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WEAK EDGE-DEGREE DOMINATION IN HYPERGRAPHS

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Abstract. In this paper we extend the notion of weak degree domination in graphs to hypergraphs and find relationships among the domination number, the weak edge-degree domination number, the independent domination number and the independence number of a given hypergraph.

Keywords: hypergraph, weak degree domination number, independent domination number, graph theory

MSC 2000: 05C

0. Introduction

For all terminology and notation in hypergraph theory we refer the reader to C. Berge [8]. All hypergraphs considered in this paper are simple, finite and loop-free.

Given a hypergraph $H = (X, E)$ and $D \subseteq X$, we call D a *dominating set* (or simply a *domset*) of H if for every vertex $y \in X - D$ there exist $x \in D$ and $e \in E$ such that $x, y \in e$ (cf., [5]) and a *weakly edge-degree dominating* (or briefly, WEDD-) *set* of H if for every vertex $y \in X - D$ there exists $x \in D$ such that (i) $x, y \in e$ for some $e \in E$ (i.e., D is a domset of H) and (ii) $|E_x| \leqslant |E_y|$, where E_a denotes the set of edges containing the vertex a. The *domination* (*weak edge-degree domination*) *number* $\gamma(H)$ (respectively, $\gamma_w(H)$) of H is then defined as the least cardinality of a domset (WEDD-set) of H. Further, let $\gamma_i(H)$ and $\beta_0(H)$ denote respectively the least and the largest cardinality of a maximal independent (or, strongly stable as in [8], p. 448) set of H. We have the well-known inequality

$$\gamma(H) \leqslant \gamma_i(H) \leqslant \beta_0(H). \tag{1}$$

What can one say about $\gamma_w(H)$ with regard to (1)? We have examples of hypergraphs H for which each of the possibilities (i) $\gamma_w(H) < \gamma_i(H)$, (ii) $\gamma_w(H) = \gamma_i(H)$ and (iii) $\gamma_w(H) > \gamma_i(H)$ may occur (see Fig. 1 for illustrative examples). In this paper we give a class of hypergraphs in which for every hypergraph H, $\gamma_w(H)$ lies between $\gamma(H)$ and $\gamma_i(H)$, as well as a class in which for every hypergraph H, $\gamma_w(H)$ lies between $\gamma_i(H)$ and $\beta_0(H)$. In fact, we have a *conjecture* that for any hypergraph H with *cyclomatic number* (see [3], [4], [6], [9], [10], [11], [17] $\mu(H)$ equal to zero

$$\gamma_i(H) \leqslant \gamma_w(H) \leqslant \beta_0(H). \tag{2}$$

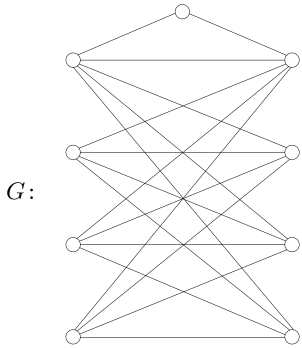
The notion of a WEDD-set in a graph was first introduced by E. Sampathkumar and L. Pushpa Latha [16] who conjectured that (2) must hold for any tree H and this was almost instantly proved to be true by J. H. Hattingh and R. C. Laskar [12] and the present authors [7] (see [13], [15]). In substantiation of our more general conjecture as mentioned above, we shall show in this paper that for any hypertree (i.e., a connected acyclic hypergraph) H the inequalities in (2) hold.

It is not hard to find hypergraphs with nonzero cyclomatic number which still satisfy (2) (e.g., see Fig. 2 (a)), as also such hypergraphs that do not satisfy (2) (e.g., see Fig. 1 (b), (c)), thus pointing at the interesting *open problem of characterizing hypergraphs H that satisfy* (2) as well as the importance of settling our above conjecture.

1. Some general results

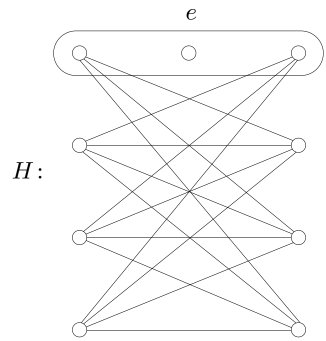
Given a hypergraph $H = (X, E)$, by the *edge-degree* $\mathrm{ed}(x)$ of a vertex $x \in X(H)$ we mean the cardinality $|E_x|$ of the *edge-neighbourhood* E_x of x. Hence, by a *pendant vertex* we mean a vertex x in H for which $\mathrm{ed}(x) = 1$; that is, x is contained in exactly one edge of H (see [1], [2]). Clearly, every vertex of an edge e of H is pendant if and only if e is a component of H. The set of pendant vertices in H is denoted by $\P(H)$. We shall call an edge of H a *pendant edge* if it contains exactly one "nonpendant" vertex (i.e., a vertex x for which $\mathrm{ed}(x) \geqslant 2$). A nonpendant vertex of H is called a *support* if it is contained in a pendant edge. By the *removal* of an edge e from H we shall mean the operation of deleting e from E and deletion of the pendant vertices contained in e from X so that the hypergraph obtained by *removing* e from H is given by $H - e = (X - e^0, E - \{e\})$ where e^0 denotes the set of pendant vertices in the edge e.

Next, for any $u \in X(H)$, let $N_w(u) = \{v \in X(H)\colon\ u, v \in e$ for some $e \in E(H)$ and $\mathrm{ed}(u) \geqslant \mathrm{ed}(v)\}$ denote the *weak edge-degree vertex neighbourhood* of u; any particular element of $N_w(u)$ is called a *weak neighbour* of u. Clearly, $D \subseteq X(H)$ is



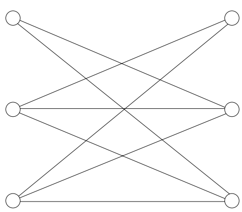
A graph G with
$\gamma(G) = \gamma_w(G) = 3 < 4 = \gamma_i(G) = \beta_0(G)$

(a)



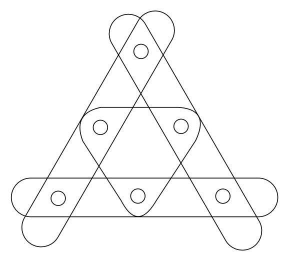
A hypergraph H with
$\gamma_w(H) = 2 < 3 = \gamma_i(H)$

(b)



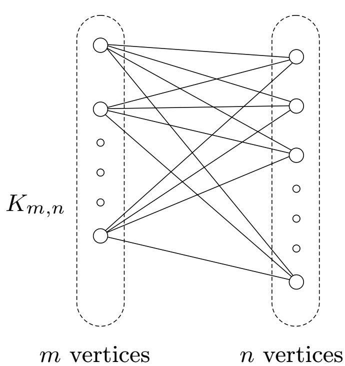
A graph G with
$\gamma(G) = \gamma_w(G) = \gamma_i(G) = 2 < 3 = \beta_0(G)$

(c)



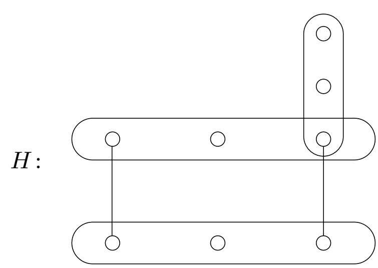
A hypergraph H with $\gamma_w(H) = \gamma_i(H) = 2$

(d)



A graph $G = K_{m,n}$ with
$\gamma_i(G) = m < n = \gamma_w(G)$ for $m < n$

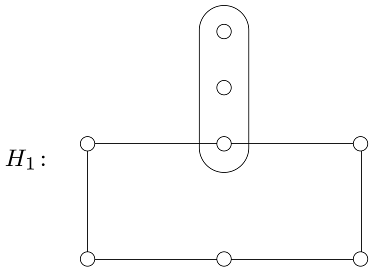
(e)



A hypergraph H with
$\gamma_i(H) = 2 < 3 = \gamma_w(H)$

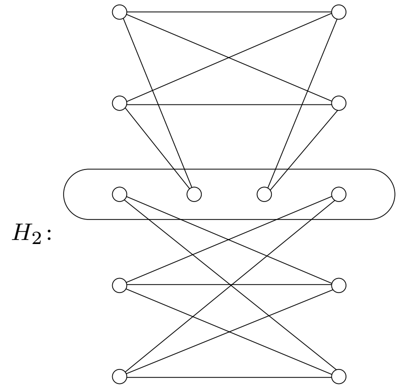
(f)

Figure 1



A hypergraph $H_1 = (X, \mathcal{E})$ with cyclomatic number $\mu(H_1) = \sum_{E \in \mathcal{E}} |E| - |X| - m(H_1) = 15-8-6 = 1 > 0$ and $2 = \gamma_i(H_1) < 3 = \gamma_w(H_1) < \beta_0(H_1) = 4$

(a)



A non-edge-degree regular hypergraph H_2 with $\gamma_w(H_2) = 4 < 5 = \gamma_i(H_2)$

(b)

Figure 2

a WEDD-set of H if and only if

$$|D \cap N_w(u)| \geqslant 1 \quad \text{for every } u \in X - D. \tag{3}$$

The set of all WEDD-sets of H will be denoted by $\boldsymbol{D}_w(H)$. The following results can be easily established in the same way as their analogues in graph theory (see [7]).

Theorem 1. *Let $H = (X, E)$ be any hypergraph and $D \in \boldsymbol{D}_w(H)$. Then D is a minimal* WEDD*-set of H if and only if for each $d \in D$ one of the following conditions is satisfied:*

(i) $D \cap N_w(d) = \emptyset$,

(ii) $\exists\, x \in X - D$ *such that* $D \cap N_w(x) = \{d\}$.

Theorem 2. *In any hypergraph $H = (X, E)$, $N_w(u) = \emptyset \Leftrightarrow u \in \bigcap_{S \in \boldsymbol{D}_w(H)} S$; that is, u has no weak neighbour if and only if u lies in every* WEDD*-set of H.*

Theorem 3. *In any hypergraph $H = (X, E)$, every* WEDD*-set D contains a vertex of minimum edge-degree $\delta_e(H)$.*

Theorem 4. *In any hypergraph $H = (X, E)$, the set $W^0 = \{x \in X\colon\ N_w(x) = \emptyset\}$ is an independent subset of every* WEDD*-set of H.*

2. Main results

A connected acyclic hypergraph is called a *hypertree*. Clearly, any hypertree $H = (X, E)$ satisfies the "Linearity Property", viz.,

$$|e_1 \cap e_2| \leqslant 1 \quad \text{for any two } e_1, e_2 \in E \tag{4}$$

(e.g., see C. Berge [9], p. 8). Next, let $(u_0, e_1, u_1, e_2, u_2, e_3, u_3, \ldots, u_{k-1}, e_k, u_k)$ be a diametrical path in H. Then the following observations are straightforward:

O1. e_1 is a pendant edge and every vertex of e_1 except u_1 is a pendant vertex of H.

O2. Every edge which has a nonempty intersection with e_2, except possibly e_3, is a pendant edge. If $k = 3$, then e_3 also becomes a pendant edge.

O3. $k = 2$ if and only if $a_1 \cap a_2 = \{u_1\}$ for all edges $a_1, a_2 \in E$.

O4. $e_i \cap e_j = \emptyset$ for all distinct $i, j \in \{1, 2, \ldots, k\}$ with $|i - j| > 1$.

The following general observations are also useful in our investigation.

Lemma 1. *If S is any* WEDD*-set of a hypergraph H and if e is any edge of H with $e^0 \neq \emptyset$ then $S \cap e^0 \neq \emptyset$; further, if S is a minimal* WEDD*-set of H then $|S \cap e^0| = 1$.*

A hypergraph is *edge-degree regular* if all its vertices have the same edge-degree and hence the following result is easy to see.

Theorem 5. *For any edge-degree regular hypergraph $H = (X, E)$, $\gamma_w(H) = \gamma(H) \leqslant \gamma_i(H)$.*

Fig. 2 (b) exhibits an edge-degree regular hypergraph H such that $\gamma_w(H) < \gamma_i(H)$ as well as a hypergraph which is not edge-degree regular but still satisfies the inequality of Theorem 5; the latter example illustrates that edge-degree regularity is a sufficient condition for H to satisfy the conclusion of Theorem 5 but it is not necessary. Thus, it is important to characterize hypergraphs H for which $\gamma_w(H) = \gamma_i(H)$.

We shall now proceed to establish the following first main result of the paper.

Theorem 6. *For any hypertree $H = (X, E)$, $\gamma_i(H) \leqslant \gamma_w(H)$.*

Proof. We shall prove the result by induction on the *size* (i.e., the number of edges) of hypertrees.

If H is any hypertree of size $q = 1$, then H consists of just one edge whence every vertex in it is pendant so that each vertex constitutes a minimal independent domset as well as a minimum WEDD-set of H; thus, we have $\gamma_i(H) = 1 = \gamma_w(H)$ and the result follows.

Next, let H be any hypertree of size $q = 2$ with $E = \{e_1, e_2\}$. Then $e_1 \cap e_2 = \{u_1\}$ for some vertex u_1 whence, again, $\gamma_i(H) = 1 < 2 = \gamma_w(H)$ and the result follows.

If H is any hypertree of size $q = 3$, let $E = \{e_1, e_2, e_3\}$. Then either $e_1 \cap e_2 \cap e_3 = \{u_1\}$ for some vertex u_1 or there exist vertices u_1 and u_2 such that $e_1 \cap e_2 = \{u_1\}$, $e_2 \cap e_3 = \{u_2\}$ and $e_1 \cap e_3 = \emptyset$. In the first case, $\{u_1\}$ is a minimal independent domset whence, again, $\gamma_i(H) = 1$. Further, any set consisting of exactly one pendant vertex from each of the edges e_1, e_2 and e_3 is a minimum WEDD-set of H so that $\gamma_w(H) = 3$, implying the result. Consider the latter case. If e_2 has a pendant vertex then $\gamma_i(H) = 2 < 3 = \gamma_w(H)$ and if e_2 has no pendant vertex then any set consisting of one pendant vertex v_1 chosen from e_1 and one pendant vertex v_3 chosen from e_3 turns out to be both a minimal independent domset and a minimum WEDD-set of H, whence we get $\gamma_i(H) = 2 = \gamma_w(H)$. Thus, the result is seen to hold in this case as well.

Hence, suppose the result holds for all hypertrees of size less than an arbitrarily given positive integer q. Let H be any hypertree of size $q > 3$. Let $(u_0, e_1, u_1, e_2, u_2, e_3, u_3, \ldots, u_{k-1}, e_k, u_k)$ be a diametrical path in H. If $k \leqslant 3$ then the above arguments and observations O1–O4 yield the desired result. Hence, we let $k \geqslant 4$.

Case 1: Suppose H contains a WEDD-set S of cardinality $\gamma_w(H)$ (or, henceforth, a "$\gamma_w(H)$-set" for brevity), which contains a support vertex x of H. Then we let $H' = (X', E')$ denote the subhypergraph of H obtained after removing all the pendant edges in E_x. Since H' is a hypertree with $|E'| < |E|$, by the induction hypothesis we get

$$\gamma_i(H') \leqslant \gamma_w(H'). \tag{5}$$

Let $P(x)$ denote the set of pendant edges in H with x as their support. Then

$$|(e - \{x\}) \cap S| = 1 \quad \text{for every } e \in P(x). \tag{6}$$

Hence, if $S' = \bigcup\limits_{e \in P(x)} ((e - \{x\}) \cap S)$ then it is easy to see that the set $S - S'$ is a WEDD-set of H', whence we get

$$\gamma_w(H') \leqslant \gamma_w(H) - |P(x)|. \tag{7}$$

Next, let T be a $\gamma_i(H')$-set. If $x \in T$, then T is a maximal independent set of H. Also, if $x \notin T$ then $T \cup S'$ is a maximal independent set of H. Thus, in either case, we have

$$\gamma_i(H) \leqslant \gamma_i(H') + |P(x)|. \tag{8}$$

The relations (5), (7) and (8) yield the desired result in this case.

Case 2: Next, consider the case when H does not contain any $\gamma_w(H)$-set that contains the support of H.

Subcase 1: e_2 contains a pendant vertex (i.e., $e_2^0 \neq \emptyset$).

Let S be any $\gamma_w(H)$-set and let H' be the subhypergraph obtained after removing all edges with their supports in e_2, one at a time successively. Let the set of edges so removed be denoted by $P(e_2)$. Clearly, for each $e \in P(e_2)$, Lemma 1 implies that $|S \cap e^0| = 1$. Hence we observe that $S - \bigcup_{e \in P(e_2)} (S \cap e)$ is a WEDD-set of H', irrespective of whether or not u_2 belongs to S. Thus, it follows that

$$\gamma_w(H') \leqslant \gamma_w(H) - |P(e_2)|. \tag{9}$$

Next, let T be a $\gamma_i(H')$-set. Since $T \cap e_2 \neq \emptyset$ and $|T \cap e_2| = 1$, let $\{w_0\} = T \cap e_2$. If $w_0 \in e_2^0$ in H then $T \cup T''$ where T'' is a set of pendant vertices, one chosen arbitrarily from each member of $P(e_2)$, is a maximal independent set of H and hence

$$\gamma_i(H) \leqslant \gamma_i(H') + |P(e_2)|. \tag{10}$$

If $w_0 \in e_2 - e_2^0$ then $T \cup T'$ where T' is a set of pendant vertices, one chosen arbitrarily from each member of $P(e_2 - \{w_0\})$, is a maximal independent set of H and hence

$$\gamma_i(H) \leqslant \gamma_i(H') + |P(e_2 - \{w_0\})|. \tag{11}$$

Thus, in every case we have

$$\gamma_i(H) \leqslant \gamma_i(H') + |P(e_2)|. \tag{12}$$

Since H' is a hypertree of size less than q the induction hypothesis implies that $\gamma_i(H') \leqslant \gamma_w(H')$, whence the inequality (12) yields the desired result in this subcase.

Subcase 2: e_2 does not contain a pendant vertex.

If u_2 is a support, then let H' be the subhypergraph obtained by removing e_2 and all pendant edges with their supports in $e_2 - \{u_2\}$, one by one in succession. By the assumption of the case, $u_2 \notin S$. Also, for every $e \in P(e_2 - \{u_2\})$, $|S \cap e| = 1$. Then $S - \bigcup_{e \in P(e_2 - \{u_2\})} (S \cap e)$ is a WEDD-set of H' and so

$$\gamma_w(H') \leqslant \gamma_w(H) - |P(e_2 - \{u_2\})|. \tag{13}$$

Also, if T is a $\gamma_i(H')$-set then $T \cup T'$ where T' is a set of pendant vertices, one chosen arbitrarily from each member of $P(e_2 - \{u_2\})$, is a maximal independent set of H and hence $\gamma_i(H) \leqslant \gamma_i(H') + |P(e_2 - \{u_2\})| \leqslant \gamma_w(H') + |P(e_2 - \{u_2\})| \leqslant \gamma_w(H)$, as desired.

Next, suppose u_2 is not a support. Let H' be the subhypergraph obtained after removing all members of $P(e_2)$ along with e_2. Then, whether $u_2 \in S$ or $u_2 \notin S$, it is not hard to see that $\gamma_i(H) \leqslant \gamma_i(H') + |P(e_2)| \leqslant \gamma_w(H') + |P(e_2)| \leqslant \gamma_w(H)$, as desired. This completes the proof. □

The second main result is the following one.

Theorem 7. *For any hypertree $H = (X, E)$, $\gamma_w(H) \leqslant \beta_0(H)$.*

Proof. We shall prove the result by induction on the size of hypertrees. If H is any hypertree of size $q = 1$, then H consists of just one edge, whence $\gamma_w(H) = 1 = \beta_0(H)$ and the result follows. Next, let H be any hypertree of size $q = 2$ with $E = \{e_1, e_2\}$. Then, since H is connected, there must exist a vertex u_1 such that $e_1 \cap e_2 = \{u_1\}$. Then any set consisting of exactly one pendant vertex from each of the two edges forms a maximum independent set which is also a minimum WEDD-set of H, so that $\gamma_w(H) = 2 = \beta_0(H)$ implying the result. Further, let H be any hypertree of size $q = 3$ and let $E = \{e_1, e_2, e_3\}$. Then either $e_1 \cap e_2 \cap e_3 = \{u_1\}$ for some vertex u_1 or there exist vertices u_1 and u_2 such that $e_1 \cap e_2 = \{u_1\}$, $e_2 \cap e_3 = \{u_2\}$ and $e_1 \cap e_3 = \emptyset$. In the former case, $\operatorname{diam}(H)$ (i.e., the largest length of a shortest path between any two vertices in H) $= 2$ and in the latter case $\operatorname{diam}(H) = 3$. It is easily verified that in the former case $\gamma_w(H) = 3 = \beta_0(H)$ and in the latter case $\gamma_w(H) = 3 = \beta_0(H)$ or $\gamma_w(H) = 2 = \beta_0(H)$ according to whether e_2 is a pendant edge of H or not. Thus, the result is seen to hold in this case as well.

Hence, suppose the result holds for all hypertrees of size less than an arbitrarily given size $q \geqslant 4$. Let $(u_0, e_1, u_1, e_2, u_2, e_3, u_3, \ldots, u_{k-1}, e_k, u_k)$ be a diametrical path in H (i.e., $k = \operatorname{diam}(H)$). Trivially, $k > 1$ since $q \geqslant 4$. If $k = 2$ then $\gamma_w(H) = |P(u_1)| = |E| = \beta_0(H)$. If $k = 3$, then $\gamma_w(H) = |P(e_2)| = \beta_0(H)$ when e_2 has no pendant vertices and $\gamma_w(H) = |P(e_2)| + 1 = \beta_0(H)$ when e_2 has a pendant vertex, again implying the desired result in either case.

Hence, let H be any hypertree of size $q \geqslant 4$ and $k \geqslant 4$.

Case 1: e_2 does not contain a pendant vertex.

Let H' be the subhypergraph of H obtained after removing e_2 and all pendant edges having their supports in $e_2 - \{u_2\}$ one by one in succession. Let S be any $\gamma_w(H')$-set.

Subcase 1 (a): $u_2 \notin S$

Let S' be formed by choosing one vertex each arbitrarily from each member of $P(e_2 - \{u_2\})$. Then $S \cup S'$ is a WEDD-set of H, whence we see that $\gamma_w(H) \leqslant \gamma_w(H') + |S'|$. Further, let T be a $\beta_0(H')$-set. Then $T \cup S'$ is a maximal independent set of H and, therefore, $\beta_0(H) \geqslant \beta_0(H') - |S'|$. Also, by the induction hypothesis, we have $\gamma_w(H') \leqslant \beta_0(H')$. The above inequalities yield $\gamma_w(H) \leqslant \beta_0(H)$.

Subcase 1 (b): $u_2 \in S$

Since S is a minimal WEDD-set of H, there exists a subset A of $X' - S$ such that $S \cap N_w(u) = \{u_2\}$ for every $u \in A$. Further, since in H' not every vertex u in any edge other than e_3 can satisfy $S \cap N_w(u) = \{u_2\}$, it follows that $A \subseteq e_3$. Now, $d_{H'}(u) \geqslant d_{H'}(u_2)$ for every $u \in A$. However, $d_{H'}(u) = d_H(u)$ for every $u \in A$ because $A \subseteq e_3$. Let $A_1 \subseteq A$ be such that $d_{H'}(u) = d_{H'}(u_2)$ for every $u \in A_1$ and $d_{H'}(v) > d_{H'}(u_2)$ for every $v \in A - A_1$. Since $d_{H'}(u_2) + 1 = d_H(u_2)$ we see that $d_H(u_2) \leqslant d_H(v)$ for every $v \in A - A_1$. Also, for every $u \in A_1$, $d_{H'}(u) = d_H(u) = d_H(u_2) - 1$, which yields $d_H(u) < d_H(u_2)$ for every $u \in A_1$. Now, choose a vertex $w_0 \in A_1$. Then for every $u \in A_1 - \{w_0\}$, $d_H(u) = d_H(w_0)$ and for every $v \in A - A_1$ we have $d_H(v) \geqslant d_H(\{u_2\}) > d_H(w_0)$. Thus, $d_H(w_0) < d_H(v)$ for every $v \in (A - \{w_0\}) \cup \{u_2\}$ and so $(S - \{u_2\}) \cup \{w_0\} \cup S'$ is a WEDD-set of H and hence $\gamma_w(H) \leqslant \gamma_w(H') + |P(e_2 - \{u_2\})|$. The other part, viz., $\beta_0(H') + |P(e_2 - \{u_2\})| \leqslant \beta_0(H)$, follows as in the Subcase 1 (a), whence we get $\gamma_w(H) \leqslant \beta_0(H)$ as desired.

Case 2: e_2 has a pendant vertex.

Let H'' be the subhypergraph of H obtained after removing all pendant edges having their supports in $e_2 - \{u_2\}$ one by one in succession. In H'', e_2 is a pendant edge. Let S be any $\gamma_w(H'')$-set. Then $S \cap (e_2 - \{u_2\}) = \emptyset$ and, in fact, this set consists of a single vertex, say x_0. Without loss of generality, we may assume that x_0 is a pendant vertex also in H then $x_0 \in e_2^0$, since otherwise $S_y = (S - \{x_0\}) \cup \{y\}$ for some $y \in e_2^0$ would also be a $\gamma_w(H')$-set. Let S' be a set of pendant vertices, one chosen from each member of $P(e_2 - \{u_2\})$. Then $S \cup S'$ is a WEDD-set of H, whence we get $\gamma_w(H) \leqslant \gamma_w(H') + |P(e_2 - \{u_2\})|$. Now, let T be a $\beta_0(H')$-set. Then $|T \cap e_2| = 1$. Let $T \cap e = \{w\}$. If $w = u_2$, then T along with S' is a maximal independent set of H. On the other hand, if $w \neq u_2$, then $w \in e - \{u_2\}$ and w is a pendant vertex in H''. Without loss of generality, we may assume that w is a pendant vertex also in H; that is, $w \in e_2^0$. Then $T \cup S'$ is a maximal independent set of H, whence we see that $\beta_0(H) \geqslant \beta_0(H'') + |P(e_2 - \{u_2\})|$. Now, by the induction hypothesis, recall that $\gamma_w(H'') \leqslant \beta_0(H'')$; this inequality, together with the foregoing ones, yields the result that $\gamma_w(H) \leqslant \beta_0(H)$, as desired. □

3. Concluding remarks

Thus, we have shown the following result:

Theorem 8. *For any hypertree H, $\gamma_i(H) \leqslant \gamma_w(H) \leqslant \beta_0(H)$.*

Fig. 2 (a) exhibits a hypergraph which is not a hypertree but still satisfies the inequalities of Theorem 8.

It is strongly believed that the conclusion of Theorem 8 must also hold for any hypergraph whose cyclomatic number is zero. Nevertheless, as illustrated already in Fig. 2 (a), there do exist hypergraphs with nonzero cyclomatic number that satisfy relation (2). This raises a natural problem of characterizing in general the hypergraphs that satisfy (2). However, a solution of this problem appears to be complex and hence may force a step-by-step approach to solve it finally. A natural next step would be to attempt settling the problem for hypergraphs without significant cycles (cf. [14]), which is being tried presently.

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