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C—S-MAXIMAL SUPERASSOCIATIVE SYSTEMS. II

HELMUT LÄNGER

Two basic concepts in mathematics are that of associativity and that of a function. It is well known that both these concepts are connected by the fact that up to isomorphism all semigroups are given by all algebras of unary functions (over some suitable set) together with composition. In order to generalize the concept of associativity to n-ary operations ($n>2$) K. Menger introduced the concept of superassociativity (cf. [8], [9]). For investigations concerning superassociative systems (i.e. algebras with one superassociative operation) see also [1], [4], [5], [6], [7] (chapter 3) and [12]. In fact, superassociativity turns out to be quite a natural generalization of associativity since up to isomorphism all n-dimensional superassociative systems (i. e. superassociative systems of the type $n+1$) are given by all algebras of n-ary functions (over some suitable set) together with composition (cf. [1]). Hence studying superassociative systems is very important in order to generalize results on semigroups on the one hand and to get results on function algebras on the other hand. One of the questions in this field is that of classifying all superassociative systems. Since this question seems to be unsolvable in full generality one can try to classify certain classes of superassociative systems. A result in this direction was proved by H. Skala ([12]). Although the structure of quasi-trivial semigroups (i. e. semigroups any subset of which is a subsemigroup) turns out to be comparatively simple (for investigations concerning the varieties of idempotent semigroups cf., e. g., [2]) the class of all quasi-trivial superassociative systems (i. e. superassociative systems any subset of which is a subalgebra) has still not been classified. However, there are partial solutions of this problem (cf. [5], [6]). For investigations concerning the structure of quasi-trivial superassociative systems the concept of C—S-maximality introduced in [4] proves to be very useful since C—S-maximal quasi-trivial superassociative systems only consist of constants and selectors and thus have a trivial structure. Examples for C—S-maximal superassociative systems are the full function algebras over some at least three-element set.

In the following let n be some fixed positive integer and let k be some fixed cardinal.

Definition 1. *Let (A, f) be some fixed algebra of the type $n+1$.*

(A, f) is called an n-dimensional superassociative system if

$$f(f(x_0, x_1, \ldots, x_n), x_{n+1}, \ldots, x_{2n}) =$$
$$= f(x_0, f(x_1, x_{n+1}, \ldots, x_{2n}), \ldots, f(x_n, x_{n+1}, \ldots, x_{2n}))$$
$$\text{for any } x_0, \ldots, x_{2n} \in A .$$

$\boldsymbol{C}(A, f) := \{x \in A \mid f(x, x_1, \ldots, x_n) = x$ for any $x_1, \ldots, x_n \in A\}$ The elements of $\boldsymbol{C}(A, f)$ are called constants of (A, f).

$\boldsymbol{S}_i(A, f) := \{x \in A \mid f(x, x_1, \ldots, x_n) = x_i$ for any $x_1, \ldots, x_n \in A\}$ for any $i = 1, \ldots, n$. The elements of $\boldsymbol{S}_i(A, f)$ are called the i-th selectors of (A, f).

$\boldsymbol{S}(A, f) := \boldsymbol{S}_1(A, f) \cup \ldots \cup \boldsymbol{S}_n(A, f)$. The elements of $\boldsymbol{S}(A, f)$ are called selectors of (A, f).

(A, f) is called quasi-trivial if $f(x_0, \ldots, x_n) \in \{x_0, \ldots, x_n\}$ for any $x_0, \ldots, x_n \in A$.

(A, f) is called C—S-maximal if the quasi-trivial subalgebra $(\boldsymbol{C}(A, f) \cup \cup \boldsymbol{S}(A, f), f)$ of (A, f) is a maximal quasi-trivial subalgebra of (A, f).

Let K_S denote the class of all n-dimensional superassociative systems (B, g) such that $|\boldsymbol{C}(B, g)| = k$.

Let K_{CS} denote the class of all C—S-maximal algebras in K_S.

In the present paper for several sub-classes L of K_S the following problem is completely solved: Does there exist some non-C—S-maximal n-dimensional superassociative system in L? That is, to find necessary and sufficient conditions on (n, k) for $L \subseteq K_{CS}$.

Theorem 2 (cf. [4]). *Put $L := \{(A, f) \in K_S \mid \boldsymbol{S}_1(A, f), \ldots, \boldsymbol{S}_n(A, f) \neq \emptyset\}$. Then $L \subseteq K_{CS}$ iff ($n = 2 < k$ or ($n = 3$ and $k \geqslant 1$) or $n > 3$).*

Theorem 3 (cf. [4]). *Put $L := \{(A, f) \in K_S \mid$there exists some $(x_1, \ldots, x_n) \in \in \boldsymbol{S}_1(A, f) \times \ldots \times \boldsymbol{S}_n(A, f)$ such that $f(x, x_1, \ldots, x_n) = x$ for any $x \in A\}$. Then $L \subseteq K_{CS}$ iff ($n = 2 < k$ or ($n = 3$ and $k \geqslant 1$) or $n > 3$).*

Theorem 4. *Put $L := \{(A, f) \in K_S \mid$if $x, y \in A$ and if $x \neq y$, then there exist $x_1, \ldots, x_n \in \boldsymbol{C}(A, f)$ such that $f(x, x_1, \ldots, x_n) \neq f(y, x_1, \ldots, x_n)\}$. Then $L \subseteq K_{CS}$* iff there is not $k = 2 \leqslant n$.

Proof. Case 1. $k < 2$.

Assume $D = (D, h) \in L$. Suppose $|D| > 1$. Then there exist $a, b \in D$ such that $a \neq b$. Since $D \in L$ there exist $a_1, \ldots, a_n \in \boldsymbol{C}(D)$ such that $h(a, a_1, \ldots, a_n) \neq h(b, a_1, \ldots, a_n)$. Since $a_1, \ldots a_n \in \boldsymbol{C}(D)$ there are also $h(a, a_1, \ldots, a_n)$, $h(b, a_1, \ldots, a_n) \in \boldsymbol{C}(D)$. Hence $k = |\boldsymbol{C}(D)| \geqslant 2$ contradicting the assumption of Case 1. Therefore $|D| \leqslant 1$ whence $\boldsymbol{C}(D) \cup \boldsymbol{S}(D) = D$. Hence $D \in K_{CS}$, which proves $L \subseteq K_{CS}$.

Case 2. $k = 2$ and $n = 1$.

Assume $L \nsubseteq K_{CS}$. Then there exists some $D = (D, h) \in L \setminus K_{CS}$. Since $D \notin K_{CS}$

there exists some set E, $\boldsymbol{C}(D)\cup\boldsymbol{S}(D)\subset E\subseteq D$, such that $E=(E, h)$ is a quasi--trivial subalgebra of D. Let $c\in E\setminus(\boldsymbol{C}(D)\cup\boldsymbol{S}(D))$. Since $c\notin\boldsymbol{S}(D)$ there exists some $d\in D$ such that $h(c, d)\neq d$. Since $D\in L$ there exists some $e\in\boldsymbol{C}(D)$ such that $h(h(c, d), e)\neq h(d, e)$. Since $e\in\boldsymbol{C}(D)$ there is also $h(d, e)\in\boldsymbol{C}(D)\subseteq E$, which together with $c\in E$, together with $h(c, h(d, e))=h(h(c, d), e)\neq h(d, e)$ and together with the quasi-triviality of E implies $h(c, h(d, e))=c$. Since $h(d, e)\in\boldsymbol{C}(D)$ there is also $h(c, h(d, e))\in\boldsymbol{C}(D)$ whence $c\in\boldsymbol{C}(D)$ contradicting the choice of c. Hence $L\subseteq K_{CS}$.

Case 3. $k=2$ and $n>1$.

Put $B:=\{1, 2, 3\}$ and define $g\colon B^{n+1}\to B$ as follows: $g(x_0, \ldots, x_n):=x_0$ or 1 or 2 if $x_0\in\{1, 2\}$ or $(x_0, \ldots, x_n) = (3, 1, \ldots, 1)$ or $(3, 2)\in\{(x_0, x_1), \ldots, (x_0, x_n)\}$, respectively and $g(x_0, \ldots, x_n):=3$ otherwise $(x_0, \ldots, x_n\in B)$. Then $(B, g)\in L\setminus K_{CS}$ whence $L\nsubseteq K_{CS}$.

Case 4. $k>2$.

Assume $l\nsubseteq K_{CS}$. Then there exists some $D=(D, h)\in L\setminus K_{CS}$. Since $D\notin K_{CS}$ there exists some set F, $\boldsymbol{C}(D)\cup\boldsymbol{S}(D)\subset F\subseteq D$, such that $F=(F, h)$ is a quasi-trivial subalgebra of D. Let $e'\in F\setminus(\boldsymbol{C}(D)\cup\boldsymbol{S}(D))$, let $e''\in\boldsymbol{C}(F)$ and let $e'''\in\boldsymbol{C}(D)$. Since $\boldsymbol{C}(D)\subseteq F$ we have $f(e'', e''', \ldots, e''')=e''$ and therefore $f(e'', x_1, \ldots, x_n)$ $= f(f(e'', e''', \ldots, e'''), x_1, \ldots, x_n) = f(e'', f(e''', x_1, \ldots, x_n), \ldots, f(e''', x_1, \ldots, x_n))$ $= f(e'', e''', \ldots, e''')=e''$ for any $x_1, \ldots, x_n\in D$ whence $e''\in\boldsymbol{C}(D)$. Hence $\boldsymbol{C}(F)\subseteq\boldsymbol{C}(D)$. Since obviously $\boldsymbol{C}(D)\subseteq\boldsymbol{C}(F)$ we have $\boldsymbol{C}(D)=\boldsymbol{C}(F)$. Therefore $e'\in F\setminus\boldsymbol{C}(F)$, which together with the quasi-triviality of F and together with $|\boldsymbol{C}(F)|=|\boldsymbol{C}(D)|=k>2$ implies $e'\in\boldsymbol{S}(\boldsymbol{C}(F)\cup\{e'\}, h)$ by a theorem of H. Skala ([12]). Hence there exists some integer j, $1\leqslant j\leqslant n$, such that $e'\in\boldsymbol{S}_j(\boldsymbol{C}(F)\cup\{e'\}, h) = \boldsymbol{S}_j(\boldsymbol{C}(D)\cup\{e'\}, h)$. Since $e'\notin\boldsymbol{S}(D)$ we have $e'\notin\boldsymbol{S}_j(D)$ and therefore there exist $d_1, \ldots, d_n\in D$ such that $h(e', d_1, \ldots, d_n)\neq d_j$. Since $D\in L$ there exist $e_1, \ldots, e_n\in\boldsymbol{C}(D)$ such that $h(h(e', d_1, \ldots, d_n), e_1, \ldots, e_n)\neq h(d_j, e_1, \ldots, e_n)$. Since $e_1, \ldots, e_n\in\boldsymbol{C}(D)$ there are also $h(d_1, e_1, \ldots, e_{.}), \ldots, h(d_n, e_1, \ldots, e_n)\in\boldsymbol{C}(D)$, which together with $h(e', h(d_1, e_1, \ldots, e_n), \ldots, h(d_n, e_1, \ldots, e_n)) = h(h(e', d_1, \ldots, d_n), e_1, \ldots, e_n)\neq h(d_j, e_1, \ldots, e_n)$ contradicts $e'\in\boldsymbol{S}_j(\boldsymbol{C}(D)\cup\{e'\}, h)$. Hence $L\subseteq K_{CS}$.

Theorem 5. *Put* $L:=\{(A, f)\subseteqq K_S\mid$ *if* $x, y\in A$ *and if* $x\neq y$, *then there exist* $x_1, \ldots, x_n\in A$ *such that* $f(x, x_1, \ldots, x_n)\neq f(y, x_1, \ldots, x_n)\}$. *Then* $L\nsubseteq K_{CS}$.

Proof. Case 1. $k=0$.

Put $B:=\{1, 2, 3\}$ and define $g\colon B^{n+1}\to B$ as follows $g(x_0, \ldots, x_n):=x_0$ if $x_1=1$ and $g(x_0, \ldots, x_n):=x_1$ otherwise $(x_0, \ldots, x_n\in B)$. Then $(B, g)\in L\setminus K_{CS}$ whence $L\nsubseteq K_{CS}$.

Case 2. $k>0$.

Let D be some set of cardinality k such that $1, 2\notin D$, put $B:=D\cup\{1, 2\}$ and define $g\colon B^{n+1}\to B$ as follows: $g(x_0, \ldots, x_n):=x_0$ or 2 if $x_0\in D$ or $(x_0, x_1)=(2, 1)$, respectively and $g(x_0, \ldots, x_n) := x_1$ otherwise $(x_0, \ldots, x_n \in B)$. Then $(B, g) \in L \setminus K_{CS}$ whence $L \nsubseteq K_{CS}$.

Theorem 6. *Put* $L := \{(A, f) \in K_S \mid f(x, \ldots, x) = x$ *for any* $x \in A\}$. *Then* $L \nsubseteq K_{CS}$.

Proof. It runs along the same lines as that of the foregoing theorem.

Theorem 7. *Put* $L := \{(A, f) \in K_S \mid \boldsymbol{S}(A, f) = \emptyset$. *Then* $L \nsubseteq K_{CS}$.

Proof. Case 1. $k = 0$.

Put $B := \{1, 2, 3\}$ and define $f: B^{n+1} \to B$ as follows: $g(x_0, \ldots, x_n) := 1$ if $x_1 = \ldots = x_n = 1$ and $g(x_0, \ldots, x_n) := 2$ otherwise $(x_0, \ldots, x_n \in B)$. Then $(B, g) \in L \setminus K_{CS}$ whence $L \nsubseteq K_{CS}$.

Case 2. $k > 0$.

Let D be some set of cardinality k such that $1, 2 \notin D$, put $B := D \cup \{1, 2\}$ and define $g: B^{n+1} \to B$ as follows: $g(x_0, \ldots, x_n) := x_0$ or 1 if $x_0 \in D$ or $(x_0, x_1) \in \{(1, 2), (2, 2)\}$, respectively and $g(x_0, \ldots, x_n) := x_1$ otherwise $(x_0, \ldots, x_n \in B)$. Then $(B, g) \in L \setminus K_{CS}$ whence $L \nsubseteq K_{CS}$.

Theorem 8. *Put* $L := \{(A, f) \in K_S \mid f(x_0, x_1, \ldots, x_n) = f(x_0, x_{\pi 1}, \ldots, x_{\pi n})$ *for any* $x_0 \in A$, *for any* $x_1, \ldots, x_n \in \boldsymbol{C}(A, f)$ *and for any* $\pi \in \mathrm{Sym}\,\{1, \ldots, n\}\}$. *Then* $L \subseteq K_{CS}$ *iff* $n \geqslant 2 < k$.

Proof. Case 1. $k = 0$.

The example of Case 1 of the proof of Theorem 7 shows $L \nsubseteq K_{CS}$.

Case 2. $n = 1$ and $k > 0$.

The special case $n = 1$ of the example of Case 2 of the proof of Theorem 5 shows $L \nsubseteq K_{CS}$.

Case 3. $n > 1$ and $k = 1$.

Put $B := \{1, 2\}$ and define $g: B^{n+1} \to B$ as follows: $g(x_0, \ldots, x_n) := 2$ if $x_0 = \ldots x_n = 2$ and $g(x_0, \ldots, x_n) := 1$ otherwise $(x_0, \ldots, x_n \in B)$. Then $(B, g) \in L \setminus K_{CS}$ whence $L \nsubseteq K_{CS}$.

Case 4. $n > 1$ and $k = 2$.

The example of Case 3 of the proof of Theorem 4 shows $L \nsubseteq K_{CS}$.

Case 5. $n > 1$ and $k > 2$.

Assume $L \nsubseteq K_{CS}$. Then there exists some $D = (D, h) \in L \setminus K_{CS}$. Since $D \notin K_{CS}$ there exists some set F, $\boldsymbol{C}(D) \cup \boldsymbol{S}(D) \subset F \subseteq D$, such that $F = (F, h)$ is a quasi-trivial subalgebra of D. Let $a \in F \setminus (\boldsymbol{C}(D) \cup \boldsymbol{S}(D))$. Analogously to Case 4 of the proof of Theorem 4 one concludes that there exists some integer j, $1 \leqslant j \leqslant n$, such that $a \in \boldsymbol{S}_j(\boldsymbol{C}(D) \cup \{a\}, h)$. Let $k \in \{1, \ldots, n\} \setminus \{j\}$ and let $a_1, \ldots, a_n \in \boldsymbol{C}(D)$ such that $a_j \neq a_k$. Then $h(a, a_1, \ldots, a_n) = a_j \neq a_k = a_{(jk)j} = f(a, {}_{(jk)1}, \ldots, a_{(jk)n})$ contradicting $D \in L$. Hence $L \subseteq K_{CS}$.

Theorem 9. *Put* $L := \{(A, f) \in K_S \mid f(x_0, x_1, \ldots, x_n) = f(x_0, x_{\pi 1}, \ldots, x_{\pi n})$ *for any* $x_0, \ldots, x_n \in A$ *and for any* $\pi \in \mathrm{Sym}\,\{1, \ldots, n\}\}$. *Then* $L \subseteq K_{CS}$ *iff* $n \geqslant 2 < k$.

Proof. It follows immediately from the proof above.

Theorem 10. *Put* $L := \{(A, f) \in K_S \mid (A, f)$ *simple*$\}$. *Then* $L \subseteq K_{CS}$ *iff* $(n = 1 < k$ or $k > 2)$.

Proof. Case 1. $k = 0$.

According to [10] (Corollary 1.7) there exists some congruence-free inverse semigroup $(B, \cdot)$ such that the semilattice of all idempotents of $(B, \cdot)$ coincides with the semilattice of all rational numbers (with the natural ordering). Now define $g: B^{n+1} \to B$ by $g(x_0, \ldots, x_n) := x_0 x_1 (x_0, \ldots, x_n \in B)$. Then $(B, g) \in L \setminus K_{CS}$ whence $L \not\subset K_{CS}$.

Case 2. $k = 1$.

Put $B := \{1, 2, 3, 4, 5\}$ and define $g: B^{n+1} \to B$ as follows: $g(x_0, \ldots, x_n) := x_0 + x_1 - 2$ or $x_0 + x_1 - 5$ if $(x_0, x_1) \in \{2, 3\} \times \{2, 4\}$ or $(x_0, x_1) \in \{4, 5\} \times \{3, 5\}$, respectively and $g(x_0, \ldots, x_n) := 1$ otherwise $(x_0, \ldots, x_n \in B)$. Then $(B, g) \in L \setminus K_{CS}$ whence $L \not\subseteq K_{CS}$.

Case 3. $k > 1$ and $n = 1$.

Assume $L \not\subseteq K_{CS}$. Then there exists some $D = (D, h) \in L \setminus K_{CS}$. Since $D \notin K_{CS}$ we have $\boldsymbol{C}(D) \subset D$ which together with $|\boldsymbol{C}(D)| = k > 1$ and together with the fact that $\boldsymbol{C}(D)$ is an ideal of D would imply that $(\boldsymbol{C}(D))^2 \cup (\operatorname{diag} D^2)$ is a non-trivial congruence on D contradicting $D \in L$. Hence $L \subseteq K_{CS}$.

Case 4. $k = 2$ and $n > 1$.

The example of Case 3 of the proof of Theorem 4 shows $L \not\subseteq K_{CS}$.

Case 5. $k > 2$ and $n > 1$.

Assume there exists some $D \in L$ and assume there exist $a, b \in D$ such that $a \neq b$ and such that $h(a, x_1, \ldots, x_n) = h(b, x_1, \ldots, x_n)$ for any $x_1, \ldots, x_n \in \boldsymbol{C}(D)$. Then $h(D, \boldsymbol{C}(D), \ldots, \boldsymbol{C}(D)) \subseteq \boldsymbol{C}(D)$ would imply that $\cap \{\ker (h(., a_1, \ldots, a_n)) \mid a_1, \ldots, a_n \in \boldsymbol{C}(D)\}$ is a non-trivial congruence on D contradicting $D \in L$. Hence $L \subseteq \{(A, f) \subseteqq K_S \mid$ if $x, y \in A$ and if $x \neq y$, then there exist $x_1, \ldots, x_n \in \boldsymbol{C}(A, f)$ such that $f(x, x_1, \ldots, x_n) \neq f(y, x_1, \ldots x_n)\}$. Now $L \subseteq K_{CS}$ by Theorem 4.

Remark. In order to see that for $n, k > 1$ the class L of Theorem 10 contains algebras (A, f) with $|A| > 1$ 1 consider the full n-place function algebra over some set of cardinality k (cf. [11]; for a short proof of the simplicity of this algebra see [3]).

In the following let M be some fixed non-empty set.

Definition 11. $F_n(M) := M^{M^n}$.

Let $k': (F_n(M))^{n+1} \to F_n(M)$ be defined as follows: $(k'(f_0, f_1, \ldots, f_n))(x_1, \ldots, x_n) := f_0(f_1(x_1, \ldots, x_n), \ldots, f_n(x_1, \ldots, x_n))$ for any $f_0, \ldots, f_n \in F_n(M)$ and for any $x_1, \ldots, x_n \in M$.

$\boldsymbol{S}_n(M) := \{f \in F_n(M) \mid f(x_1, \ldots, x_n) = f(x_{\pi 1}, \ldots, x_{\pi n})$ for any $x_1, \ldots, x_n \in M$ and for any $\pi \in \operatorname{Sym}\{1, \ldots, n\}\}$.

Lemma 12.

(i) $(F_n(M), k') \in K_S$ with $k = |M|$.
(ii) $k'((S_n(M))^{n+1}) \subseteq S_n(M)$.
(iii) $(S_n(M), k') \in K_S$ *with* $k = |M|$.
(iv) $\boldsymbol{C}(S_n(M), k') = \{(x_1, \ldots, x_n) \mapsto a \mid a \subseteqq M\}$.
(v) *If* $\min(|M|, n) > 1$, *then* $\boldsymbol{S}(\boldsymbol{S}_n(M), k') = \emptyset$.

Proof. Easy.

Theorem 13 (cf. [4]). *$(F_n(M), k')$ is C—S-maximal iff there is not* $|M| = n = 2$.

Theorem 14. *$(\boldsymbol{S}_n(M), k')$ is C—S-maximal iff there is not* $|M| = 2 \leq n$.

Proof. Put $\boldsymbol{S} := \boldsymbol{S}_n(M) = (\boldsymbol{S}_n(M), k')$.

Case 1. $|M| = 1$.
Then $|S| = 1$ and hence $\boldsymbol{C}(S) \cup \boldsymbol{S}(S) = S$ whence S is C—S-maximal.
Case 2. ($|M| = 2$ and $n = 1$) or $|M| > 2$.

Then S is C—S-maximal because of Theorem 4.

Case 3. $|M| = 2$ and $n > 1$.
Let $a, b \in M$ such that $\neq b$ and define $g\colon M^n \to M$ as follows: $g(x_1, \ldots, x_n) := a$ if $x_1 = \ldots = x_n = a$ and $g(x_1, \ldots, x_n) := b$ otherwise $(x_1, \ldots, x_n \in M)$. Then $g \notin \boldsymbol{C}(S) \cup \boldsymbol{S}(S)$ and $(\boldsymbol{C}(S) \cup \boldsymbol{S}(S) \cup \{g\}, k')$ is a quasi-trivial subalgebra of S. Hence S is not C—S-maximal.

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C—S-МАКСИМАЛЬНЫЕ СУПЕРАССОЦИАТИВНЫЕ СИСТЕМЫ. II

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Резюме

Пусть n-положительное целое число и $A=(A, f)$ — алгебра типа $n+1$. A называется суперассоциативной системой, если $f(f(x_0, x_1, \ldots, x_n), x_{n+1}, \ldots, x_{2n}) = f(x_0, f(x_1, x_{n+1}, \ldots, x_{2n}), \ldots, f(x_n, x_{n+1}, \ldots, x_{2n}))$ для всех $x_0, \ldots, x_{2n} \in A$. Пусть $C(A) := \{x \in A) \mid f(x, x_1, \ldots, x_n) = x$ для всех $x_1, \ldots, x_n \in A\}$ и $S(A) := \{x \in A \mid$ существует $i \in \{1, \ldots, n\}$ такое, что $f(x, x_1, \ldots, x_n) = x_i$ для всех $x_1, \ldots, x_n \in A\}$. A называется квазитривиальной, если $f(x_0, \ldots, x_n) \in \{x_0, \ldots, x_n\}$ для всех $x_0, \ldots, x_n \in A$. Для некоторых классов K суперассоциативных систем найдены все пары (n, k), для которых все алгебры B класса K типа $n+1$, для которых $|C(B)| = k$, обладают свойством, что $C(B) \cup S(B)$ является максимальной квазитривиальной подалгеброй алгебры B.