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ON TWO STRENGHTENINGS OF REGULARITY OF MEASURES

ZDENA RIEČANOVÁ

The aim of this paper is to generalize the approximation property, which in the theory of Baire and Borel measures is called completion regularity ([1] p. 230). In an abstract set we define the approximation property (α) (of two σ-rings) in terms of small systems (introduced in [8] and studied also in [3], [4], [6], [9], [10]). We show some equivalent properties (Theorem 1). Applications of Theorem 1 for some types of set functions are contained in corollaries 1, 2, 3 and for Borel measures in Theorem 2. In Section 3 we study the existence of unique extension and the validity of property (α) for some types of set functions defined on σ-rings.

1

Let X be an abstract set. Let $\boldsymbol{D}$, $\boldsymbol{V}$, $\boldsymbol{Z}$, $\boldsymbol{S}$ be systems of subsets of X, $\{\mathcal{N}_n\}_{n=1}^{\infty}$ a sequence of subsystems of the system $\boldsymbol{S}$ fulfilling the following conditions:

(i) $\boldsymbol{Z}$ is a σ-ring, $\boldsymbol{D}\subset\boldsymbol{Z}$ and $\boldsymbol{V}\subset\boldsymbol{Z}$.

(ii) $\boldsymbol{S}$ is a σ-ring such that $\boldsymbol{Z}\subset\boldsymbol{S}$ and if $E\in\boldsymbol{S}$, then there is $F\in\boldsymbol{Z}$ such that $E\subset F$.

(iii) For any positive integer n, if $E\in\mathcal{N}_n$, $F\subset E$, $F\in\boldsymbol{S}$, then $F\in\mathcal{N}_n$.

(iv) For any positive integer n there are positive integers m, k such that $M\in\mathcal{N}_m$, $K\in\mathcal{N}_k$ implies $M\cup K\in\mathcal{N}_n$.

(v) If $E\in\boldsymbol{Z}$ and n is any positive integer, then there are $D\in\boldsymbol{D}$, $V\in\boldsymbol{V}$ such that $D\subset E\subset V$, $E-D\in\mathcal{N}_n$, $V-E\in\mathcal{N}_n$.

Remark 1. From the condition (iv) the following property follows:

(vi) If $A, B\in\bigcap_{n=1}^{\infty}\mathcal{N}_n$, then $A\cup B\in\bigcap_{n=1}^{\infty}\mathcal{N}_n$.

Theorem 1. *Let* $\boldsymbol{D}$, $\boldsymbol{V}$, $\boldsymbol{Z}$, $\boldsymbol{S}$, $\{\mathcal{N}_n\}_{n=1}^{\infty}$ *have the properties (i)—(v). The following conditions are equivalent:*

(α) *If* $E\in\boldsymbol{S}$, *then there exist sets* $G, F\in\boldsymbol{Z}$ *such that* $G\subset E\subset F$ *and* $F-G\in\bigcap_{n=1}^{\infty}\mathcal{N}_n$.

(β) *If* $E\in\boldsymbol{S}$ *and n is any positive integer, then there is* $D\in\boldsymbol{D}$ *such that* $D\subset E$ *and* $E-D\in\mathcal{N}_n$.

(γ) *If* $E\in\boldsymbol{S}$ *and n is any positive integer, then there is* $V\in\boldsymbol{V}$ *such that* $E\subset V$ *and* $V-E\in\mathcal{N}_n$.

(δ) *If $E \in \boldsymbol{S}$ and n is any positive integer, then there are $D \in \boldsymbol{D}$, $V \in \boldsymbol{V}$ such that $D \subset E \subset V$ and $V - D \in \mathcal{N}_n$.*

Proof. $(\alpha) \Rightarrow (\beta)$, (γ), (δ)

Suppose that $E \in \boldsymbol{S}$ is any set. There exist F, $G \in \boldsymbol{Z}$ such that $G \subset E \subset F$ and $F - G \in \bigcap_{n=1}^{\infty}$. By the property (iii) $E - G, F - E \in \mathcal{N}_k$ holds for every positive integer k. Let n be an arbitrary positive integer. Choose m, k making use of (iv). By the property (v) there are sets $D \in \boldsymbol{D}$, $V \in \boldsymbol{V}$ such that $D \subset G$, $F \subset V$, $G - D \in \mathcal{N}_m$, $V - F \in \mathcal{N}_m$ and hence

$$E - D = (E - G) \cup (G - D) \in \mathcal{N}_n,$$
$$V - E = (V - F) \cup (F - E) \in \mathcal{N}_n.$$

Now choose sets $D_1 \in \boldsymbol{D}$, $V_1 \in \boldsymbol{V}$ such that $D_1 \subset E \subset V_1$ and $V_1 - E \in \mathcal{N}_m$, $E - D_1 \in \mathcal{N}_k$. Then $V_1 - D_1 = (V_1 - E) \cup (E - D_1) \in \mathcal{N}_n$.

$(\beta) \Rightarrow (\alpha)$

Choose any $E \in \boldsymbol{S}$. By (ii) there exists $H \in \boldsymbol{Z}$ such that $E \subset H$. For every positive integer n there are $D_n \in \boldsymbol{D}$ such that $D_n \subset H - E$ and $(H - E) - D_n \in \mathcal{N}_n$. Hence by (iii) $(H - E) - \bigcup_{k=1}^{\infty} D_k \in \mathcal{N}_n$, too. Denote $F = H - \bigcup_{k=1}^{\infty} D_k$. Evidently $E \subset F$ and

$$F - E = \left(H - \bigcup_{k=1}^{\infty} D_k\right) - E \in \bigcap_{n=1}^{\infty} \mathcal{N}_n.$$

Similarly for every positive integer n there are $C_n \in \boldsymbol{D}$ such that $C_n \subset E$ and $E - C_n \in \mathcal{N}_n$. Denote $G = \bigcup_{k=1}^{\infty} C_k$. Then $G \subset E$ and $E - G \in \bigcap_{n=1}^{\infty} \mathcal{N}_n$.

Evidently G, $F \in \boldsymbol{Z}$ and by (vi) $F - G = (F - E) \cup (E - G) \in \bigcap_{n=1}^{\infty} \mathcal{N}_n$.

$(\gamma) \Rightarrow (\alpha)$

Choose any $E \in \boldsymbol{S}$. By (ii) there exists $H \in \boldsymbol{Z}$ such that $E \subset H$. For every positive integer n there are $V_n \in \boldsymbol{V}$ such that $H - E \subset V_n$ and $V_n - (H - E) \in \mathcal{N}_n$. Denote $G = \bigcup_{i=1}^{\infty} (H - V_i)$. Then $G \subset E$, $G \in \boldsymbol{Z}$ and

$$E - G = E - \bigcup_{i=1}^{\infty} (H - V_i) \subset E - (H - V_n) \subset V_n - (H - E)$$

for every n. By (iii) $E - G \in \bigcap_{n=1}^{\infty} \mathcal{N}_n$.

Similarly for every positive integer n there are $U_n \in \boldsymbol{V}$ such that $E \subset U_n$ and $U_n - E \in \mathcal{N}_n$. Denote $F = \bigcap_{i=1}^{\infty} U_i$. Then $E \subset F$, $F \in \boldsymbol{Z}$ and $F - E = \bigcap_{i=1}^{\infty} U_i - E \subset U_n - E$ for every n. Hence $F - E \in \bigcap_{n=1}^{\infty} \mathcal{N}_n$.

By the property (vi) $F-G=(F-E)\cup(E-G)\in\bigcap_{n=1}^{\infty}\mathcal{N}_n$.

$(\delta)\Rightarrow(\beta)$, (γ)

Choose any $E\in\boldsymbol{S}$ and let n be an arbitrary positive integer. There are $C\in\boldsymbol{D}$, $U\in\boldsymbol{V}$ such that $C\subset E\subset U$ and $U-C\in\mathcal{N}_n$. Hence by (iii) $U-E, E-C\in\mathcal{N}_n$, too.

2

Let X be an abstract set. Let $\boldsymbol{D}$, $\boldsymbol{V}$, $\boldsymbol{Z}$, $\boldsymbol{S}$ be systems of subsets of the set X fulfilling the conditions (i) and (ii) from Section 1. We shall study the following types of set functions $\mu\colon \boldsymbol{S}\to\langle 0,\infty\rangle$:

(1) μ is positive, monotone, subadditive and $\mu(\emptyset)=0$ (we shall call it a submeasure).

(2) μ is positive, additive and $\mu(\emptyset)=0$ (we shall call it a finitely additive measure).

(3) μ is positive, monotone, subadditive, continuous from below and $\mu(\emptyset)=0$ (we shall call it a continuous from below submeasure).

(4) μ is positive, countably additive and $\mu(\emptyset)=0$ (we shall call it a countably additive measure).

The sequence $\{\mathcal{N}_n\}_{n=1}^{\infty}$ of subsystems of $\boldsymbol{S}$ defined by

$$\mathcal{N}_n=\left\{E\in\boldsymbol{S}\colon \mu(E)<\frac{1}{n}\right\}$$

(where $\mu\colon \boldsymbol{S}\to\langle 0,\infty\rangle$ is an arbitrary set function of one of the types (1)—(4)) satisfies the properties (iii) and (iv) from Section 1. Hence Theorem 1 from Section 1 has the following corollaries:

Corollary 1. *Let $\boldsymbol{D}$, $\boldsymbol{V}$, $\boldsymbol{Z}$, $\boldsymbol{S}$ have the properties (i) and (ii). Let $\mu\colon \boldsymbol{S}\to\langle 0,\infty\rangle$ be an arbitrary set function of one of the types (1)—(4) listed above. Let there hold for every set $E\in\boldsymbol{Z}$*

$$\inf\{\mu(E-D)\colon E\supset D\in\boldsymbol{D}\}=\inf\{\mu(V-E)\colon E\subset V\in\boldsymbol{V}\}=0.$$

Then the following conditions are equivalent:

(a_1) *If $E\in\boldsymbol{S}$, then there exist sets $G, F\in\boldsymbol{Z}$ such that $G\subset E\subset F$ and $\mu(F-G)=0$.*

(b_1) $\inf\{\mu(E-D)\colon E\supset D\in\boldsymbol{D}\}=0$ *for all $E\in\boldsymbol{S}$.*

(c_1) $\inf\{\mu(V-E)\colon E\subset V\in\boldsymbol{V}\}=0$ *for all $E\in\boldsymbol{S}$.*

(d_1) $\inf\{\mu(V-C)\colon C\subset E\subset V,\ C\in\boldsymbol{D},\ V\in\boldsymbol{V}\}=0$ *for all $E\in\boldsymbol{S}$.*

Corollary 2. *Let $\boldsymbol{D}$, $\boldsymbol{V}$, $\boldsymbol{Z}$, $\boldsymbol{S}$ have the properties (i) and (ii). Let $\mu\colon \boldsymbol{S}\to\langle 0,\infty)$ be a finitely additive, resp. countably additive measure. Let there hold for every $E\in\boldsymbol{Z}$*

$$\mu(E)=\inf\{\mu(V)\colon E\subset V\in\boldsymbol{V}\}=\sup\{\mu(D)\colon E\supset D\in\boldsymbol{D}\}.$$

Then the following conditions are equivalent:

(a_2) *If $E \in \boldsymbol{S}$, then there exist sets $G, F \in \boldsymbol{Z}$ such that $G \subset E \subset F$ and $\mu(F-G)=0$.*

(b_2) $\mu(E)=\sup\{\mu(D): E \supset D \in \boldsymbol{D}\}$ *for all* $E \in \boldsymbol{S}$.

(c_2) $\mu(E)=\inf\{\mu(V): E \subset V \in \boldsymbol{V}\}$ *for all* $E \in \boldsymbol{S}$.

(d_2) $\inf\{\mu(V-C): C \subset E \subset V, C \in \boldsymbol{D}, V \in \boldsymbol{V}\}=0$ *for all* $E \in \boldsymbol{S}$.

Remark 2. The version of Corollary 2 in the case where μ is a submeasure or a continuous from below submeasure on $\boldsymbol{S}$ is false. See the following example.

Example 1. Let card $X=\aleph_1$. Let $x_0 \in X$, $Y=X-\{x_0\}$. We equip Y with the discrete topology and let the topology for X be the one point compactification of Y. Let $\boldsymbol{D}$ be the class of all compact G_δ's in X, $\boldsymbol{Z}$ be the class of all Baire sets, $\boldsymbol{S}$ be the class of all Borel sets and $\boldsymbol{V}$ be the class of all open Baire sets.

Evidently $\boldsymbol{Z}$ consists of all finite or countable subsets of Y and all complements in X of finite or countable subsets of Y. $\boldsymbol{S}$ consists of all subsets of X (because if $E \subset X$, then $E \cup \{x_0\}$ is a compact set). The set $\{x_0\}$ is not Baire.

Define $\mu(E)$ to be 1 if $x_0 \in E$ or if $x_0 \notin E$ and E is not a finite or a countable set and define $\mu(E)$ to be 0 in the other cases. Then μ is a continuous from below submeasure on $\boldsymbol{S}$ and

$$\mu(E)=\inf\{\mu(V): E \subset V \in \boldsymbol{V}\}$$

for all $E \in \boldsymbol{S}$. Hence μ has the property (c_2). But it has not the properties (a_2), (b_2) and (d_2), because $\mu(\{x_0\})=1$ and if $F \in \boldsymbol{Z}$, $F \subset \{x_0\}$, then $F=\emptyset$. (Note that the restriction of μ to $\boldsymbol{Z}$ is a Baire measure.)

Remark 3. If $\mu: \boldsymbol{S} \to \langle 0, \infty \rangle$ is an arbitrary set function of one of the types (1)—(4) listed above and for every set $E \in \boldsymbol{Z}$ holds

$$\inf\{\mu(E-D): E \supset D \in \boldsymbol{D}\}=\inf\{\mu(V-E): E \subset V \in \boldsymbol{V}\}=0,$$

then (a_2) implies (b_2), (c_2) and (d_2).

Corollary 3. *Let $\boldsymbol{D}$, $\boldsymbol{V}$, $\boldsymbol{Z}$, $\boldsymbol{S}$ have the properties (i) and (ii). Let $\mu: \boldsymbol{S} \to \langle 0, \infty \rangle$ be a σ-finite countably additive measure on $\boldsymbol{S}$. Let there hold for every set $E \in \boldsymbol{Z}$*

$$\inf\{\mu(E-D): E \supset D \in \boldsymbol{D}\}=\inf\{\mu(V-E): E \subset V \in \boldsymbol{V}\}=0,$$

then the conditions (a_2), (b_2), (c_2), (d_2) are equivalent.

Proof.

(a_2) $\Rightarrow$ (b_2), (c_2), (d_2) (see Remark 3)

(d_2) $\Rightarrow$ (a_2) (see Corollary 1)

(b_2) $\Rightarrow$ (a_2), (c_2) $\Rightarrow$ (a_2)

Choose any set $E \in \boldsymbol{S}$. There are sets $E_n \in \boldsymbol{S}$ ($n=1, 2, \ldots$) such that $E=\bigcup_{n=1}^{\infty} E_n$ and $\mu(E_n)<\infty$ for all n. Given $\varepsilon>0$. By (b_2) there are $D_n \in \boldsymbol{D}$, $D_n \subset E_n$ such that $\mu(E_n-D_n) < \frac{\varepsilon}{2^{n+1}}$. Put $G=\bigcup_{n=1}^{\infty} D_n$. Then $G \subset E$, $G \in \boldsymbol{Z}$ and $\mu(E-G)<\frac{\varepsilon}{2}$.

Choose $F \in \boldsymbol{Z}$ by (ii) so that $E \subset F$. Now, by (b_2) there is $D \in \boldsymbol{D}$ such that $D \subset F-E$ and $\mu[(F-E)-D]<\frac{\varepsilon}{2}$. Clearly $E \subset F-D$, $F-D \in \boldsymbol{Z}$ and $\mu[(F-D)-E]<\frac{\varepsilon}{2}$. Summarizing $G \subset E \subset F-D$, G, $F-D \in \boldsymbol{Z}$ and $\mu[(F-D)-G]<\varepsilon$. From this (a_2) easily follows.

Suppose now (by (c_2)) that $V_n \in \boldsymbol{V}$ such that $E_n \subset V_n$ and $\mu(V_n-E_n)<\frac{\varepsilon}{2^{n+1}}$ for all n. Put $V=\bigcup_{n=1}^{\infty} V_n$. Then $E \subset V$, $V \in \boldsymbol{Z}$ and $\mu(V-E)<\frac{\varepsilon}{2}$. Clearly there is (by ($c_2$)) $U \in \boldsymbol{V}$ such that $V-E \subset U$ and $\mu[U-(V-E)]<\frac{\varepsilon}{2}$. Hence $V-U \subset E$, $V-U \in \boldsymbol{Z}$ and $\mu[E-(V-U)]<\frac{\varepsilon}{2}$. Summarizing $V-U \subset E \subset V$, $V-U$, $V \in \boldsymbol{Z}$ and $\mu[V-(V-U)]<\varepsilon$. From this (a_2) easily follows.

Remark 4. In the case of X being a locally compact Hausdorff topological space, $\boldsymbol{D}$ the class of all compact G_δ's, $\boldsymbol{V}$ the class of all open Baire sets, $\boldsymbol{Z}$ the class of all Baire sets and $\boldsymbol{S}$ the class of all Borel sets in X, we get the following application of Corollary 3 for Baire and Borel measures (in the terminology of [1]).

Theorem 2. *If μ is a Borel measure on a locally compact Hausdorff topological space X, the following conditions are equivalent:*

(a) *μ is completion regular.*

(b) *$\mu(E)=\inf\{\mu(V): E \subset V$, V is an open Baire set$\}$ for all Borel sets E.*

(c) *$\mu(E)=\sup\{\mu(D): E \supset D$, D is a compact G_δ set$\}$ for all Borel sets E.*

(d) *$\inf\{\mu(V-D): D \subset E \subset V$, D is a compact G_δ, V is an open Baire set$\}=0$ for all Borel sets E.*

3

Let $\mu: \boldsymbol{Z} \to \langle 0, \infty\rangle$ be an arbitrary set function of one of the types (1)—)(4) listed above. The approximation property (a_1), resp. (a_2), implies that the set function μ has at most one extension to a set function of the same type defined on $\boldsymbol{S}$. (Indeed,

$$\mu(F) \leqq \mu(G)+\mu(F-G)=\mu(G)=\varkappa(G) \leqq \varkappa(E) \leqq \varkappa(F)=\mu(F)$$

for all $\varkappa$ extending μ.)

The converse of this proposition is false e.g. in the case of μ being a countably additive measure (see [11] and also [2], [5], [7]).

Of course, in the case when $\boldsymbol{Z}$ and $\boldsymbol{S}$ are σ-algebras and μ is a finite, finitely additive measure on $\boldsymbol{Z}$, the approximation property (a_1) is equivalent to the existence of a unique extension μ to $\boldsymbol{S}$. (This assertion follows from Theorem 2 of [7].)

Proposition 1. *Every finite continuous from below submeasure on* $\boldsymbol{Z}$ *satisfies the approximation property* (a_1) *iff* $\boldsymbol{Z}=\boldsymbol{S}$.

Proof. In the case when $\boldsymbol{Z}\neq\boldsymbol{S}$ there is a set $C\in\boldsymbol{S}$ such that $C\notin\boldsymbol{Z}$. We define the continuous from below submeasure v on $\boldsymbol{Z}$ in the following way: $v(E)=1$ if $E-C\neq\emptyset$ and $v(E)=0$ if $E\subset C$. Evidently, for v the approximation property (a_1) is not true. (Because if F, $G\in\boldsymbol{Z}$, $G\subset C\subset F$, then $v(F-G)=1$.)

Proposition 2. Every *finite continuous from below submeasure on* $\boldsymbol{Z}$ *has at most one extension to a continuous from below submeasure on* $\boldsymbol{S}$ *iff* $\boldsymbol{Z}=\boldsymbol{S}$.

Proof. In the case when $\boldsymbol{Z}\neq\boldsymbol{S}$ there is $C\in\boldsymbol{S}$ such that $C\notin\boldsymbol{Z}$. Then the set functions μ_1, μ_2 defined on $\boldsymbol{S}$ by

$\mu_1(E)=1$ or 0 according to $E-C\neq\emptyset$ or $E\subset C$

$\mu_2(E)=0$ or 1 according to whether there is $F\in\boldsymbol{Z}$ such that $E\subset F\subset C$ or not

have the same restriction on $\boldsymbol{Z}$. But $0=\mu_1(C)\neq\mu_2(C)=1$.

The versions of propositions 1 and 2 in the case of a countably additive set function are false.

Remark 5. It may happen that every finite, countably additive measure on $\boldsymbol{Z}$ has at most one extension to a countably additive measure on $\boldsymbol{S}$, but $\boldsymbol{Z}=S$ need not hold. (See the following example and also [2] and [5].)

Remark 6. It may happen that every finite, countably additive measure on $\boldsymbol{S}$ satisfies the approximation property (a_1), but $\boldsymbol{Z}=\boldsymbol{S}$ need not hold. (See the following example.)

Example. Let card $X=\aleph_1$. Let $\boldsymbol{Z}$ consist of all finite or countable subsets of X and of all complements in X of finite or countable subsets of X. Let $\boldsymbol{S}$ consist of all subsets of X. Evidently $\boldsymbol{Z}$ and $\boldsymbol{S}$ have the properties (i) and (ii), $\boldsymbol{Z}\neq\boldsymbol{S}$ and $X\in\boldsymbol{Z}$. Let μ be an arbitrary finite, countably additive measure on $\boldsymbol{S}$. The set $F=\{x\in X: \mu(\{x\})>0\}$ is finite or countable, hence $F\in\boldsymbol{Z}$ and $X-F\in\boldsymbol{Z}$. By the Ulam Theorem ([12], Theorem 2, p. 141) there holds $\mu(X-F)=0$. If $E\in\boldsymbol{S}$, then $F\cap E\subset E\subset(E\cap F)\cup(X-F)$, where $F\cap E$, $(F\cap E)\cup(X-F)\in\boldsymbol{Z}$ and $\mu([(E\cap F)\cup(X-F)]-(E\cap F))=\mu(X-F)=0$. Hence μ has the property (a_1).

Let v be an arbitrary finite, countably additive measure on $\boldsymbol{Z}$ and let it have an extension to a countably additive set fuction $\varkappa$ on $\boldsymbol{S}$. Then $\varkappa$ is finite, hence $\varkappa$ has the property (a_1), hence it is unique.

Proposition 3. *If every finite countably additive measure on* $\boldsymbol{S}$ *satisfies the condition* (a_2), *then every* σ*-finite countably additive measure on* $\boldsymbol{S}$ *satisfies* (a_2), *too.*

Proof. Let μ be σ-finite. Choose any $E\in\boldsymbol{S}$. There exists a sequence $E_n\in\boldsymbol{S}$ such that $E=\bigcup_{n=1}^{\infty}E_n$ and $\mu(E_n)<\infty$. Define $\mu_n(A)=\mu(A\cap E_n)$ for all $A\in\boldsymbol{S}$ and all n.

Because μ_n is finite, there are $G_n \in \boldsymbol{Z}$, $G_n \subset E_n$ and $\mu_n(E_n - G_n) = 0$ for all n. Put $G = \bigcup_{n=1}^{\infty} G_n$. Clearly $G \in \boldsymbol{Z}$ and

$$\mu(E - G) \leqq \sum_{n=1}^{\infty} \mu(E_n - G) \leqq \sum_{n=1}^{\infty} \mu(E_n - G_n) = \sum_{n=1}^{\infty} \mu_n(E_n - G_n) = 0.$$

By the property (ii) there is $F \in \boldsymbol{Z}$ such that $E \subset F$. Now there is $H \in \boldsymbol{Z}$, $H \subset F - E$ and $\mu[(F - E) - H] = 0$. Hence $E \subset F - H$ and $\mu[(F - H) - E] = 0$. Summarizing $G \subset E \subset F - H$ and $\mu[(F - H) - G] = 0$. From this (a_2) follows.

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О ДВУХ УСИЛЕНИЯХ РЕГУЛЯРНОСТИ МЕР

Здена Риечанова

Резюме

Целью настоящей статьи является обобщение (при помощи т.н. малых систем) одного аппроксимационного свойства, в теории мер Бэра и Борела называемого полной регулярностью. Приводятся приложения к некоторым типам множественных функций.