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ON EDGE INDEPENDENCE NUMBERS AND EDGE COVERING NUMBERS OF k-UNIFORM HYPERGRAPH

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In general, we follow the notation and terminology of book [1]. By a hypergraph $\boldsymbol{H}$ is meant a couple $\langle \boldsymbol{X}, \mathscr{E} \rangle$, where $\boldsymbol{X}$ is a finite set of elements called vertices and $\mathscr{E} = \{\boldsymbol{E}_1, \ldots, \boldsymbol{E}_m\}$ is a finite system of non-empty subsets of $\boldsymbol{X}$ called edges, where $\boldsymbol{E}_i \neq \boldsymbol{E}_j$ and $|\boldsymbol{E}_i| > 1$ for $i, j \in \{1, \ldots, m\}$, $i \neq j$, or $\mathscr{E}$ is an empty set. If $\mathscr{E}$ is an empty set, call a hypergraph $\boldsymbol{H}$ empty. (By $|\boldsymbol{E}_i|$ the cardinality of the set $\boldsymbol{E}_i$ is denoted.)

By the degree $d(x)$ of the vertex x we mean the cardinality of the set of all edges of the hypergraph $\boldsymbol{H}$ such that the vertex x of $\boldsymbol{H}$ belongs to all of them. The vertex x is isolated in $\boldsymbol{H}$ if $d(x) = 0$. Two edges $\boldsymbol{E}_i$, $\boldsymbol{E}_j \in \mathscr{E}$ are disjoint if $\boldsymbol{E}_i \cap \boldsymbol{E}_j = \emptyset$. A hypergraph $\boldsymbol{H}\langle \boldsymbol{N} \rangle = \langle \boldsymbol{N}, \mathscr{E}_N \rangle$ is said to be a subhypergraph of a hypergraph $\boldsymbol{H} = \langle \boldsymbol{X}, \mathscr{E} \rangle$ induced by a set $\boldsymbol{N}$ if $\boldsymbol{N} \subseteq \boldsymbol{X}$ and $\mathscr{E}_N$ is the system of all edges $\boldsymbol{E}_i \in \mathscr{E}$ such that $\boldsymbol{E}_i \subseteq \boldsymbol{N}$. A hypergraph is said to be k-uniform, $k > 1$, if all its edges have cardinality k. A 2-uniform hypergraph is called graph. In all the following consideration we will suppose that $|\boldsymbol{X}| \geqslant k \geqslant 3$.

A k-uniform hypergraph with n vertices is called complete if its set of edges has the cardinality $\binom{n}{k}$. The complement of a k-uniform hypergraph $\boldsymbol{H} = \langle \boldsymbol{X}, \mathscr{E} \rangle$ is the k-uniform hypergraph $\bar{\boldsymbol{H}} = \langle \boldsymbol{X}, \bar{\mathscr{E}} \rangle$ if $|\mathscr{E} \cup \bar{\mathscr{E}}| = \binom{n}{k}$ and $\mathscr{E} \cap \bar{\mathscr{E}} = \emptyset$.

A set $\boldsymbol{P} \subseteq \mathscr{E}$ is called an edge covering of $\boldsymbol{H}$ if for any non-isolated vertex $x \in \boldsymbol{X}$ there exists an edge $\boldsymbol{E}_i \in \boldsymbol{P}$ such that $x \in \boldsymbol{E}_i$. The cardinality of a minimum set which is an edge covering of $\boldsymbol{H}$ is called the edge covering number $\alpha_1(\boldsymbol{H})$ of $\boldsymbol{H}$.

A set $\boldsymbol{N} \subseteq \mathscr{E}$ is called an edge independent set of $\boldsymbol{H}$ if edges of $\boldsymbol{N}$ are pairwise disjoint. The cardinality of a maximum set which is an edge independent set of $\boldsymbol{H}$ is called the edge independence number $\beta_1(\boldsymbol{H})$ of $\boldsymbol{H}$.

The following lemma, proved in [6], deals with a relation between the edge covering number and the edge independence number in a k-uniform hypergraph $\boldsymbol{H}$ without isolated vertices.

Lemma 1. *For a k-uniform hypergraph* $\boldsymbol{H}$ *with n vertices without isolated vertices the following inequalities hold*

(1) $$\alpha_1(\boldsymbol{H}) + (k-1)\beta_1(\boldsymbol{H}) \leqslant n$$

(2) $$\beta_1(\boldsymbol{H}) + (k-1)\alpha_1(\boldsymbol{H}) \geqslant n.$$

Remark 1. (1) and (2) are generalizations of Gallai's [4] relations for graphs.

Theorem 1. *For a k-uniform hypergraph* $\boldsymbol{H} = \langle \boldsymbol{X}, \mathscr{E} \rangle$ *with n vertices and its complement* $\bar{\boldsymbol{H}} = \langle \boldsymbol{X}, \bar{\mathscr{E}} \rangle$

(3) $$\left\lfloor \frac{n}{k} \right\rfloor \leqslant \beta_1(\boldsymbol{H}) + \beta_1(\bar{\boldsymbol{H}}) \leqslant 2\left\lfloor \frac{n}{k} \right\rfloor$$

(4) $$0 \leqslant \beta_1(\boldsymbol{H})\cdot\beta_1(\bar{\boldsymbol{H}}) \leqslant \left\lfloor \frac{n}{k} \right\rfloor^2$$

holds. ($\lfloor x \rfloor$ *denotes the greatest integer* $\leqslant x$.)

Proof. The upper bounds in (3) and (4) follow from the inequalities

$$\beta_1(\boldsymbol{H}) \leqslant \left\lfloor \frac{n}{k} \right\rfloor \quad \text{and} \quad \beta_1(\bar{\boldsymbol{H}}) \leqslant \left\lfloor \frac{n}{k} \right\rfloor.$$

Let $\beta_1(\boldsymbol{H}) = r$, i.e. in a hypergraph $\boldsymbol{H}$ there exists the edge independent set $\boldsymbol{N}$ cardinality r. If $r = \left\lfloor \frac{n}{k} \right\rfloor$ the lower bound of (3) holds. Let $\boldsymbol{V}(\boldsymbol{N})$ be a set of vertices incident with edges from $\boldsymbol{N}$. Let $r < \left\lfloor \frac{n}{k} \right\rfloor$; then $\bar{\boldsymbol{H}}\langle \boldsymbol{X} - \boldsymbol{V}(\boldsymbol{N})\rangle$ is a complete subhypergraph of a hypergraph $\bar{\boldsymbol{H}}$, so

$$\beta_1(\bar{\boldsymbol{H}}) \geqslant \left\lfloor \frac{n - |\boldsymbol{V}(\boldsymbol{N})|}{k} \right\rfloor.$$

From this there follows

$$\beta_1(\boldsymbol{H}) + \beta_1(\bar{\boldsymbol{H}}) \geqslant r + \left\lfloor \frac{n - k\cdot r}{k} \right\rfloor = \left\lfloor \frac{n}{k} \right\rfloor,$$

which is the lower bound of (3). The lower bound of (4) is trivial.

Remark 2. The equality in the lower bounds (3) and (4) holds for every complete k-uniform hypergraph. Clearly, for any $n > k$ there exist k-uniform hypergraphs with n vertices such that the equality in the upper bound (3) and (4) holds.

Theorem 2. *For a k-uniform hypergraph* $\boldsymbol{H} = \langle \boldsymbol{X}, \mathscr{E} \rangle$ *and its complement* $\bar{\boldsymbol{H}} = \langle \boldsymbol{X}, \bar{\mathscr{E}} \rangle$ *where neither* $\boldsymbol{H}$ *nor* $\bar{\boldsymbol{H}}$ *have isolated vertices*

(5) $$\left\lfloor \frac{n}{k} \right\rfloor + 1 \leqslant \beta_1(\boldsymbol{H}) + \beta_1(\bar{\boldsymbol{H}}) \qquad \text{for} \quad n > k,\ n \neq 2k$$

(6) $$\left\lfloor \frac{n}{k} \right\rfloor \leqslant \beta_1(\boldsymbol{H}) + \beta_1(\bar{\boldsymbol{H}}) \qquad \text{for} \quad n = 2k$$

holds.

Proof. If $n < 2k$, the bound of (5) is 2. In this case the assertion of (5) holds, because $\beta_1(\boldsymbol{H}) \geqslant 1$ and $\beta_1(\bar{\boldsymbol{H}}) \geqslant 1$. If $n = 2k$, then the bound of (6) follows from the theorem 1.

Let $n \geqslant 2k + 1$. Suppose in fact that the assertion (5) does not hold, i.e. a k-uniform hypergraph $\boldsymbol{H}$ such that

(7) $$\beta_1(\boldsymbol{H}) + \beta_1(\bar{\boldsymbol{H}}) = \left\lfloor \frac{n}{k} \right\rfloor$$

exists. If $\beta_1(\boldsymbol{H}) = \left\lfloor \frac{n}{k} \right\rfloor$ then $\beta_1(\bar{\boldsymbol{H}}) \geqslant 1$, which is a contradiction to (7), thus for hypergraphs $\boldsymbol{H}$ such that $\beta_1(\boldsymbol{H}) = \left\lfloor \frac{n}{k} \right\rfloor$ or $\beta_1(\bar{\boldsymbol{H}}) = \left\lfloor \frac{n}{k} \right\rfloor$ the assertion of (5) holds:

Let $\beta_1(\boldsymbol{H}) \leqslant \left\lfloor \frac{n}{k} \right\rfloor - 1$ and $\boldsymbol{N}$ be an edge independent set of $\boldsymbol{H}$ cardinality $\beta_1(\boldsymbol{H})$. From (7) it follows that $\boldsymbol{H}\langle \boldsymbol{V}(\boldsymbol{N})\rangle$ is a complete subhypergraph of a hypergraph $\boldsymbol{H}$ and if $|\boldsymbol{X} - \boldsymbol{V}(\boldsymbol{N})| \geqslant k$, is $\bar{\boldsymbol{H}}\langle \boldsymbol{X} - \boldsymbol{V}(\boldsymbol{N})\rangle$ a complete subhypergraph of a hypergraph $\bar{\boldsymbol{H}}$.

Let us analyse three possibilities:

I. Let $\beta_1(\boldsymbol{H}) \geqslant 2$ and $\beta_1(\bar{\boldsymbol{H}}) \geqslant 2$, thus $n \geqslant 4k$. We consider the set of vertices $\boldsymbol{M} \subseteq \boldsymbol{X}$ such that $|\boldsymbol{M} \cap \boldsymbol{V}(\boldsymbol{N})| = 2k$ and $|\boldsymbol{M}| = 4k$. Let $\boldsymbol{M} = \boldsymbol{K}_1 \cup \boldsymbol{K}_2 \cup \boldsymbol{K}_3 \cup \boldsymbol{K}_4$, where $|\boldsymbol{K}_i| = k$ and $\left\lfloor \frac{k}{2} \right\rfloor \leqslant |\boldsymbol{K}_i \cap \boldsymbol{V}(\boldsymbol{N})| \leqslant \left\lceil \frac{k}{2} \right\rceil$ for $i \in \{1, 2, 3, 4\}$. Two of the sets $\boldsymbol{K}_1, \boldsymbol{K}_2, \boldsymbol{K}_3, \boldsymbol{K}_4$ form edges in $\boldsymbol{H}$ and two in $\bar{\boldsymbol{H}}$, because

$$\beta_1(\boldsymbol{H}\langle \boldsymbol{V}(\boldsymbol{N}) - \boldsymbol{M}\rangle) = \beta_1(\boldsymbol{H}) - 2$$

and

$$\beta_1(\bar{\boldsymbol{H}}\langle \boldsymbol{X} - \boldsymbol{V}(\boldsymbol{N}) - \boldsymbol{M}\rangle) = \beta_1(\bar{\boldsymbol{H}}) - 2.$$

Let $\boldsymbol{K}_1, \boldsymbol{K}_2 \in \mathscr{E}$ and $\boldsymbol{K}_3, \boldsymbol{K}_4 \in \bar{\mathscr{E}}$. If $|(\boldsymbol{K}_1 \cup \boldsymbol{K}_2) \cap \boldsymbol{V}(\boldsymbol{N})| \leqslant k$ then $\beta_1(\boldsymbol{H}\langle \boldsymbol{V}(\boldsymbol{N}) - (\boldsymbol{K}_1 \cup \boldsymbol{K}_2)\rangle) = \beta_1(\boldsymbol{H}) - 1$, thus $\beta_1(\boldsymbol{H}\langle \boldsymbol{V}(\boldsymbol{N}) \cup \boldsymbol{K}_1 \cup \boldsymbol{K}_2\rangle) = \beta_1(\boldsymbol{H}) + 1$, which is a contradiction. It means that for k even k-uniform hypergraph such that the assertion (7) is valid does not exist. Let k be odd and $|(\boldsymbol{K}_1 \cup \boldsymbol{K}_2) \cap \boldsymbol{V}(\boldsymbol{N})| > k$. We can suppose that $|\boldsymbol{K}_1 \cap \boldsymbol{V}(\boldsymbol{N})| \geqslant |\boldsymbol{K}_2 \cap \boldsymbol{V}(\boldsymbol{N})|$ holds. Let $\boldsymbol{E}_1, \boldsymbol{E}_2 \subseteq \boldsymbol{M}$ be two k-tuples such that $\boldsymbol{E}_1 \cap \boldsymbol{K}_1 = \emptyset$, $\boldsymbol{E}_2 \cap \boldsymbol{K}_1 = \emptyset$, $(\boldsymbol{E}_1 \cup \boldsymbol{E}_2) \cap \boldsymbol{K}_2 = \boldsymbol{K}_2$, $\boldsymbol{E}_1 \cap \boldsymbol{E}_2 = \emptyset$, $|\boldsymbol{E}_1 \cap \boldsymbol{V}(\boldsymbol{N})| + |\boldsymbol{K}_1 \cap \boldsymbol{V}(\boldsymbol{N})| \leqslant k$, $|\boldsymbol{E}_2 \cap \boldsymbol{V}(\boldsymbol{N})| + |\boldsymbol{K}_1 \cap \boldsymbol{V}(\boldsymbol{N})| \leqslant k$. If at least one of the k-tuples E_1, E_2 (e.g. E_1) is edge of H then $\beta_1(H\langle(V(N)\cap M)-(E_1\cup K_1)\rangle)=1$, thus $\beta_1(H)=|N|+1$, which is a contradiction, because N is an edge independent set of H with a maximum cardinality. If E_1, $E_2\in\bar{\mathscr{E}}$, then $M-(E_1\cup E_2\cup K_1)\in\bar{\mathscr{E}}$, which is a contradiction to (7). Thus in a case when $\beta_1(H)\geqslant 2$ and $\beta_1(\bar{H})\geqslant 2$, a k-uniform hypergraph such that the assertion (7) is valid does not exist, the bound of (5) holds.

II. Let $n\geqslant 3k$ and $\beta_1(H)=1$. We will prove an assertion (**A**): *If* (7) *holds and in a hypergraph* H *there exist edges* E_1, E_2 *such that* $E_1\cap E_2=\{x\}$, *then* x *is an isolated vertex of a hypergraph* $\bar{H}$.

Proof of (**A**). For each k-tuple E such that $E\cap E_2=\emptyset$ or $E\cap E_1=\emptyset$ there is $E\in\bar{\mathscr{E}}$. Let the assertion (**A**) be not valid, thus there exists an edge $K_0\in\bar{\mathscr{E}}$ so that $x\in K_0$. Then the k-tuple E_0 such that $E_0\cap E_2=\emptyset$, $E_0\cap K_0=\emptyset$, $|K_0\cap E_1|+|E_0\cap E_1|=k$ is from $\bar{\mathscr{E}}$ too. From this follows that E_0 and K_0 can belong to an edge independent set of a hypergraph $\bar{H}$. Since $\bar{H}\langle X-E_1\rangle$ is a complete subhypergraph of $\bar{H}$ then

$$\beta_1(\bar{H})\geqslant\left\lfloor\frac{|X-E_1-K_0-E_0|}{k}\right\rfloor+2=\left\lfloor\frac{n-2k}{k}\right\rfloor+2=\left\lfloor\frac{n}{k}\right\rfloor,$$

which is a contradiction to (7). Thus the auxiliary assertion is proved.

Let $M\in\mathscr{E}$. We consider two k-tuples K_1, $K_2\subseteq X$ such that $(K_1\cup K_2)\cap M=M$, $K_1\cap K_2=\emptyset$, $K_1\cap M\neq\emptyset$, $K_2\cap M\neq\emptyset$. K_1, K_2 cannot simultaneously belong to $\bar{\mathscr{E}}$, because it is a contradiction to (7) and cannot simultaneously belong into $\mathscr{E}$ because $\beta_1(H)=1$. Let $E_1\subseteq X$ be a k-tuple such that $|E_1\cap K_1|=r-1$, $|E_1\cap M|=r-1$, $|E_1\cap M\cap K_1|=r-1$. Let $R\subseteq X$ be a k-tuple such that $R\cap K_2=\emptyset$, $R\cap E_1=\emptyset$, $(M\cap K_1)-E_1=R\cap M$. Clearly $R\in\bar{\mathscr{E}}$, which follows from the assertion (**A**), because $|M\cap R|=1$ and H does not contain isolated vertices. But $R\cap E_1=\emptyset$ and $\beta_1(H)=1$, then $E_1\in\mathscr{E}$. We consider the k-tuple E_2 such that $|E_2\cap E_1|=r-2$, $|E_2\cap M|=r-2$ and $|E_2\cap E_1\cap M|=r-2$. Analogously as for E_1 we prove that $E_2\in\mathscr{E}$. We proceed analogously in the next steps, till we obtain an k-tuple E_{r-1} such that $|E_{r-1}\cap E_{r-2}\cap M|=1$, $|E_{r-1}\cap E_{r-2}|=1$, $|E_{r-1}\cap M|=1$ and $E_{r-1}\in\mathscr{E}$. From the auxiliary assertion (**A**) it follows that the vertex of $E_{r-1}\cap M$ is an isolated vertex in a hypergraph $\bar{H}$, which is a contradiction to the assumption of theorem 2. It means that in the case $\beta_1(H)=1$ and $n\geqslant 3k$ a hypergraph such that (7) holds does not exist, thus the bound of (5) is valid.

III. Let $\beta_1(H)=1$ and $2k<n<3k$. In this case a lower bound from (5) equals 3. Let it be not valid, thus a k-uniform hypergraph H such that $\beta_1(H)+\beta_1(\bar{H})=2$ and $\beta_1(H)=1$ exists. Let $M\in\mathscr{E}$, then $|X-M|\geqslant k+1$. First we indicate that if such a hypergraph exists, then an edge which has just one vertex in an edge M exists. Let E_1, E_2 be two k-tuples such that $E_1\cap E_2=\emptyset$ and $(E_1\cup E_2)\cap M=M$. Let $E_1\in\mathscr{E}$, $E_2\in\bar{\mathscr{E}}$ and $|E_1\cap M|=r>1$. We consider a k-tuple K_1 such that $|K_1 \cap E_1| = k - 1$, $|K_1 \cap M| = r - 1$. If $K_1 \in \bar{\mathscr{E}}$, then $|X - (K_1 \cup E_1)| = n - (k + 1) \geqslant k$ and $\bar{H}\langle X - (K_1 \cup E_1)\rangle$ is a complete subhypergraph of $\bar{H}$, $\beta_1(\bar{H}) = 2$, which is a contradiction to the assumption. Thus $K_1 \in \mathscr{E}$. Proceeding analogously we indicate that in $\mathscr{E}$ there exist edges that in a set M have $r - 2, r - 3, \ldots, 2, 1$ vertices. Thus in a hypergraph H there exists at least one edge E such that $|E \cap M| = 1$. Let $E \cap M = \{x\}$. Then $|X - (E \cup M)| \geqslant 2$. Let $x_1, x_2 \in X - (E \cup M)$. We the consider k-tuples $F_1 = \{x_1\} \cup M - \{x\}$ and $F_2 = \{x_2\} \cup E - \{x\}$. Then $F_1 \cap F_2 = \emptyset$, $F_1 \cap E = \emptyset$, $F_2 \cap M = \emptyset$, which is a contradiction to the fact that $\beta_1(H) = 1$ and $\beta_1(\bar{H}) = 1$. Thus a hypergraph H with n vertices, $2k < n < 3k$, such that $\beta_1(H) + \beta_1(\bar{H}) = 2$ and the assumptions of theorem 2 fulfills does not exist. The proof of theorem 2 is now complete.

Remark 3. The equality in the bound (5) holds for an arbitrary k-uniform hypergraph H such that all edges have at least one vertex x in common for which $d(x) < \binom{n-1}{k-1}$ in a hypergraph H. The equality in (6) holds for any k-uniform hypergraph H such that $E \in \mathscr{E} \Leftrightarrow (X - E) \in \bar{\mathscr{E}}$.

Theorem 3. *For a k-uniform hypergraph* $H = \langle X, \mathscr{E}\rangle$ *and its complement* $\bar{H} = \langle X, \bar{\mathscr{E}}\rangle$ *where neither* H *nor* $\bar{H}$ *have isolated vertices and* $n \neq 2k$

$$2\left\lceil \frac{n}{k} \right\rceil \leqslant \alpha_1(H) + \alpha_1(\bar{H}) \leqslant 2n - (k-1)\left\lfloor \frac{n}{k} \right\rfloor - k + 1 \tag{8}$$

$$\left\lceil \frac{n}{k} \right\rceil^2 \leqslant \alpha_1(H)\cdot\alpha_1(\bar{H}) \leqslant \frac{1}{4}\left(2n - (k-1)\left\lfloor \frac{n}{k} \right\rfloor - k + 1\right)^2 \tag{9}$$

holds. ($\lceil x\rceil$ *denotes the smallest integer* $\geqslant x$.)

Proof. The lower bounds of (8) and (9) follow from the fact that for each k-uniform hypergraph without isolated vertices $\alpha_1(H) \geqslant \left\lceil \frac{n}{k} \right\rceil$ holds. From lemma 1 it follows that

$$\alpha_1(H) \leqslant n - (k-1)\beta_1(H)$$

$$\alpha_1(\bar{H}) \leqslant n - (k-1)\beta_1(\bar{H}).$$

Adding these inequalities we obtain

$$\alpha_1(H) + \alpha_1(\bar{H}) \leqslant 2n - (k-1)(\beta_1(H) + \beta_1(\bar{H})).$$

From (5) if follows that

$$\alpha_1(H) + \alpha_1(\bar{H}) \leqslant 2n - (k-1)\left(\left\lfloor \frac{n}{k} \right\rfloor + 1\right),$$

which is the upper bound from (8). The upper bound in (9) follows from the upper bound in (8).

Remark 4. *a*) If in the assumption from theorem 3 we omit the condition that neither $\boldsymbol{H}$ nor $\bar{\boldsymbol{H}}$ contains isolated vertices, the lower bound in (8) changes into the form $\left\lceil \frac{n}{k} \right\rceil$, in (9) into 0 and the upper bound in (8) and (9) does not change.

b) The equality in the lower bounds (8) and (9) holds for a k-uniform hypergraph such that $\alpha_1(\boldsymbol{H}) = \alpha_1(\bar{\boldsymbol{H}}) = \left\lceil \frac{n}{k} \right\rceil$. Clearly, such hypergraphs exist.

c) The equality in the upper bound (8) is attained, e.g. for hypergraphs $\boldsymbol{H} = \langle \boldsymbol{X}, \mathscr{E} \rangle$ with the following structure: $\mathscr{E}$ consists of all k-tuples which contain $(k - 1)$ firmly chosen vertices and $n \equiv 0 \pmod{k}$. Then $\alpha_1(\boldsymbol{H}) = n - k + 1$ and $\alpha_1(\bar{\boldsymbol{H}}) = \left\lceil \frac{n}{k} \right\rceil$.

The inequalities for edge covering numbers and edge independence numbers for undirected graphs are investigated in [2], [3] and [5].

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О ЧИСЛЕ РЕБЕРНОЙ НЕЗАВИСИМОСТИ И РЕБЕРНОГО ПОКРЫТИЯ k-УНИФОРМНЫХ ГИПЕРГРАФОВ

František Olejník

Резюме

В этой работе приведены верхние и нижние оценки суммы и произведения числа реберной независимости для k-униформного гиперграфа $\boldsymbol{H}$ и его дополнения $\bar{\boldsymbol{H}}$. То же самое сделано для числа реберного покрытия.