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Professor Tibor Neubrunn (1929-1990)

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PROFESSOR TIBOR NEUBRUNN (1929–1990)

A significant Czechoslovak mathematician and pedagogue professor Tibor Neubrunn, died in Bratislava on November 21st, 1990. He was born on the 2nd of August, 1929 in Veľká Hradná. In 1950 he took his leaving examination in Trenčín and in 1954 he graduated in mathematics at the Comenius University in Bratislava. Since 1953 until his death he was an assistant, assoc. professor (1965) and professor (1980) at the Comenius University. In the year 1967/68 he was a professor at the University of Baghdad and in the years 1972/73 and 1973/74 at the University of St. Salvador in Brasil. In 1964 he was awarded the PhD. degree and in 1979 the DrSc. degree.

The research interests of T. Neubrunn were very large. His first papers dealt with the measure theory. In [1] he improved a result of R. E. Zink and gave a weaker condition for inner regularity. Paper [4] presents an excellent remarkable generalization of the construction of a measure from a content, when instead of a locally compact space an abstract space is considered. In [2] following M. Kvačko a generalization of the Jęgorov Theorem is given for sequences of functions with values in some types of topological spaces. In [7] a metric space is studied associated with an arbitrary measure space and some relations to the metric space given by E. Marczewski and H. Steinhaus are discussed.

In [10] a vector measure exhaustion principle is formulated and its equivalence with the axiom of choice is proved.

In [15], following E. Marczewski and C. Ryll-Nardzewski, the σ -additivity of set functions on the product spaces is studied for not necessarily finite measures.

Following [5] and the habilitation dissertation of T. Neubrunn in [22] a systematical approach to the measurability of functions on the product space is given. The notion of the \mathcal{P} -system plays here the central role.

The papers [9], [13], [16], [26], [34] deal with the absolute continuity and dominancy of measures. While in [13] the product measures are considered, in [9] an arbitrary σ -ideal (instead of the σ -ideal of sets of the measure zero) and in [16] the so-called small system are considered. The paper [26] deals with the question, whether or not the given measure dominates a given collection. Since he studies sets of σ -ideals instead of sets of measures, his results can be applied to the case of measures as well as to the case of submeasures. Paper [34] deals with the Radon-Nikodym Theorem. Many authors followed these investigations important also for mathematical statistics (recall at least V. Ficker, L. Drewnowski, J. Lloyd, Z. Lipiecki).

In an excellent way T. Neubrunn contributed to the development of the theory of real functions. His merit is not only in the achievement of his significant original results, but also in his initiative stimulating research into the theory of real functions in our country.

The study of sets of distances belongs to his first interests. It was devoted a great effort to this studies after the second world war. These studies had a remarkable sense from the point of view of the structure of the properties of metric spaces as well as the possibility of considerable applications (e. g. in the theory of functional equations and the theory of bases of real numbers). The papers [3], [17] belong to this group.

Another domain of Professor Neubrunn's interests were functions with closed graphs. This class is related to another (e. g. the Baire class 1) and has been studied up to the present day. The papers [6], [47] belong to this sphere.

A very large sphere of results of T. Neubrunn is represented by various generalizations of the notion of continuity. This study is related to the study of the properties of topological and metric spaces, which are the domains of considered functions. To this area there belong the articles [19], [29], [31], [39], [43] as well as the articles [35], [40], [41], [47], [48], [50] devoted to the generalized continuity of multifunctions. To these types of functions the so-called Blumberg sets of functions are related. It was known before many years ago that to every function $f: \mathbf{R} \rightarrow \mathbf{R}$ there exists such a set A (depending on f) that A is dense in \mathbf{R} and the restriction $f|_A$ is continuous. The set A is called the Blumberg set of the function f . This notion was extended for functions from a topological space to another and some related types of sets were introduced for these functions. The study of the properties of these sets for various classes of functions and the extension of the problems of multifunctions are in the papers [36], [50], [51].

Very important results are in [38], where in several known assertions of the theory of stochastic processes, continuity is substituted by quasicontinuity.

Another group of his scientific articles is devoted to the structure of spaces of functions ([11], [12], [14], [19], [24], [43]), some of them with some applications in other areas (e. g. [11] has applications in the theory of infinite series).

He was interested also in the problems of various types of the convergence of functional sequences ([23], [27], [32]).

Professor T. Neubrunn's great merit is his initiation of the study of multifunctions in our country. He and his colleagues and students contributed to the theory by many results ([36], [40], [41], [50], [54]). Because of his pedagogical activity (the last 10 years he was chairman of the department of probability and mathematical statistics) these results had an influence on the theory of random sets and fuzzy random variables. Professor Neubrunn's activity in quantum logic theory had a similar importance.

The papers [18], [20], [21] belong among the first papers devoted to the study of quantum logic theory in Czecho-Slovakia. Especially the compatibility of observables was studied ([20]). In [18] Gudder's q - σ -algebra is considered and some sufficient conditions are given for the equality of the generated q - σ -algebra and the generated σ -algebra. This result evoked much interest (e. g. S. P. Gudder, A. S. Lima, A. Mukherjee, N. A. Tserpes, B. J. Pettis, P. Pták, J. Šipoš, J. E. Zerbe). In this relation recently V. Olejček solved one of Professor Neubrunn's problems proving that the q - σ -algebra generated by the family of all closed circles in the plane coincides with the σ -algebra generated by this family.

The papers [33], [37], [42], [49], [55] and [56] deal also with the quantum logic theory. The paper [38] presents a review of various modifications of the notion of compatibility in quantum logics, which is an important tool in this theory. In [55] Professor Neubrunn gives a counterexample to the Alexandrov theorem (with respect to a result of O. Beaver and T. Cook) and he presents a correct version of the theorem. The paper [56] deals with regular finitely additive states in the Hilbert space logic.

The extent of the research interests of Professor Neubrunn can be demonstrated by the papers devoted to non-standard analysis and its use in the pedagogical process ([44], [46], [61] and the book [79]).

Professor Neubrunn educated 12 new scientists (to his pupils J. Dravecký, Ľ. Holá, J. Malík, M. Matejdes, O. Náther, V. Toma belong). His modesty was apparent also in his scientific communication. He was able to suggest inconspicuously some new ideas. On the other hand, he accepted ideas of other people very sincerely.

T. Neubrunn was able to gather a great group of mathematicians for scientific cooperation. This fact was a consequence not only of his scientific level, but in a great measure it was a

consequence of his personality.

In his own scientific work T. Neubrunn succeeded to do penetrate to the nucleus of the studied problem, to generalize it and to obtain a new, unifying point of view of the studied domain. He was also able to find promptly the topical trends in the development of mathematics and to react on them by stimulating of interests of his colleagues and of his own interest.

The papers [55] and [56] are the final research papers by Professor Tibor Neubrunn. Unfortunately, he did not wait to see them because of his death. Of course, they did not mean the end of his research work. Though seriously ill he finished the work on the monograph [85]. Then he wrote three articles ([67], [68], [69]) with philosophical, pedagogical and historical aspects.

“What is the soul or the spirit of mathematics?”, he asks in [68] and he answers: “One of the elements explaining this question is the education of man to the desire to look for truth.” This was also the creed of Tibor Neubrunn’s exemplary life.

B. Riečan and T. Šalát

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