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ON SOME PROPERTIES OF THE EQUATION $y''(x) + f(x)y^{\alpha}(x) = 0,\ 0 < \alpha < 1$

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The above equation, denote it by (r), has been studied with $f(x) > 0$ [4], [5]. In this paper two theorems will be proved when $f(x)$ is a continuous not necessarily nonnegative function and some theorems in the case of $f(x) > 0$.

A solution $y(x)$ of equation (r) will be called oscillatory if it has at least one zero in the interval (x, ∞) for an arbitrary x. If for every $x > x_1$ $y(x) \neq 0$, then the solution will be called nonoscillatory. The number α is assumed to be from the interval $0 < \alpha < 1$ and „odd", i. e. of the form $\alpha = p/q$, p and q are odd integers.

Theorem 1. *Let the function $f(x)$ be continuous in the interval $\langle x_0, \infty)$. Then every solution of equation* (r) *can be extended to the whole interval $\langle x_0, \infty)$.*

Proof. Let $y(x)$ be a solution of (r) defined in an interval $\langle x_1, x_2)$ $(x_1 \geqq x_0)$, such that $y(x_1) = y_0$, $y'(x_1) = y_1$. Then from (r) for $x \in (x_1, x_2)$ we obtain

$$y(x) = y_0 + y_1(x - x_1) - \int_{x_1}^{x} (x - t)f(t)y^{\alpha}(t)\mathrm{d}t.$$

Wherefrom we have for $x - x_1 \geqq 1$

$$|y(x)| \leqq (x - x_1)\left(|y_0| + |y_1| + \int_{x_1}^{x} |f(t)|\ |y(t)|^{\alpha}\ \mathrm{d}t\right). \tag{1}$$

In the case $x - x_1 < 1$ we have the following estimate

$$|y(x)| \leqq |y_0| + |y_1| + \int_{x_1}^{x} |f(t)|\ |y(t)|^{\alpha}\mathrm{d}t$$

and we proceed in the same way. From the inequality (1) we have

$$\frac{|f(x)|\ |y(x)|^{\alpha}}{\left(|y_0| + |y_1| + \int_{x_1}^{x} |f(t)|\ |y(t)|^{\alpha}\mathrm{d}t\right)^{\alpha}} \leqq (x - x_1)^{\alpha}\ |f(x)|.$$

Hence by integration in the interval $\langle x_1, x\rangle$ we have

$$\left(|y_0| + |y_1| + \int_{x_1}^{x} |f(t)|\,|y(t)|^\alpha \mathrm{d}t\right)^{1-\alpha} \leqq (1-\alpha)\int_{x_1}^{x} (t-x_1)^\alpha |f(t)|\,\mathrm{d}t + $$
$$+ (|y_0| + |y_1|)^{1-\alpha}$$

and finally we receive an inequality

$$(2)\quad |y(x)| \leqq (x-x_1)\left\{(1-\alpha)\int_{x_1}^{x} (t-x_1)^\alpha |f(t)|\mathrm{d}t + (|y_0| + |y_1|)^{1-\alpha}\right\}^{1/(1-\alpha)}.$$

Because the right-hand side of the last inequality is defined and continuous for every $x \geqq x_1$, then the solution $y(x)$ increases more slowly than the function on the right-hand side. Since the inequality

$$|y'(x)| \leqq |y_1| + \int_{x_1}^{x} |f(t)|\,|y(t)|^\alpha \mathrm{d}t,$$

is also valid, hence $y'(x)$ is bounded for every x and thus the solution $y(x)$ can be extended to the whole interval $\langle x_1, \infty)$. A similar consideration is possible also in the interval $(x_2, x_1\rangle$. The theorem is thus proved.

Remark. From the inequality (2) the following estimate follows: Provided that $\int^{\infty} x^\alpha |f(x)|\mathrm{d}x < \infty$, then for every solution $y(x)$ of the equation (r) there exists a constant K, so that for every $x \geqq x_1$ $y(x) < Kx$ is valid.

Theorem 2. *Let the function $f(x)$ be continuous in the interval $\langle x_0, \infty)$ and let*

$$\int^{\infty} x^\alpha |f(x)|\mathrm{d}x < \infty.$$

Then for every solution $y(x)$ of equation (r) *there exists $\lim_{x\to\infty} y'(x) = c$. Initial conditions can be chosen such that $c \neq 0$. If in addition*

$$\int^{\infty} x^{\alpha+1}|f(x)|\mathrm{d}x < \infty,$$

then any solution of the equation (r) is of the form

$$y(x) = c_2 x + c_1 + o(1),$$

where c_1 and c_2 are suitable constants.

Proof. From (r) we have

$$y'(x) = y'(x_1) - \int_{x_1}^{x} f(t)y^{\alpha}(t)\mathrm{d}t. \tag{3}$$

By the above remark $|y(x)| < Kx$, therefore

$$\left|\int_{x_1}^{x} f(t)y^{\alpha}(t)\mathrm{d}t\right| \leqq K^{\alpha}\int_{x_1}^{\infty} |f(x)|x^{\alpha}\mathrm{d}x < \infty.$$

It follows from this that the integral $\int_{x_1}^{\infty} f(t)y^{\alpha}(t)\mathrm{d}t$ exists and by (3) the $\lim_{x\to\infty} y'(x)$ exists. Further we prove that the initial conditions can be chosen such that $c > 0$. But this follows from the next inequality

$$y'(x) = y'(x_1) - \int_{x_1}^{x} f(t)y^{\alpha}(t)\mathrm{d}t \geqq y'(x_1) - K^{\alpha}\int_{x_1}^{\infty} |f(x)|x^{\alpha}\mathrm{d}x > 0,$$

which is true for a sufficient large x_1. The first part of this theorem is thus proved.

To prove the second part, let us suppose that $\int^{\infty} x^{\alpha+1}|f(x)|\mathrm{d}x < \infty$. Then according to the first part for $x \to \infty$, $y'(x)$ has a limit. Denote it by c_2. Integrating (r) in the interval $\langle x, \infty)$ we get

$$y'(x) = c_2 + \int_{x}^{\infty} f(t)y^{\alpha}(t)\mathrm{d}t.$$

From this we have by integration in the interval $\langle x_1, x\rangle$

$$y(x) = c_2x + y(x_1) - c_2x_1 + \int_{x_1}^{\infty} (t - x_1)f(t)y^{\alpha}(t)\mathrm{d}t + \int_{x}^{\infty} (x - t)f(t)y^{\alpha}(t)\mathrm{d}t.$$

As

$$\left|\int_{x_1}^{x} (t - x_1)f(t)y^{\alpha}(t)\mathrm{d}t\right| \leqq K^{\alpha}\int_{x_1}^{\infty} x^{\alpha+1}|f(x)|\mathrm{d}x < \infty,$$

is valid, then the integral $\int_{x_1}^{\infty} (t - x_1)f(t)y^{\alpha}(t)\mathrm{d}t$ exists and

$$\int_{x}^{\infty} (x - t)f(t)y^{\alpha}(t)\mathrm{d}t = o(1)$$

is obvious.

If we denote

$$y(x_1) - c_2x_1 + \int_{x_1}^{\infty} (t - x_1)f(t)y^\alpha(t)\mathrm{d}t = c_1,$$

then we can write

$$y(x) = c_2x + c_1 + o(1).$$

Thus the second part is proved.

In paper [4] it has been proved that provided $\int^{\infty} xf(x)\mathrm{d}x < \infty$, the equation (r) has two types of nonoscillatory solutions. Bounded solutions and solutions of the type $y(x) \sim cx$. The following theorem asserts that the equation (r) has no other nonoscillatory solutions.

Theorem 3. *Let the function $f(x)$ be nonnegative, continuous in the interval $\langle x_0, \infty)$ and such that*

$$\int^{\infty} xf(x)\mathrm{d}x < \infty.$$

Then any nonoscillatory solution of equation (r) *is either bounded or of the form* $y(x) \sim cx$ $(c \neq 0)$.

Proof. Let $y(x)$ be a nonoscillatory solution of (r), then for a sufficiently large x we have $y(x) \neq 0$. Let $y(x)$ be positive for $x > a$ (likewise for $y(x) < 0$). From (r) for $b > a$ we have

$$y(x) = y(b) + (x - b)y'(x) + \int_b^x (t - b)f(t)y^\alpha(t)\mathrm{d}t.$$

Let us suppose that $y(x)$ is not a bounded solution of (r), then for a sufficiently large b we have $y(x) > 1$ and

$$y(x) \leqq y(b) + xy'(x) + y(x)\int_b^x tf(t)\mathrm{d}t.$$

From it we have

$$1 \leqq \frac{y(b)}{y(x)} + \frac{xy'(x)}{y(x)} + \int_b^{\infty} xf(x)\mathrm{d}x. \tag{4}$$

Let ε be an arbitrary positive number, then it is possible to choose a number $b_1 \geqq b$ so that $\int_{b_1}^{\infty} xf(x)\mathrm{d}x < \varepsilon$. Thus from (4) we get

$$1 - \varepsilon \leqq \frac{y(b)}{y(x)} + \frac{xy'(x)}{y(x)},$$

from this it follows that

$$\liminf_{x\to\infty} \frac{xy'(x)}{y(x)} \geqq 1 - \varepsilon. \tag{5}$$

Let us choose $\varepsilon < 1/3$, a number $b_2 \geqq b_1$ such that in the interval $\langle b_2, \infty)$ the inequality $xy'(x) \gtreqless (1 - 2\varepsilon)y(x)$ is fulfilled. With regard to the inequality (5) such a choice is possible. Az $y'(x)$ is a decreasing function we have from (r)

$$(1 - 2\varepsilon)(y'(b_2) - y'(x)) = (1 - 2\varepsilon) \int_{b_1}^{x} f(t)y^{\alpha}(t)\mathrm{d}t \leqq$$

$$\leqq \int_{b_2}^{\infty} (1 - 2\varepsilon)y(t)f(t)\mathrm{d}t \leqq y'(b_2) \int_{b_2}^{\infty} xf(x)\mathrm{d}x < \varepsilon y'(b_2).$$

From the last inequality if follows that

$$0 < \frac{1 - 3\varepsilon}{1 - 2\varepsilon} y'(b_2) < y'(x),$$

and by this the $\lim_{x\to\infty} y'(x) = c > 0$, i. e. $y(x) \sim cx$.

Theorem 4. *Let there exist a number β, $0 < \beta \leqq 1$ such that*

$$\liminf_{x\to\infty} x^{\beta} \int_{x}^{\infty} f(t)\mathrm{d}t = k > 0$$

and let the function $f(x)$ be nonnegative and continuous in the interval $\langle x_0, \infty)$. Then for any solution $y(x)$ of (r) *$y(x) \not\equiv 0$ in the interval $\langle a, \infty)$ $(a \geqq x_0)$ there exists a constant c such that for any $x \geqq a$ the following inequality holds*

$$|y(x)| \geqq c(x - a)^{(1-\beta)/(1-\alpha)}.$$

Proof. We can consider only such a solution $y(x)$ of (r) that is positive for $x \geqq a$. By integrating (r) in the interval $\langle x, b\rangle$ and then in $\langle a, x\rangle$ $(b > a)$ we have

$$y(x) = y(a) + (x - a)y'(b) + \int_{a}^{x} (t - a)f(t)y^{\alpha}(t)\mathrm{d}t + (x - a)\int_{x}^{b} f(t)y^{\alpha}(t)\mathrm{d}t.$$

Because the function $f(x)$ is nonnegative and for any $b > a$ there is $y'(b) \geqq 0$, we get the following inequality

$$y(x) \geqq (x - a) \int_{x}^{b} f(t)y^{\alpha}(t)\mathrm{d}t \geqq (x - a)y^{\alpha}(x) \int_{x}^{b} f(t)\mathrm{d}t.$$

And thus we have for any $b > a$

$$(y(x))^{1-\alpha}/(x - a)^{1-\beta} \geqq (x - a)^{\beta} \int_{x}^{b} f(t)\mathrm{d}t.$$

Since the function on the left-hand side of the last inequality is bounded from below by a positive constant on every finite interval $\langle a, a_1 \rangle$, then it follows from this and from the assumption of the theorem that there exists a constant $c > 0$ so that

$$y(x) \geqq c(x - a)^{(1-\beta)/(1-\alpha)}.$$

Thus the theorem is proved.

In the following two theorems we shall state conditions under which equation (r) has nontrivial oscillatory solutions together with nonoscillatory solutions and conditions under which all solutions of (r) are nonoscillatory.

Theorem 5. *Let the function $f(x)$ be positive and continuous in the interval $\langle x_0, \infty)$ $(x_0 > 0)$. Besides let the function*

$$f(x)x^{(3+\alpha)/2}$$

be nonincreasing and bounded from below with a positive constant k. Then equation (r) *has both nontrivial oscillatory nad nonoscillatory solutions.*

Proof. 1. We shall prove the existence of an oscillatory solution of (r). The change of variables $\lg x = t$, $y = x^{1/2}u$ transforms (r) into

$$\ddot{u}(t) - u(t)/4 + f(x)x^{(3+\alpha)/2}u^{\alpha}(t) = 0 \tag{6}$$

$$\left(\dot{u} = \frac{\mathrm{d}u}{\mathrm{d}t},\ \lg x = t\right).$$

Let us find a solution of equation (6) with the initial conditions

$$u(a) = 0,\ 0 < \dot{u}^2(a) < (4k)^{(1+\alpha)/(1-\alpha)}k(1 - \alpha)/(1 + \alpha). \tag{7}$$

We shall show that this solution is oscillatory. Multiply (6) by $2\dot{u}(t)$ and integrate it in the interval $\langle a, t \rangle$, we have

$$\dot{u}^2(t) - u^2(t)/4 + 2 \int_{a}^{t} f(\mathrm{e}^{\nu})\mathrm{e}^{(3+\alpha)\nu/2}u^{\alpha}(\nu)\dot{u}(\nu)\mathrm{d}\nu = \dot{u}^2(a).$$

If we use the second mean value theorem for integrals after an arrangement we obtain

$$\dot{u}^2(t) - u^2(t)/4 + 2/(\alpha + 1) f(\mathrm{e}^{t})\mathrm{e}^{(3+\alpha)t/2}u^{\alpha+1}(t) \leqq \dot{u}^2(a). \tag{8}$$

The solution with initial conditions (7) remains for $t \geqq a$ in the region

$$(9) \qquad -(4k)^{1/(1-\alpha)} < u(t) < (4k)^{1/(1-\alpha)}.$$

In the opposite case the point $t_0 > a$ would exist such that $u(t_0) = (4k)^{1/(1-\alpha)}$. Then from (8) it follows

$$\dot{u}^2(t_0) \leqq \dot{u}^2(a) + (4k)^{(1+\alpha)/(1-\alpha)}\{k - 2/(1+\alpha)f(e^{t_0})e^{(3+\alpha)t_0/2}\} \leqq \dot{u}^2(a) -$$
$$- (4k)^{(1+\alpha)/(1-\alpha)}k(1-\alpha)/(1+\alpha) < 0,$$

but this is a contradiction. If $u(t)$ is a solution of (6), then $-u(t)$ is also a solution of (6) and thus we can consider only $u(t) \geqq 0$. Hence inequality (9) it true.

We shall show further that this solution $u(t)$ has no last zero. Let the poin- $\bar{t} > a$ be the last zero of $u(t)$ and for $t > \bar{t}$ $u(t) > 0$. From (6) it is evident thas for any $t > \bar{t}$ $\ddot{u}(t) < 0$. In the opposite case a num ber$t_1 > \bar{t}$ would exist such that $\ddot{u}(t_1) \geqq 0$, i. e.

$$\ddot{u}(t_1) = u^{\alpha}(t_1)\{u^{1-\alpha}(t_1)/4 - 2/(\alpha+1)f(e^{t_1})e^{(3+\alpha)t_1/2}\} \geqq 0.$$

Hence we have

$$u^{1-\alpha}(t_1)/4 \geqq 2/(\alpha+1)f(e^{t_1})e^{(3+\alpha)t_1/2} > k$$

and finally $u(t_1) \geqq (4k)^{1/(1-\alpha)}$, which contradicts (9).
Thus $u(t)$ is concave in the interval $\langle \bar{t}, \infty)$ and since it is a positive function, there exists the $\lim\limits_{t\to\infty} u(t) = c > 0$. From (6) we obtain for $t \to \infty$

$$0 = -c/4 + \lim_{t\to\infty} f(e^t)e^{(3+\alpha)t/2}\, c^{\alpha} \geqq c^{\alpha}(-c^{1-\alpha}/4 + k).$$

Hence we have $c \geqq (4k)^{1/(1-\alpha)}$. The case $c > (4k)^{1/(1-\alpha)}$ contradicts (9). If $c = (4k)^{1/(1-\alpha)}$, then from (8)we obtain

$$-(1/4)(4k)^{2/(1-\alpha)} + 2/(\alpha+1)\, k(4k)^{(1+\alpha)/(1-\alpha)} \leqq \dot{u}^2(a),$$

and hence after an arrangement

$$(4k)^{(1+\alpha)/(1-\alpha)}k(1-\alpha)/(1+\alpha) \leqq \dot{u}^2(a).$$

But this contradits (7). Thus the solution $u(t)$ has no last zero and by the above transformation $y(x)$ is oscillatory.

2. We shall prove the existence of a nonoscillatory solution. Since the function $f(x)x^{(3+\alpha)/2}$ is nonincreasing, there exi*s*ts a constant K such that for any $x \in \langle x_0, \infty)$ the following inequality holds $f(x)x^{\alpha} < Kx^{(\alpha-3)/2}$. From this it follows that for an arbitrary positive number $\varepsilon < 1$, we can find a number $a > x_0$ such that $\int\limits_a^{\infty} f(x)x^{\alpha}dx < \varepsilon < 1$. Consider the initial value problem of

(r) $y(a) = 0$, $y'(a) \geqq 1$. This solution $y(x)$ has no zero in the interval (a, ∞). In the opposite case a number $b > a$ would exist such that $y(b) = 0$ and $y(x) > 0$ in the interval (a, b). The solution $y(x)$ satisfies the following inequality

$$y(x) \leqq y'(a)(x - a) \tag{10}$$

for all x in the interval (a, b). Integrating (r) and by (10) we obtain

$$y'(a) \leqq y'(x) + \int_a^x f(t)y'^{\alpha}(a)(t - a)^{\alpha}\mathrm{d}t \leqq y'(x) + y'(a)\int_a^{\infty} f(t)t^{\alpha}\mathrm{d}t \leqq$$
$$\leqq y'(x) + y'(a)\varepsilon.$$

Finally we have the inequality

$$0 < y'(a)(1 - \varepsilon) \leqq y'(x).$$

We see that $y(x)$ is an increasing function in the interval (a, b) and thus we have a contradiction. Hence the solution has no zero in the interval (a, ∞) and thus is nonoscillatory.

It is obvious that in both cases there exists an infinite number of such solutions and this proves the theorem.

Theorem 6. *Let the function $f(x)$ be positive and continuous in the interval $\langle x_0, \infty)$ $(x_0 > 0)$. Let there exist a number β, $0 < \beta < (1 - \alpha)/2$ such that the function*

$$f(x)x^{(3+\alpha)/2+\beta}$$

is nondecreasing and bounded from above by a positive constant k. Then all solutions of (r), *besides the trivial one, are nonoscillatory.*

Proof. Let there be $\delta = 1/2 + \beta/(\alpha - 1)$. The change of variables $\lg x = t$, $y = x^{\delta}u$ transforms (r) into

$$\ddot{u}(t) + (2\delta - 1)\dot{u}(t) + \delta(\delta - 1)u(t) + f(x)x^{(\alpha+3)/2+\beta}u^{\alpha}(t) = 0 \tag{11}$$

$$\left(\dot{u} = \frac{\mathrm{d}u}{\mathrm{d}t}, \lg x = t\right),$$

where $2\delta - 1 < 0$, $\delta(\delta - 1) < 0$.

If we multiply (11) by $2\dot{u}(t)$, integrate in the interval $\langle t_0, t\rangle$ and use the second mean value theorem for integrals we obtain

$$\dot{u}^2(t) + 2(2\delta - 1)\int_{t_0}^{t}\dot{u}^2(\nu)\mathrm{d}\nu + \delta(\delta - 1)u^2(t) + \tag{12}$$
$$+ 2f(\mathrm{e}^t)\mathrm{e}^{((3+\alpha)/2+\beta)t}\int_{\xi}^{t} u^{\alpha}(\nu)\dot{u}(\nu)\mathrm{d}\nu = \dot{u}^2(t_0) + \delta(\delta - 1)u^2(t_0) \quad (t_0 \leqq \xi \leqq t).$$

Since $f(x)x^{(3+\alpha)/2+\beta} < k$ for all x of the interval $\langle x_0, \infty)$, then after an arrangement from (12) it follows

$$\dot{u}^2(t) + \delta(\delta - 1)u^2(t) + 2/(\alpha + 1)ku^{\alpha+1}(t) > \dot{u}^2(t_0) + \delta(\delta - 1)u^2(t_0). \tag{13}$$

Let us suppose that the equation (11) has an oscillatory solution $u(t) \not\equiv 0$. If we denote by $\{t_n\}_{n=0}^{\infty}$ the sequence of zeros of the solution $u(t)$ and by $\{t'_n\}_{n=0}^{\infty}$ the sequence of zeros of $\dot{u}(t)$, then from (13) we can see that $\{|\dot{u}(t_n)|\}_{n=0}^{\infty}$ increase. Besides, $u(t)$ satisfies the inequality

$$-\{2k/((\alpha + 1)\delta(1 - \delta))\}^{1/(1-\alpha)} \leqq u(t) \leqq \{2k/((\alpha + 1)\delta(1 - \delta))\}^{1/(1-\alpha)},$$

since in the opposite case the point t'_k would exist such that the following holds

$$u(t'_k) > \{2k/((\alpha + 1)\delta(1 - \delta))\}^{1/(1-\alpha)}, \quad u(t'_k) = 0.$$

Hence

$$\delta(\delta - 1)u^{1-\alpha}(t'_k) < -2k/(1 + \alpha) < -k < -f(e^{t'_k})e^{((3+\alpha)/2+\beta)t'_k}.$$

Then from (11) we obtain

$$\ddot{u}(t'_k) = -u^{\alpha}(t'_k)\{\delta(\delta - 1)u^{1-\alpha}(t'_k) + f(e^{t'_k})e^{((\alpha+3)/2+\beta)t'_k}\} > 0,$$

i. e. the solution $u(t)$ at a neighbourhood of the point t'_k is a convex function, but this is a contradiction.

As $\{|\dot{u}(t_n)|\}$ increases, then either $\lim_{n\to\infty} |\dot{u}(t_n)| = \infty$, or $\lim_{n\to\infty} |\dot{u}(t_n)| = L > |\dot{u}(t_1)| > 0$. If in the first case we put $t_0 = t_k$ and $t = t'_k$ so that $|\dot{u}(t_k)$ is sufficiently large we obtain from (13)

$$\delta(\delta - 1)u^2(t'_k) + 2/(\alpha + 1)\, ku^{\alpha+1}(t'_k) > \dot{u}^2(t_k).$$

But this contradicts the boundedness of $u(t)$. In the second case $\ddot{u}(t)$ is bounded. This follows from (11) since $u(t)$ and $\dot{u}(t)$ are bounded for every oscillatory solution. Besides, it is evident from (12) that $\int_{t_0}^{\infty} \dot{u}^2(v)\mathrm{d}v < \infty$. Hence it follows that $\lim_{t\to\infty} \dot{u}(t) = 0$ (cf. [3] p. 185), but this is a contradiction. We have thus proved that the solution $u(t) \not\equiv 0$ of the equation (11) cannot be oscillatory. Thus any nontrivial solution of (r) is nonoscillatory. This proves the theorem.

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