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Martin Bača

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MARTIN BAČA

1. Introduction

Various types of labellings of graphs have been intensively studied by combinatorialists for some time. The notion of magic labellings has its origin in very classical Chinese mathematics. Only recently have these labellings been investigated by using modern notions of the graph theory. The notion of magic labellings of plane graphs was defined by Ko-Wei Lih in [1], where magic labellings of type (1, 1, 0) for wheels, friendship graphs and prisms are given.

This paper describes a magic labelling of type (1, 1, 0) for graphs of convex polytopes.

2. Terminology and notation

G is a finite connected plane graph without loops or multiple edges, $V(G)$ is its vertex set and $E(G)$ is its edge set. A labelling of type (1, 1, 0) assigns labels from the set $\{1, 2, \ldots, |V(G)| + |E(G)|\}$ to the vertices and edges of a plane graph G in such a way that each vertex and edge receives exactly one label and each number is used exactly once as a label. If we label only vertices or only edges, we call such a labelling a vertex labelling or an edge labelling, respectively. The weight of a face under a labelling is the sum of the labels of vertices and edges surrounding that face.

A labelling is *magic* if, for every integer s, all s-sided faces have the same weight [1].

We allow different weights for different values of s.

A labelling is *jump-magic* if, for every integer s, there exists a finite subset T^s of integer numbers such that the weight of each s-sided face is an element of T^s. We allow different sets T^s for different values of s. Two *labellings* f and f' are *complementary* if, for every integer s, the sum of the f-weight and f'-weight of each s-sided face is a constant.

We make the convention that $x_{j,n+1} = x_{j,1}$ and we shall use the expressions $\alpha = \dfrac{(-1)^j + 1}{2}$ and $\beta = \dfrac{(-1)^{j+1} + 1}{2}$ (for $j = 0, 1, 2, \ldots, m$) to simplify later notations.

3. Labellings of m-prisms

For $m \geq 1$ and $n \geq 3$ let R_n^m be the Cartesian product $P_{m+1} \times C_n$ of a path on $m + 1$ vertices with a cycle on n vertices, embedded in the plane and labelled as in Fig. 1. We will call the plane graph R_n^m an *m-prism*.

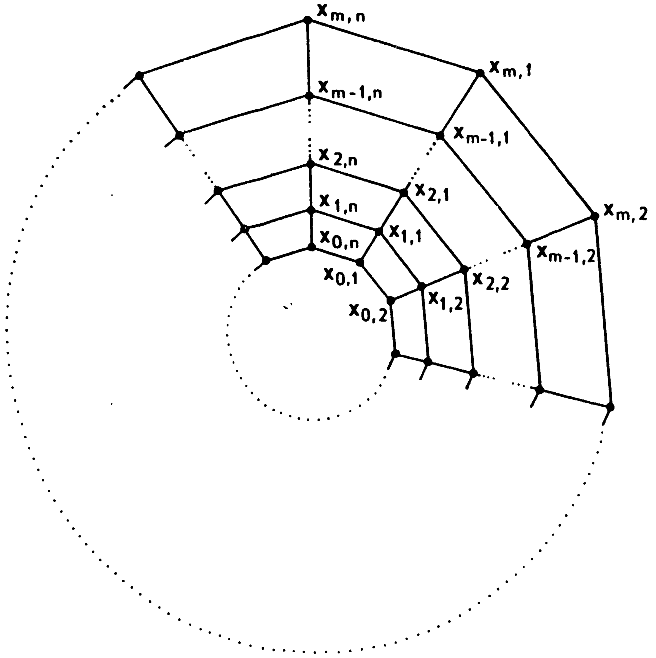

Fig. 1

Define the vertex labelling f_1 as follows.

$f_1(x_{j,i}) = \alpha[(j+1)n + 1 - i] + \beta(jn + i)$ for $1 \leq i \leq n$ and $0 \leq j \leq m$.

Theorem 1. *The vertex labelling f_1 of R_n^m is jump-magic if $m \geq 1$ and $n \geq 3$, $n \neq 4$.*

Proof. The weights of all 4-sided faces constitute the set

$$W^4 = \{w_0^4, w_1^4, \dots, w_{m-1}^4\},$$

where for $1 \leq i \leq n$ and $0 \leq j \leq m - 1$

$$f_1(x_{j,i}) + f_1(x_{j,i+1}) + f_1(x_{j+1,i}) + f_1(x_{j+1,i+1}) = w_j^4.$$

The weights of both n-sided faces constitute the set

$$W^n = \{w_0^n, w_m^n\} \quad \text{where} \quad w_0^n = \sum_{i=1}^{n} f_1(x_{0,i})$$

and $w_m^n = \sum_{i=1}^{n} f_1(x_{m,i})$.

It is simple to verify that the vertex labelling f_1 is jump-magic.

Define the edge labelling f_2 as follows.

$$f_2(x_{j,i}x_{j,i+1}) = \begin{cases} n\left(m - j + \left[\frac{j}{n}\right]\right) + i - j & \text{if} \quad i \geq k+1 \\ n\left(m - j + \left[\frac{j}{n}\right] + 1\right) + i - j & \text{if} \quad i < k+1 \end{cases}$$

$$f_2(x_{j,i}x_{j+1,i}) = \begin{cases} n\left(2m + 1 - j - \left[\frac{j}{n}\right]\right) - i + j + 2 & \text{if} \quad i \geq k+2 \\ n\left(2m - j - \left[\frac{j}{n}\right]\right) - i + j + 2 & \text{if} \quad i < k+2, \end{cases}$$

where $k = j - \left[\frac{j}{n}\right]n$, $1 \leq i \leq n$, $0 \leq j \leq m$ and the expression $\left[\frac{j}{n}\right]$ denotes the greatest integer less than or equal to $\frac{j}{n}$.

Theorem 2. *The edge labelling f_2 of R_n^m is jump-magic if $m \geq 1$, $n \geq 3$, $n \neq 4$ and it is complementary to the jump-magic vertex labelling f_1.*

Proof. The weights of all 4-sided (n-sided) faces constitute sets

$$U^4 = \{u_0^4, u_1^4, \ldots, u_{m-1}^4\}, \quad U^n = \{u_0^n, u_m^n\},$$

where for $1 \leq i \leq n$ and $0 \leq j \leq m - 1$

$$f_2(x_{j,i}x_{j,i+1}) + f_2(x_{j+1,i}x_{j+1,i+1}) + f_2(x_{j,i}x_{j+1,i}) + f_2(x_{j,i+1}x_{j+1,i+1}) = u_j^4$$

and

$$u_0^n = \sum_{i=1}^{n} f_2(x_{0,i}x_{0,i+1}), \; u_m^n = \sum_{i=1}^{n} f_2(x_{m,i}x_{m,i+1}).$$

The edge labelling f_2 is jump-magic. Since $w_j^4 + u_j^4 = 4|V(R_n^m)| + |E(R_n^m)| + 4$ for each $1 \leq i \leq n$, $0 \leq j \leq m - 1$ and $w_0^n + u_0^n = w_m^n + u_m^n = n[|V(R_n^m)| + 1]$ it follows that the labellings f_1 and f_2 are complementary.

Our previous results lead to the following theorem.

Theorem 3. *For $m \geq 1$, $n \geq 3$, $n \neq 4$ the graph of the m-prism R_n^m has a magic labelling of type* (1, 1, 0).

Proof. Label the vertices and the edges of R_n^m by f_1 and $|V(R_n^m)| + f_2$, respectively. The resulting labelling is a labelling of type (1, 1, 0) with labels from the set $\{1, 2, \ldots, |V(R_n^m)| + |E(R_n^m)|\}$ and the common weight for all 4-sided faces is $8|V(R_n^m)| + |E(R_n^m)| + 4$ and for both n-sided faces it is $n[2|V(R_n^m)| + 1]$.

REFERENCES

[1] LIH KO-WEI.: On magic and consecutive labelings of plane graphs. Utilitas Math. 24, 1983, 165—197.

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Katedra matematiky
Vysokej školy technickej
Švermova 9
042 00 Košice

О МАГИЧЕСКИХ РАЗМЕТКАХ m-ПРИЗМ

Martin Bača

Резюме

Пусть G — связный плоский граф с $|V(G)|$ вершинами и $|E(G)|$ ребрами.

Разметка типа (1, 1, 0) приписывает метки из множества $\{1, 2, \ldots, |V(G)| + |E(G)|\}$ вершинам и ребрам таким образом, что каждой вершине и ребру приписывается только одна метка, причем каждая метка используется только один раз.

Вес грани относительно данной разметки равен сумме меток, приписанных ее вершинам и ребрам.

Разметка называется магической, если все грани с одним и тем же числом сторон имеют один и тот же, зависящий от числа сторон, вес.

В работе построены магические разметки типа (1, 1, 0) для одного класса графов выпуклых многогранников.