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COMPATIBLE MAPPINGS OF TYPE (B) AND COMMON FIXED POINT THEOREMS IN SAKS SPACES

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Abstract. In this paper we first introduce the concept of compatible mappings of type (B) and compare these mappings with compatible mappings and compatible mappings of type (A) in Saks spaces. In the sequel, we derive some relations between these mappings. Secondly, we prove a coincidence point theorem and common fixed point theorem for compatible mappings of type (B) in Saks spaces.

Keywords: Saks spaces, compatible mappings of type (A), compatible mappings of type (B), coincidence, common fixed points and compatible mappings

MSC 2000: 54H25

I. Introduction

In 1968, Goebel [6] proved a coincidence theorem that received some attention. Several years later, Okada [22], Singh-Virendra [27], Kulshrestha [16] and Naimpally-Singh-Whitfield [20] extended Goebel's results to L-spaces, metric spaces, 2-metric spaces and multivalued contraction mappings on metric space, respectively.

In 1976, Jungck [8] initially gave a common fixed point theorem for commuting mappings, which generalized the well-known Banach fixed point theorem. Jungck's theorem was generalized, extended and unified in various ways by many authors. S. Sessa [26] defined a generalization of commuting mappings which is called weakly commuting mappings.

Recently, Jungck [9] introduced more generalized commuting mappings called compatible mappings which are more general than weakly commuting mappings. In general, commuting mappings are weakly commuting and weakly commuting mappings

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are compatible, but the converses are not necessarily true ([9] and [26]). Several authors proved common fixed point theorems using this concept ([10], [11] and [13]–[15]).

Further, Jungck-Murthy-Cho [12] defined the concept of compatible mappings of type (A) which is equivalent to the concept of compatible mappings under some conditions, and proved a common fixed point theorem for compatible mappings of type (A) in a metric space. Pathak-Khan [21] introduced more generalized compatible mappings called compatible of type (B) and compared these mappings with compatible mappings and compatible mappings of type (A). Also, they derived some relations between these mappings and proved a common fixed point theorem for compatible mappings of type (B) in metric spaces.

On the other hand, Cho-Singh [2]–[3], Murthy-Sharma [18] and many others have studied the aspects of coincidence and common fixed point theorems in the setting of Saks spaces. They have been motivated by various concepts already known in ordinary matric spaces and have thus introduced analogues of various concepts in the framework of the Saks spaces. Especially, Cho-Singh [2] and Murthy-Sharma [18] introduced the concepts of commuting and weakly uniformly contraction pair of mappings, respectively, and have proved several fixed point theorems by using these concepts. Further, Murthy-Sharma-Cho [19] introduced the concept of compatible mappings of type (A) in Saks spaces and proved some coincidence and common fixed point theorems.

In this paper we introduce the concept of compatible mappings of type (B) and compare these mappings and compatible mappings of type (A) in the Saks space. In the sequel, we derive some relations between these mappings. Also, we prove a coincidence point theorem and a common fixed point theorem for compatible mappings of type (B) in Saks spaces. Our theorem extends, generalizes and improves the results of several authors.

Throughout this paper, $(X_S, d) = (X, N_1, N_2)$ denotes a Saks space, and N_1 is equivalent to N_2 on X. In brief we shall define X as a Saks space.

The following lemma due to Orlicz [24] is useful for the proof of our main theorem:

Lemma 1.1. *Let X be a Saks space. Then the following statements are equivalent:*

(1) *N_1 is equivalent to N_2 on X.*
(2) *(X, N_1) is a Banach space and $N_1 \leqslant N_2$ on X.*
(3) *(X, N_2) is a Frechet space and $N_2 \leqslant N_1$ on X.*

The general information on Saks spaces may be found in ([1], [23]–[25]).

In this section we introduce the concept of compatible mappings of type (B) and show that these mappings are equivalent to compatible mappings and compatible mappings of type (A) under some conditions in Saks spaces.

Now, we shall give two definitions ([19]):

Definition 2.1. Let S and T be mappings from a Saks space X into itself. The mappings S and T are said to be compatible if

$$\lim_{n\to\infty} N_2(STx_n - TSx_n) = 0,$$

whenever $\{x_n\}$ is a sequence in X such that $\lim\limits_{n\to\infty} Sx_n = \lim\limits_{n\to\infty} Tx_n = t$ for some $t \in X$.

Definition 2.2. Let S and T be mappings from a Saks space X into itself. The mappings S and T are said to be compatible of type (A) if

$$\lim_{n\to\infty} N_2(TSx_n - SSx_n) = 0 \text{ and } \lim_{n\to\infty} N_2(STx_n - TTx_n) = 0,$$

whenever $\{x_n\}$ is sequence in X such that $\lim\limits_{n\to\infty} Sx_n = \lim\limits_{n\to\infty} Tx_n = t$ for some $t \in X$.

We introduce the following

Definition 2.3. Let S and T be mappings from Saks space X into itself. The mappings S and T are said to be compatible of type (B) if

$$\lim_{n\to\infty} N_2(STx_n - TTx_n) \leqslant \tfrac{1}{2}\big[\lim_{n\to\infty} N_2(STx_n - St) + \lim_{n\to\infty} N_2(St - SSx_n)\big]$$

and

$$\lim_{n\to\infty} N_2(TSx_n - SSx_n) \leqslant \tfrac{1}{2}\big[\lim_{n\to\infty} N_2(TSx_n - Tt) + \lim_{n\to\infty} N_2(Tt - TTx_n)\big],$$

whenever $\{x_n\}$ is a sequence in X such that $\lim\limits_{n\to\infty} Sx_n = \lim\limits_{n\to\infty} Tx_n = t$ for some $t \in X$.

The following Propositions 2.1, 2.3 show that under some conditions ([12]) Definitions 2.1 and 2.2 are equivalent:

Proposition 2.1. *Let S and T be continuous mappings of a Saks space X into itself. If S and T are compatible, then they are compatible of type* (A).

Proposition 2.2. *Let S and T be compatible mappings of type* (A) *from a Saks space X into itself. If one of S and T is continuous, then S and T are compatible.*

From Proposition 2.1 and 2.2 we have

Proposition 2.3. *Let S and T be continuous mappings from a Saks space S into itself. Then S and T are compatible if and only if they are compatible of type* (A).

Remarks 2.1. By suitable examples, G. Jungck, P. P. Murthy and Y. J. Cho [12] have shown that Proposition 2.3 does not hold if S and T are not continuous.

Proofs of the following propositions follow the same lines as suggested in ([21]).

The next propositions show that Definitions 2.1, 2.2 and 2.3 are equivalent under some conditions:

Proposition 2.4. *Every pair of compatible mappings of type* (A) *is compatible of type* (B).

Proposition 2.5. *Let S and T be continuous mappings of a Saks space X into itself. If S and T are compatible of type* (B), *then they are compatible of type* (A).

Proposition 2.6. *Let S and T be continuous mappings of a Saks space X into itself. If S and T are compatible of type* (B), *then they are compatible.*

As a direct consequence of Proposition 2.1 and 2.4 we have

Proposition 2.7. *Let S and T be continuous mappings from a Saks space X into itself. If S and T are compatible, then they are compatible of type* (B).

By unifying Propositions 2.4 $\sim$ 2.7 we have

Proposition 2.8. *Let S and T be continuous mappings from a Saks space X into itself. Then*

(1) *S and T are compatible if and only if they are compatible of type* (B).

(2) *S and T are compatible of type* (A) *if and only if they are compatible of type* (B).

Remark 2.2. In H. K. Pathak and M. S. Khan [21] we may find some examples to show the fact that Proposition 2.8 is not true if S and T are not continuous.

Now, we give the following properties of compatible mappings of type (B) for our main theorems:

Proposition 2.9. *Let S and T be compatible mappings of type* (B) *from a Saks space X into itself. If $St = Tt$ for some $t \in X$, then $STt = SSt = TTt = TSt$.*

Proposition 2.10. *Let S and T be compatible mappings of type* (B) *from a Saks space X into itself. Suppose that $\lim\limits_{n\to\infty} Sx_n = \lim\limits_{n\to\infty} Tx_n = t$ for some $t \in X$. Then we have*

(1) $\lim\limits_{n\to\infty} TTx_n = St$ *if S is continuous at t.*
(2) $\lim\limits_{n\to\infty} SSx_n = Tt$ *if T is continuous at t.*
(3) $STt = TSt$ *and* $St = Tt$ *if S and T are continuous at t.*

III. A common fixed point theorem

Let $\mathbb{R}^+$ be the set of non-negative real numbers and F the family of mappings $\varphi\colon (\mathbb{R}^+)^9 \to \mathbb{R}^+$ such that each φ is upper semicontinuous, non-decreasing in each coordinate variable, and for any $t > 0$,

$$\varphi(t,t,t,t,0,\alpha t,0,\alpha t,0) \leqslant \beta t \text{ and}$$
$$\varphi(t,t,t,t,\alpha t,0,0,0,\alpha t) \leqslant \beta t,$$

where $\beta = 1$ for $\alpha = 2$, and $\beta < 1$ for $\alpha < 2$,

$$\nu(t) = \varphi(t,t,t,t,a_1 t,a_2 t,a_3 t,a_4 t,a_5 t) < t,$$

where $\nu\colon \mathbb{R}^+ \to \mathbb{R}^+$ is a mapping and $a_1 + a_2 + a_3 + a_4 + a_5 = 7$.

Let A, B, S and T be mappings from a Saks space X into itself such that

$$A(X) \cup B(X) \subset S(X) \cap T(X), \tag{3.1}$$

$$\begin{aligned} N_2^2(Ax - By) \leqslant \varphi\Big(&\max\big\{N_2^2(Sx-Ty), N_2^2(Sx-Ax), N_2^2(Ty-By)\big\}, \\ &N_2(Sx-Ax)\cdot N_2(Ty-By), \\ &N_2(Sx-Ty)\cdot N_2(Sx-Ax), N_2(Sx-Ty)\cdot N_2(Ty-By), \\ &N_2(Sx-Ty)\cdot N_2(Sx-By), N_2(Sx-Ty)\cdot N_2(Ty-Ax), \\ &N_2(Sx-By)\cdot N_2(Ty-Ax), N_2(Sx-Ax)\cdot N_2(Ty-Ax), \\ &N_2(Sx-By)\cdot N_2(Ty-By)\Big) \end{aligned} \tag{3.2}$$

for all x, y in X and $\varphi \in F$.

Then, by (3.1), since $A(X) \subset T(X)$, for an arbitrary point $x_0 \in X$ there exists a point $x_1 \in X$ such that $Ax_0 = Tx_1$. Since $B(X) \subset S(X)$, for this point x_1 we can choose a point $x_2 \in X$ such that $Bx_1 = Sx_2$ and so on. Inductively, we can define a sequence $\{y_n\}$ in X such that

$$y_{2n} = Tx_{2n+1} = Ax_{2n} \text{ and } y_{2n+1} = Sx_{2(n+1)} = Bx_{2n+1} \tag{3.3}$$

for every $n = 0, 1, 2, \ldots$.

For our main theorems, we need the following lemmas:

Lemma 3.1. ([28]) *For any $t > 0$, $\nu(t) < t$ if and only if $\lim_{n\to\infty} \nu^n(t) = 0$, where ν^n denotes the n-times composition of ν with itself.*

Lemma 3.2. *Let A, B, S and T be mappings from a Saks space (X, d) into itself satisfying the conditions* (3.1) *and* (3.2)*. Then*

(1) *$\{N_2(y_n - y_{n+1}\}$ is a non-increasing sequence in $\mathbb{R}^+$,*

(2) *$\{y_n\}$ is a Cauchy sequence in X, where $\{y_n\}$ is the sequence in X defined by* (3.3).

Proof. (1) We shall prove that $\{N_2(y_n - y_{n+1}\}$ for $n = 0, 1, 2, \ldots$ is a non-increasing sequence in $\mathbb{R}^+$. By (3.2) and (3.3) we have

$$\begin{aligned}
N_2^2(y_{2n} - y_{2n+1}) &= N_2^2(Ax_{2n} - Bx_{2n+1}) \\
&\leqslant \varphi\Big(\max\{N_2^2(y_{2n-1} - y_{2n}), N_2^2(y_{2n-1} - y_{2n}), N_2^2(y_{2n} - y_{2n+1})\}, \\
&\qquad\qquad N_2(y_{2n-1} - y_{2n}) \cdot N_2(y_{2n} - y_{2n+1}), \\
&\quad N_2(y_{2n-1} - y_{2n})N_2(y_{2n-1} - y_{2n}), N_2(y_{2n-1} - y_{2n}) \cdot N_2(y_{2n} - y_{2n+1}), \\
&\quad N_2(y_{2n-1} - y_{2n}) \cdot N_2(y_{2n-1} - y_{2n+1}), N_2(y_{2n-1} - y_{2n}) \cdot N_2(y_{2n} - y_{2n}), \\
&\quad N_2(y_{2n-1} - y_{2n+1}) \cdot N_2(y_{2n} - y_{2n}), N_2(y_{2n-1} - y_{2n}) \cdot N_2(y_{2n} - y_{2n}), \\
&\quad N_2(y_{2n-1} - y_{2n+1}) \cdot N_2(y_{2n} - y_{2n+1})\Big) \\
&\leqslant \varphi\Big(\max\{N_2^2(y_{2n-1} - y_{2n}), N_2^2(y_{2n-1} - y_{2n}), N_2^2(y_{2n} - y_{2n+1})\}, \\
&\qquad\qquad N_2(y_{2n-1} - y_{2n}) \cdot N_2(y_{2n} - y_{2n+1}), \\
&\quad N_2(y_{2n-1} - y_{2n})N_2(y_{2n-1} - y_{2n}), N_2(y_{2n-1} - y_{2n}) \cdot N_2(y_{2n} - y_{2n+1}), \\
&\quad N_2(y_{2n-1} - y_{2n}) \cdot \big(N_2(y_{2n-1} - y_{2n}) + N_2(y_{2n} - y_{2n+1})\big), 0, \\
&\quad 0, 0, \big(N_2(y_{2n-1} - y_{2n}) + N_2(y_{2n} - y_{2n+1})\big) \cdot N_2(y_{2n} - y_{2n+1})\Big).
\end{aligned}$$

Suppose that $N_2(y_{2n-1} - y_{2n}) < N_2(y_{2n} - y_{2n+1})$ for some n. Then, for some $\alpha < 2$, $N_2(y_{2n-1} - y_{2n}) + N_2(y_{2n} - y_{2n+1}) = \alpha N_2(y_{2n} - y_{2n+1})$. Since φ is non-decreasing in each variable and $\beta < 1$ for $\alpha < 2$, we have

$$\begin{aligned}
N_2^2(y_{2n} - y_{2n+1}) &\leqslant \varphi\big(N_2^2(y_{2n} - y_{2n+1}), N_2^2(y_{2n} - y_{2n+1}), N_2^2(y_{2n} - y_{2n+1}), \\
&\qquad N_2^2(y_{2n} - y_{2n+1}), N_2^2(y_{2n} - y_{2n+1}), 0, 0, 0, \\
&\qquad N_2^2(y_{2n} - y_{2n+1})\big) \\
&\leqslant \beta N_2(y_{2n} - y_{2n+1}) < N_2(y_{2n} - y_{2n+1}), \text{ a contradiction.}
\end{aligned}$$

Therefore, $\{N_2(y_n - y_{n+1})\}$ is a non-increasing sequence in $\mathbb{R}^+$.

(2) Let $\{N_2(y_n - y_{n+1})\}$ be a non-increasing sequence in $\mathbb{R}^+$. Now, again by (3.2) and (3.3), we have

$$\begin{aligned}
N_2^2(y_1 - y_2) &= N_2^2(Ax_2 - Bx_1) \\
&\leqslant \varphi\Big(\max\big\{N_2^2(y_0 - y_1), N_2^2(y_1 - y_2), N_2^2(y_0 - y_1)\big\}, \\
&\qquad\qquad\qquad N_2(y_1 - y_2) \cdot N_2(y_0 - y_1)\big\}, \\
&\quad N_2(y_0 - y_1) \cdot N_2(y_1 - y_2), N_2^2(y_0 - y_1), 0, \\
&\quad N_2(y_0 - y_1) \cdot \big(N_2(y_0 - y_1) + N_2(y_1 - y_2)\big), 0, \\
&\quad N_2(y_1 - y_2) \cdot \big(N_2(y_0 - y_1) + N_2(y_1 - y_2)\big), 0\Big) \\
&\leqslant \varphi\big(N_2^2(y_0 - y_1), N_2^2(y_0 - y_1), N_2^2(y_0 - y_1), N_2^2(y_0 - y_1), \\
&\quad N_2^2(y_0 - y_1), 2N_2^2(y_0 - y_1), \\
&\quad N_2^2(y_0 - y_1), 2N_2^2(y_0 - y_1), N_2^2(y_0 - y_1)\big) \\
&= \nu\big(N_2^2(y_0 - y_1)\big).
\end{aligned}$$

In general, we have $N_2^2(y_n - y_{n+1}) \leqslant \nu^n\big(N_2^2(y_0 - y_1)\big)$, which implies that, by Lemma 3.1, we have

$$\lim_{n\to\infty} N_2^2(y_n - y_{n+1}) \leqslant \lim_{n\to\infty} \nu^n\big(N_2^2(y_0 - y_1)\big) = 0.$$

Therefore, we have $\lim\limits_{n\to\infty} N_2(y_n - y_{n+1}) = 0$. This shows that $\{y_n\}$ is a Cauchy sequence with respect to N_2 in $S(X) \cap T(X)$. This completes the proof. $\square$

Now, we are ready to present our main theorems:

Theorem 3.1. *Let A, B, S and T be mappings from a Saks space X into itself satisfying the conditions* (3.1), (3.2) *and* (3.10)*:*

(3.10) $\qquad S(X) \cap T(X)$ *is a closed subspace of X.*

Then (1) *A and S have a coincidence point in X, and* (2) *B and T have a coincidence point in X.*

P r o o f. By Lemma 3.2 the sequence $\{y_n\}$ defined by (3.3) is a Cauchy sequence in $S(X) \cap T(X)$ with respect to N_1, since N_1 is equivalent to N_2 on X. So by Lemma 1.1, (X, N_1) is a Banach space and hence $\{y_n\}$ converges to a point w in $S(X) \cap T(X)$. On the other hand, since the subsequences $\{y_{2n}\}$ and $\{y_{2n+1}\}$ of $\{y_n\}$ are also Cauchy sequences in $S(X) \cap T(X)$, they also converge to the same limit w. Hence there exist two points u, v in X such that $Su = w$ and $Ty = w$. By (3.2) we have

$$\begin{aligned}
N_2^2(Au - y_{2n+1}) &= N_2^2(Au - Bx_{2n+1}) \\
&\leqslant \varphi\Big(\max\big\{N_2^2(Su - y_{2n}), N_2^2(Su - Au), N_2^2(y_{2n} - y_{2n+1})\big\}, \\
&\qquad\qquad\qquad\qquad N_2(Su - Au) \cdot N_2(y_{2n} - y_{2n+1}), \\
&\quad N_2(Su - y_{2n}) \cdot N_2(Su - Au), N_2(Su - y_{2n}) \cdot N_2(y_{2n} - y_{2n+1}), \\
&\quad N_2(Su - y_{2n}) \cdot N_2(Su - y_{2n+1}), N_2(Su - y_{2n}) \cdot N_2(y_{2n} - Au), \\
&\quad N_2(Su - y_{2n+1}) \cdot N_2(y_{2n} - Au), N_2(Su - Au) \cdot N_2(y_{2n} - Au), \\
&\quad N_2(Su - y_{2n+1}) \cdot N_2(Su - y_{2n+1})\Big).
\end{aligned}$$

Since $\lim\limits_{n\to\infty} \nu^n(t) = 0$ as in the proof of Lemma 3.2, letting $n \to \infty$ we have

$$N_2^2(Au - w) \leqslant \varphi\big(0, 0, 0, 0, 0, 0, 0, N_2^2(w - Au), 0\big),$$

which is a contradiction. Hence $Au = w = Su$, that is, u is a coincidence point of A and S. Similarly, we can show that v is a coincidence point of B and T. This completes the proof. □

Putting $A = B$ in Theorem 3.1, we have the following

Corollary 3.2. *Let A, S and T be mappings from a Saks space X into itself satisfying the conditions* (3.1), (3.2) *and* (3.10).

Then (1) A and S have a coincidence point in X and (2) A and T have a coincidence point in X. Indeed, A, S and T have a coincidence point in X if A is one-to-one.

P r o o f. By Theorem 3.1 with $A = B$, we have the direct proofs of (1) and (2). As in the proof of Theorem 3.1 we have $Au = Su = Av = Tv$. Hence since A is one-to-one, $u = v$ is a coincidence point of A, S and T. This completes the proof. □

IV. Another common fixed point theorem

In this section we prove a common fixed point theorem of Greguš type for compatible mappings of type (B) in a Saks space by employing Theorem 3.1.

Theorem 4.1. *Let A, B, S and T be mappings from a Saks space X into X satisfying the conditions* (3.1), (3.2), (3.10) *and*

(4.1) *the pairs A, S and B, T are compatible mappings of type* (B).

Then A, B, S and T have a unique common fixed point in X.

Proof. By Theorem 3.1, there exist two points u and v in X such that $Au = Su = w$ and $Bv = Tv = w$. Since A and S are compatible mappings of type (B), by Proposition 2.9, $ASu = SSu = SAu = AAu$, which implies that $Aw = Sw$. Similarly, since B and T are compatible mappings of type (B), we have $Bw = Tw$. Now we prove that $Aw = w$. If $Aw \neq w$, then by (3.2) we have

$$\begin{aligned}
N_2^2(Au - y_{2n+1}) &= N_2^2(Aw - Bx_{2n+1}) \\
&\leqslant \varphi\Big(\max\big\{N_2^2(Sw - y_{2n}), N_2^2(Sw - Aw), N_2^2(y_{2n} - y_{2n+1})\big\}, \\
&\qquad\qquad N_2(Sw - Aw)\cdot N_2(y_{2n} - y_{2n+1}), \\
&\quad N_2(Sw - y_{2n})\cdot N_2(Sw - Aw), N_2(Sw - y_{2n})\cdot N_2(y_{2n} - y_{2n+1}), \\
&\quad N_2(Sw - y_{2n})\cdot N_2(Sw - y_{2n+1}), N_2(Sw - y_{2n})\cdot N_2(y_{2n} - Aw), \\
&\quad N_2(Sw - y_{2n+1})\cdot N_2(y_{2n} - Aw), N_2(Sw - Aw)\cdot N_2(y_{2n} - Aw), \\
&\quad N_2(Sw - y_{2n+1})\cdot N_2(y_{2n} - y_{2n+1})\Big).
\end{aligned}$$

Letting $n \to \infty$, we have

$$\begin{aligned}
N_2^2(Aw - w) &\leqslant \varphi\big(N_2^2(Sw - w), 0, N_2(Sw - w)\cdot N_2(Sw - Aw), 0, N_2^2(Sw - w), \\
&\qquad N_2(Sw - w)\cdot N_2(w - Aw), N_2(Sw - w)\cdot N_2(w - Aw), \\
&\qquad N_2(Sw - Aw)\cdot N_2(w - Aw), 0\big) \\
&= \varphi\big(N_2^2(Aw - w), 0, 0, 0, N_2^2(Aw - w), N_2^2(Aw - w), 0, 0\big) \\
&< N_2^2(Aw - w),
\end{aligned}$$

which is a contradiction. Hence have $Aw = w = Sw$. Similarly, we have $Bw = w = Tw$. This means that w is a common fixed point of A, B, S and T. The uniqueness of the fixed point w follows easily from (3.2). □

Remark 4.1. Since it is possible to replace the condition of commuting mappings, weakly commuting mappings, compatible mappings, or compatible mappings of type (A) by compatible mappings of type (B), Theorem 4.1 extends, generalizes and improves a number of fixed point theorem already known in ordinary metric spaces ([4], [5], [7], [9], [14] and [17]), and in Saks spaces ([18] and [19]).

Remark 4.2. Our result derives a common fixed point theorem for four mappings which are not necessarily continuous.

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