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DISTANCE IN STRATIFIED GRAPHS

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Abstract. A graph G is stratified if its vertex set is partitioned into classes, called strata. If there are k strata, then G is k-stratified. These graphs were introduced to study problems in VLSI design. The strata in a stratified graph are also referred to as color classes. For a color X in a stratified graph G, the X-eccentricity $e_X(v)$ of a vertex v of G is the distance between v and an X-colored vertex furthest from v. The minimum X-eccentricity among the vertices of G is the X-radius $\operatorname{rad}_X G$ of G and the maximum X-eccentricity is the X-diameter $\operatorname{diam}_X G$. It is shown that for every three positive integers a, b and k with $a \leqslant b$, there exist a k-stratified graph G with $\operatorname{rad}_X G = a$ and $\operatorname{diam}_X G = b$. The number s_X denotes the minimum X-eccetricity among the X-colored vertices of G. It is shown that for every integer t with $\operatorname{rad}_X G \leqslant t \leqslant \operatorname{diam}_X G$, there exist at least one vertex v with $e_X(v) = t$; while if $\operatorname{rad}_X G \leqslant t \leqslant s_X$, then there are at least two such vertices. The X-center $C_X(G)$ is the subgraph induced by those vertices v with $e_X(v) = \operatorname{rad}_X G$ and the X-periphery $P_X(G)$ is the subgraph induced by those vertices v with $e_X(G) = \operatorname{diam}_X G$. It is shown that for k-stratified graphs $H_1, H_2, \ldots, H_k$ with colors $X_1, X_2, \ldots, X_k$ and a positive integer n, there exists a k-stratified graph G such that $C_{X_i}(G) \cong H_i$ $(1 \leqslant i \leqslant k)$ and $d(C_{X_i}(G), C_{X_j}(G)) \geqslant n$ for $i \neq j$. Those k-stratified graphs that are peripheries of k-stratified graphs are characterized. Other distance-related topics in stratified graphs are also discussed.

1. Introduction

Graphs are often useful mathematical models for structures and relationships that occur in real-life phenomena. Design of a *Very Large Scale Integrated Circuit* (VLSI) chip involves many complex processes. Currently, a typical VLSI chip consists of millions of transistors assembled through layering of various materials in a silicon base. At some point during this process the designer of an integrated circuit (IC) transforms a circuit description into a geometric description, which is known as a

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layout. The process of converting the specifications of an electrical circuit into a layout is called the *physical design.* Due to the large number of components and the exacting details required, the physical design is not practical without automation. VLSI design automation is the study of algorithms and data structures employed in the physical design process [11].

Many of the problems encounterd in the physical design process are modeled by graphs. Various problems [9, 10] and via minimization problems [3,8] are among such problems. In recent years, advances in VLSI fabrication technology have made it possible to use more than two routing layers for interconnection. In fact, the two most popular processors on the market today, the PowerPC chip designed by Motorolla, IBM, and Apple, as well as the Pentium processor designed and manufactured by the Intel Corporation, use three or more layers. Figure 1 depicts a 3-layer vertical-horizontal-vertical (VHV) routing problem in a standard cell architecture. In the design of algorithms to solve the multilayer routing problems encountered in this process, it is desirable to use graphs in which the vertices are partitioned into classes.

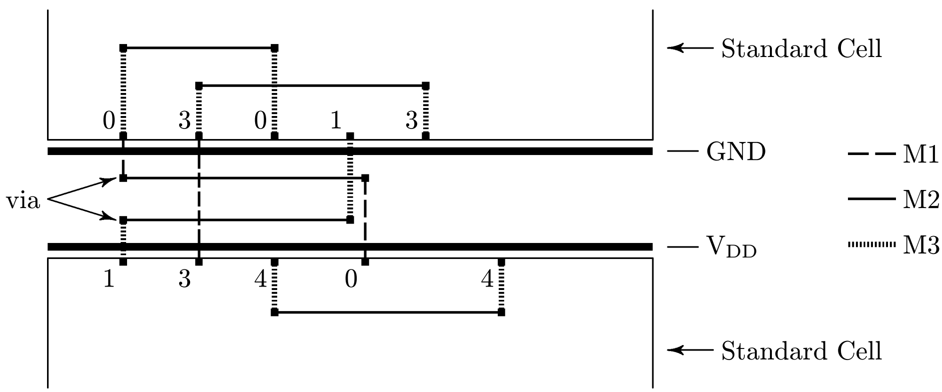


Figure 1. VHV

Dividing the vertex set of a graph into classes according to some prescribed rule is a fundamental process in graph theory. The vertices of a graph can be divided into cut-vertices and non-cut-vertices. Equivalently, the vertices of a tree are divided into its leaves and non-leaves. The vertex set of a graph is partitioned according to the degrees of its vertices. When studying distance, the vertices of a connected graph are partitioned according to their eccentricities. Probably the best known example of this process is graph coloring, where the vertex set of a graph is partitioned into classes each of which is independent in the graph.

In VLSI design, the design of computer chips often yields a division of the nodes into several layers each of which must induce a planar subgraph. So here too the vertex set of a graph is divided into classes. Motivated by these observations, Rashidi [7] defined a graph to be a stratified graph if its vertex set is partitioned into classes.

Formally, then, a graph G is a *stratified graph* if its vertex set $V(G)$ is partitioned into classes, called *strata*. Each class then is a *stratum*. If there are k strata, then G is called a *k-stratified graph*. A 1-stratified graph is the simply a graph. Indeed, an n-stratified graph of order n is essentially a graph as well. Normally, we denote the strata of a k-stratified graph by $S_1, S_2, \ldots, S_k$. The strata are also referred to as color classes, where the vertices of S_i are colored $X_i (1 \leqslant i \leqslant k)$. When specific colors are employed, we use red for X_1, blue of X_2, and yellow for X_3. So the vertices of S_1 are colored red.

In this paper our emphasis is on distance in stratified graphs, either presenting analogues of theorems on graphs or presenting theorems that illustrate differences between graphs and stratified graphs.

2. Distance in stratified graphs

Let G be a connected k-stratified graph. The *distance* $d(u, v)$ between two vertices u and v of G is the length of a shortest $u-v$ path. For a vertex v of G, the *eccentricity* $e(v)$ is the distance between v and a vertex farthest from v. For a color X (one of the k colors), the *X-eccentricity* $e_X(v)$ of a vertex v of G is the distance between v and an X-colored vertex farthest from v. The minimum X-eccentricity among the vertices of G is called the *X-radius* $\operatorname{rad}_X G$, while the maximum X-eccentricity is the X-diameter $\operatorname{diam}_X G$.

Figure 2 shows a 2-stratified graph G (actually a path) whose vertices are therefore colored red (R) and blue (B). The vertices of G are labeled with their red eccentricities. Consequently, $\operatorname{rad}_R G = 2$ and $\operatorname{diam}_R G = 5$. Hence, note that while $\operatorname{rad} G \leqslant \operatorname{diam} G \leqslant 2\operatorname{rad} G$ for ordinary radius and diameter in a graph, such is not the case for $\operatorname{rad}_R G$ and $\operatorname{diam}_R G$ in 2-stratified graphs or indeed for $\operatorname{rad}_X G$ and $\operatorname{diam}_X G$ for a color X in a k-stratified graph G, as we now show.

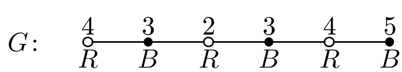


Figure 2.

Theorem 1. *For every three positive integers, a, b, and k with $a \leqslant b$ and $k \geqslant 2$, there exists a k-stratified graph G and a color X such that $\operatorname{rad}_X G = a$ and $\operatorname{diam}_X G = b$.*

Proof. Suppose first that a and b are positive integers such that $a \leqslant b < 2a$. We begin by identifying an end-vertex of the path P_{b-a+1} with a vertex of C_{2a} and to the end-vertex of G as well. The remaining vertices of G are then colored arbitrarily with the remaining $k-1$ colors so that each color has been used. If k is too large to do this, then pendant edges are added to G at its vertex of degree 3 and these new vertices are colored with the remaining colors. The resulting graph G' has X-radius a and X-diameter b. This construction is illustrated in Figure 3 for $a = 4$, $b = 7$, and $k = 5$.

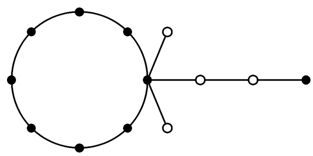


Figure 3.

Next, suppose that a and b are positive integers with $b \geqslant 2a$. Define G to be the path P_{b+1} of order $b+1$, say $v_0, v_1, \ldots, v_b$. The vertices v_0 and v_{2a} are assigned the color X and the remaining $k-1$ colors are distributed among the remaining $b-1$ vertices. If k is too large to do this, then pendant edges are added to G at v_{2a-1} and the new vertices are colored with the remaining colors. The resulting graph G' has X-radius a and X-diameter b. This construction is illustrated in Figure 4 for $a = 3$, $b = 8$, and $k = 9$. □

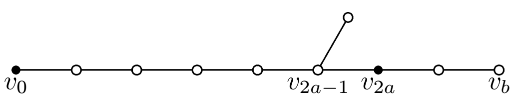


Figure 4.

We now establish some basic results concerning X-eccentricities of vertices in a k-stratified graph.

Theorem 2. *Let X be a color in a connected k-stratified graph G. If $uv \in E(G)$, then $|e_X(u) - e_X(v)| \leqslant 1$.*

P r o o f. Suppose that $e_X(v) \leqslant e_X(u) = t$. We show that $e_X(v) \geqslant t-1$, which will then complete the proof. Let w be an X-colored vertex such that $e_X(u) = d(u,w)$. Thus

$$t = d(u,w) \leqslant d(u,v) + d(v,w) \leqslant 1 + e_X(v).$$

□

We now state some immediate consequences of Theorems 1 and 2.

Corollary 3. *Let G be a connected k-stratified graph and let t be an integer such that $\operatorname{rad}_X G \leqslant t \leqslant \operatorname{diam}_X G$. Then there exists a vertex v of G such that $e_X(v) = t$.*

Let G be a connected k-stratified graph with X as one of its colors. The X-*eccentricity set* is the set of X-eccentricities of the vertices of G.

Corollary 4. *Let G be a connected k-stratified graph with X as one of its colors such that $\operatorname{rad}_X G = r$ and $\operatorname{diam}_X G = d$. Then the X-eccentricity set of G is $\{r, r+1, \ldots, d\}$.*

Corollary 5. *Let $S = \{r, r+1, \ldots, d\}$ be a set of positive integers with $r \leqslant d$. Then S is the X-eccentricity set of some k-stratified graph.*

If G is a connected graph and t is an integer with $\operatorname{rad} G < t \leqslant \operatorname{diam} G$, then Lesniak [5] showed that there are in fact at least two vertices of G having eccentricity t. The corresponding statement for stratified graphs is not true, however; that is, if G is a k-stratified graph and t is an integer such that $\operatorname{rad}_X G < t \leqslant \operatorname{diam}_X G$, then G need not contain two vertices having X-eccentricity t (although by Corollary 3 there must be at least one such vertex). This is illustrated in the 2-stratified graph G of Figure 5, where $\operatorname{rad}_X G = 3$ and $\operatorname{diam}_X G = 9$ but there is only one vertex having X-eccentricity 7,8, or 9.

Let G be a k-stratified graph (with X as one of its colors) and define

$$s_X = \min\{e_X(v)\}$$

over all X-colored vertices v of G. Certainly, $s_X = 0$ if and only if G contains exactly one X-colored vertex. In the stratified graph of Figure 5, $s_X = 6$. We now show that there is an analogous theorem to Lesniak's if $\operatorname{diam}_X G$ is replaced by s_X.

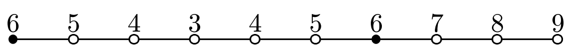
G:


Figure 5.

Theorem 6. *Let G be a connected k-stratified graph with X as one of its colors. If t is an integer such that $\operatorname{rad}_X G < t \leqslant s_X$, then there exist at least two vertices with X-eccentricity t.*

P r o o f. Let u be a vertex with $e_X(u) = t$. Then there is an X-colored vertex v with $d(u, v) = t$. Let w be a vertex such that $e_X(w) = \operatorname{rad}_X G$, and let P be a w–v path in G of length $d(w, v)$. Since v is an X-colored vertex, $e_X(v) \geqslant s_X$. Thus $\operatorname{rad}_X G < t \leqslant s_X \leqslant e_X(v)$. By Corollary 4, there is a vertex z ($\neq w$) on P such that $e_X(z) = t$. Hence $d(w, v) > d(z, v)$. Consequently,

$$d(u, v) = t > \operatorname{rad}_X G \geqslant d(w, v) > d(z, v);$$

so $d(u, v) > d(z, v)$. This implies that u and z are distinct vertices with X-eccentricity t. $\square$

One of the most studied subgraphs of a connected graph is its center. In this section, we introduce the center, indeed a total of k centers, in a connected k-stratified graph and describe some of their properties.

For a color X in a k-stratified graph G, the X-*center* $C_X(G)$ of G is the subgraph induced by those vertices v with $e_X(v) = \mathrm{rad}_X\, G$. Figure 6 shows three 2-stratified graphs G_1, G_2, and G_3 whose vertices are colored red (R) and blue (B), together with their red centers $C_R(G_1)$, $C_R(G_2)$, and $C_R(G_3)$.

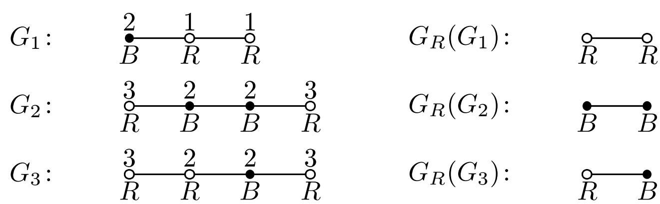


Figure 6.

A well-known property of the center of a connected graph G, due to Harary and Norman [4], is that it always lies in a single block of G. The same is true for k-stratified graphs.

Theorem 7. *Let G be a connected k-stratified graph, $k \geqslant 1$. For each color X of G, the X-center $C_X(G)$ lies in a single block of G.*

P r o o f. Suppose, to the contrary, that for some color X, the X-center $C_X(G)$ does not lie in a single block of G. Then G contains at least two X-colored vertices and there is a cut-vertex v of G such that distinct components of $G - v$ contain vertices of $C_X(G)$. Let u be an X-colored vertex of G such that $e_X(v) = d(u, v)$. Let w be a vertex of $C_X(G)$ belonging to a component of $G - v$ distinct from that containing u. Then

$$e_X(w) \geqslant d(w, u) = d(w, v) + d(v, u) > e_X(v).$$

Thus $e_X(v) < \mathrm{rad}_X\, G$, producing a contradiction. $\square$

We have now an analogue of a well-known result on trees.

Corollary 8. *Let T be a k-stratified tree, $k \geqslant 1$. For each color X, either $C_X(T) \cong K_1$ or $C_X(T) \cong K_2$.*

Hedetniemi (see [2]) showed that every graph is the center of some connected graph. Using this proof technique we establish the corresponding result for stratified graphs.

Theorem 9. *For every k-stratified graph H and each color X of H, there exists a k-stratified graph G such that $C_X(G) = H$.*

P r o o f. Since the result is true for $k = 1$, we assume that $k \geqslant 2$. To construct G, we add four X-colored vertices u_1, v_1, u_2, v_2 to H and join $v_i (i = 1, 2)$ to all vertices of H as well as add the edges $u_i v_i (i = 1, 2)$. Then $e_X(u_i) = 4$ and $e_X(v_i) = 3$ for $i = 1, 2$; while $e_X(w) = 2$ for all vertices w of H. Thus $C_X(G) = \langle V(H) \rangle = H$. (See Figure 7 of an illustration of this construction.) $\square$

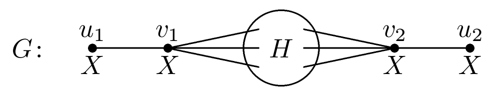


Figure 7.

We now show that it is possible to prescribe all k centers of a k-stratified graph simultaneously. Indeed, we show that every two of these centers can be arbitrarily far apart (if desired).

The *distance between two subgraphs* G_1 and G_2 of a graph G is defined by

$$d(G_1, G_2) = \min\{d(v_1, v_2) | v_1 \in V(G_1), v_2 \in V(G_2)\}.$$

Theorem 10. *Let $H_1, H_2, \ldots, H_k$ be k-stratified graphs with colors $X_1, X_2, \ldots, X_k$. For each integer $n \geqslant 2$, there exists a k-stratified graph G such that $C_{X_i}(G) \cong H_i$ for $i = 1, 2, \ldots, k$ and $d(C_{X_i}(G), C_{X_j}(G)) = n$ for $1 \leqslant i < j \leqslant k$.*

P r o o f. We begin with a copy of each of the k-stratified graphs $H_1, H_2, \ldots, H_k$. For each integer i $(1 \leqslant i \leqslant k)$, add a vertex z_i and all edges of the form $z_i v$ for all $v \in V(H_i)$. Further, for distinct integers i and j with $1 \leqslant i, j \leqslant k$, the vertices z_i and z_j are connected by a path of length $n - 2$, each internal vertex of which belongs to no other path. The colors of the vertices z_i $(1 \leqslant i \leqslant k)$ and all other added vertices are chosen arbitrarily. Next for each i $(1 \leqslant i \leqslant k)$ we add a vertex u_i and all edges $u_i v$ for all $v \in V(H_i)$. For $1 \leqslant i \leqslant k - 1$, the vertex u_i is colored X_{i+1}, and u_k is colored X_1. Finally, for each i $(1 \leqslant i \leqslant k)$, we add a path of length n at u_i, where each vertex along the path is colored X_i. (See Figure 8 for an example having $k = 3$ and $n = 4$.) This completes the construction of the k-stratified graph G, which then has the desired properties. $\square$

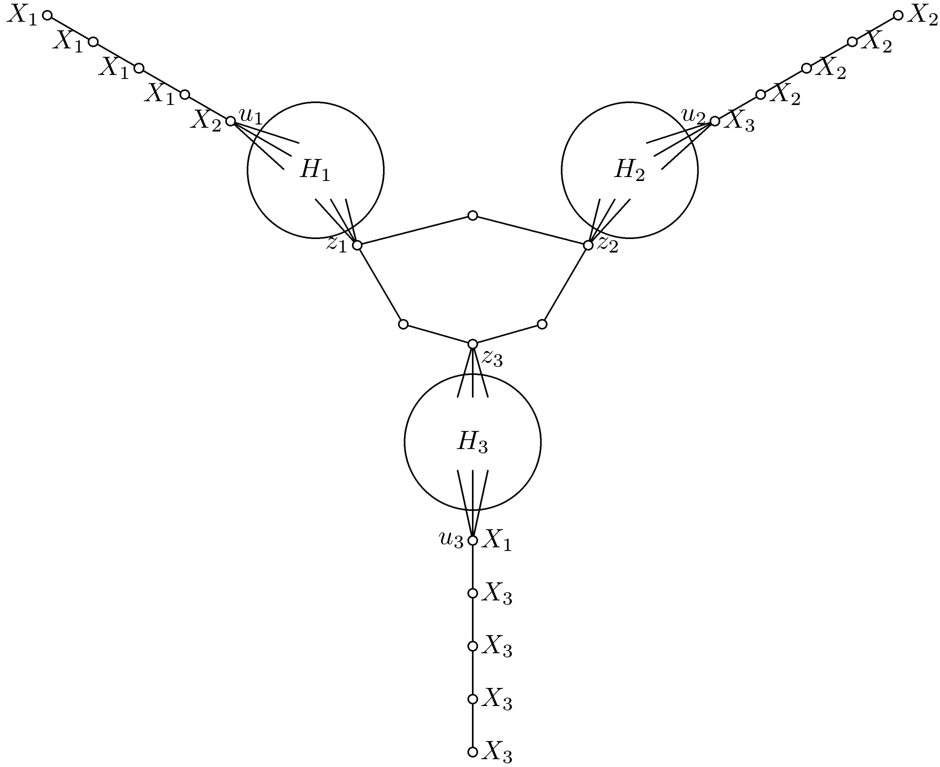


Figure 8.

4. The periphery of a stratified graph

The concept opposite to the center of a connected graph is the periphery. The *periphery* $P(G)$ of a connected graph G is the subgraph of G induced by those vertices v with $e(v) = \operatorname{diam} G$. Bielak and Syslo [1] showed that a graph G is the periphery of a connected graph if and only if every vertex of G has eccentricity 1 or no vertex of G has eccentricity 1.

For a color X of a k-stratified graph G, the X-*periphery* $P_X(G)$ is the subgraph of G induced by those vertices v with $e_X(v) = \operatorname{diam}_X G$. For the 2-stratified graph G of Figure 9, the red eccentricity of each vertex is shown along with the red periphery.

We now present a characterization of those k-stratified graphs that are the periphery of some connected k-stratified graph.

Theorem 11. *A k-stratified graph G is the X-periphery of a k-stratified graph if and only if every vertex of G has X-eccentricity* 1 *or no X-colored vertex of G has X-eccentricity* 1*.*

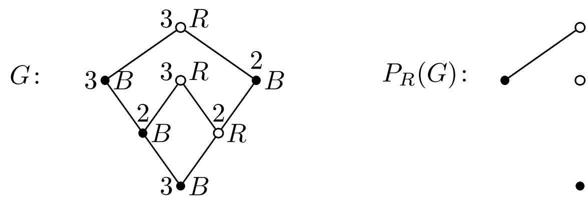


Figure 9.

P r o o f. If every vertex of G has X-eccentricity 1, then G is the X-periphery of itself. Suppose, next, that G is a k-stratified graph in which no X-colored vertex of G has X-eccentricity 1. In this case, define H to be that k-stratified graph obtained by adding an X-colored vertex v to G and joining v to all vertices of G that do not have X-eccentricity 1. Then v has X-eccentricity 1 and every vertex of G has X-eccentricity 2; so G is the X-periphery of H.

For the converse, suppose, to the contrary, that there is a k-stratified graph G for which some X-colored vertex u has X-eccentricity 1 but not all vertices of G have X-eccentricity 1 and such that G is the X-periphery of some k-stratified graph H. Since not all vertices of G have the same X-eccentricity, G is a proper induced subgraph of H. Suppose that $\operatorname{diam}_X H = d$, where, then, $d \geqslant 2$. In H, $e_X(u) = d$. Hence there exists an X-colored vertex w such that $d(u, w) = d$. Since in G, $e_X(u) = 1$, it follows that u is adjacent to all other X-colored vertices of G. This implies that w does not belong to G. However, since $d(w, u) = d$ and u is X-colored, it follows that $e_X(w) = d$ and so w belongs to the X-periphery of H, but this is a contradiction. □

5. Proximity and seclusion in stratified graphs

In this section we introduce concepts that have no natural analogue in graphs but which have applications to other areas. Suppose that a city councilperson in a large city is looking for a location for his or her office. This public servant, of course, wishes to serve and be available to all of the various ethnic groups within the city. Typically, ethnic groups live in clusters of neighborhoods in various parts of the city. A particular ethnic group would like to feel that their concerns are of sufficient importance that the councilperson's office will be located in close proximity to some neighborhood in which the given ethnic group lives. If we think of the street intersections of the city as vertices, street segments as edges, and a vertex colored according to the ethnic group most notably represented by the particular neighborhood involved, we are led to a new concept in our study of stratified graphs.

For a vertex v in a k-stratified graph G, it may be of interest to know the minimum distance from v to a vertex in some prescribed stratum. It is this fact that leads us to define the concepts in this section.

For a vertex v of a k-stratified graph G and a color X of G, the *X-proximity* $\delta_X(v)$ of v is the distance between v and an X-colored vertex closest to v. If v itself is an X-colored vertex, then, of course, $\delta_X(v) = 0$. This concept is, in a sense, opposite to that of the X-eccentricity.

If the vertices of a graph are not partitioned, such as in a 1-stratified or ordinary graph, then the proximity of every vertex is 0. Hence, in the domain of ordinary graphs, proximity is not an interesting concept for study. A basic result regarding the X-proximity of adjacent vertices in a k-stratified graph G is given below. It parallels Theorem 2.

Theorem 12. *Let X be a color in a connected k-stratified graph. If $uv \in E(G)$, then $|\delta_X(u) - \delta_X(v)| \leqslant 1$.*

P r o o f. Suppose that $\delta_X(u) \geqslant \delta_X(v) = t$. We show that $\delta_X(u) \leqslant t + 1$, which will then complete the proof. Let w be the closest X-colored vertex to v; so $d(v, w) = t$. Thus

$$\delta_X(u) \leqslant d(u, w) \leqslant d(u, v) + d(v, w) \leqslant 1 + t.$$

□

For a color X in a k-stratified graph G, the *maximum X-proximity* $\Delta_X(G)$ is the greatest X-proximity among all vertices of G. This concept is analogous to that of diameter in graphs. The minimum X-proximity among all vertices of G is always 0, and this value is attained by all X-colored vertices in G. We define the *X-seclusion* $S_X(G)$ of G to be the subgraph induced by those vertices v of G with $\delta_X(v) = \Delta_X(G)$. For the 2-stratified graph of Figure 10 the red proximity of each vertex is shown along with the red seclusion $S_R(G)$.

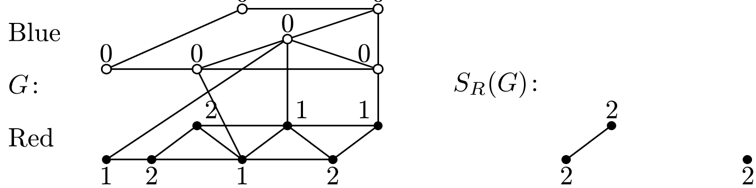


Figure 10.

Theorem 13. *For positive integers a and b, there exists a 2-stratified graph G with $\Delta_R(G) = a$ and $\Delta_B(G) = b$.*

Proof. Consider a path of order $a+b$ such that the first a vertices are colored red and the remaining b vertices are colored blue. In this path one end-vertex is colored red and the other is colored blue. The distance from one end-vertex to the closest vertex of the other color is a or b, depending on the end-vertex selected. Figure 11 illustrates a 2-stratified path with $\Delta_B(G) = \delta_B(u) = 5$ and $\Delta_R(G) = \delta_R(v) = 7$. □

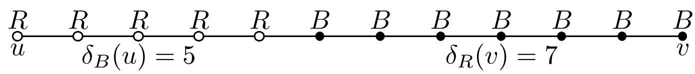


Figure 11.

The preceding result naturally gives rise to a question: Given an integer $k \geqslant 3$, for which positive integers a_i $(1 \leqslant i \leqslant k)$ does there exist a k-stratified graph such that $\Delta_{X_i} = a_i$?

We now present a result concerning X-seclusions.

Theorem 14. *For every ℓ-stratified graph H and every integer k where $k > \ell$, there exists a k-stratified graph G and a color X of G that is not a color of H such that $S_X(G) = H$.*

Proof. If X is a color of a k-stratified graph G $(k \geqslant 2)$, then, necessarily, no vertex of $S_X(G)$ is X-colored, that is, $S_X(G)$ is ℓ-stratified for some $\ell < k$. Now, let H be an ℓ-stratified graph, let k be an integer with $k > \ell$, and let $X_1, X_2, \ldots, X_{k-\ell} = X$ be $k-\ell$ colors not used in H. Further, let P: $v_1, v_2, \ldots v_{k-\ell}$ be a path of order $k-\ell$, where v_i is colored X_i. We construct G from H and P by joining v_1 to every vertex of H. Then in G, $\delta_X(v) = k-\ell$ for every vertex v in H; while $\delta_X(v_i) = k-\ell-i$ for $1 \leqslant i \leqslant k-\ell$. Thus $\Delta_X(G) = k-\ell$ and $S_X(G) = H$. (See Figure 12 for an illustration of this construction.) □

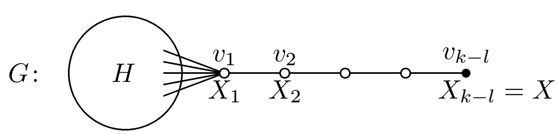


Figure 12.

Corollary 15. *Every k-stratified graph G is the X-seclusion of some $(k+1)$-stratified graph H, where X is a color in H that is not in G.*

Proof. To construct H, we add a vertex u to G, assign color X to u, and join u to every vertex of G. It remains only to observe that the X-seclusion of H is G. □

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