

László Zsilinszky

Baire spaces and weak topologies generated by gap and excess functionals

Mathematica Slovaca, Vol. 49 (1999), No. 3, 357--366

Persistent URL: <http://dml.cz/dmlcz/132825>

Terms of use:

© Mathematical Institute of the Slovak Academy of Sciences, 1999

Institute of Mathematics of the Academy of Sciences of the Czech Republic provides access to digitized documents strictly for personal use. Each copy of any part of this document must contain these *Terms of use*.



This paper has been digitized, optimized for electronic delivery and stamped with digital signature within the project *DML-CZ: The Czech Digital Mathematics Library* <http://project.dml.cz>

BAIRE SPACES AND WEAK TOPOLOGIES GENERATED BY GAP AND EXCESS FUNCTIONALS

LÁSZLÓ ZSILINSZKY

(Communicated by *Lubica Holá*)

ABSTRACT. Let (X, d) be a separable Baire metric space or a completely metrizable space. It is shown that the family $CL(X)$ of all nonempty closed subsets of X endowed with the finite Hausdorff topology is a Baire space. Baireness of other weak topologies on $CL(X)$ generated by gap and excess functionals is also investigated.

Introduction

Hyperspace topologies, i.e. topologies on the nonempty closed subsets $CL(X)$ of a topological space X has been studied from the beginning of our century. Historically there have been two topologies of particular importance: the *Hausdorff metric topology* and the *Vietoris topology*. The Hausdorff metric topology is probably the best-studied hyperspace topology owing to its applicability to various areas of mathematics (see [Be1; p. 308]). The Vietoris topology τ_V first appeared explicitly in [Vi], and then it has been very thoroughly studied through the century.

A fundamental article on the above topologies is Michael's paper [Mi], which set the agenda for study of hyperspaces for the next 30 years. Basic topological properties, such as separation axioms, countability axioms, compactness, connectedness, metrizability of the Vietoris topology are due to Michael, but other interesting results can be found in [Ke], [Ve] on normality of τ_V , in [Mc] on the Baireness of τ_V , in the book [Ku] (a very full discussion of τ_V for a metric base space, where it coincides with the Hausdorff metric topology) or more recently in [KT] and [Be1].

AMS Subject Classification (1991): Primary 54B20; Secondary 54E52.

Key words: weak hyperspace topologies, proximal hit-and-miss topologies, finite Hausdorff topology, gap and excess functionals, Baire spaces, feeble homeomorphism.

In their usual presentations, the Hausdorff metric topology is an example of a functional space topology, whereas the Vietoris topology is an example of a hit-and-miss topology. In general, a particular hyperspace topology will fit into at least one of these (overlapping) categories.

A *hit-and-miss hyperspace topology* has a subbase that can be split into two parts ([Be1]). One kind of subbasic open sets consist of all closed sets that hit an open set $U \subset X$ — it is of the form $U^- = \{A \in CL(X) : A \cap U \neq \emptyset\}$; whereas the other kind of subbasic open set consists of all closed sets that avoid a particular kind of closed sets — it is of the form $(B^c)^+ = \{A \in CL(X) : A \cap B = \emptyset\}$, where B ranges over a given family $\Delta \subset CL(X)$.

Beside the Vietoris topology (which is the hit-and-miss topology with $\Delta = CL(X)$), there is one more topology of this kind, the so-called *Fell topology* or *topology of closed convergence* τ_F , which has been given a considerable attention; it is the hit-and-miss topology with $\Delta =$ closed compact subsets of X . The Fell topology was introduced by Fell in [Fe] and has a remarkable property: τ_F is always compact on $CL(X) \cup \{\emptyset\}$, regardless of the base space X . This compactness property makes the Fell topology highly useful in various applications ([At], [No]). For further results on this topology see e.g. [KT], [Be1] and more recently [HLP] on normality of τ_F .

The general study of hyperspaces of the hit-and-miss type seems to have only been undertaken in the seminal papers of Poppe [Po1-2]. The so-called *proximal hit-and-miss topology* or *hit-and-far topology* ([Be1]) is related to the hit-and-miss one; it was formulated by Naimpally and his collaborators ([BLLN], [DNS]) and was studied in full generality by Beer and Tamaki in [BT1-2] (see also [Be1], [DH], [Zs1]). This general approach of (proximal) hit-and-miss topologies is however exceptional, since the recent research for the most part has been descriptive of individual new topologies that arise in applications (e.g. the slice topology, the linear topology, the ball (proximal) topology — see [Be1] for a survey).

Another way of constructing hyperspace topologies is to define them as *weak topologies*, i.e. define them as the weakest topology on the hyperspace that makes certain set functionals (such as e.g. the distance, gap or excess functionals) continuous with respect to this topology. Probably the most important hyperspace topology of this sort, the *Wijsman topology* τ_W , is the weak topology on $CL(X)$ generated by the distance functionals viewed as functionals of set argument. It was introduced by Wijsman [Wi] in the context of convex analysis, and then studied in abstract by a number of authors ([FLL], [LL], [He], [BLLN], [Be1-2]).

A general program aimed at the characterization of topologies in terms of the continuity of set functionals was carried out by Beer and his collaborators over the last decade ([BLLN], [BL], [Be1]). The interest for these weak topologies arises among others from the fact that under certain natural conditions on the

base space and on the generating family of set functionals they are measurably compatible, i.e. their Borel field is the Effros σ -algebra generated by $\{U^- : U \text{ open in } X\}$. This, in turn, allows us to express multifunction measurability as ordinary measurability of an associated single-valued function (see [Be2], [Be1; Chapter 6]).

As for the recent research in the above mentioned fields, it has focused e.g. on comparison of diverse hyperspace topologies ([DN], [HLu1], [CLZ]), on expressing hypertopologies as weak topologies ([BLN], [BL], [Be2]) or as suprema or infima of (parts) of the Wijsman, Hausdorff, the Vietoris and other important topologies ([BLN], [CLP], [LLP]). This latter research exposed the fundamental role of the Wijsman topology in the construction of the lattice of hyperspace topologies.

Until recently there has been given a little attention to the general approach to (proximal) hit-and-miss topologies. In [BT1-2] several basic results e.g. on uniformizability and the mutual coincidence of hit-and-miss resp. hit-and-far topologies was settled, further in [DH], [HLe] other basic properties were established (related results can be found in [Zs3-4]).

Another interesting problem was to find Polish (i.e. separable completely metrizable) hyperspace topologies. The answer was found by Beer and Costantini in [Be3], [Co1] (see also [Zs2] for a short proof), calling attention again to the Wijsman topology; namely, it was shown that the Wijsman topology is Polish provided the base space is Polish. More recently Hola and Luchetti [HLu2] have found other Polish weak topologies.

A natural question in this connection is to ask when these topologies give rise to Baire hyperspaces, i.e. to hyperspaces where any countable intersection of dense open sets is dense. It is the purpose of this paper to answer this question. We shall make use of results of [Zs1], where the above problem is settled for the hit-and-miss and proximal hit-and-miss topologies as well as for the Wijsman topology. The relevant result for the Wijsman topology claims, that if X is a completely metrizable space or a separable Baire space, then $(CL(X), \tau_W)$ is a Baire space. It is worth mentioning in this respect the result of Costantini [Co2], who has constructed a completely metrizable space with non-Čech-complete hyperspace endowed with the Wijsman topology.

It turns out that the method of establishing the Baireness of the Wijsman topology can be used for our immediate purposes, too. More precisely, we shall define an auxiliary proximal hit-and-miss topology the Baireness of which is inferable from our previous results in [Zs1] and which is a “Baire topology” if and only if the investigated weak topology has the same property.

Preliminaries

In what follows (X, d) will be a metric space. The symbols $CL(X)$, $K(X)$, $F(X)$ will stand for the family of all nonempty closed, compact and finite subsets of X , respectively. The open (resp. closed) ball about x of radius ε will be denoted by $S(x, \varepsilon)$ (resp. $B(x, \varepsilon)$). The *distance* of x from a nonempty set A is defined as

$$d(x, A) = \inf\{d(x, a); a \in A\}.$$

The open (resp. closed) ε -*enlargement* of A is the set

$$S(A, \varepsilon) = \{x \in X; d(x, A) < \varepsilon\} \quad (B(A, \varepsilon) = \{x \in X; d(x, A) \leq \varepsilon\}).$$

We shall frequently use the fact that

$$S(S(A, \alpha), \beta) \subset S(B(A, \alpha), \beta) \subset S(A, \alpha + \beta)$$

for every nonempty $A \subset X$ and all $\alpha, \beta > 0$. Given two nonempty sets A, B we define the *gap* between them as

$$D(A, B) = \inf\{d(a, b); a \in A, b \in B\}$$

and the *excess* of A over B as

$$e(A, B) = \sup\{d(a, B); a \in A\}$$

and, finally, the *Hausdorff distance* between A and B as

$$H(A, B) = \max\{e(A, B), e(B, A)\}.$$

It is well-known that

$$|D(A, B) - D(A, C)| \leq H(B, C) \quad \text{and} \quad |e(A, B) - e(A, C)| \leq H(B, C)$$

for every $A, B, C \in CL(X)$ ([Be1; Proposition 3.2.5]).

Let Δ_1, Δ_2 be (possibly empty) subfamilies of $CL(X)$. In what follows $\tau = \tau_{\Delta_1, \Delta_2}$ will stand for the weak topology on $CL(X)$ generated by the family

$$\mathcal{R} = \{D(\cdot, B); B \in \Delta_1\} \cup \{e(\cdot, B); B \in \Delta_2\}$$

of gap and excess functionals (cf. [Be1]). If $\Delta_2 = \emptyset$, resp. $\Delta_2 = \Delta_1$ we obtain the topologies $\tau_{\Delta_1}^G$ and $\tau_{\Delta_1}^{GE}$ considered in [HLu2]. In particular, if $\Delta_1 = \Delta_2 = K(X)$ we obtain the so-called *finite Hausdorff topology* τ_{fH} ([Be1-2]), which is measurably compatible for a separable base space X .

For an arbitrary nonempty $A \subset X$ define

$$A^{++} = \{B \in CL(X); D(B, A^c) > 0\} = \{B \in CL(X); \exists \varepsilon > 0: S(B, \varepsilon) \subset A\},$$

$$A^- = \{B \in CL(X); A \cap B \neq \emptyset\},$$

where A^c stands for the complement of A in X . The *proximal hit-and-miss topology* τ_{Δ}^{++} has as a subbase all sets of the form $(B^c)^{++}$, where B ranges over a fixed nonempty subfamily $\Delta \subset CL(X)$, together with all sets of the form U^- , where U is an arbitrary nonempty open subset of X (cf. [Be1]). We shall need the following subfamilies of $CL(X)$:

$$B(\Delta_1) = \{B(A, \varepsilon); A \in \Delta_1, \varepsilon > 0\},$$

$$S(\Delta_2) = \{(S(A, \varepsilon))^c; A \in \Delta_2, \varepsilon > 0\} \quad \text{and} \quad \Delta = B(\Delta_1) \cup S(\Delta_2).$$

We shall denote by $\Sigma(\Delta)$ the set of all finite unions of members of Δ . The symbol $\text{cl}(\mathcal{A})$ (resp. $\text{cl}_{++}(\mathcal{A})$) will stand for the closure of $\mathcal{A} \subset CL(X)$ in $(CL(X), \tau)$ (resp. $(CL(X), \tau_{\Delta}^{++})$).

A topological space Y is called *quasi-regular* provided each of its nonempty open subset contains a closed subset with nonempty interior. Following [FK], we shall adopt the informal definition of a *complete space*, i.e. a topological space that can be proved to be Baire by an argument similar to that of the Baire Category Theorem. Examples of such spaces are completely metrizable spaces, locally compact regular (Hausdorff) spaces, pseudo-complete spaces, Čech-complete spaces and (weakly) α -favorable spaces (see [AL], [HM], [Wh]).

A mapping f from a topological space X onto a topological space Y is said to be *feebly continuous* (*feebly open*) if $\text{int } f^{-1}(V) \neq \emptyset$ ($\text{int } f(U) \neq \emptyset$) for an arbitrary nonempty open $V \subset Y$ ($U \subset X$). A *feeble homeomorphism* is a feebly continuous feebly open bijection (cf. [HM]).

Main Results

LEMMA. *Suppose that either Δ_1 or Δ_2 contains $F(X)$. Then*

- (i) $\tau \subset \tau_{\Delta}^{++}$;
- (ii) *for each $\emptyset \neq U \in \tau_{\Delta}^{++}$ there exists $\emptyset \neq U_0 \in \tau$ such that $U_0 \subset U$.*

Proof.

(i): Observe that whenever $0 \leq \alpha < \beta$ and $A_s \in \Delta_s$ ($s = 1, 2$), then

$$D^{-1}(\cdot, A_1)(\alpha, \beta) = \left(\bigcup_{n=1}^{\infty} \left((B(A_1, \alpha + \frac{1}{n}))^c \right)^{++} \right) \cap (S(A_1, \beta))^- \in \tau_{B(\Delta_1)}^{++} \subset \tau_{\Delta}^{++}$$

and, further,

$$e^{-1}(\cdot, A_2)(\alpha, \beta) = \left(\bigcup_{n=1}^{\infty} \left(S(A_2, \beta - \frac{1}{n}) \right)^{++} \right) \cap ((B(A_2, \alpha))^c)^- \in \tau_{S(\Delta_2)}^{++} \subset \tau_{\Delta}^{++}.$$

(ii): Pick a nonempty $\mathbf{U} = \bigcap_{j \in J} (B_j^c)^{++} \cap \bigcap_{k \in K} (E_k^c)^{++} \cap \bigcap_{i \in I} U_i^- \in \tau_{\Delta}^{++}$, where $B_j = B(D_j, \varepsilon_j) \in B(\Delta_1)$ ($j \in J$, J finite), $E_k = (S(F_k, \delta_k))^c \in S(\Delta_2)$ ($k \in K$, K finite) and without loss of generality U_i are pairwise disjoint nonempty open subsets of X for all $i \in I$ (I finite). If $A_0 \in \mathbf{U}$ then $A_0 \cap U_i \neq \emptyset$ for each $i \in I$, further $D(A_0, B_j) > 0$ and $D(A_0, E_k) > 0$ for every $j \in J$, $k \in K$.

Accordingly, there exists $a_i \in A_0 \cap U_i$ with $S(a_i, \gamma) \subset U_i$ for some $\gamma > 0$, and, further, $d(a_i, D_j) > \varepsilon_j$ and $d(a_i, F_k) < \delta_k$ for all i, j, k . Put $A^* = \{a_i; i \in I\}$, $\varepsilon = \min_{j \in J} D(A^*, D_j)$ and $\delta = \max_{k \in K} e(A^*, F_k)$. Then $\varepsilon > \varepsilon_j$ for all $j \in J$ and $\delta < \delta_k$ for all $k \in K$.

Then we have $D^{-1}(\cdot, D_j)(\frac{\varepsilon + \varepsilon_j}{2}, +\infty) \subset (B_j^c)^{++}$, furthermore $e^{-1}(\cdot, F_k)[0, \frac{\delta + \delta_k}{2}) \subset (S(F_k, \frac{\delta + \delta_k}{2}))^{++} \subset (S(F_k, \delta_k))^{++} = (E_k^c)^{++}$ for all j, k . Define

$$\mathbf{W} = \begin{cases} \bigcap_{i \in I} D^{-1}(\cdot, \{a_i\})[0, \gamma) & \text{if } F(X) \subset \Delta_1, \\ e^{-1}(\cdot, A^*)[0, \gamma) \cap \bigcap_{i \in I} e^{-1}(\cdot, A^* \setminus \{a_i\})(\gamma, +\infty) & \text{if } F(X) \subset \Delta_2. \end{cases}$$

In both cases it follows that $\mathbf{W} \in \tau$, $A^* \in \mathbf{W}$ and $\mathbf{W} \subset \bigcap_{i \in I} U_i^-$. Consequently

$$\mathbf{U}_0 = \bigcap_{j \in J} D^{-1}(\cdot, D_j)(\frac{\varepsilon + \varepsilon_j}{2}, +\infty) \cap \bigcap_{k \in K} e^{-1}(\cdot, F_k)[0, \frac{\delta + \delta_k}{2}) \cap \mathbf{W} \in \tau$$

and $\mathbf{U}_0 \subset \mathbf{U}$. Finally, observe that \mathbf{U}_0 is nonempty since it contains A^* . \square

Remark. In view of [Be1; Exercise 4.1.2.(a)] if Δ_1 contains the singletons then the Lemma holds.

THEOREM 1. *Suppose that either Δ_1 or Δ_2 contains $F(X)$. Then $(CL(X), \tau_{\Delta}^{++})$ is a quasi-regular space.*

Proof. Pick any nonempty $\mathbf{U} \in \tau_{\Delta}^{++}$. Then, by the Lemma, there exists $\emptyset \neq \mathbf{U}_0 \in \tau$ with $\mathbf{U}_0 \subset \mathbf{U}$. Because τ is a weak topology, it is completely regular and, consequently $\text{cl}(\mathbf{U}_1) \subset \mathbf{U}_0$ for some $\emptyset \neq \mathbf{U}_1 \in \tau$. Finally, according to the Lemma, $\tau \subset \tau_{\Delta}^{++}$, so $\mathbf{U}_1 \in \tau_{\Delta}^{++}$. Further

$$\emptyset \neq \text{cl}_{++}(\mathbf{U}_1) \subset \text{cl}(\mathbf{U}_1) \subset \mathbf{U}_0 \subset \mathbf{U}.$$

\square

THEOREM 2. *Suppose that either Δ_1 or Δ_2 contains $F(X)$. Then $(CL(X), \tau)$ is a Baire space if and only if $(CL(X), \tau_{\Delta}^{++})$ is.*

Proof. By the Lemma, the identity mapping $\text{id}: (CL(X), \tau_{\Delta}^{++}) \rightarrow (CL(X), \tau)$ is a feeble homeomorphism and feeble homeomorphisms preserve Baire spaces (see [Ne]). \square

THEOREM 3. *Suppose that either Δ_1 or Δ_2 contains $F(X)$.*

- (i) *Suppose that both Δ_1 and Δ_2 are separable with respect to the induced Hausdorff metric H . If X is a Baire space then so is $(CL(X), \tau)$.*
- (ii) *If X is a complete space, then $(CL(X), \tau)$ is a Baire space.*

P r o o f .

(i): According to our Theorem 1, Theorem 2 and [Zs1; Corollary 4.2] it suffices to prove that X is separable and there exist countable families $\Gamma_1 \subset B(\Delta_1)$ and $\Gamma_2 \subset S(\Delta_2)$ such that whenever $B = B_1 \cup B_2$, $B_1 \in B(\Delta_1)$, $B_2 \in S(\Delta_2)$ and $F \neq \emptyset$ is a finite subset of B^c , then there exists $D \in \Sigma(\Gamma_1 \cup \Gamma_2)$ with $B \subset D$ and $F \subset D^c$.

Let Δ'_s be a countable dense set in (Δ_s, H) ($s = 1, 2$). Then choosing a singleton from every member of $\Delta'_1 \cup \Delta'_2$ we get a countable subset of X , which, by our conditions on Δ_1, Δ_2 , is also dense in X . Accordingly, X is separable.

Now define the following countable families:

$$\Gamma_1 = \{B(E, r); E \in \Delta'_1, r \text{ is a positive rational}\},$$

$$\Gamma_2 = \{(S(E, r))^c; E \in \Delta'_2, r \text{ is a positive rational}\}.$$

Let $B = B(A_1, \varepsilon_1) \cup (S(A_2, \varepsilon_2))^c$ and $F \subset B^c$ be finite, where $A_s \in \Delta_s$ ($s = 1, 2$). Then $\delta_1 = D(F, A_1) - \varepsilon_1 > 0$ and $\delta_2 = \varepsilon_2 - e(F, A_2) > 0$, hence there exist sets $E_s \in \Delta'_s$ such that $H(A_s, E_s) < \frac{\delta_s}{4}$ for $s = 1, 2$. Choose positive rationals r_1, r_2 such that

$$\varepsilon_1 + \frac{\delta_1}{4} < r_1 < D(F, A_1) - \frac{\delta_1}{4} \quad \text{and} \quad e(F, A_2) + \frac{\delta_2}{4} < r_2 < \varepsilon_2 - \frac{\delta_2}{4}.$$

Take any $z \in B(A_1, \varepsilon_1)$. Then $d(z, E_1) \leq d(z, A_1) + H(A_1, E_1) < \varepsilon_1 + \frac{\delta_1}{4} < r_1$, so $z \in B(E_1, r_1)$ whence $B(A_1, \varepsilon_1) \subset B(E_1, r_1) \in \Gamma_1$. Moreover, $D(F, E_1) \geq D(F, A_1) - H(A_1, E_1) \geq D(F, A_1) - \frac{\delta_1}{4} > r_1$, thus $F \subset (B(E_1, r_1))^c$. A similar argument establishes that $(S(A_2, \varepsilon_2))^c \subset (S(E_2, r_2))^c \in \Gamma_2$ and $F \subset S(E_2, r_2)$. This means that for $D = B(E_1, r_1) \cup (S(E_2, r_2))^c \in \Sigma(\Gamma_1 \cup \Gamma_2)$ we have $B \subset D$ and $F \subset D^c$.

(ii): This follows directly from our Theorem 1, Theorem 2 and [Zs1; Theorem 4.3]. □

COROLLARY 1. *Let (X, d) be a metric space. Suppose that $\Delta \subset CL(X)$ contains the singletons.*

- (i) *Suppose that Δ is separable with respect to the induced Hausdorff metric and X is a Baire space. Then $(CL(X), \tau_\Delta^G)$ and $(CL(X), \tau_\Delta^{GE})$ are Baire spaces.*
- (ii) *If X is a complete space then $(CL(X), \tau_\Delta^G)$ and $(CL(X), \tau_\Delta^{GE})$ are Baire spaces.*

COROLLARY 2. *If X is a separable Baire metric space or a complete space, then $(CL(X), \tau_{fH})$ is a Baire space.*

REFERENCES

- [AL] AARTS, L. M.—LUTZER, D. J.: *Completeness properties designed for recognizing Baire spaces*, *Dissertationes Math.* **116** (1974), 1–48.
- [At] ATTOUCH, H.: *Variational Convergence for Functions and Operators*, Pitman, Boston, 1990.
- [Be1] BEER, G.: *Topologies on Closed and Closed Convex Sets*, Kluwer, Dordrecht, 1993.
- [Be2] BEER, G.: *Topologies on closed and closed convex sets and the Effros measurability of set valued functions*, *Sémin. Anal. Convex Montpellier* **21** (1991), Exp. No. 2.
- [Be3] BEER, G.: *A Polish topology for the closed subsets of a Polish space*, *Proc. Amer. Math. Soc.* **113** (1991), 1123–1133.
- [BL] BEER, G.—LUCCHETTI, R.: *Weak topologies for the closed subsets of a metrizable space*, *Trans. Amer. Math. Soc.* **335** (1993), 805–822.
- [BLLN] BEER, G.—LECHICKI, A.—LEVI, S.—NAIMPALLY, S.: *Distance functionals and suprema of hyperspace topologies*, *Ann. Mat. Pura Appl. (4)* **162** (1992), 367–381.
- [BT1] BEER, G.—TAMAKI, R.: *On hit-and-miss hyperspace topologies*, *Comment. Math. Univ. Carolin.* **34** (1993), 717–728.
- [BT2] BEER, G.—TAMAKI, R.: *The infimal value functional and the uniformization of hit-and-miss hyperspace topologies*, *Proc. Amer. Math. Soc.* **122** (1994), 601–612.
- [CLP] COSTANTINI, C.—LEVI, S.—PELANT, J.: *Infima of hyperspace topologies*, *Matematika* **42** (1995), 67–86.
- [CLZ] COSTANTINI, C.—LEVI, S.—ZIEMINSKA, J.: *Metrics that generate the same hyperspace convergence*, *Set-Valued Anal.* **1** (1993), 141–157.
- [Co1] COSTANTINI, C.: *Every Wijsman topology relative to a Polish space is Polish*, *Proc. Amer. Math. Soc.* **123** (1995), 2569–2574.
- [Co2] COSTANTINI, C.: *On the hyperspace of a non-separable metric space*, *Proc. Amer. Math. Soc.* **126** (1998), 3393–3396.
- [DNS] DI CONCILIO, A.—NAIMPALLY, S.—SHARMA, P.: *Proximal Hypertopologies*. *Proc. Sixth Brazilian Topology Meeting, Campinas, Brasil, 1988.*
- [DH] DI MIAO, G.—HOLÁ, E.: *On hit-and-miss topologies*, *Rend. Accad. Sci. Fis. Mat. Napoli (4)* **62** (1995), 103–124.
- [DN] DI MAIO, A.—NAIMPALLY, S.: *Comparison of hypertopologies*, *Rend. Istit. Mat. Univ. Trieste* **22** (1990), 140–161.
- [Fe] FELL, J.: *A Hausdorff topology for the closed subsets of locally compact non-Hausdorff space*, *Proc. Amer. Math. Soc.* **13** (1962), 472–476.
- [FLL] FRANCAVIGLIA, S.—LECHICKI, A.—LEVI, S.: *Quasi-uniformization of hyperspaces and convergence of nets of semicontinuous multifunctions*, *J. Math. Anal. Appl.* **112** (1985), 347–370.

- [FK] FLEISSNER, W. G.—KUNEN, K.: *Barely Baire spaces*, *Fund. Math.* **101** (1978), 229–240.
- [He] HESS, C.: *Quelques résultats sur la mesurabilité des multifonctions à valeurs dans un espace métrique séparable*, *Sémin. Anal. Convexe* **16** (1986), Exp. No. 1.
- [HLe] HOLÁ, E.—LEVI, S.: *Decomposition properties of hyperspaces topologies*, *Set-Valued Anal.* **5** (1997), 309–321.
- [HLP] HOLÁ, E.—LEVI, S.—PELANT, J.: *Normality and paracompactness of the Fell topology*, *Proc. Amer. Math. Soc.* **127** (1999), 2193–2197.
- [HLu1] HOLÁ, E.—LUCCHETTI, R.: *Equivalence among hypertopologies*, *Set-Valued Anal.* **3** (1995), 339–350.
- [HLu2] HOLÁ, E.—LUCCHETTI, R.: *Polishness of weak topologies generated by gap and excess functionals*, *J. Convex Anal.* **3** (1996), 283–294.
- [HM] HAWORTH, R. C.—MCCOY, R. A.: *Baire spaces*, *Dissertationes Math.* **141** (1977), 1–77.
- [Ke] KEESLING, J.: *On the equivalence of normality and compactness in hyperspaces*, *Pacific J. Math.* **33** (1970), 657–667.
- [KT] KLEIN, E.—THOMPSON, A.: *Theory of Correspondences*, Wiley, New York, 1975.
- [Ku] KURATOWSKI, K.: *Topology I*, Academic Press, New York, 1966.
- [LL] LECHICKI, A.—LEVI, S.: *Wijsman convergence in the hyperspace of a metric space*, *Boll. Un. Mat. Ital. B* (7) **1** (1987), 439–451.
- [LLP] LEVI, S.—LUCCHETTI, R.—PELANT, J.: *On the infimum of the Hausdorff and Vietoris topologies*, *Proc. Amer. Math. Soc.* **118** (1993), 971–978.
- [Mc] MCCOY, R. A.: *Baire spaces and hyperspaces*, *Pacific J. Math.* **58** (1975), 133–142.
- [Mi] MICHAEL, E.: *Topologies on spaces of subsets*, *Trans. Amer. Math. Soc.* **71** (1951), 152–182.
- [No] NORBERG, T.: *On Vervaat's sup vague topology*, *Ark. Mat.* **28** (1990), 139–144.
- [Ne] NEUBRUNN, T.: *A note on mappings of Baire spaces*, *Math. Slovaca* **27** (1977), 173–176.
- [Pol] POPPE, H.: *Eine Bemerkung über Trennungsaxiome in Räumen von abgeschlossenen Teilmengen topologischer Räume*, *Arch. Math.* **16** (1965), 197–199.
- [Po2] POPPE, H.: *Einige Bemerkungen über den Raum der abgeschlossenen Mengen*, *Fund. Math.* **59** (1966), 159–169.
- [Vi] VIETORIS, L.: *Stetige Mengen*, *Monatsh Math. Phys.* **31** (1921), 173–204.
- [Wh] WHITE JR., H. E.: *Topological spaces that are α -favourable for a player with perfect information*, *Proc. Amer. Math. Soc.* **50** (1975), 477–482.
- [Wi] WIJSMAN, R.: *Convergence of sequences of convex sets, cones and functions II*, *Trans. Amer. Math. Soc.* **123** (1966), 32–45.
- [Zs1] ZSILINSZKY, L.: *Baire spaces and hyperspace topologies*, *Proc. Amer. Math. Soc.* **124** (1996), 3175–3184.
- [Zs2] ZSILINSZKY, L.: *Polishness of the Wijsman topology revisited*, *Proc. Amer. Math. Soc.* **126** (1998), 3763–3765.

LÁSZLÓ ZSILINSZKY

[Zs3] ZSILINSZKY, L.: *On separation axioms in hyperspaces*, Rend. Circ. Mat. Palermo (2) **45** (1996), 75–83.

Received June 29, 1997

*Department of Mathematics
University of South Carolina
Columbia, SC 29208
U. S. A
E-mail: zsilinsz@math.sc.edu*

*Present address:
Department of Mathematics
and Computer Science
University of North Carolina at Pembroke
Pembroke, NC 28303
U. S. A.
E-mail: laszlo@nat.uncp.edu*