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UNIFORM EXPONENTIAL ERGODICITY OF STOCHASTIC  
DISSIPATIVE SYSTEMS

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*Dedicated to Ivo Vrkoč on the occasion of his 70th birthday*

*Abstract.* We study ergodic properties of stochastic dissipative systems with additive noise. We show that the system is uniformly exponentially ergodic provided the growth of nonlinearity at infinity is faster than linear. The abstract result is applied to the stochastic reaction diffusion equation in  $\mathbb{R}^d$  with  $d \leq 3$ .

*Keywords:* dissipative system, compact semigroup, exponential ergodicity, spectral gap

*MSC 2000:* 60H15, 60J99, 37A30, 47A35

## 1. INTRODUCTION

In this paper we deal with a semilinear stochastic equation

$$(1.1) \quad \begin{cases} dX = (AX + F(X)) dt + \sqrt{Q} dW, \\ X(0) = x \in E, \end{cases}$$

in a separable Banach space  $(E, \|\cdot\|)$  continuously embedded into a separable Hilbert space  $H$  with the inner product  $\langle \cdot, \cdot \rangle$  and the norm  $|\cdot|$ . We assume that  $F: E \rightarrow E$  is a nonlinear mapping,  $(W_t)$  is a standard cylindrical Wiener process in  $H$  defined on a probability space  $(\Omega, \mathcal{F}, (\mathcal{F}_t), \mathbb{P})$  and  $Q = Q^* \in \mathcal{L}(H)$  is nonnegative. Under

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the assumptions stated below equation (1.1) has a unique solution which defines a Markov  $E$ -valued process with the transition semigroup

$$P_t\varphi(x) = \mathbb{E}_x\varphi(X(t)),$$

and moreover, it has a unique invariant measure  $\mu$ . In this paper we provide conditions under which the convergence to an invariant measure is uniformly ergodic in the following sense: There exist positive constants  $C$  and  $\gamma$  such that

$$(1.2) \quad \|P_t^*\nu - \mu\|_{\text{var}} \leq Ce^{-\gamma t}\|\nu - \mu\|_{\text{var}} \leq 2Ce^{-\gamma t},$$

for any Borel probability measure  $\nu$  on  $E$ , where  $\|\cdot\|_{\text{var}}$  denotes the norm of total variation of measures and  $P_t^*$  is the adjoint Markov semigroup (in some papers  $P_t^*\nu$  is also denoted by  $\nu P_t$ ). This result is known as the uniform exponential ergodicity of the Markov process associated with the transition semigroup  $(P_t)$ . Note that the convergence in (1.2) is uniform with respect to all initial probability measures. This property is rather unusual on a non-compact state space. For example, if  $F = 0$  then (1.2) never holds. However, we assume below that the growth of  $F$  is faster than linear at infinity (since  $\varepsilon > 0$  in Hypothesis 1.3), and it turns out that (1.2) is satisfied, that is  $\|P_t^*\nu - \mu\|_{\text{var}}$  is small for large values of  $t$ , even if  $\nu = \delta_a$  (say) with the  $\|a\|$  arbitrarily large.

The strong (variational) convergence of  $P_t^*\nu$  to the invariant measure for stochastic evolution equations has been investigated in numerous papers (see [17]–[20], the monograph [9] and the references therein or the survey paper [21]). The geometric ergodicity (which corresponds to the convergence (1.2) where the constant  $C$  may depend on the initial measure  $\nu$ ) was studied in [15] and [27]. If the diffusion process  $X$  is reversible, then as a corollary of (1.2) we obtain

$$(1.3) \quad \int_E |P_t\varphi(x) - \langle\varphi, \mu\rangle|^2\mu(dx) \leq e^{-\gamma t} \int_E |\varphi(x)|^2\mu(dx),$$

where  $\langle\varphi, \mu\rangle = \int \varphi d\mu$ . Existence of the spectral gap for dissipative system (1.1) and for other infinite dimensional Markov processes has been recently an object of intense study, see for example [1], [6], [7], [10], [29], [30], [25].

We will formulate now the main assumptions of the paper.

**Hypothesis 1.1.** *There exists an operator  $A_0$  in  $H$  such that  $A_0$  is an infinitesimal generator of a  $C_0$ -semigroup  $\mathcal{S} = (S(t))$  on  $H$  and  $A$  is a part of  $A_0$  in  $E$ , that is*

$$\text{dom}(A) = \{x \in \text{dom}(A_0) \cap E : A_0x \in E\},$$

and  $A = A_0|_{\text{dom}(A)}$ . Moreover, we assume that  $A$  generates a compact  $C_0$ -semigroup in  $E$  (which we again denote by  $\mathbf{S}$ ) and

$$(1.4) \quad \int_0^T t^{-\alpha} \|S(t)Q^{1/2}\|_{\text{HS}}^2 dt < \infty,$$

for certain  $\alpha, T > 0$ , where  $\|B\|_{\text{HS}}$  stands for a Hilbert-Schmidt norm of an operator  $B \in \mathcal{L}(H)$ .

It follows from Hypothesis 1.1 that the stochastic convolution integral

$$Z(t) = \int_0^t S(t-s)\sqrt{Q} dW(s), \quad t \geq 0,$$

is well defined and has an  $H$ -continuous version.

Our next assumption concerns regularity of the process  $Z$ .

**Hypothesis 1.2.** *There exists  $E$ -valued,  $E$ -continuous version of the process  $Z$ , such that*

$$(1.5) \quad \sup_{t \geq 0} \mathbb{E}\|Z(t)\|^2 < \infty$$

Our next hypothesis is basically a condition on the nonlinear term  $F$ . By  $\langle \cdot, \cdot \rangle_{E, E^*}$  we denote the duality between  $E$  and  $E^*$  and by  $\partial\|\cdot\|$  the subdifferential of the norm  $\|\cdot\|$ .

**Hypothesis 1.3.** *The mapping  $F: E \rightarrow E$  is Lipschitz continuous on bounded sets and for each  $x \in \text{dom}(A)$  there exists  $x^* \in \partial\|x\|$  such that for some  $k_1, k_2, k_3 > 0$*

$$(1.6) \quad \langle Ax, x^* \rangle_{E, E^*} \leq 0,$$

$$(1.7) \quad \langle F(x+y), x^* \rangle_{E, E^*} \leq -k_1\|x\|^{1+\varepsilon} + k_2\|y\|^s + k_3, \quad y \in E.$$

The solution to equation (1.1) is defined as an  $E$ -continuous adapted process  $X$  satisfying the integral equation

$$(1.8) \quad X(t) = S(t)x + \int_0^t S(t-s)F(X(s)) ds + Z(t), \quad t \geq 0.$$

**Proposition 1.4.** *Assume that Hypotheses 1.1, 1.2 and 1.3 hold. Then for each  $x \in E$  there exists a unique solution  $X$  to equation (1.1). Moreover, the equation (1.1) defines an  $E$ -valued Markov process in the usual way.*

**P r o o f.** The existence and uniqueness of solutions to (1.1) follows immediately from Theorem 7.10 in [8]. The Markov property may be shown as in [8].  $\square$

Let  $(P_t)$  be the Markov semigroup associated to equation (1.1), that is

$$(1.9) \quad P_t \varphi(x) = \mathbb{E}_x \varphi(X(t)), \quad x \in E, \quad t \geq 0, \quad \varphi \in \mathcal{M}(E),$$

where  $\mathbb{E}_x$  denotes the expectation corresponding to the initial condition  $X(0) = x$  and  $\mathcal{M}(E)$  denotes the space of bounded measurable functions on  $E$ . Set

$$P(t, x, \Gamma) = P_t I_\Gamma(x), \quad x \in E, \quad \Gamma \in \mathcal{B}(E),$$

where  $\mathcal{B}(E)$  stands for the Borel  $\sigma$ -algebra on  $E$ . Let  $\mathcal{P}$  be the set of Borel probability measures on  $E$  and let  $(P_t^*)$  denote the adjoint Markov semigroup acting on measures, i.e.,

$$P_t^* \nu(\Gamma) = \int_E P(t, x, \Gamma) \nu(dx), \quad t \geq 0, \quad \Gamma \in \mathcal{B}(E), \quad \nu \in \mathcal{P}.$$

Recall that an invariant measure  $\mu \in \mathcal{P}$  is defined as a stationary point of the dynamical system  $(P_t^*)$ , that is  $P_t^* \mu = \mu$  for all  $t \geq 0$ . Further, recall that the Markov semigroup  $(P_t)$  is called strongly Feller on  $E$  if  $P_t(\mathcal{M}) \subset C_b(E)$  for each  $t > 0$  (or, alternatively, if the mapping  $x \rightarrow P(t, x, \Gamma)$  is continuous on  $E$  for each  $t > 0$  and  $\Gamma \in \mathcal{B}(E)$ ), and  $(P_t)$  is called topologically irreducible if  $P(t, x, U) > 0$  for each  $t > 0$ ,  $x \in E$  and every open set  $U \subset E$ . Our last assumption is

**Hypothesis 1.5.** *The Markov semigroup  $(P_t)$  associated to the solution of equation (1.1) is strongly Feller and topologically irreducible.*

In Propositions 2.7–2.9 below, sufficient conditions for the strong Feller property and topological irreducibility are expressed in terms of coefficients of equation (1.1). Basically, they are reformulations of known results from [17], [18] and [20] (see also the monographs [8] and [9]). It is well known that Hypothesis 1.5 and the existence of an invariant measure  $\mu \in \mathcal{P}$  yield  $P_t^* \nu \rightarrow \mu$  as  $t \rightarrow \infty$  in the total variation norm for every initial measure  $\nu \in \mathcal{P}$  (see e.g. [26]).

## 2. UNIFORM EXPONENTIAL ERGODICITY AND SOME AUXILIARY RESULTS

**Proposition 2.1.** *Assume Hypotheses 1.1, 1.2 and 1.3. Then there exists  $M > 0$  such that*

$$(2.1) \quad \sup_{x \in E} \sup_{t \geq 1} \mathbb{E}_x \|X(t)\| \leq M.$$

*Proof.* Let us note first that in virtue of the Fernique theorem Hypothesis 1.2 implies

$$\sup_{t \geq 0} \mathbb{E} \|Z(t)\|^p < \infty,$$

for all  $p > 0$ , since the process  $Z$  is Gaussian in  $E$ . For  $x \in E$  set  $Y^x(t) = X(t) - Z(t)$ , where  $X$  is the solution to (1.1) starting from  $X(0) = x$  so that

$$(2.2) \quad Y^x(t) = S(t)x + \int_0^t F(Y^x(s) + Z(s)) \, ds, \quad t \geq 0.$$

We will prove first that for each  $x \in E$  and  $p \geq \frac{1}{2}$ ,

$$(2.3) \quad \sup_{t \leq T} \mathbb{E} \|Y^x(t)\|^{2p} < \infty$$

for arbitrary fixed  $T > 0$ . In the proof of (2.3) we follow similar proofs (see Theorem 7.10 of [8] or Lemma 2.2 of [13]), so we omit some details. For  $\alpha > 0$  we define  $R(\alpha) = \alpha(\alpha I - A)^{-1}$ , and

$$(2.4) \quad Y_\alpha^x(t) = R(\alpha)S(t)x + \int_0^t R(\alpha)S(t-s)F(Y_\alpha^x(s) + Z(s)) \, ds, \quad t \leq T$$

(note that (1.6) implies contractivity of  $S$ , so  $R(\alpha)$  is well defined for each  $\alpha > 0$ ). It is well known that

$$(2.5) \quad Y_\alpha^x \rightarrow Y^x, \quad \frac{dY_\alpha^x}{dt} - AY_\alpha^x - F(Y_\alpha^x + Z(t)) = \delta_\alpha^x \rightarrow 0,$$

uniformly in  $t \leq T$  as  $\alpha \rightarrow \infty$  (cf. p. 201 of [8]). Also,

$$(2.6) \quad \begin{aligned} \frac{d^-}{dt} \|Y_\alpha^x(t)\|^{2p} &= 2p \|Y_\alpha^x(t)\|^{2p-1} \frac{d^-}{dt} \|Y_\alpha^x(t)\| \\ &\leq 2p \|Y_\alpha^x(t)\|^{2p-1} (k_2 \|Z(t)\|^s + k_3 + \|\delta_\alpha^x(t)\|), \end{aligned}$$

by Hypothesis 1.3. Therefore, for  $t \leq T$ ,

$$(2.7) \quad \|Y_\alpha^x(t)\|^{2p} \leq \|Y_\alpha^x(0)\|^{2p} + \int_0^t 2p \|Y_\alpha^x(u)\|^{2p-1} (k_2 \|Z(u)\|^s + k_3 + \|\delta_\alpha^x(u)\|) \, du.$$

Taking  $p = \frac{1}{2}$  and passing with  $\alpha$  to infinity we obtain

$$(2.8) \quad \|Y^x(t)\| \leq \|Y^x(0)\| + \int_0^t (k_2 \|Z(u)\|^s + k_3) \, du,$$

and (2.3) follows for  $p = \frac{1}{2}$ . By (2.5) and (2.7) we can see also that for  $t \leq T$  the norm  $\|Y_\alpha^x(t)\|$  is bounded uniformly in  $\alpha$ . Hence, passing with  $\alpha$  to infinity in (2.7) we arrive at

$$(2.9) \quad \|Y^x(t)\|^{2p} \leq \|Y^x(0)\|^{2p} + \int_0^t 2p \|Y^x(u)\|^{2p-1} (k_2 \|Z(u)\|^s + k_3) \, du.$$

Now it is easy to prove (2.3) for arbitrary  $p > 0$  by induction (with the induction step  $\frac{1}{2}$ ) using 1.2 and the Hölder inequality on the right hand side of (2.9) (cf. Lemma 2.2 of [13]). Using Hypothesis 1.3 we find that

$$(2.10) \quad \frac{d^-}{dt} \|Y_\alpha^x(t)\| \leq -k_1 \|Y_\alpha^x(t)\|^{1+\varepsilon} + k_2 \|Z(t)\|^s + k_3 + \|\delta_\alpha(t)\|,$$

and proceeding as above we obtain

$$(2.11) \quad \begin{aligned} \|Y^x(t)\| &\leq \|Y^x(\tau)\| - k_1 \int_\tau^t \|Y^x(u)\|^{1+\varepsilon} du \\ &\quad + k_2 \int_\tau^t \|Z(u)\|^s du + k_3(t - \tau), \end{aligned}$$

for  $0 \leq \tau \leq t$ , which by the Jensen inequality yields

$$(2.12) \quad \begin{aligned} \mathbb{E}\|Y^x(t)\| &\leq \mathbb{E}\|Y^x(\tau)\| - k_1 \int_\tau^t (\mathbb{E}\|Y^x(u)\|)^{1+\varepsilon} du \\ &\quad + C(t - \tau), \quad t \geq \tau \geq 0, \end{aligned}$$

for a certain  $C > 0$ . Note that by (2.3) the random variables  $\|Y^x(t)\|$ ,  $t \leq T$ , are uniformly integrable, hence the function

$$\varphi(t) = \mathbb{E}\|Y^x(t)\|$$

is continuous. A standard comparison theorem yields

$$(2.13) \quad \varphi(t) \leq y(t), \quad t \geq 0,$$

where  $y$  solves the equation

$$(2.14) \quad \begin{cases} \dot{y} = -k_1 y^{1+\varepsilon} + C, & t \geq 0 \\ y(0) = \|x\|. \end{cases}$$

By (2.13) and (2.14) it follows that

$$(2.15) \quad \mathbb{E}\|Y^x(t)\| \leq \max\left(\left(\frac{2C}{k_1}\right)^{1+\varepsilon}, \left(\frac{2}{k_1\varepsilon} + 2\right)^{1/\varepsilon}\right), \quad t \geq 1, \in E,$$

which together with Hypothesis 1.2 completes the proof of (2.1). □

**Lemma 2.2.** *Assume Hypotheses 1.1, 1.2 and 1.3. Then there exist a compact  $K \subset E$  and  $\kappa > 0$  such that*

$$(2.16) \quad \inf_{x \in E} P(2, x, K) \geq \kappa.$$

*Proof.* Step 1. We will show first that the set of probability laws

$$\mathcal{P}(r) = \{\mathcal{L}(Y^x(1) + Z(1)) : \|x\| \leq r\},$$

is relatively compact in  $E$  for each  $r > 0$ . Indeed, since the semigroup  $\mathcal{S}$  is compact in  $E$ , the set

$$K_1 = \{\mathcal{S}(1)y : \|y\| \leq r\}$$

is relatively compact in  $E$ . Moreover, the operator

$$L^2(0, 1; E) \ni f \rightarrow Tf = \int_0^1 \mathcal{S}(1-u)f(u) du \in E,$$

where the integral is defined in the Bochner sense, is compact. Therefore, putting

$$\tilde{B}(r_1) = \{f \in L^2(0, 1; E) : \|f\|_{L^2(0,1;E)} \leq r_1\},$$

we find that  $T(\tilde{B}(r_1))$  is relatively compact in  $E$ . Let

$$\Omega(r_2) = \left\{ \omega : \sup_{t \leq 1} \|Z(t)\| \leq r_2 \right\}.$$

If  $\omega \in \Omega(r_2)$  and  $\|x\| \leq r$  then invoking (2.8) we obtain

$$\|Y^x(t)\| \leq r + \int_0^t (k_2 r_2^s + k_3) du = r + k_2 r_2^s + k_3, \quad t \leq 1,$$

and since  $F$  is bounded on bounded sets of  $E$ ,

$$\sup_{x \in B(r), \omega \in \Omega(r_2)} \sup_{t \leq 1} \|F(Y^x(t) + Z(t))\| \leq \sup_{y \in B(R)} \|F(y)\| < \infty,$$

where  $R = r + r_2 + k_2 r_2^s + k_3$ . Let

$$f_{x,\omega}(t) = F(Y^x(t) + Z(t)).$$

Then

$$\mathcal{U}(r) = \{f_{x,\omega} : x \in B(r), \omega \in \Omega(r_2)\} \subset \tilde{B}(R)$$



and therefore the set  $K_2 = T\mathcal{U}(r)$  is relatively compact in  $E$ . For a given  $\eta \in (0, 1)$  we choose  $r_2$  in such a way that

$$\mathbb{P}(\Omega(r_2)) \geq 1 - \frac{1}{2}\eta.$$

Let  $K_3 \subset E$  be such a compact set that

$$\mathbb{P}(Z(1) \in K_3) \geq 1 - \frac{1}{2}\eta,$$

and let  $\Omega_1 = \{\omega : Z(1) \in K_3\}$ . Finally, let  $K(r) = K_1 + K_2 + K_3$ . Then, for  $x \in B(r)$ ,

$$\mathbb{P}(Y^x(1) + Z(1) \in K_1 + K_2 + K_3) \geq \mathbb{P}(\Omega(r_2) \cap \Omega_1) \geq 1 - \eta.$$

Step 2. Conclusion. It follows from Step 1 that for each  $\eta \in (0, 1)$  and  $r > 0$  there exists a compact set  $K(r) \subset E$  such that

$$\inf_{\|y\| \leq r} P(1, y, K(r)) > 1 - \eta.$$

Moreover, (2.1) yields the existence of  $R > 0$  such that

$$P(1, x, B(R)) \geq 1 - \eta, \quad x \in E.$$

Then by the Chapman-Kolmogorov equality

$$P(2, x, K(r)) \geq \int_{B(R)} P(1, y, K(R))P(1, x, dy) \geq (1 - \eta)^2,$$

which completes the proof of the lemma.  $\square$

Let us recall some basic concepts of Ergodic Theory of Markov chains. Let  $(X_i)$  be an  $E$ -valued Markov chain with the transition kernel  $P^m(x, \Gamma)$ ,  $m \in \mathbb{N}$ ,  $x \in E$ ,  $\Gamma \in \mathcal{B}(E)$ , and let  $\varphi \geq 0$  be a nontrivial measure on  $\mathcal{B}(E)$ . The chain  $(X_i)$  is called  $\varphi$ -irreducible if for each  $\Gamma \in \mathcal{B}(E)$  with  $\varphi(\Gamma) > 0$  we have

$$(2.17) \quad \sum_{i=1}^{\infty} P^i(x, \Gamma) > 0, \quad x \in E.$$

Recall that a set  $\Pi \in \mathcal{B}(E)$  is called a small set if there exist  $m \in \mathbb{N}$  and a nontrivial measure  $\lambda \geq 0$  such that

$$(2.18) \quad \inf_{x \in \Pi} P^m(x, \cdot) \geq \lambda(\cdot).$$

We will need the following result which is an immediate consequence of Lemma 2 in [14], see also Theorem 5.2.2 in [22].

**Lemma 2.3.** *Let  $(X_i)_{i \in \mathbb{N}}$  be  $\varphi$ -irreducible. Then there exists a small set  $\Pi \in \mathcal{B}(E)$  such that  $\varphi(\Pi) > 0$ .*

**Theorem 2.4.** *Assume Hypotheses 1.1–1.3 and 1.5. Then there exists an invariant measure  $\mu \in \mathcal{P}$  such that for certain constants  $C > 0$ , and  $\gamma > 0$  we have*

$$(2.19) \quad \|P_t^* \nu - \mu\|_{\text{var}} \leq C e^{-\gamma t} \|\nu - \mu\|_{\text{var}} \leq 2C e^{-\gamma t}$$

for all  $t > 0$  and  $\nu \in \mathcal{P}$ , where  $\|\cdot\|_{\text{var}}$  stands for the norm of total variation of measures.

*Proof.* Consider the skeleton chain  $(X_n)$ , where  $X_i = X(i)$  and for a fixed  $x_0 \in E$  set  $\varphi(\cdot) = P(1, x_0, \cdot)$ . It is well known that by Hypothesis 1.5 the measures  $\{P(t, x, \cdot) : t > 0, x \in E\}$  are equivalent hence the chain  $(X_n)$  is  $\varphi$ -irreducible and by Lemma 2.3 there exists a set  $\Pi \in \mathcal{B}(E)$  such that

$$(2.20) \quad P(1, x_0, \Pi) > 0,$$

and

$$(2.21) \quad \inf_{x \in \Pi} P(m, x, \Gamma) \geq \lambda(\Gamma), \quad \Gamma \in \mathcal{B}(E),$$

for some  $m \in \mathbb{N}$  and a nontrivial measure  $\lambda$ . By (2.16) we have

$$(2.22) \quad \begin{aligned} \inf_{x \in E} P(m+3, x, \Gamma) &\geq \inf_{x \in E} \int_{\Pi} P(3, x, dy) P(m, y, \Gamma) \\ &\geq \lambda(\Gamma) \inf_{x \in E} P(3, x, \Pi) \\ &= \lambda(\Gamma) \inf_{x \in E} \int_E P(1, y, \Pi) P(2, x, dy) \\ &\geq \lambda(\Gamma) \inf_{x \in E} \int_K P(1, y, \Pi) P(2, x, dy) \\ &\geq \kappa \lambda(\Gamma) \inf_{y \in K} P(1, y, \Pi). \end{aligned}$$

By (2.20) and the equivalence of transition measures  $P(1, y, \Pi) > 0$  for all  $y \in E$ . Since the function  $y \rightarrow P(1, y, \Pi)$  is continuous by the Strong Feller Property and  $K$  is compact, we obtain

$$(2.23) \quad \inf_{x \in E} P(m+3, x, \Gamma) \geq \kappa \delta \lambda(\Gamma), \quad \Gamma \in \mathcal{B}(E),$$

for a certain  $\delta > 0$ . For  $T = m + 3$  it follows that

$$(2.24) \quad P_{t+T}^* \nu(\Gamma) = \int_E \int_E P(T, y, \Gamma) P(t, x, dy) \nu(dx) \geq \tilde{\mu}(\Gamma), \quad t \geq 0, \nu \in \mathcal{P},$$

where  $\tilde{\mu}(\cdot) = \kappa \delta \lambda(\cdot)$ . Hence,  $\tilde{\mu}$  is a nontrivial lower bound measure and it follows that there exists an invariant measure  $\mu \in \mathcal{P}$  (see e.g. [16]). To prove the exponential convergence, take arbitrary  $\delta_1, \delta_2 \in \mathcal{P}$  and set  $\delta = \delta_1 - \delta_2$ . We will denote by  $\zeta^+$  and  $\zeta^-$  the positive and negative part respectively of a signed measure  $\zeta$ . Obviously, we have

$$(2.25) \quad \eta := \delta^+(E) = \delta^-(E) = \frac{1}{2} \|\delta\|_{\text{var}},$$

and without loss of generality we can assume  $\eta > 0$ . Then

$$(2.26) \quad \|P_t^* \delta\|_{\text{var}} = \eta \left\| \left( P_t^* \left( \frac{1}{\eta} \delta^+ \right) - \tilde{\mu} \right) - \left( P_t^* \left( \frac{1}{\eta} \delta^- \right) - \tilde{\mu} \right) \right\|_{\text{var}}, \quad t \geq 0.$$

Furthermore, by (2.24) the measures  $P_T^* \left( \frac{1}{\eta} \delta^+ \right) - \tilde{\mu}$  and  $P_T^* \left( \frac{1}{\eta} \delta^- \right) - \tilde{\mu}$  are nonnegative, thus

$$(2.27) \quad \left\| P_T^* \left( \frac{1}{\eta} \delta^+ \right) - \tilde{\mu} \right\|_{\text{var}} = P_T^* \left( \frac{1}{\eta} \delta^+ \right)(E) - \tilde{\mu}(E) = 1 - \tilde{\mu}(E),$$

and similarly

$$(2.28) \quad \left\| P_T^* \left( \frac{1}{\eta} \delta^- \right) - \tilde{\mu} \right\|_{\text{var}} = 1 - \tilde{\mu}(E),$$

which by (2.25) and (2.26) yields

$$(2.29) \quad \|P_T^* \delta\|_{\text{var}} \leq \eta(2 - 2\tilde{\mu}(E)) = (1 - \tilde{\mu}(E)) \|\delta\|_{\text{var}}.$$

For  $q = 1 - \tilde{\mu}(E) \in (0, 1)$  the semigroup property of  $P_t^*$  yields

$$(2.30) \quad \|P_{nT}^* \delta\|_{\text{var}} \leq q^n \|\delta\|_{\text{var}}, \quad n \geq 1,$$

which is the geometric ergodicity for the chain  $(X_{nT})$ . Set  $\beta = -\log q > 0$  and let  $[\alpha]$  stand for the integer part of the real number  $\alpha$ . Then

$$(2.31) \quad \begin{aligned} \|P_t^* \delta\|_{\text{var}} &\leq \|P_{t-[\frac{t}{T}]T}^* P_{[\frac{t}{T}]T}^* \delta\|_{\text{var}} \leq \|P_{[\frac{t}{T}]T}^* \delta\|_{\text{var}} \\ &\leq e^{-\beta[\frac{t}{T}]} \|\delta\|_{\text{var}} \leq e^{-\frac{\beta}{T}t} e^{\frac{\beta}{T}(t-[\frac{t}{T}]T)} \|\delta\|_{\text{var}} \leq e^{\beta} e^{-\frac{\beta}{T}t} \|\delta\|_{\text{var}}, \end{aligned}$$

and (2.19) follows with  $\gamma = \frac{\beta}{T}$  and  $C = e^{\beta}$ . □

As a corollary of Theorem 2.4 we obtain the following property.

**Corollary 2.5.** *Assume Hypotheses 1.1–1.3 and 1.5 and let the Markov semigroup  $(P_t)$  be symmetric in  $L^2(E, \mu)$ . Then there exist constants  $C, \gamma > 0$  such that*

$$(2.32) \quad \begin{aligned} \|P_t \varphi - \langle \varphi, \mu \rangle\|_{L^2(E, \mu)} &\leq e^{-\gamma t} \|\varphi - \langle \varphi, \mu \rangle\|_{L^2(E, \mu)} \\ &\leq e^{-\gamma t} \|\varphi\|_{L^2(E, \mu)} \end{aligned}$$

for all  $\varphi \in L^2(E, \mu)$  and  $t \geq 0$ .

*P r o o f.* The proof follows easily from Theorem 2.4 and [3], Theorem 1.2 (see also [24]).  $\square$

**Remark 2.6.** (i) In [10], Section 6.2, estimate (2.32) is obtained essentially for a strongly dissipative symmetric system provided  $Q$  is boundedly invertible. Then, in Section 6.3, an analogue of (2.32) is obtained for systems with the nonlinearity  $F = F_0 + F_1$  with  $F_0$  strongly dissipative and  $F_1$  bounded and  $Q$  still boundedly invertible. However, in the latter case estimate (2.32) holds in  $L^2(E, \mu_0)$ , where  $\mu_0$  is the unique invariant measure of equation (1.1) with  $F_1 = 0$ . Hence, our result and the result from [10] are not exactly comparable. Finally, let us note that in our case (2.32) holds even if  $Q$  is not boundedly invertible, provided  $F = QDG$ , where  $DG$  is the gradient of the mapping  $G: E \rightarrow \mathbb{R}$ .

(ii) Due to [3] the first inequality in (2.32) is equivalent (for symmetric  $(P_t)$ ) to (2.19) which however must be satisfied only if  $\nu \ll \mu$ ,  $\frac{d\nu}{d\mu} \in L^2(E, \mu)$ , and with  $C$  possibly dependent on  $\nu$ . Thus the statement of Theorem 2.4 is essentially stronger than the convergence (2.32).

For the reader's convenience we will amend this section with three propositions which are minor modifications of earlier results [20], [18] and [4], in which Hypothesis 1.5 (strong Feller property and irreducibility) is verified.

**Proposition 2.7.** *Assume Hypotheses 1.1, 1.2 and 1.3. Let*

$$Q_t = \int_0^t S(s)QS^*(s) ds,$$

and let

$$(2.33) \quad S(t)(E) \subset Q_t^{1/2}(H), \quad t > 0.$$

If there exists a mapping  $u \in C(E, H)$  which is bounded on bounded sets and such that  $F = Q^{1/2}u$  then the solution to (1.1) is strong Feller and irreducible

i.e. Hypothesis 1.5 holds. In particular, if  $Q$  is boundedly invertible and  $Q^{1/2} \in \mathcal{L}(E, H)$  then the above conditions hold with  $u = Q^{-1/2}F$ .

**P r o o f.** The Strong Feller Property follows from Theorem 3.1 of [20], where applicability of the Girsanov Theorem to equation (1.1) is also proved. Since (2.33) implies topological irreducibility for the linear equation ( $F = 0$ ), the solution to (1.1) is irreducible as well.  $\square$

Proposition 2.7 is applicable basically (though not exclusively, cf. [20]) to the cases when  $Q$  is boundedly invertible. In the following two statements  $Q^{-1}$  may be unbounded.

**Proposition 2.8.** *Let  $Q > 0$ , assume Hypotheses 1.1, 1.2 and 1.3 and let one of the following conditions be satisfied: either*

- (i)  $S(t)H \subset E$  for  $t > 0$  and  $\|S(t)\|_{H \rightarrow E} \leq q(t)$  with a certain  $q \in L^2(0, T)$ , or
- (ii)  $Q^{1/2} \in \mathcal{L}(H, E)$  and  $Q^{1/2}(H) = E$ .

*Then the solution to equation (1.1) is topologically irreducible.*

**P r o o f.** See Propositions 2.7, 2.8 and 2.11 and Lemma 2.6 of [19].  $\square$

**Proposition 2.9.** *Assume Hypotheses 1.1, 1.2 and 1.3. Moreover, assume that for each  $n \in \mathbb{N}$  there exists a  $k_n < \infty$  such that*

$$(2.34) \quad |F(x) - F(y)| \leq k_n|x - y|, \quad \|x\| + \|y\| \leq n,$$

*(that is  $F$  is Lipschitz continuous on bounded sets of  $E$  with respect to the norm in  $H$ ),  $S(t)(H) \subset Q_t^{1/2}(H)$  for  $t > 0$ , and*

$$(2.35) \quad \int_0^T \|Q_t^{-1/2}S(t)\|_{\mathcal{L}(H)} dt < \infty,$$

*for a certain  $T > 0$ . Then the solution to (1.1) is strongly Feller.*

**P r o o f.** The proof is a simple combination of arguments from [18] and [4] so it is only sketched. Let  $c > 0$  be the norm of the embedding  $j: E \rightarrow H$ . For  $m \geq 1$ , and  $x \in E$  set

$$(2.36) \quad F_m(x) = \begin{cases} F(x) & \text{if } |x| \leq cm, \\ F\left(\frac{cmx}{|x|}\right) & \text{if } |x| > cm. \end{cases}$$

By (2.34)  $F_m$  is uniquely extendible to a bounded, globally Lipschitz function on  $H$  for each  $m \in \mathbb{N}$ . Therefore, the solution to the equation

$$(2.37) \quad \begin{cases} dX_m(t) = (AX_m(t) + F_m(X_m(t))) dt + \sqrt{Q} dW(t), \\ X_m(0) = x, \end{cases}$$

is strongly Feller for each  $m \in \mathbb{N}$  by [4]. Since the paths of solutions to (2.37) and (1.1) coincide with high probability if  $m$  is large, it follows easily from the proof of Proposition 2.1 (i) that

$$(2.38) \quad \lim_{m \rightarrow \infty} \sup_{\|x\| \leq R} \|P_m(t, x, \cdot) - P(t, x, \cdot)\|_{\text{var}} = 0,$$

for all  $t > 0$ ,  $R > 0$  where  $P_m$  denotes the transition kernel associated with (2.37). Hence the solution to (1.1) is strongly Feller as well.  $\square$

**Remark 2.11.** (i) The existence of an invariant measure for (1.1) has been proved independently in [11] (cf. also [12]) by a method based on a version of the Krylov-Bogolyubov argument.

(ii) Note that in the proof of Theorem 2.4 we have proved that the whole space  $E$  is a small set for the chain  $(X_n)$  (cf. (2.22), (2.23)). The exponential ergodicity of the chain  $(X_n)$  follows also by this fact and Theorem 16.2.2 in [22]. However, in the respective part of the proof of Theorem 2.4 we prove the exponential ergodicity directly using a simple argument.

(iii) The method used in the paper can be also easily applied to some cases of stochastic evolution equations with non-additive noise term; basically to the case when the diffusion coefficient is bounded and has bounded inverse and the semigroup  $\mathcal{S}$  is exponentially stable. For example, if the nonlinear drift term  $F$  obeys Hypothesis 1.3 and the conditions (C1)–(C5) from the paper [20] are satisfied the proof of Theorem 2.4 can be repeated (with obvious modifications).

### 3. EXAMPLE

Consider a stochastic parabolic equation

$$(3.1) \quad \begin{cases} \frac{\partial}{\partial t} u(t, \xi) = \Delta u(t, \xi) + f(t, \xi) + \eta(t, \xi), & (t, \xi) \in \mathbb{R}_+ \times D, \\ u(0, \xi) = x(\xi), & \xi \in D, \\ u(t, \xi) = 0, & (t, \xi) \in \mathbb{R}_+ \times \partial D, \end{cases}$$

on a bounded domain  $D \subset \mathbb{R}^d$ ,  $d \leq 3$ , with a smooth boundary  $\partial D$ , where  $f: \mathbb{R} \rightarrow \mathbb{R}$  is locally Lipschitz and  $\eta$  symbolically denotes a noise white in time and, in general, dependent on the space variable  $\xi$ . The system (3.1) is rewritten in a usual manner as an equation of the form (1.1) where we put  $H = L^2(D)$ ,  $E = C_0(D)$ ,  $A_0 = \Delta$  with  $\text{dom}(A_0) = H_0^1(D) \cap H^2(D)$ , and  $F: E \rightarrow E$  is defined as the superposition operator,  $F(y)(\xi) := f(y(\xi))$ , where  $y \in E$ ,  $\xi \in D$ . The noise  $\eta$  is modelled in the

equation (1.1) by the Wiener process  $W_t$  and the covariance operator  $Q \in \mathcal{L}(H)$ , formally we have  $\eta = Q^{1/2}dW/dt$ . It is well known that the operator  $A_0$  generates a strongly continuous semigroup on  $H$ , its part  $A$  on the space  $E$  generates a strongly continuous semigroup on  $E$  as required in Hypothesis 1.1. Assume that there exist positive constants  $c_1, c_2, c_3, s$  and  $\varepsilon$  such that

$$(3.2) \quad f(\alpha + \beta) \operatorname{sgn} \alpha \leq -c_1|\alpha|^{1+\varepsilon} + c_2|\beta|^s + c_3, \quad \alpha, \beta \in \mathbb{R},$$

holds.

Note that  $F$  does not map  $E$  into  $E$ . To address this difficulty we proceed as follows. Let  $f_0(\xi) = f(\xi) - f(0)$  and let  $F_0(x)(\xi) = f_0(x(\xi))$ . Then  $f_0$  satisfies (3.2) and the mapping  $F_0: E \rightarrow E$  is well defined. Equation (1.1) can be rewritten in the form

$$(3.3) \quad \begin{aligned} X(t) = & S(t)x + \int_0^t S(t-s)F_0(X(s)) ds + \int_0^t S(t-s)m ds \\ & + \int_0^t S(t-s)\sqrt{Q} dW(s), \end{aligned}$$

where  $x \in E$  and  $m(\xi) = f(0)$ . Then

$$\int_0^t S(t-s)m ds \in \operatorname{dom}(A) \subset E,$$

and putting

$$Z_m(t) = \int_0^t S(t-s)m ds + \int_0^t S(t-s)\sqrt{Q} dW(s),$$

and  $Y(t) = X(t) - Z_m(t)$  we can rewrite (3.3) in the form

$$Y(t) = S(t)x + \int_0^t S(t-s)F_0(Y(s) + Z_m(s)) ds.$$

Now it is clear, that the proof of existence and uniqueness of solutions provided in the proof of Theorem 7.10 in [8] applies in the present case. Moreover,

$$\int_0^\infty \|S(t)m\| dt < \infty,$$

and therefore the proof of Proposition 2.1 and, consequently, all remaining statements in Section 2 remain valid as well.

Note that (3.2) is satisfied when  $f$  is a polynomial of odd degree larger than one with a negative leading coefficient. It is well known ([28], Theorem 2.2) that the subdifferential of the norm  $\partial\|x\|$  at a point  $x \in E$  contains the Dirac measures  $\delta_{\xi_1}$  or  $-\delta_{\xi_2}$ , if  $\|x\| = x(\xi_1)$  or  $\|x\| = -x(\xi_2)$ , respectively, hence it is easily seen that (3.2) implies (1.7) and Hypothesis 1.3 is verified. The remaining assumptions depend on the covariance operator  $Q$ . Assume at first that  $Q$  is boundedly invertible, that is,  $Q$  is an injection and  $Q^{-1} \in \mathcal{L}(H)$ . Then we have to verify (1.4) with  $Q = I$ . By the well known estimates on the Green functions [2] it follows that

$$\|S(t)\|_{\text{HS}} \leq Ct^{-d/4}, \quad t \in (0, 1],$$

hence (1.4) is satisfied if the dimension  $d$  is one. In fact, for  $d > 1$  and  $Q$  boundedly invertible even the Ornstein-Uhlenbeck process  $Z$  does not take values in  $H$ , so these cases cannot be considered in the present framework. If  $d = 1$ , however, it is easy to see that all remaining conditions are satisfied. Proceeding as in Theorem 4.1 in [23] we easily see that  $Z$  has an  $E$ -valued modification. By the Sobolev embedding theorem, for each  $\delta > \frac{1}{4}$  there exists a constant  $c_\delta < \infty$  such that

$$\|x\| \leq c_\delta |(-A)^\delta x|, \quad x \in \text{dom}((-A_0)^\delta).$$

Take  $\delta \in (\frac{1}{4}, \frac{1}{2})$  and  $p > 2$ ; since  $S$  is exponentially stable we obtain for some constants  $c_1, c_2, c_3$  and  $\omega > 0$

$$\begin{aligned} \sup_{t \geq 0} \mathbb{E} \left\| \int_0^t S(t-r)Q^{1/2} dW_r \right\|^p &\leq c_1 \sup_{t \geq 0} \mathbb{E} \left| (-A_0)^\delta \int_0^t S(t-r)Q^{1/2} dW(r) \right|^p \\ &\leq c_2 \sup_{t \geq 0} \left( \int_0^t \|(-A_0)^\delta S(r)\|_{\text{HS}}^2 dr \right)^{p/2} \\ &\leq c_3 \left( \int_0^\infty r^{-2\delta} e^{-2\omega r} dr \right)^{p/2} < \infty. \end{aligned}$$

The same estimates hold for the process  $Z_m$  and Hypothesis 1.2 is verified. Hypothesis 1.5 (strong Feller property and topological irreducibility) is satisfied in the present case (see e.g. [20]) and we can conclude that in the one-dimensional case if  $Q$  is boundedly invertible (in particular, if  $Q = I$  which corresponds to the case of space-time white noise) and the growth condition (3.2) is satisfied Theorem 2.4 is applicable.

Now we will examine some cases when the covariance  $Q$  may be degenerate. In order to obtain easily verifiable conditions we only consider the so-called diagonal case. We assume that there exists an orthonormal basis  $(e_n)$  in  $H = L^2(D)$  such



that  $e_n \in E$  and for a certain  $C < \infty$

$$\sup_{\xi \in D} |e_n(\xi)| < C, \quad \sup_{\xi \in D} |\nabla e_n(\xi)| < C\sqrt{\alpha_n}, \quad n \geq 1,$$

and such that  $(e_n)$ ,  $(\alpha_n)$  are the respective eigenvectors and eigenvalues of the operator  $-A_0$ ,  $\alpha_n > \omega > 0$ . We assume that the covariance operator  $Q$  has the same eigenvectors  $e_n$  with the respective eigenvalues  $0 < \lambda_n \leq \lambda_0 < \infty$ , that is,

$$Qe_n = \lambda_n e_n, \quad n \geq 1.$$

We again impose condition (3.2) on  $f$ . As in the previous case, we just have to check (1.4) and Hypotheses 1.2 and 1.5. For  $\gamma \in (0, 1)$  we have

$$\begin{aligned} \|S(t)Q^{1/2}\|_{\text{HS}}^2 &= \sum_{n=1}^{\infty} \lambda_n e^{-2\alpha_n t} \leq \sup_{n \geq 1} \alpha_n^{1-\gamma} e^{-2\alpha_n t} \sum_{n=1}^{\infty} \frac{\lambda_n}{\alpha_n^{1-\gamma}} \\ &\leq \frac{\text{const}}{t^{1-\gamma}} \sum_{n=1}^{\infty} \frac{\lambda_n}{\alpha_n^{1-\gamma}} \end{aligned}$$

thus (1.4) is satisfied with  $0 < \delta < \gamma$  provided

$$(3.4) \quad \sum_{n=1}^{\infty} \frac{\lambda_n}{\alpha_n^{1-\gamma}} < \infty$$

holds for some  $\gamma > 0$ . Hypothesis 1.2 has been verified under condition (3.4) in Theorems 5.2.9 and 11.3.1 of [9].

The strong Feller property can be verified by Proposition 2.9. The assumption (2.34) is obviously satisfied by the local Lipschitz continuity of  $f$ . The condition (2.35) is equivalent to

$$(3.5) \quad \sup_{n \in N} \left( \frac{\alpha_n}{\lambda_n} (1 - e^{-2\alpha_n t})^{-1} \right)^{1/2} \in L^1(0, T).$$

It remains to verify that the solution to (3.1) is topologically irreducible. To this end, we will use Proposition 2.11 of [18] according to which, in the present case, it suffices to verify that  $\text{im}(\mathcal{K})$  is dense in  $C_0 := \{y \in C([0, T], E), y(0) = 0\}$  where

$$\mathcal{K}: L^2(0, T, H) \rightarrow C_0, \quad \mathcal{K}u(t) := \int_0^t S(t-r)Q^{1/2}u(r) dr, \quad t \in [0, T].$$

The well known estimates on the Green kernel for a parabolic problem [2] yield

$$(3.6) \quad \|S(t)\|_{\mathcal{L}(H, E)} \leq \frac{\text{const}}{t^{d/4}}, \quad t \in [0, T],$$

hence  $\|S(\cdot)\|_{\mathcal{L}(H,E)}$  is integrable for  $d \leq 3$  and the density follows from [18], Proposition 2.8 and Remark 2.9.

We can summarise that Theorem 1.6 is applicable to the system (3.1) under conditions (3.2), (3.4), (3.5) provided  $d \leq 3$ . In particular, if there exist  $a \geq b \geq 0$  and constants  $k_1, k_2$  such that

$$k_1 \alpha_n^{-a} \leq \lambda_n \leq k_2 \alpha_n^{-b}, \quad n \geq 1,$$

or, equivalently, if

$$K_1 n^{-2a/d} \leq \lambda_n \leq K_2 n^{-2b/d}, \quad n \geq 1,$$

then it is easy to check that the condition (3.4) is satisfied if  $b > \frac{d}{2} - 1$  while (3.5) holds true if  $a < 1$ . So in this case our results are applicable if

$$(3.7) \quad \frac{d}{2} - 1 < b \leq a < 1.$$

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