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NEWS AND NOTICES

SOME NOTES TO AN IMPROBABLE ANNIVERSARY

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One can hardly believe that more than twenty five years have passed since one of the authors of this paper met for the first time Professor Alois Kufner. However, the language of numbers is inexorable.

Alois Kufner was born in Plzeň, attended primary and secondary school in Stříbro. After the final examination in 1953 he entered the Faculty of Mathematics and Physics of Charles University in Prague, majoring in applied mathematics. He graduated in 1958 and on September 1 he joined the Mathematical Institute of the Czechoslovak Academy of Sciences where he has been working till now. At first he was member of the Department of Partial Differential Equations the head of which was Prof. Ivo Babuška, later he joined, together with Prof. J. Nečas, the newly established Department of the Theory of Partial Differential Equations. He obtained his Candidate of Science (CSc) degree in 1965 and habilitated as Associated Professor (Dozent) at Charles University in 1968. In 1981 he defended his dissertation for the

Doctor of Science (DrSc) degree and four years later was appointed Professor at the School of Mechanical and Electrical Engineering (now West-Bohemian University) in Plzeň. From 1979 till 1990 he was Director of the Mathematical Institute of the Academy.

Kufner's field of scientific interest was determined by the subject of his CSc dissertation, in which he studied, under J. Nečas' supervision, the embedding of Sobolev spaces with weights and its application to the theory of weak solutions of the Dirichlet problem for elliptic PDE's. Problems from this field were studied also in his habilitation thesis, DrSc dissertation and the majority of his 42 original papers and 13 talks at conferences.

The first papers [3, 4] stemming from his CSc dissertation present the results of his study of Sobolev spaces with a weight which is given by a power of the distance from the boundary or from a fixed point on the boundary of a lipschitzian domain. The main tool is the well known Hardy inequality

$$\int_0^\infty F(t)^p t^{\varepsilon-p}dt \leqslant \left(\frac{p}{|\varepsilon-p+1|}\right)^p \int_0^\infty |f(t)|^p t^\varepsilon dt,$$

which holds for $1 < p < \infty$, $\varepsilon \neq p-1$ and for all functions f for which the right hand side of the inequality is finite; here

$$F(t) = \begin{cases} \displaystyle\int_0^t |f(s)|ds & \text{for} \quad \varepsilon < p-1 \\ \displaystyle\int_t^\infty |f(s)|ds & \text{for} \quad \varepsilon > p-1 \end{cases}$$

This research is continued in important and still cited papers [6–9] which resulted from fruitful cooperation with J. Kadlec and O.V. Besov on the problem of approximation of functions from Sobolev spaces with a power or general weight by smooth functions and smooth functions with compact supports. It was here that the Hardy inequality was extended to the important singular case $\varepsilon = p-1$.

Another personal friendship connected with scientific cooperation concerned H. Triebel and resulted in joint papers [12, 13] generalizing the Hardy inequality in two directions. First, more general weights expressed in terms of the exponential and a continuously differentiable function were considered. For such weights a Hardy-type inequality was proved. Second, the authors also dealt with a Hardy inequality in which the derivative of the function on the right hand side was replaced by a difference quotient. In this connection one usually speaks about fractional derivatives since such expressions occur in the interpolation of Sobolev spaces.

It was the application of the Hardy inequality in the theorems on embeddings of Sobolev spaces with weights that served as the main motivation for A. Kufner to study it. Provided the domain on which the spaces are considered is lipschitzian and the weight is of power type, the embedding theorems are deduced using the local description of the boundary of the domain and the parameters in them agree with those in the Hardy inequality. However, there is no such agreement for domains with less regular boundaries. Sufficient conditions for domains with hölderian boundaries are established in [14]. This paper closes one period of Kufner's research work. Its results are collected in the monograph [45] which was submitted as his DrSc dissertation.

An important feature of the following period is Kufner's cooperation with B. Opic. The techniques of proofs in the above mentioned papers of course have their limits given by the application of the one-dimensional Hardy inequality to domains in spaces of higher dimensions. Therefore a natural question arises whether there exists a multidimensional analogue of the Hardy inequality which would facilitate an unmediated approach to problems in higher dimensions. In [18] the authors used Sobolev spaces equipped with a system of weight functions (generally different weights correspond to different derivatives) to solve the Dirichlet problem for degenerate and singular linear elliptic equations. They extended their results later [25] to nonlinear equations with both weak and strong degeneracies and singularities, and dealt also with anisotropic cases.

This approach is directly connected with the multidimensional Hardy inequality

$$\int_\Omega |u(x)|^p v_0(x)\,\mathrm{d}x \leqslant C \sum_{i=1}^N \int_\Omega \left|\frac{\partial u}{\partial x_i}\right|^p v_i(x)\,\mathrm{d}x\,.$$

A sufficient condition for its validity was found in [19] via methods of the calculus of variations. The condition is formulated as a requirement on the solution of the PDE in which the weight functions occur as coefficients. Another approach based on integral estimates made it possible, in the paper [29] written together with another Kufner's student P. Gurka, to find an explicit sufficient condition of Muckenhoupt type for the validity of a double-weight inequality with different exponents on the left and right sides:

$$\left(\int_Q |u(x)|^q w(x)\,\mathrm{d}x\right)^{1/q} \leqslant C \left(\sum_{i=1}^N \int_Q \left|\frac{\partial u(x)}{\partial x_i}\right|^p v(x)\,\mathrm{d}x\right)^{1/p}.$$

Another interesting result obtained jointly with H.-P. Heinig [31] gives conditions of validity of a double-weight Hardy inequality of higher order

$$\left(\int_0^\infty |u(x)|^q w_0(x)\,\mathrm{d}x\right)^{1/q} \leqslant C \left(\int_0^\infty |u^{(m+n)}(x)|^p w_{mn}(x)\,\mathrm{d}x\right)^{1/p},$$

where the function u together with its derivatives of orders up to $m-1$ vanishes at the point 0 while the derivatives of orders $m, \dots, m+n-1$ vanish at infinity. Again this period of Kufner's scientific activity is concluded by a monograph [47]. In it the authors covered in detail a very extensive field of problems connected with the Hardy inequality.

The last period covers the nineties and starts with Kufner's release from the burden of administrative duties. Both the number of cooperators and the field of interest are growing [32–42]. The Hardy inequality remains the leitmotiv but other related notions also appear. Kufner more and more frequently returns to problems of boundary value problems for elliptic equations which formed the starting point of his scientific career.

For the sake of completeness let us add that we have not mentioned a number of papers connected more or less closely with the main stream of results concerning the Hardy inequality. Of course, Kufner's scientific activities have brought about the corresponding response, as is corroborated by a number of invitations to conferences and lectures.

Alois Kufner has always combined his research work with deep interest in teaching. The long list of his book publications includes monographs, lecture notes, textbooks, handbooks, a number of translations and popular publications. They have been appreciated by a wide spectrum of readers ranging from scientific workers to students of secondary schools and talented pupils of primary schools. First of them [61] dealing with the Fourier series and written jointly with J. Kadlec, was a useful contribution to Czech mathematical literature, and also its English version has been successful. Of substantial impact was the book on modern methods of solution of nonlinear differential equations [44] the coauthor of which was S. Fučík. However, it is his comprehensive monograph [43] which is still meeting with widest acceptance and belongs to the standard equipment of all who work with function spaces or apply them in other branches.

Although being employed in an institute of the Academy of Sciences, Prof. Kufner has frequently lectured to students at the Faculty of Mathematics and Physics of Charles University and, in particular, at the School of Mechanical and Electrical Engineering, now West-Bohemian University at Plzeň. He has educated a number of graduates and doctorands and founded a sort of Czech school of the theory of function spaces. His close relation to the Department of Mathematics of the Faculty of Applied Sciences of the West-Bohemian University in his place of birth has lasted for more than two decades. He played a major part in its transformation from a service workplace providing standard training in Mathematics for engineers to a department with considerable scientific potential and rich international contacts.

Professor Kufner has been also known as a tireless organizer. He was at the birth of the series of seminars in the theory of PDE's organized yearly by the above mentioned Department of Mathematics at Plzeň for almost two decades. This series is every fourth year amended by an international spring school Nonlinear Analysis, Function Spaces and Applications—a significant event attended by outstanding specialists from all over the world [67, 69, 73, 74]. First of them took place in 1978 and was initiated by S. Fučík and A. Kufner. The list of Kufner's merits in organization of Czech mathematics would be long, let us mention just his longlasting activities in the Union of Czechoslovak Mathematicians and Physicists where he did much useful work also in the field of international contacts.

Professor Kufner reached sixty years of age on February 1, 1994. On the occasion of this very improbable anniversary we extend to him our best wishes.

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