i contains B_i as a deformation retract and $\dim N_i = m_i$ for $i = 1, \ldots, k$. Put $N = N_1 \sqcup \cdots \sqcup N_k$. Then the sum $Y \cup_B N$ of Y and N along B is a finite contractible H-CW complex with

$$(Y \cup_B N)^H = \{b_0\} \sqcup N .$$

By arguing as in the proof of Theorem A, there exists a complex H-vector bundle η over $Y \cup_B N$ such that $F_{b_0}(\eta) \cong \mathbb{C}^{n_0} \oplus W_0$ and $\eta|_{N_i} \cong \langle \mathbb{C}^{n_i} \oplus W_i \rangle$ for $i = 1, \ldots, k$. Moreover, for a complex G-representation V_0 with $V_0{}^G = \mathbb{C}^{n_0}$,

$$\mathrm{Res}^G_H(V_0) \cong \mathbb{C}^{n_0} \oplus W_0 \oplus W \oplus \mathbb{C}^r ,$$

where W is a complex H-representation without H-trivial summand and H acts trivially on $\mathbb{C}^r$ (cf. Remark 5.3). Also, the G-vector bundle $G \times_H (\eta \oplus \langle W \oplus \mathbb{C}^r \rangle)$ over $G \times_H (Y \cup_B N)$ extends to a complex G-vector bundle ξ over X, where X is obtained from $G \times_H (Y \cup_B N)$ by adding the cone C_0 over the orbit $G(b_0)$, and V_0 occurs as the fibre over the vertex v_0 of C_0. By identifying v_0 with the origin in the disk D^{m_0}, we may assume that $X^G = D^{m_0}$. Now, by contracting the cone C_0 to v_0 and replacing each N_i by $N_i \times D^{2r}$, where H acts trivially on D^{2r}, we may assume that

$$X_{(H)} = G/H \times N \times D^{2r} .$$

Set $M = X^G \sqcup X_{(H)}$. Let ν be the complex G-vector bundle over M defined by

$$\nu|_{X^G} = \langle V_0 \ominus V_0^G \rangle \qquad \text{and} \qquad \nu|_{(G/H \times N_i \times D^{2r})} = G \times_H \langle W_i \oplus W \rangle$$

for $i = 1, \ldots, k$, where $V_0 \ominus V_0{}^G$ denotes the G-nontrivial summand of V_0. Then, as G-vector bundles, $\xi|_M \cong TM \oplus \nu$ when the m_i's all are even, and $\xi|_M \cong TM \oplus \langle \mathbb{R} \rangle \oplus \nu$ when the m_i's all are odd. Thus, the Normal Bundle Condition holds for X, ξ, M, and ν. Again, by adding to ξ and ν a suitable product bundle over X and M, respectively, we may assume that the Isotropy Subgroup Condition and the General Position Condition both also hold (cf. Remark 2.3). Therefore, according to Theorem 2.1 and Remark 2.2, there exists a smooth action of G on a disk D^m containing M as an invariant smooth submanifold with equivariant normal bundle ν, such that

$$(D^m)^G = X^G = D^{m_0} \qquad \text{and} \qquad D^m_{(H)} = D(\nu|_{X^G})_{(H)} \sqcup X_{(H)} .$$

Note that the bundle TD^m (resp., $TD^m \oplus \langle\mathbb{R}\rangle$) admits the structure of a complex G-vector bundle when the m_i's all are even (resp., odd). Now, consider the equivariant double of D^m, i.e., first take $D^m \times D^1$ with the diagonal action of G, where G acts trivially on D^1, and then restrict the G-action to the boundary $\partial(D^m \times D^1) = S^m$. As a result, we get a smooth action of G on S^m such that

$$(S^m)^G = S^{m_0} \quad \text{and} \quad S^m_{(H)} = M_0 \sqcup M_1 \sqcup \cdots \sqcup M_k ,$$

where $M_0 = \partial\big(D^{m_0+1} \times D(V_0 \ominus V_0^G)\big)_{(H)}$ and $M_i = G/H \times \partial(N_i \times D^{2r+1})$ for $i = 1, \dots, k$. Clearly, $W_i \oplus W$ occurs as the normal representation over M_i for $i = 1, \dots, k$. Since $m_i = \dim N_i \geq 1$, the orbit space M_i/G is connected. Choose a point $x \in S^{m_0}$ (which is fixed under the G-action on S^m) and using the Slice Theorem, identify an invariant neighborhood of x in S^m with the closed unit disk D_x of the slice representation S_x. Since $S^m \smallsetminus \operatorname{Int} D_x \cong D^m$, we get a smooth action of G on D^m such that

$$(D^m)^G = D^{m_0} \quad \text{and} \quad D^m_{(H)} = (M_0 \smallsetminus \operatorname{Int} D_x) \sqcup M_1 \sqcup \cdots \sqcup M_k$$

with $W_i \oplus W$ occurring as the normal representation over M_i for $i = 1, \dots, k$. Finally, by adding to D^m an open equivariant collar along ∂D^m, or by identifying $D^m \smallsetminus \partial D^m$ with $\mathbb{R}^m$, or by identifying $S^m \smallsetminus \{x\}$ with $\mathbb{R}^m$, we get a smooth action of G on $\mathbb{R}^m$ such that

$$(\mathbb{R}^m)^G = \mathbb{R}^{m_0} \quad \text{and} \quad \mathbb{R}^m_{(H)} = (M_0 \smallsetminus D_x) \sqcup M_1 \sqcup \cdots \sqcup M_k$$

with $W_i \oplus W$ occurring as the normal representation over M_i for $i = 1, \dots, k$. □

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