Czechoslovak Mathematical Journal

Piotr Kot

Description of simple exceptional sets in the unit ball

Czechoslovak Mathematical Journal, Vol. 54 (2004), No. 1, 55–63

Persistent URL: `http://dml.cz/dmlcz/127863`

Czechoslovak Mathematical Journal, 54 (129) (2004), 55–63

DESCRIPTION OF SIMPLE EXCEPTIONAL SETS IN THE UNIT BALL

Piotr Kot, Kraków

(Received March 21, 2001)

Abstract. For $z \in \partial B^n$, the boundary of the unit ball in $\mathbb{C}^n$, let $\Lambda(z) = \{\lambda\colon |\lambda| \leqslant 1\}$. If $f \in \mathbb{O}(B^n)$ then we call $E(f) = \{z \in \partial B^n\colon \int_{\Lambda(z)} |f(z)|^2 \,\mathrm{d}\Lambda(z) = \infty\}$ the exceptional set for f. In this note we give a tool for describing such sets. Moreover we prove that if E is a G_δ and F_σ subset of the projective $(n-1)$-dimensional space $\mathbb{P}^{n-1} = \mathbb{P}(\mathbb{C}^n)$ then there exists a holomorphic function f in the unit ball B^n so that $E(f) = E$.

Keywords: boundary behavior of power series, exceptional set

MSC 2000: 30B30

1. Introduction

Let $\mathbb{S}$ denote the unit sphere in the complex space $\mathbb{C}^n$. Wojtaszczyk constructed in [6, Theorem 1] a sequence of homogeneous polynomials in $\mathbb{C}^n$ with special properties on the boundary of the unit ball $\mathbb{B}^n$ in $\mathbb{C}^n$. By means of those polynomials he could give an example of a function $f \in \mathbb{O}(\mathbb{B}^n)$, the space of holomorphic functions in $\mathbb{B}^n$, such that $|f|$ is not integrable with any power p, $1 \leqslant p < \infty$, on any slice of the form $\Lambda(z) = \mathbb{C}z \cap \mathbb{B}^n$, where $z \in \mathbb{S}$ (see [6]).

In this note we focus our attention on another related problem. Suppose now that f is a holomorphic function in the unit ball $\mathbb{B}^n$ in $\mathbb{C}^n$. Let Π_1 be the set of all complex one-dimensional linear subspaces of $\mathbb{C}^n$. Let

$$E(f) = \{\Lambda \in \Pi_1\colon\ f|_{\Lambda\cap\mathbb{B}^n} \ \text{ is not } \ L^2\text{-integrable on } \ \Lambda \cap \mathbb{B}^n\}.$$

It turns out that $E(f)$ is a G_δ-set in the natural topology in Π_1. Note that Π_1 can be identified with the projective $(n-1)$-dimensional space $\mathbb{P}^{n-1} = \mathbb{P}(\mathbb{C}^n)$. Now let E be a given arbitrary G_δ-subset of $\mathbb{P}^{n-1}$. We try to construct $f \in \mathbb{O}(\mathbb{B}^n)$

such that $E = E(f)$. Such function can be obtained (see Theorem 3.6 below) by means of modified Wojtaszczyk polynomials; the construction of those polynomials is performed in Theorem 3.5.

We give also examples of functions holomorphic in the unit ball with another kind of bad behavior on one-dimensional slices (Proposition 4.1).

Note that other examples of functions with bad behavior on lower-dimensional subsets of $\mathbb{B}^n$ were given by several authors; see e.g. [2], [3], [4], [6].

2. Slices

There is a natural, unitarily invariant (Lebesgue) measure on $\mathbb{S}$. We normalize it so that the measure of the whole sphere $\mathbb{S}$ equals 1 and we denote this measure by σ. Moreover there exists a natural (Lebesgue) measure on $\mathbb{P}^{n-1}$. We denote this measure by $\sigma_{\mathbb{P}}$. First we prove a result about the relation between homogeneous polynomials and slices $\Lambda(z)$.

Proposition 2.1. *Let $f \in \mathbb{O}(B^n)$ and $f(z) = \sum_{m\in\mathbb{N}} p_m(z)$ where $p_m(z)$ is a sequence of homogeneous polynomials of the degree m. If for $z \in \mathbb{S}$ we denote $\Lambda = \Lambda(z) = \mathbb{C}z \cap \mathbb{B}^n$ then*

$$\int_\Lambda |f|^2 \,\mathrm{d}\Lambda < \infty \Leftrightarrow \sum_{m=1}^{\infty} \frac{|p_m(z)|^2}{m} < \infty.$$

Moreover

$$\int_{\mathbb{B}^n} |f(y)|^2 \,\mathrm{d}y < \infty \Leftrightarrow \sum_{m=1}^{\infty} \int_{w\in\mathbb{P}^{n-1}} \frac{|p_m(w)|^2}{m} \,\mathrm{d}\sigma_{\mathbb{P}} < \infty.$$

P r o o f. For $z \in \partial\mathbb{B}^n$ let $\Lambda = \Lambda(z) = \mathbb{C}z \cap \mathbb{B}^n$. We can calculate

$$\begin{aligned} \int_\Lambda |f|^2 \,\mathrm{d}\Lambda &= \int_{|\lambda|\leqslant 1} |f(\lambda z)|^2 \,\mathrm{d}\lambda = \sum_{m=1}^{\infty} \int_{|\lambda|\leqslant 1} |p_m(z)|^2 |\lambda|^{2m} \,\mathrm{d}\lambda \\ &= 2\pi \sum_{m=1}^{\infty} \int_{0\leqslant r\leqslant 1} |p_m(z)|^2 r^{2m+1} \,\mathrm{d}r = \pi \sum_{m=1}^{\infty} \frac{|p_m(z)|^2}{m+1}. \end{aligned}$$

From this follows

$$\int_\Lambda |f|^2 \,\mathrm{d}\Lambda < \infty \Leftrightarrow \sum_{m=1}^{\infty} \frac{|p_m(z)|^2}{m} < \infty.$$

To prove the second part, first we can easily prove that there exist constants $c, \widetilde{c} > 0$ independent of the choice of the function f such that

$$c \int_{\mathbb{B}^n} |f(y)|^2 \,\mathrm{d}y \leqslant \sum_{m=1}^{\infty} \int_{w \in \mathbb{P}^{n-1}} \int_{|\lambda| \leqslant 1} |p_m(w)|^2 |\lambda|^{2m} \,\mathrm{d}\lambda \sigma_{\mathbb{P}} \leqslant \widetilde{c} \int_{\mathbb{B}^n} |f(y)|^2 \,\mathrm{d}y.$$

Now because

$$\sum_{m=1}^{\infty} \int_{w \in \mathbb{P}^{n-1}} \int_{|\lambda| \leqslant 1} |p_m(w)|^2 |\lambda|^{2m} \,\mathrm{d}\lambda \,\mathrm{d}\sigma_{\mathbb{P}} = \pi \sum_{m=1}^{\infty} \int_{w \in \mathbb{P}^{n-1}} \frac{|p_m(w)|^2}{m+1} \,\mathrm{d}\sigma_{\mathbb{P}}$$

we conclude that

$$\int_{\mathbb{B}^n} |f(y)|^2 \,\mathrm{d}y < \infty \Leftrightarrow \sum_{m=1}^{\infty} \int_{w \in \mathbb{P}^{n-1}} \frac{|p_m(w)|^2}{m} \,\mathrm{d}\sigma_{\mathbb{P}} < \infty.$$

□

Proposition 2.2. *If $f \in \mathbb{O}(\mathbb{B}^n)$ then $E(f)$ is a G_δ subset of the projective space $\mathbb{P}^{n-1}$.*

Proof. Let $f \in \mathbb{O}(B^n)$. There exists a sequence of homogeneous polynomials $p_m(z)$ such that $p_m(z)$ is of degree m for every m and $f(z) = \sum\limits_{k=0}^{\infty} p_k(z)$. Let $h_t(z) = \sum\limits_{k=1}^{\infty} |p_k(z)|^2 k^{-1} t^{2k}$. For $t < s < 1$ we define $g_s(z;\lambda) = \sum\limits_{k=0}^{\infty} p_k(zs)\lambda^k$. There exists $M_s > 0$ such that if $|\lambda| \leqslant 1$, $z \in \mathbb{B}^n$ then $|g_s(z;\lambda)| \leqslant M_s$. Therefore by Cauchy's inequality it follows that $|p_k(zs)| \leqslant M_s$ for $z \in \mathbb{B}^n$. From this it follows that $|p_k(z)| \leqslant s^{-k} M_s$ for $z \in \mathbb{B}^n$. Now, it is clear that h_t is continuous. We define $h(z) = \sup\limits_{t<1} h_t(z)$. Because $E(f) = h^{-1}(\infty)$, therefore it is enough to prove that

$$h^{-1}(\infty) = \bigcap_{M \in \mathbb{N}} \bigcup_{t<1} h_t^{-1}((M;\infty)).$$

Let $z \in \mathbb{B}^n$ be such that $h(z) = \infty$. Let $M \in \mathbb{N}$. There exists $t < 1$ such that $h_t(z) > M$ and therefore $z \in \bigcup\limits_{t<1} h_t^{-1}((M;\infty))$. Moreover if $h(z) < \infty$ for some $z \in \mathbb{B}^n$ then for $M > h(z)$ we have $z \notin \bigcup\limits_{t<1} h_t^{-1}((M;\infty))$. The proof is complete. □

3. Homogeneous polynomials

Definition 3.1. In the complex n-dimensional space $\mathbb{C}^n$ we will always consider the usual scalar product $\langle\cdot,\cdot\rangle$. On the unit sphere $\mathbb{S}$ we will consider a unitary invariant pseudo-metric ϱ:

$$\varrho(z_1, z_2) := \sqrt{1 - |\langle z_1, z_2\rangle|}.$$

As usual, we denote the open ball with center $z_0 \in \mathbb{S}$ and radius r

$$B(z_0; r) := \{z \in \mathbb{S} \colon \varrho(z_0, z) < r\}.$$

There is a natural, unitarily invariant (Lebesgue) measure on $\mathbb{S}$. We normalize it so that the measure of the whole sphere $\mathbb{S}$ equals 1 and we denote this measure by σ. As in the paper [6] using (1.4.5) of [5] we easily compute that

$$r^{2n-2} \leqslant \sigma(B(z_0; r)) \leqslant 2^{n-1} r^{2n-2}.$$

A subset $A \subset \mathbb{S}$ is called α-separated if $\varrho(z_1, z_2) > \alpha$ for all distinct elements z_1 and z_2 of A.

Lemma 3.2. *Suppose that $\{\zeta_1, \ldots, \zeta_s\}$ is a $C/\sqrt{N}$-separated subset of $\mathbb{S}$. Then for $C > 2$ we have $s \leqslant N^{n-1}$.*

P r o o f. Since the balls $B(\zeta_j; C/(2\sqrt{N}))$ are disjoint we get

$$s\,\frac{C^{2n-2}}{2^{2n-2}N^{n-1}} \leqslant \sum_{j=1}^{s} \sigma\left(B\left(\zeta_j; \frac{C}{2\sqrt{N}}\right)\right) \leqslant 1$$

so $s \leqslant N^{n-1}$. $\square$

Now we need the following Lemmas from the paper [6]:

Lemma 3.3 [6, Lemma 2]. *If $A \subset \mathbb{S}$ is $\alpha/\sqrt{N}$-separated then for each $\beta > \alpha$ there exists an integer $K = K(\alpha, \beta)$ such that A can be partitioned into K disjoint $\beta/\sqrt{N}$-separated sets.*

Lemma 3.4 [6, Proposition 1]**.** *There exists a constant $C > 2$ such that for all integers N large enough, for each $C/\sqrt{N}$-separated subset $\{\xi_1, \ldots, \xi_s\}$ of $\mathbb{S}$ and each integer k with $N \leqslant k \leqslant 2N$ the polynomial*

$$p_k(z) := \sum_{j=1}^{s} \langle z, \xi_j \rangle^k$$

satisfies

1. $|p_k(z)| \leqslant 2$ *for all* $z \in \mathbb{S}$,
2. $|p_k(z)| \geqslant 0.5$ *for each* $z \in \mathbb{S}$ *such that* $\varrho(z, \xi_j) \leqslant 1/(4\sqrt{N})$ *for some* $j = 1, \ldots, s$.

Now we are ready to prove the following result (compare: [6, Theorem 1]).

Theorem 3.5. *There exists $K \in \mathbb{N}$ such that for $0 < \varepsilon < 1$ and for each pair of closed subsets D, T of $\mathbb{S}$ such that $\varrho(z, w) > 0$ for all $z \in D$ and all $w \in T$ we can choose $m_0 = m_0(D, T, \varepsilon) \in \mathbb{N}$ and a sequence $p_m(z)$ of homogeneous polynomials of degree m which satisfy*

1. $|p_m(z)| \leqslant 2$ *for all* $z \in \mathbb{S}$, $m > m_0$,
2. $\sum\limits_{i=Km}^{K(m+1)-1} |p_i(z)| \geqslant 0.5$ *for all* $z \in T$, $m > m_0$,
3. $\sum\limits_{i=Km}^{K(m+1)-1} |p_i(z)| \leqslant 2^{-(Km)^{1-\varepsilon}}$ *for all* $z \in D$, $m > m_0$.

Proof. Let C be as in Lemma 3.4. Let $K = K(\alpha, \beta)$ be as in Lemma 3.3 for $\alpha = 0.25$ and $\beta = C$. For $N = Km$ fix a maximal $1/(4\sqrt{N})$-separated subset $A \subset T$. Using Lemma 3.3 we can divide A into at least K disjoint $C/\sqrt{N}$-separated subsets $A_0, A_1, \ldots, A_{K-1}$. We define

$$p_{Km+j}(z) := \sum_{\xi \in A_j} \langle z, \xi \rangle^{Km+j}$$

for $j = 0, 1, \ldots, K-1$. From Lemma 3.4 we infer that there exists m_0 so large that for $m > m_0$ we have $|p_{Km+j}(z)| \leqslant 2$ for all $z \in \mathbb{S}$ and $|p_{Km+j}(z)| \geqslant 0.5$ for

$$z \in \bigcup_{\xi \in A_j} B\Big(\xi; \frac{1}{4\sqrt{N}}\Big).$$

Since $A = \bigcup\limits_{l=0}^{K-1} A_l$ is a maximal $1/(4\sqrt{N})$-separated subset of T we conclude that

$$\bigcup_{j=0}^{K-1} \bigcup_{\xi \in A_j} B\Big(\xi; \frac{1}{4\sqrt{N}}\Big) = \bigcup_{\xi \in A} B\Big(\xi; \frac{1}{4\sqrt{N}}\Big) \supset T$$

and from this it follows that

$$\sum_{i=Km}^{K(m+1)-1} |p_i(z)| \geqslant 0.5 \text{ for all } z \in T, \ m > m_0.$$

Without loss of generality we can assume that m_0 is so large that $\varrho(z,w) > \sqrt{1/N^\varepsilon}$ for all $z \in D$ and $w \in T$. Using Lemma 3.2 we have for m_0 large enough, $m > m_0$, $N = Km$ and $z \in D$

$$\begin{aligned}
\sum_{j=0}^{K-1} |p_{Km+j}(z)| &\leqslant \sum_{j=0}^{K-1} \sum_{\xi \in A_j} |\langle z, \xi\rangle|^{Km+j} \leqslant \sum_{\xi \in A} |\langle z, \xi\rangle|^N \\
&\leqslant \sum_{\xi \in A} \left(1 - \frac{1}{N^\varepsilon}\right)^N \leqslant N^{n-1}\left(1 - \frac{1}{N^\varepsilon}\right)^{N^\varepsilon N^{1-\varepsilon}} \\
&\leqslant \frac{N^{n-1}}{2.5^{N^{1-\varepsilon}}} \leqslant \frac{1}{2^{N^{1-\varepsilon}}}.
\end{aligned}$$

□

Now we prove the main Theorem of this note.

Theorem 3.6. *If E is a G_δ and F_σ subset of $\mathbb{P}^{n-1}$ then we can choose a function $f \in \mathbb{O}(B^n)$ such that $E = E(f)$.*

Proof. There exist sequences D_i and S_i of closed subsets of $\mathbb{P}^{n-1}$ such that $S_i \subset S_{i+1}$, $\bigcup S_i = E$ and $D_i \subset D_{i+1}$, $\bigcup D_i = \mathbb{P}^{n-1} \setminus E$. If $z, w \in \mathbb{S}$ and $\langle z; w\rangle \in \mathbb{R}_+$ then we have

$$\begin{aligned}
\sqrt{2}\varrho(z;w) &= \sqrt{2 - 2|\langle z;w\rangle|} = \sqrt{\langle z;z\rangle + \langle w;w\rangle - 2\langle z;w\rangle} \\
&= \|z - w\|.
\end{aligned}$$

Therefore because $D_i \cap S_i = \emptyset$ we conclude that $\varrho(z;w) > 0$ for each $z \in D_i$ and each $w \in S_i$. By Theorem 3.5 there exist $K \in \mathbb{N}$, $c \in \mathbb{R}_+$, a sequence of numbers m_i such that $Km_i + K \leqslant Km_{i+1}$, and homogeneous polynomials $p_{Km_i+0}(z), \ldots, p_{Km_i+K-1}(z)$ satisfying

1. $\sum\limits_{v=Km_i}^{K(m_i+1)-1} |p_v(z)|^2 \geqslant 1$ for all $z \in S_i$,
2. $\sum\limits_{v=Km_i}^{K(m_i+1)-1} |p_v(z)|^2 \leqslant 1/2^i$ for all $z \in D_i$,
3. $|p_\nu(z)| \leqslant c$ for all $z \in \mathbb{S}$ and $\nu = Km_i + j$, $i \in \mathbb{N}$, $j = 0, 1, \ldots, K-1$.

Let $\mathbb{A} = \{Km_i + j\colon\ i \in \mathbb{N},\ j = 0, \ldots, K-1\}$. We define $f(z) := \sum\limits_{v\in\mathbb{A}} v^{1/2} p_v(z)$. Because $|p_v(z)| \leqslant c|z|^v$ for all $z \in B^n$ we have $f \in \mathbb{O}(B^n)$. If $z \notin E$ then there exists $j_0 \in \mathbb{N}$ such that $z \in D_j$ for all $j \geqslant j_0$ and therefore we have

$$\sum_{v\in\mathbb{A}} |p_v(z)|^2 \leqslant \sum_{v\in\mathbb{A}, v<Km_{j_0}} |p_v(z)|^2 + \sum_{k=j_0}^{\infty} \frac{1}{2^k} < \infty$$

and we conclude that $\int_{\mathbb{C}z\cap\mathbb{B}^n} |f|^2 < \infty$.

If $z \in E$ then there exists i_0 such that $z \in S_i$ for all $i \geqslant i_0$. Therefore:

$$\sum_{v\in\mathbb{A}} |p_v(z)|^2 \geqslant \sum_{k=i}^{\infty} 1 = \infty.$$

Now it is clear that $\int_{\mathbb{C}z\cap\mathbb{B}^n} |f|^2 = \infty$. It follows therefore that $E = E(f)$. □

4. Highly nonintegrable functions

We give a nontrivial example of a highly nonintegrable function in the unit ball as another application of Theorem 3.5 .

Proposition 4.1. *There exists a function $f \in \mathbb{O}(B^n)$ such that $f|_{\mathbb{C}z\cap\mathbb{B}^n}$ is bounded for all $z \in \mathbb{S}$ and $\int_{\mathbb{B}^n} |f|^2 = \infty$.*

P r o o f. There exists a sequence of numbers $\varepsilon_i > 0$, a sequence S_i of closed subsets, and a sequence U_i of open subsets of $\mathbb{P}^{n-1}$ which have the following properties:

1. $S_i \subset U_i$,
2. $U_i \cap U_j = \emptyset$ for $i \neq j$,
3. $\varrho(z, w) > \varepsilon_i$ for all $z \in \mathbb{S} \setminus U_i$ and $w \in S_i$,
4. $\sigma(S_j) > 0$ for all $j \in \mathbb{N}$.

If we define $D_i = \mathbb{P}^{n-1} \setminus U_i$, then D_i are closed in $\mathbb{P}^{n-1}$ and $\varrho(z, w) > 0$ for all $z \in D_i$ and all $w \in S_i$. Because

$$\sum_{i=1}^{K} a_i^2 \leqslant \left(\sum_{i=1}^{K} a_i\right)^2 = \sum_{i=1}^{K}\sum_{j=1}^{K} a_i a_j \leqslant \sum_{i=1}^{K}\sum_{j=1}^{K} (a_i^2 + a_j^2) = 2K \sum_{i=1}^{K} a_i^2$$

for $K \in \mathbb{N}$ and $a_i > 0$, by Theorem 3.5 there exist $K \in \mathbb{N}$, $c \in \mathbb{R}_+$, a sequence of numbers m_j $(j \in \mathbb{N})$ so that $Km_j + K \leqslant Km_{j+1}$ and a sequence of homogeneous polynomials $p_m(z)$ of degree m such that

1. $|p_m(z)| \leqslant c$ for all $z \in \mathbb{S}$,

2. $\sum\limits_{v=Km_j}^{K(m_j+1)-1} |p_v(z)|^2 \geqslant 1$ for all $z \in S_j$,

3. $\sum\limits_{v=Km_j}^{K(m_j+1)-1} |p_v(z)| \leqslant 2^{-\sqrt{m_j}} \leqslant m_j^{-1}$ for all $z \in \mathbb{S} \setminus U_j$.

We can assume that m_j is so large that

$$\sqrt{\sigma(S_j)}m_j \geqslant 2^j \sqrt{Km_j + K}$$

for all $j \in \mathbb{N}$.

We define

$$f(z) := \sum_{j\in\mathbb{N}} \sum_{v=Km_j}^{K(m_j+1)-1} \frac{\sqrt{v}p_v(z)}{\sqrt{\sigma(S_j)}}.$$

Because

$$\frac{\sqrt{v}|p_v(z)|}{\sqrt{\sigma(S_j)}} \leqslant \frac{c\sqrt{Km_j + K}}{\sqrt{\sigma(S_j)}}|z|^{Km_j} \leqslant \frac{cm_j}{2^j}|z|^{Km_j}$$

for $v = Km_j + i$, $i = 0, 1, \dots, K-1$, $j \in \mathbb{N}$, $z \in \mathbb{B}^n$, it is easy to see that $f \in \mathbb{O}(B^n)$. Let $z \in \mathbb{S}$, $\lambda \in \mathbb{C}$ where $|\lambda| = 1$. Because $U_i \cap U_j = \emptyset$ for $i \neq j$ there exists $j_0 \in \mathbb{N}$ so that $z \in \mathbb{S} \setminus \bigcup\limits_{j\geqslant j_0} U_j$. Now we have

$$\begin{aligned} |f(\lambda z)| - \sum_{j\leqslant j_0} \sum_{v=Km_j}^{K(m_j+1)-1} \frac{\sqrt{v}|p_v(z)|}{\sqrt{\sigma(S_j)}} &\leqslant \sum_{j\geqslant j_0} \sum_{v=Km_j}^{K(m_j+1)-1} \frac{\sqrt{v}|p_v(z)|}{\sqrt{\sigma(S_j)}} \\ &\leqslant \sum_{j\geqslant j_0} \frac{\sqrt{Km_j + K}}{m_j\sqrt{\sigma(S_j)}} \leqslant \sum_{j\geqslant j_0} \frac{1}{2^j} < \infty \end{aligned}$$

and we conclude that $f|_{\mathbb{C}z\cap\mathbb{B}^n}$ is bounded.

Moreover we can write

$$\sum_{j\in\mathbb{N}} \sum_{v=Km_j}^{K(m_j+1)-1} \int_{\mathbb{P}^{n-1}} \frac{v|p_v(z)|^2}{v\sigma(S_j)} \geqslant \sum_{j\in\mathbb{N}} \int_{S_j} \frac{1}{\sigma(S_j)} = \infty$$

and we conclude by Proposition 2.1 that $\int_{\mathbb{B}^n} |f|^2 = \infty$. $\square$

References

[1] *J. Globevink*: Holomorphic functions which are highly nonintegrable at the boundary. Israel J. Math. To appear.
[2] *J. Globevnik and E. L. Stout*: Highly noncontinuable functions on convex domains. Bull. Sci. Math. *104* (1980), 417–439.
[3] *J. Globevnik and E. L. Stout*: Holomorphic functions with highly noncontinuable boundary behavior. J. Anal. Math. *41* (1982), 211–216.
[4] *J. Siciak*: Highly noncontinuable functions on polynomially convex sets. Zeszyty Naukowe Uniwersytetu Jagiellonskiego *25* (1985), 95–107.
[5] *W. Rudin*: Function Theory in the Unit Ball of $\mathbb{C}^n$. Springer, New York, 1980.
[6] *P. Wojtaszczyk*: On highly nonintegrable functions and homogeneous polynomials. Ann. Pol. Math. *65* (1997), 245–251.

Author's address: Politechnika Krakowska, Instytut Matematyki, ul. Warszawska 24, 31-155 Kraków, Poland, e-mail: `pkot@usk.pk.edu.pl`.