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CAUCHY PROBLEM FOR SOME SEMILINEAR EVOLUTION EQUATIONS

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ABSTRACT. Well-posedness of the Cauchy problem in Sobolev spaces is studied for some semilinear evolution equations. For example, hyperbolic equations and Schrödinger equations are included in this framework. The maximum order of the derivatives in the nonlinear part which is specified is the one which yields well-posedness without any special additional assumptions, as the following counterexample shows.

In [1] I studied the Cauchy problem in H^∞ for an operator P of the form:

$$P = P_{pm}(\mathrm{D}_t, \mathrm{D}_x) + \sum_{j=r}^{m} a_j(t, x, \mathrm{D}_x)\, \mathrm{D}_t^{m-j} \tag{0.1}$$

with

$$P_{pm}(\mathrm{D}_t, \mathrm{D}_x) = \mathrm{D}_t^m + \sum_{j=1}^{m} \sum_{|\alpha|=pj} \mathring{a}_{\alpha j}\, \mathrm{D}_x^\alpha\, \mathrm{D}_t^{m-j}$$

and satisfying the following properties. Let r be the maximum multiplicity of the characteristic roots and assume that $P_{pm}(\tau, \xi)$ can be written in the form:

$$P_{pm}(\tau, \xi) = \prod_{j=1}^{r} \prod_{i=1}^{s_j} \left(\tau - \lambda_j^i(\xi)\right), \tag{0.2}$$

where λ_j^i are real-valued, $\lambda_j^i(\xi) \neq \lambda_k^h(\xi)$ if $i \neq h$ and $\xi \neq 0$, $\lambda_j^i(\xi) = \lambda_k^i(\xi)$ for some $\xi \neq 0$ and $s_r \geq s_{r-1} \geq \cdots \geq s_1$, $\sum\limits_{j=1}^{r} s_j = m$.

$$a_j(t, x, \mathrm{D}_x) = \sum_{|\alpha| \leq p(j-r)} a_{\alpha j}(t, x)\, \mathrm{D}_x^\alpha, \qquad \text{where} \quad a_{\alpha j} \in \mathcal{B}\big([-T, T]\,;\, \mathcal{B}^\infty(\mathbb{R}^n)\big). \tag{0.3}$$

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Remark. Especially (0.2) is satisfied if the characteristic roots are real with constant multiplicity, that is, $P_{pm}(\tau,\xi)$ can be written in the form:

$$P_{pm}(\tau,\xi)=\prod_{i=1}^{k}\left(\tau-\lambda^i(\xi)\right)^{r_i},$$

with $\sum_{i=1}^{k} r_i=m$, $r=r_1\geq\cdots\geq r_k$, $\lambda^i(\xi)\neq\lambda^h(\xi)$ if $i\neq h$ and $\xi\neq 0$.

The result I proved in [1] is the following:

THEOREM. *If P satisfies the assumptions* (0.1), (0.2), (0.3), *the initial data g_h are in H^∞ and $f\in\mathcal{C}\big([-T,T];H^\infty\big)$, then the Cauchy problem*

$$\begin{aligned} &Pu(t)=f(t)\,,\\ &\mathrm{D}_t^h\,u(0)=g_h\,,\qquad h=0,\ldots,m-1\,,\end{aligned}\tag{C}$$

has a solution $u(t,\cdot)\in H^\infty$ for all $t\in[-T,T]$. Moreover the following energy inequality holds for every $s\in\mathbb{N}$:

$$\|u(t,\cdot)\|_{H^s}\leq M(T)\left\{\sum_{h=0}^{m-1}\|g_h\|_{H^{s+p(m-1-h)}}+\left|\int_0^t\|f(\tau,\cdot)\|_{H^s}\,\mathrm{d}\tau\right|\right\}.$$

If $p=1$, the result above can be deduced from the literature about hyperbolic equations. As it is well-known, if the characteristic roots are not distinct, then the Cauchy problem is C^∞-well-posed only when, in general, some special conditions on the lower order terms hold. (See [4], [5], [7]). Such conditions are trivially satisfied if the maximum order of the lower order term is $m-r$.

As for non-kowalewskian equations, only results in the case $p=2$ (Schrödinger type equations) are available (see [2], for example). Well-posedness in H^∞ has been proved some time ago by T a k e u c h i if $p=2$, $r=1$ and an assumption is made on the subprincipal symbol P_{2m-1}. (See [9], [10], [11].)

Thus it seems that, in order to get H^∞-well-posedness, the highest order term which is allowed (after P_{pm}) is P_{pm-pr}. If this is not the case, we cannot expect H^∞-well-posedness to hold, in general.

The purpose of this paper is to study the Cauchy problem in H^∞ for some semilinear equations whose linearization is of the form (0.1), i.e.

$$P_{pm}(\mathrm{D}_t,\mathrm{D}_x)\big(u(t,x)\big)+f\Big(t,x,\big\{\partial_t^j\partial_x^\alpha u(t,x)\big\}_{\substack{j=0,\ldots,m-r\\|\alpha|\leq p(m-r-j)}}\Big)=0\,,\tag{0.4}$$

where P_{pm} is as above.

The main result is found in §3, where the hypotheses on f are fully detailed.

In order to prove this result a refinement of the energy estimates for the linear operator is needed. Thus §2 is devoted to resume the result obtained in [1] with a view to getting more precise estimates.

Finally §4 shows that derivatives of u of higher order are not allowed in f, in general. An example is given showing that H^∞-well-posedness may fail even in the linear case if the order of the lower order term exceeds $pm - pr$.

§1. Notation

We shall denote by S^m the class of the pseudo-differential operators $p(x, \mathrm{D}_x)$ whose symbol $p(x,\xi)$ satisfies the following condition:

$$\sup_{\alpha,\beta\in\mathbb{N}^n} \sup_{\substack{x,\xi\in\mathbb{R}^n \\ |\xi|\geq B}} \left|\partial_\xi^\alpha \mathrm{D}_x^\beta p(x,\xi)\right| \cdot \langle\xi\rangle^{|\alpha|-m} < \infty \qquad \text{for some} \quad B \geq 0 .$$

The pseudo-differential operator whose symbol is $\langle\xi\rangle$ will be denoted by $\Lambda(\mathrm{D}_x)$.

If $u \in H^s(\mathbb{R}^n)$, its Sobolev norm $\|\Lambda^s u\|_{L^2}$ will be denoted by $\|u\|_s$.

Let $M, p, s \in \mathbb{N}$ be fixed. If $u(t,x) \in \bigcap_{j=0}^{M} C^j\big([-T,T];\, H^{s+p(M-j)}(\mathbb{R}^n)\big) =:$ $\Xi_M^s\big([-T,T],\mathbb{R}^n\big)$, we shall use the following notation:

$$|||u(t)|||_{s,M}^2 = \sum_{j=0}^{M} \|\partial_t^j u(t,\cdot)\|_{s+p(M-j)}^2 \,. \tag{1.1}$$

§2. The linear equation

Consider a linear operator of the form:

$$P = P_{pm}(\mathrm{D}_t, \mathrm{D}_x) + \sum_{j=r}^{m} a_j(t,x,\mathrm{D}_x)\, \mathrm{D}_t^{m-j} \tag{2.1}$$

satisfying (0.2) and (0.3).

The following result is a refinement of [1; Proposition 6.2].

Theorem 2.1. *If P satisfies the assumptions* (2.1), (0.2), (0.3), *the initial data g_h are in H^∞ and $f \in C\big([-T,T];\, H^\infty\big)$, then the Cauchy problem*

$$\begin{aligned} &Pu(t) = f(t)\,, \\ &\mathrm{D}_t^h u(0) = g_h\,, \qquad h = 0,\ldots,m-1\,, \end{aligned} \tag{C}$$

has a solution $u(t,\cdot) \in H^\infty$ for all $t \in [-T,T]$. Moreover the following energy inequality holds for every $s \in \mathbb{N}$:

$$|||u(t)|||_{s,m-r} \le M(T)\left\{ |||u(0)|||_{s,m-1} + \left| \int_0^t \|f(\tau,\cdot)\|_s \, \mathrm{d}\tau \right| \right\}. \tag{2.2}$$

Remark 2.1. If $p = 1$ and $r = 1$, (2.2) recaptures the well-known energy inequality holding in the strictly hyperbolic case.

Let ∂_i denote $\mathrm{D}_t - \lambda_i(\mathrm{D}_x)$. If $J = (j_1,\dots,j_k)$, set $\{J\} = \{j_1,\dots,j_k\}$, $|J| = k$, $\partial_J = \partial_{j_1}\cdots\partial_{j_k}$.

LEMMA 2.1. *Assume that λ_j belong to S^p, $j = 1,\dots,s$, and there exists $\delta > 0$ such that*

$$|\lambda_j(\xi) - \lambda_i(\xi)| \ge \delta\langle\xi\rangle^p \qquad \text{for any} \quad i,j\,, \quad i \ne j\,.$$

Denote $\mathcal{I}_h = \{J = (j_1,\dots,j_h) : \ j_1 < \dots < j_h\,, \ \{J\} \subset \{1,\dots,s\}\}$ for $h = 1,\dots,s$.

Then for all $h = 0,\dots,s-1$

$$\mathrm{D}_t^{s-1-h} = \sum_{J\in\mathcal{I}_{s-1}} c_J^{(h)}(\mathrm{D}_x)\partial_J \qquad \text{for some} \quad c_J^{(h)} \in S^{-ph}\,.$$

Proof. See [1; Lemma 4.2]. □

Proof of Theorem 2.1. Arguing as in [1] we reduce our Cauchy problem to a Cauchy problem for a system with diagonal principal part. The unknown functions $\mathcal{U} = \{U_J\}_{|J|\le m-1}$ are defined as $U_0 = u$ and $U_J = \partial_J u$ if $0 < |J| \le m-1$. As we showed in [1], we are led to consider a system of the form:

$$\begin{aligned} \mathrm{D}_t\mathcal{U} - \mathcal{D}(\mathrm{D}_x)\mathcal{U} - \mathcal{B}(t,x,\mathrm{D}_x)\mathcal{U} &= \mathcal{F}(t,x)\,, \\ \mathcal{U}(t=0) &= G\,, \end{aligned}$$

where the entries of the diagonal matrix $\mathcal{D}$ are some λ_j, the entries of $\mathcal{B}$ belong to $\mathcal{B}([-T,T];\, S^0)$ and the initial values G of $\mathcal{U}$ are determined as follows:

$$\begin{aligned} &U_0(t=0) = g_0\,, \\ &U_J(t=0) = (-i)^{|J|} \sum_{\substack{k\le|J| \\ j_1,\dots,j_k\in\{J\} \\ j_1<\dots<j_k}} i^k\big(\lambda_{j_1}\circ\dots\circ\lambda_{j_k}\big)(\mathrm{D}_x)g_{|J|-k} \qquad \text{if} \quad 0 < |J| \le m-1\,. \end{aligned}$$

Thus, as in [1], we have a solution $\mathcal{U}$ such that:

$$\|\Lambda^s \mathcal{U}(t)\|_{L^2} \le C(T)\left(\|\Lambda^s G\|_{L^2} + \left|\int_0^t \|\Lambda^s \mathcal{F}(\tau,\cdot)\|_{L^2}\,\mathrm{d}\tau\right|\right),$$

which yields:

$$\sum_{|J|\le m-1} \|U_J(t)\|_s \le C'(T)\left\{\sum_{j=0}^{m-1} \|g_j\|_{s+p(m-1-j)} + \left|\int_0^t \|f(\tau,\cdot)\|_s\,\mathrm{d}\tau\right|\right\}. \tag{2.3}$$

Now observe that we can write

$$\|\partial_t^j u(t,\cdot)\|_{s+p(m-r-j)} \le \sum_{|J|\le m-1} c_J \|U_J(t)\|_s \tag{2.4}$$

for some positive constants c_J. Indeed, letting $h = m - r - j$, we can write $\left\|\partial_t^j u(t,\cdot)\right\|_{s+p(m-r-j)} = \left\|\partial_t^{\left(\sum_{k=1}^{r}(s_k-1)\right)-h} u(t,\cdot)\right\|_{s+ph}$ with s_k as in (0.2). Distributing h among the terms $s_k - 1$ so that $s_k - 1 - h_k \ge 0$ and $\sum_{k=1}^{r} h_k = h$, and applying Lemma 2.1 to each $\partial_t^{s_k-1-h_k}$, we get (2.4) immediately. Then, combining (2.3) with (2.4), we get (2.2). □

Remark 2.2. Actually (2.2) is true if we only assume $a_{\alpha j} \in \mathcal{B}([-T,T];\, \mathcal{B}^s(\mathbb{R}^n))$.

§3. The nonlinear equation

Consider the following differential equation:

$$P_{pm}(\mathrm{D}_t,\mathrm{D}_x)u(t,x) + f\left(t,x,\{\partial_t^j\partial_x^\alpha u(t,x)\}_{\substack{j=0,\dots,m-r\\ |\alpha|\le p(m-r-j)}}\right) = 0\,, \tag{3.1}$$

where $P_{pm}(\mathrm{D}_t,\mathrm{D}_x)$ satisfies (0.2).

Remark 3.1. If all the characteristic roots coincide, then $r = m$, that is f is allowed to depend only on u and not on its derivatives of any order.

Let us write v_h for $\partial_t^{j_h}\partial_x^{\alpha_h} u$ and $f(t,x,v_1(t,x),\dots,v_\ell(t,x))$ for $f\left(t,x,\{\partial_t^{j_h}\partial_x^{\alpha_h}u(t,x)\}_{h=1,\dots,\ell}\right) = f\left(t,x,\{\partial_t^j\partial_x^\alpha u(t,x)\}_{\substack{j=0,\dots,m-r\\ |\alpha|\le p(m-r-j)}}\right)$.

We assume that the function f satisfies the following hypothesis:

$$\begin{aligned} &f(t,x,v_1,\dots,v_\ell) \in \mathcal{C}([-T,T];\, \mathcal{C}^\infty(\mathbb{R}^n\times\mathbb{C}^\ell))\,,\\ &\quad(\forall\, t\in[-T,T])\,(\forall\, v\in\mathbb{C}^\ell)\,(x\mapsto f(t,x,v_1,\dots,v_\ell)\in\mathcal{B}^\infty(\mathbb{R}^n))\,,\\ &\qquad(\forall\, t\in[-T,T])\,(x\mapsto f(t,x,0,\dots,0)\in H^s(\mathbb{R}^n))\,. \end{aligned} \tag{3.2}$$

Lemma 3.1. *Let* $f(x, v_1, \dots, v_\ell) \in \mathcal{C}^\infty(\mathbb{R}^n \times \mathbb{C}^\ell)$ *with bounded derivatives of all order in* x *for fixed* $v_1, \dots, v_\ell$. *Assume that* $v_h \in H^s(\mathbb{R}^n)$, $h = 1, \dots, \ell$, *for an integer* $s \geq \left[\frac{n}{2}\right] + 2$. *Let* R *be such that* $\max_{h=1,\dots,\ell} \|v_h\|_{[\frac{n}{2}]+1} \leq R$ *and let* $U_R = \{w \in \mathbb{C}^\ell : |w_h| \leq c_n R,\ h = 1, \dots, \ell\}$, *being* c_n *the constant of Sobolev embedding. Then*

$$\left\|f\big(x, v_1(x), \dots, v_\ell(x)\big)\right\|_s \leq$$
$$\leq C_s M_{s,R}\left\{1 + \sum_{h=1}^{\ell} \|v_h\|_s \left(1 + \sum_{\substack{\sum \gamma_i \leq s \\ \gamma_h \geq 1}} \|v_h\|_{s-1}^{\gamma_h - 1} \prod_{i \neq h} \|v_i\|_{s-1}^{\gamma_i}\right)\right\}.$$

Here $M_{s,R}$ *denotes* $\sup_{\substack{x \in \mathbb{R}^n \\ v \in U_R}} \max_{|\beta| + \sum_{h=1}^{\ell} \gamma_h \leq s} \left|\partial_x^\beta \partial_{v_1}^{\gamma_1} \dots \partial_{v_\ell}^{\gamma_\ell} f(x, v_1, \dots, v_\ell)\right|$.

Proof. It is a slight modification of similar Lemmata in [5; Chapt. V]. □

Lemma 3.2. *Let* f *satisfy* (3.1), (3.2) *and let* $u \in \Xi^s_{m-r}([-T,T], \mathbb{R}^n)$ *for an integer* $s \geq \left[\frac{n}{2}\right] + 2$. *Let* R *be such that* $\sup_{t \in [-T,T]} |||u(t, \cdot)|||_{[\frac{n}{2}]+1, m-r} \leq R$. *Then*

$$\left\|f\left(t, x, \{\partial_t^j \partial_x^\alpha u(t,x)\}_{\substack{j=0,\dots,m-r \\ |\alpha| \leq p(m-r-j)}}\right)\right\|_s$$
$$\leq \tilde{C}_s M_{s,R}\left\{1 + |||u(t)|||_{s,m-r}\left(1 + |||u(t)|||_{s-1,m-r}^{s-1}\right)\right\}.$$

Proof. It follows easily from Lemma 3.1. □

Let us now turn to the equation (3.1). Combining Lemma 3.2. with (2.2) we obtain

$$|||u(t)|||_{s,m-r} \leq C_R^*\Big\{|||u(0)|||_{s,m-1} + M_{s,R}^*|t| \cdot$$
$$\cdot \left(1 + \sup_\tau |||u(\tau)|||_{s,m-r}\left(1 + \sup_\tau |||u(\tau)|||_{s-1,m-r}^{s-1}\right)\right)\Big\}. \tag{3.3}$$

Thus we can finally prove the main result.

Theorem 3.1. *Assume that* (3.1), (3.2) *hold. Then any Cauchy problem for* (3.1) *with initial data at* $t = 0$ *assigned in* $H^\infty(\mathbb{R}^n)$, *has a local (in time) solution* $u(t, \cdot) \in H^\infty(\mathbb{R}^n)$.

Remark 3.2. Actually (2.2) implies H^s-well-posedness for any s in the linear problem, and (3.3) yields H^s-well-posedness for any sufficiently large s in the semilinear problem.

§4. The example

Consider an operator with constant coefficients of the following form:

$$P = P_{pm} + Q\,, \tag{4.1}$$

where

$$P_{pm}(\mathrm{D}_x, \mathrm{D}_t) = \mathrm{D}_t^m + \sum_{j=1}^{m} \sum_{|\alpha|=pj} \mathring{a}_{\alpha j}\, \mathrm{D}_x^\alpha\, \mathrm{D}_t^{m-j}$$
$$= \prod_{j=1}^{k} (\mathrm{D}_t - \lambda_j(\mathrm{D}_x))^{r_j}\,,$$

with $\lambda_j(\xi)$ real and distinct when $\xi \neq 0$, and $Q(\mathrm{D}_x) = \sum_{|\alpha|=pm-q} a_{\alpha j}\, \mathrm{D}_x^\alpha$ for some q, $pr/2 < q < pr$, where $r = \max\{r_j : j = 1, \ldots, k\}$. Suppose $r_1 = r$.

By applying Fourier transform to $Pu(t) = 0$, we get:

$$\mathrm{e}^{-\mathrm{i}t\lambda_1(\xi)}\, P(\xi, \mathrm{D}_t)\tilde{u}(t,\xi) = \{\Pi^*(\xi, \lambda_1(\xi)) + R(\xi, \mathrm{D}_t)\}\, \mathrm{D}_t^r\, v(t,\xi) + Q(\xi) v(t,\xi) = 0\,, \tag{4.2}$$

where $v(t,\xi) = \mathrm{e}^{-\mathrm{i}t\lambda_1(\xi)}\,\tilde{u}(t,\xi)$, $\Pi^*(\xi,\tau) = \prod_{j\neq 1} (\tau - \lambda_j(\xi))^{r_j}$ and $R(\xi, \mathrm{D}_t) = \prod_{j=2}^{k} \Big(\sum_{\substack{h_j \le r_j \\ (h_2,\ldots,h_k)\neq(0,\ldots,0)}} \binom{r_j}{h_j} (\lambda_1(\xi) - \lambda_j(\xi))^{r_j - h_j}\, \mathrm{D}_t^{h_j} \Big)$.

Dividing (4.2) by $\Pi^*(\xi, \lambda_1(\xi))$ and letting $q(\xi) = \frac{Q(\xi)}{\Pi^*(\xi,\lambda_1(\xi))}$, $r(\xi, \mathrm{D}_t) = \frac{R(\xi,\mathrm{D}_t)}{\Pi^*(\xi,\lambda_1(\xi))}$, we have:

$$\mathrm{D}_t^r\, v + q(\xi) v = -r(\xi, \mathrm{D}_t)\, \mathrm{D}_t^r\, v\,. \tag{4.3}$$

In what follows we shall assume that $q(\xi)$ is such that:

$$(\exists\, \xi_0 \in \mathbb{R}^n)\,(|\xi_0| = 1 \quad \& \quad \mathrm{i}^r q(\xi_0) < 0)\,. \tag{4.4}$$

Let $\tau_i(\xi)$, $i = 1, \ldots, r$, denote the roots of $\tau^r + \mathrm{i}^r q(\xi) = 0$ and let τ_r be such that $\tau_r(\xi_0) = \sqrt[r]{|q(\xi_0)|}$. Then there exists a conic neighbourhood Γ of ξ_0 in which:

$$\mathcal{R}\tau_r(\xi) = \max_{1\le j\le k} \mathcal{R}\tau_j(\xi) \ge \delta' |\xi|^{p-q/r} \qquad \text{for some} \quad \delta' > 0\,; \tag{4.5}$$

$$(i \neq j) \implies |\tau_i(\xi) - \tau_j(\xi)| \ge \delta |\xi|^{p-q/r} \qquad \text{for some} \quad \delta > 0\,. \tag{4.6}$$

Given a "large" positive integer N, take a continuous not identically vanishing function g with $\mathrm{supp}(g) \subset B(N\xi_0, \varepsilon) \Subset \Gamma$ and let $v_0(t,\xi) = g(\xi)\, \mathrm{e}^{t\tau_r(\xi)}$. By the method of successive approximations we can construct a solution $v(t,\xi)$ of the Cauchy problem (4.3) with initial data $\partial_t^j v(0,\xi) = \partial_t^j v_0(0,\xi)$ such that:

$$|\partial_t^j v(t,\xi)| \leq \left(A|\xi|^{p-q/r}\right)^j |g(\xi)| \cdot \exp\left(c|t||\xi|^{p-q/r}\right) \qquad \text{for some} \quad A, c > 0\,; \tag{4.7}$$

$$|v(t,\xi)| \geq \frac{1}{2}|v_0(t,\xi)| \tag{4.8}$$

for large $|\xi|$. Indeed, if we set $v(t,\xi) = \sum_{k=0}^{\infty} v_k(t,\xi)$, where v_k, $k = 1, \ldots$, is the solution of the Cauchy problem:

$$\begin{aligned} \mathrm{D}_t^r(t,\xi) + q(\xi)v_k(t,\xi) &= -r(\xi,\mathrm{D}_t)\,\mathrm{D}_t^r\, v_{k-1}(t,\xi)\,, \\ \partial_t^j v_k(t,\xi) &= 0\,, \qquad 0 \leq j \leq r-1\,, \end{aligned}$$

then, in view of (4.5) and (4.6), we have:

$$|\partial_t^j v_k(t,\xi)| \leq 2^{-k-1}\left(A|\xi|^{p-q/r}\right)^j |v_0(t,\xi)|$$

for sufficiently large $|\xi|$. Then (4.7) and (4.8) hold.

Now $\tilde{u}(t,\xi)$ is a solution of

$$\begin{aligned} &P(\xi,\mathrm{D}_t)\tilde{u}(t,\xi) = 0\,, && t \in [0,T]\,, \\ &\partial_t^j \tilde{u}(0,\xi) = \sum_{h=0}^{j} \left(\mathrm{i}\,\lambda_1(\xi)\right)^h \partial_t^{j-h} v(0,\xi)\,, && 0 \leq j \leq m-1\,. \end{aligned} \tag{4.9}$$

Let us assume that any forward Cauchy problem for P is well-posed in H^∞. Then there exists $s \in \mathbb{N}$ such that:

$$\|\tilde{u}(t,\xi)\|_{L^2(\mathbb{R}^n)} \leq M_s(T) \sum_{j=0}^{m-1} \||\xi|^s \partial_t^j \tilde{u}(0,\xi)\|_{L^2(\mathbb{R}^n)} \leq \tilde{M}_s(T) N^{s+p(m-1)} \|g\|_{L^2(\mathbb{R}^n)} \tag{4.10}$$

for some $M_s(T)$, $\tilde{M}_s(T) > 0$. On the other hand,

$$\|\tilde{u}(t,\xi)\|_{L^2(\mathbb{R}^n)} \geq \frac{1}{2}\,\mathrm{e}^{\delta' t N^{p-q/r}}\, \|g\|_{L^2(\mathbb{R}^n)} \tag{4.11}$$

in view of (4.8). However, (4.10) and (4.11) are incompatible if N is large enough.

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